What is the Effect of Imbalanced Sex Ratios on Women and Men in China?

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Abstract

The ratio of men relative to women has surged up significantly during the last 30 years in China. What is the effect of imbalanced sex ratios on women and men? In this paper, I quantify the effect of an increased sex ratio on marriage and labor market outcomes of men and women in China. Specifically, I develop a unified model of marriage and divorce, imbalanced sex ratios and intra-household bargaining. The model highlights two channels through which the sex ratio affects men and women: the bargaining channel and the marriage channel. First, since the sex ratio affects the outside option of marriage differently across gender, it alters the bargaining power within the household, which in turn changes intra-household allocations. Second, females have a higher probability of meeting a husband, and given more choices, females can find better husbands. Therefore, the marriage channel directly affects marriage decisions and thus intra-household allocations through the income effect. The model is calibrated to match Chinese labor and marriage market data moments in 1990. A quantitative exercise of increasing the sex ratio from 1.072 to 1.2 as in 2007 shows that total female working hours drop by 5.1% and total male working hours rise by 5.2%. The change comes from the decreased married female working hours and the increased female married fraction, given that married females work less than single females. Both predictions are in line with the actual data trend. Although the fraction of men that are married falls from 76.7% to 69.6%, they become more willing to get married. The increased sex ratio also leads to a higher degree of assortative mating. Overall, females are 9.5% better off while males are 14.42% worse off. Decomposing the results, I find that the channel of a wider choice of husbands contributes most in explaining the change of welfare, the bargaining channel plays a less important role, and the better husbands channel is a supplement to the other two channels.

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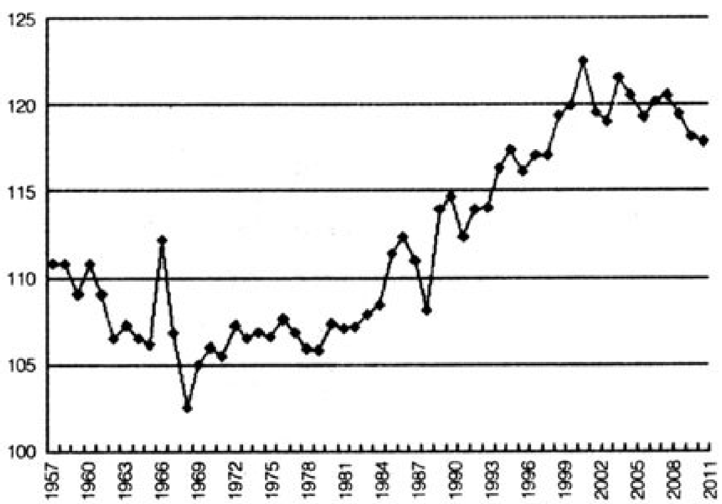
1 Introduction

The sex ratio, which is defined by the number of males per 100 females, has increased significantly in China since 1980. The driving force comes from the male-biased sex ratio at birth especially after the implementation of the One-child Policy in 1978.[1] Figure 1 illustrates this trend, where the sex ratio at birth increased from the natural level around 105 in the 1980s to over 120 in the 2000s. According to the 2015 China Statistical Yearbook, there are 33 million more men than women in China. Such a gap could generate a significant impact on China's marriage and labor market. What is the effect of the imbalanced sex ratio on women and men, especially their marriage market and labor market outcomes?

Figure 1: Sex ratio at birth in China



Source: China population Census and 1% population sample surveys

To answer this question, in this paper I develop a unified model with 3 main features: a standard matching process of the marriage market, imbalanced sex ratios, and an intra-household decision making process of married households. Firstly, the marriage market is

[1]The reason why the sex ratio is increasing is not the focus of this paper, but Li, Yi, and Zhang (2011) and Bulte, Heerink, and Zhang (2011) find that 50% to 90% of the imbalanced sex ratio can be explained by the implement of the One Child Policy in China. Qian (2008) argues that the increased male to female income worsens female survival rates in China.

defined by a random matching process, where single agents meet randomly depending on the meeting probability, and marriage occurs if and only if both agree to marry. Divorce happens when one of the spouses wants to divorce. Secondly, the meeting probability depends on the population size of singles, which is a function of the sex ratio. Lastly, the intra-household allocation is through a Nash bargaining process, with being single as the outside option. Since divorce is a credible threat, when being single is more attractive for the female, it raises her bargaining power.[2]

The impact of the imbalanced sex ratio on the marriage market and labor market passes through two channels, namely the bargaining channel and the marriage channel. The bargaining channel works as follows: Given a larger male population size, when divorce is feasible, a married female may credibly threaten to get divorced. In addition, since females are scarcer, it is harder for the husband to find a wife once he is divorced. Therefore such threat can provide a better outside option for the female, which increases her bargaining power.[3] The higher bargaining power enables the female to enjoy more leisure and work less. It also improves the attractiveness of marriage for females, since they can obtain more resources from the marriage.

