

Heat Equation

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We consider heat propagation in a medium (e.g. a solid) far away from any state transition and we define:

ϵ : internal energy density

c : specific heat

ρ : mass density

T : temperature

By virtue of the first thermodynamic principle
($\Delta U = \Delta Q - L = \Delta Q$):

$$d \left[\int_{\Omega} c \rho T \, dV \right] = \left[- \int_{\partial\Omega} \vec{J} \cdot \vec{n} \, dS \right] dt \quad (1)$$

$$d \left[\int_{\Omega} c \rho T \, dV \right] = \left[- \int_{\partial\Omega} \vec{J} \cdot \vec{n} \, dS \right] dt$$

Newton-Fourier Law, heat spreads and temperature becomes uniform throughout the medium:

$$\vec{J} = -k \vec{\nabla} T \quad (2)$$

By substituting equation (2) in (1) and applying the *divergence theorem* one gets the **heat equation**:

$$\frac{\partial T}{\partial t} - \frac{k}{c\rho} \Delta T = 0 \quad (3)$$

$$\frac{\partial T}{\partial t} - \frac{k}{c\rho} \Delta T = f(\vec{x}, t)$$

- ▶ First order in time
- ▶ Second order in space
- ▶ If we consider a heating system, we must add $f(\vec{x}, t)$

$$\frac{\partial T}{\partial t} - \frac{k}{c\rho} \Delta T = f(\vec{x}, t)$$

Discretize space and time: forward difference for time,
central difference for space:

$$T_{i,j}^{n+1} = T_{i,j}^n + \eta [T_{i+1,j}^n + T_{i-1,j}^n + T_{i,j+1}^n + T_{i,j-1}^n - 4T_{i,j}^n] \quad (4)$$

where n is the time index, i, j are x-index and y-index,

$$\eta = \frac{k\Delta t}{c\rho(\Delta x)^2}.$$

General Structure

Heat Equation

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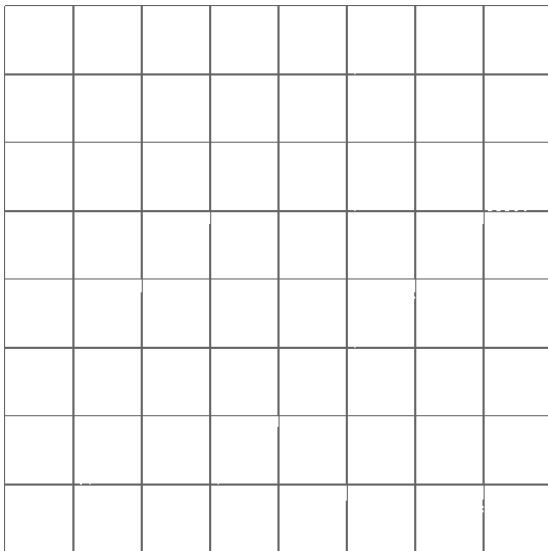
kjh

Shared Memory

Random errors

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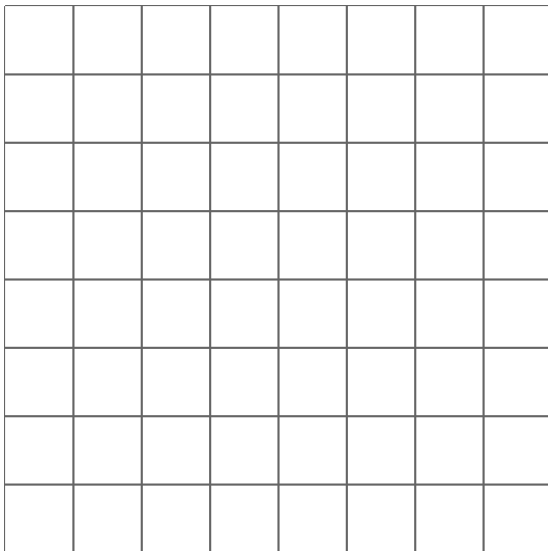


Shared Memory

Synchronized threads

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descrizione della gpu usata (numero blocchi e thread eccecc)

Performances

Execution time vs blocks number

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Performances

Execution time vs loops number

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Performances

Execution time GPU vs CPU

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Performances

Different GPUs

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