The marriage channel affects females and males more broadly. When there are more males available, females have a wider choice of husbands. The likelihood of meeting a spouse in the marriage market is higher for females, so it is more likely for females to get married. But given a wider choice of males, females may also wait longer for a better husband to marry. They are more picky in choosing their partners, which drives the marriage rate down. So the female marriage decision further depends on whether a wider

[2]In China the crude divorce rate, which is defined by the number of divorces per 1000 population, has increased from 0.7 in 1990 to 2.8 in 2015. In U.S. this number in 2014 is 3.2, suggesting that divorce is valid threat in China.

[3]Chiappori, Fortin, and Lacroix (2002) study the intra-household allocation under a collective model framework. They validate the sex ratio in the marriage market as a distribution factor, which affects intra-household bargaining position.

choice of husbands dominates the willingness of finding a better husband. The marriage channel also affects labor supply. On the one hand, married females work less than single females; on the other hand, those who have a better husband are able to enjoy more leisure due to the income effect.

The model is calibrated to match China labor and marriage market statistics in 1990 with the minimum distance estimation method as the benchmark. Then the sex ratio is adjusted exogenously to the 2007 level, while other parameters are held constant. To investigate the effect of the imbalanced sex ratio on the marriage market and the labor market, I compare model predictions for 2007 with the corresponding 1990 outcomes. The imbalanced sex ratio has a significant impact on labor market and marriage market outcomes. Total female working hours fall by 5.1% and total male working hours increase by 5.2%. Two reasons can explain the change: First, married female daily working hours drop from 7.71 hours to 7.38 hours. Second, the fraction of females that are married increases from 80.94% to 83.54%. Given that married females work less than single females, the compositional effect from the change of married share further reduces total female working hours. The model predictions of changes in married female working hours and the fraction of females that are married are in line with the actual data trend. In contrast, married male working hours rise from 7.96 hours to 8.7 hours per day, and the fraction of men that are married falls from 76.65% to 69.62%. However, males' willingness of getting married increases, which means they are less picky in choosing the spouse. The increased sex ratio also leads to a higher degree of assortative mating, where the fraction of married couples with the same education level increases by 8.22%. On average, females are 9.5% better off in consumption equivalent units, while males are 14.42% worse off.

The change of female welfare comes from 3 channels: a higher bargaining power, a higher probability of meeting a husband and marrying a better husband. Then a decomposition exercise is done by turn on each of the 3 channels. Results suggest that the higher probability of meeting a husband plays the most important role, which accounts for about 60% of the increase. While it barely affects married female labor choice, it raises the fraction of females that are married by 3.68 percentage points. The bargaining channel also explains about one-third of the welfare change. Females benefit from an average 12.05% increase in female bargaining power, which enables them to enjoy more leisure and marry more. On average, married women reduce working hours by 0.26 hours per day. The increased attractiveness of marriage due to a higher bargaining power contributes 0.32 percentage points to the increased married fraction of females. Females who find a better husband also contribute to 0.12% increase in female welfare. Due to the income effect, married females reduce their working hours by 0.07 hours. In terms of the marriage market outcomes, it leads to 0.16 percentage points decrease in the fraction of females that are married, because females are more picky in choosing the husband. The effect on male welfare works the other way round through the three channels.

This paper contributes to the literature about sex ratios, marriage decision and intrahousehold allocations. In the vein of the effect of sex ratios on marriage market outcomes, Becker (1973, 1974) first establish the theory of the marriage market, where he uses a transferable utility model to argue that the change of sex ratios affects relative supply and demand in the marriage market, which further determines the relative share of marital income in the marriage. Such results are supported by recent empirical studies. Angrist (2002) documents that the imbalanced sex ratio in the 2nd generation immigrants in U.S. leads to a higher likelihood of marriage for females. Abramitzky, Delavande, and Vasconcelos (2011) use WWI as an exogenous negative mortality shock to male population, and find that in France men had a higher likelihood of marriage in the post-WWI period. Similar results are found in China, that the scarcer females are more likely to get married when sex ratio is more imbalanced (Edlund, Li, Yi, and Zhang (2013), Porter (2016)). My quantitative result conforms to their empirical findings.

The other strand of literature studies the impact of sex ratios on intra-household allocations and other economic outcomes. Chiappori, Fortin, and Lacroix (2002) extend the Beckerian framework into a collective model, and the sex ratio is one of the distribution factors that affects intra-household bargaining position. I extend the model to a Nash bargaining framework, and the results are consistent with their findings. Siow (1998) investigates theoretically the effect of relatively scarcer fecund females on gender roles. Du and Wei (2010) investigate theoretical that the imbalanced sex ratio leads to a competitive savings behavior among males to increase their attractiveness in the marriage market, which contributes to China's high savings rate. Empirically, the negative effect of sex ratios on female labor force participation has been identified in U.S., Australia and China (Angrist (2002), Grosjean and Khattar (2015) and Edlund, Li, Yi, and Zhang (2013)). Wei and Zhang (2011) further validates Du and Wei (2010)'s theory empirically, suggesting that the imbalanced sex ratio effect passes through the marriage market and has more extensive impact on other economic outcomes. The focus of this paper is marriage and labor market outcomes.

To my knowledge, this is the first paper that provides a unified model with imbalanced sex ratios, marriage and divorce decisions and an intra-household bargaining process. The matching model of the marriage market has been applied in many quantitative studies (Aiyagari, Greenwood, and Guner (2000), Greenwood, Guner, Kocharkov, and Santos (2016)), but they usually treat a married household as one decision unit and ignore intra-household decision making. Greenwood, Guner, and Knowles (2003) incorporate Nash bargaining into the married household decision problem to quantify the marriage and fertility effect on the distribution of income. The intra-household bargaining is further extended and implemented in quantitative works by Siegel (2013), Knowles (2013) and Voena (2015), but they all assume a one-to-one matching between two genders in the marriage market. As far as I know, the only quantitative studies with imbalanced sex ratios are Knowles and Vandenbroucke (2015) and Rıos-Rull, Seitz, and Tanaka (2016), but both papers only study marriage market outcomes and ignore intra-household decisions.

The rest of the paper is organized as follows: Section 2 describes the model setup. Section 3 presents the calibration procedure. The experiment of an increased sex ratio is discussed in Section 4. Section 5 decomposes the effect on welfare into 3 channels. Section 6 concludes.

2 Model setup

2.1 Demographics

The economy is populated by males m and females f, who are different in population sizes. They are either single or married (S and M). Denote the single female population size to be S_f, and the number of single males to be S_m. Similarly the married counterpart is given by M_f and M_m.[4] The sex ratio is imbalanced, denoted by $\phi > 1$, which changes exogenously. The female population is normalized to be 1, then the male population is ϕ.

Agents face a mortality risk at the rate of δ. It is assumed to be the same for all agents. For married couples, they die together. Upon death, the individual will be replaced by a new-born doppelganger.[5] The new-born individual is endowed with the same productivity (z_m and z_f) as the dead counterpart, and the productivity levels are drawn from distri-

[4]The time index is suppressed for simplicity.

[5]Each period δ of females are dead and being replaced, while the replaced male population size is $\delta\phi$ due to the imbalanced sex ratio.

butions $z_f \in Z_f$ and $z_m \in Z_m$. Once the productivity is drawn, it remains constant for his/her life time.

2.2 Preferences

Agents derive utility from consumption c and leisure. Each person has one unit of time endowment each period, and it can be divided between market work ℓ and leisure. Consumption is assumed to be a public good for the couple to capture the economies of scale of a married family.[6]

The utility function is separable and constant relative risk aversion (CRRA) over consumption and disutility of labor.[7] It takes the functional form of:

$$U(c, \ell) = \frac{c^{1-\gamma}}{1-\gamma} - \psi_g \frac{\ell^{1+\mu}}{1+\mu},$$

where γ is the inverse of consumption elasticity, μ is the Fisch elasticity of labor supply, and ψ_g captures the disutility of working, which is gender specific.[8]

[6]This assumption assumes that consumption is jointly utilized by both spouses. Examples of such kind of goods include house, children and home appliances, which can be non-exclusively shared by both spouses. Public consumption within married households is considered to be a large component of household consumption (Lam (1988), Lundberg and Pollak (2007) Becker and Becker (2009) and Browning, Chiappori, and Lewbel (2013)).

[7]Home production is ignored in the model for two reasons. First is the data limitation. There is no data available on home production time before 2010, which is not the period of interest. In addition, my mechanism still works even without home production. As will be discussed later, wives work less. But the reduced working time is used for leisure instead of home production because females have a higher bargaining power. With the higher bargaining power, females can enjoy more leisure instead of working more, neither in the market, nor at home.

[8]I assume the disutility of working parameter ψ_g to be gender specific, because as will be shown later, I assume that there is a gender wage gap between two genders, and it will generate a much lower married female working hours than male in the model. But in the data married females work almost the same amount of time as males, so I need at least another parameter to lower female working hours to match the data. The disutility of working parameter is the most suitable choice.

2.3 Budgets

Denote the exogenous wage rate to be w. Assume that there is a gender wage gap κ between males and females such that females only earn a fraction of male wage rate:

$$w_m = w$$
$$w_f = \kappa w_m$$

Each period, the agent consumes all the labor income, so the budget constraint of a single agent with gender g is given by:

$$c = w_g z_g \ell.$$

For married households, they pool their income together for their public consumption:

$$c = w_m z_m \ell_m + w_f z_f \ell_f$$

2.4 Marriage

The marriage market is characterized by a random match process between singles with the opposite sex. Each period, a single agent has a chance of meeting a potential spouse in the marriage market. The meeting probabilities depend on the share of the single population of the opposite sex in the total single population (Schoen (1981)):

$$\omega_f = \frac{S_m}{S_f + S_m}$$
$$\omega_m = \frac{S_f}{S_f + S_m}.$$

Note that the meeting probability is endogenous as a function of the distribution of singles, which is determined by the sex ratio. When the sex ratio is more imbalanced, single females will face a higher likelihood of meeting a partner while single males are more likely to find no match. If singles do not meet anyone, they stay in the single pool.

Once two potential spouses meet, they draw a match-specific marriage quality shock $q \in Q$ from a distribution $F(q)$. To be more specific, $q \sim N(q_s, \sigma_s^2)$, where q_s and σ_s represent the mean and standard deviation of the marriage quality shock distribution for an unmarried couple. Within the marriage, the marriage quality shock is assumed to follow the distribution $G(q'|q)$, which evolves according to an autoregressive progress given by:

$$q' = (1 - \rho_m)q_m + \rho_m q + \sigma_m \sqrt{1 - \rho_m^2}\sigma,$$

where q_m and σ_m represent the long-run mean and standard deviation, and ρ_m is the autocorrelation coefficient, and $\sigma \sim N(0, 1)$. If both agents accept the marriage, they get married, but if any of the two rejects, they have to stay single. The incentive of getting married comes from economies of scale due to public consumption and the marriage quality shock.

When the marriage quality shock for the married households updates, the married couple decide whether to stay in marriage or get divorced. If one of them wants to get divorced, they become single.

2.5 Intra-household allocations

The consumption and labor choice of single households is static without savings. For a given gender g, he/she maximizes his/her period utility given the budget constraint:

$$\begin{aligned} U(c,\ell) &= \max_{c,\ell} \frac{c^{1-\gamma}}{1-\gamma} - \psi \frac{\ell^{1+\mu}}{1+\mu} \\ s.t. &\qquad c = w_g z_g \ell \end{aligned} \tag{1}$$

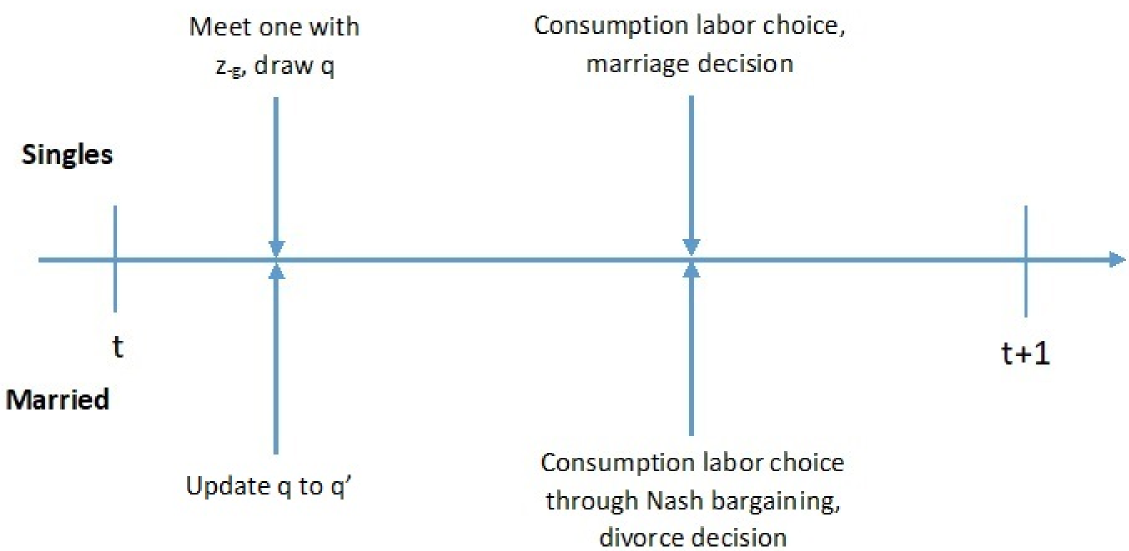
For married households, the intra-household allocation is through a Nash bargaining process. Since divorce is a credible threat, being single is the outside option of the bargaining. Therefore, the consumption and labor decision of married households is given by:

$$\begin{aligned} \max_{c^M,\ell_f,\ell_m} (V_m^M(z_g, z_{-g}, q; S_g, S_{-g}) - V_m^S(z_m; S_g, S_{-g}))(V_f^M(z_g, z_{-g}, q; S_g, S_{-g}) - V_f^S(z_f; S_g, S_{-g})) \\ s.t.\ c^M = w_m z_m \ell_m + w_f z_f \ell_f, \end{aligned} \tag{2}$$

where V^M and V^S are value functions of being married and being single respectively, which will be defined later. Note that the value of being single is a function of the meeting probability ω_g, which depends on the distribution of single populations. A higher sex ratio will increase the single male population and thus the likelihood of meeting a spouse for a female, and this raises her value of being single given the benefit of marriage. Therefore the female face a better outside option, which increases her bargaining power, and she can obtain more resources within the household in the form of enjoying more leisure.

The timing within a period is illustrated by Figure 2. At any period, a single agent with gender $g \in \{f, m\}$ and productivity z_g first meets the potential spouse of productivity type z_{-g} with probability ω_g. If they meet, they draw the marriage quality shock q. If he/she does not meet anyone, he/she makes consumption and labor choice as a single household by equation (1). For those who meet, they decide whether to get married or not, by comparing the value of being married, given the married consumption and labor choice through a Nash bargaining process defined by equation (2), with the value of being single. For those who are already married, they first update their marriage quality shock from q to q', then renegotiate the intra-household allocation together with the divorce decision. The marriage decision and intra-household bargaining take place at the same time, such that the threat of divorce is credible.

Figure 2: Timing of the model



2.6 Value functions

2.6.1 Value of being single

Now let me define the value function of being single. Consider a single agent of gender $g \in \{f, m\}$ with productivity type z_g and aggregate states of the single population distribution S_g and S_{-g}. Given the optimal allocation of single households in the current period, which generates the indirect utility $U_g^{S*}(z_g; S_g, S_{-g})$, the individual enters the next period with a discount factor β and the survival probability $1-\delta$. In the next period, the individual has $\omega_g(S_g, S_{-g})$ probability of meeting the potential spouse given the normalized distribution of the partner $\hat{S}_{-g}(z_{-g}; S_g, S_{-g})$, as will be defined later. Once they meet, they draw a common marriage quality shock q from the distribution $F(q)$. They will get married if and only if both of them are better off being married than being single:

$$V_g^M(z_g, z_{-g}, q; S_g, S_{-g}) \geq V_g^S(z_g; S_g, S_{-g}) \; and \; V_{-g}^M(z_g, z_{-g}, q; S_g, S_{-g}) \geq V_{-g}^S(z_{-g}; S_g, S_{-g}) \tag{3}$$

Let the indicator function $\mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g})$ represent the marriage decision. It equals 1 if both want to get married and equals 0 if one of them rejects.

$$\mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g}) = \begin{cases} 1, & if \ (3) \ holds, \\ 0, & otherwise \end{cases} \tag{4}$$

Note that $\mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g}) = \mathbb{1}_{-g}(z_g, z_{-g}, q; S_g, S_{-g})$ given that the marriage decision is a joint decision of both spouses.

If the individual does not meet anyone with probability $1-\omega_g(S_g, S_{-g})$, he/she has to stay single.

Therefore, for gender $g \in \{f, m\}$, the value function of being single is given by:

$$\begin{aligned} V_g^S(z_g; S_g, S_{-g}) \;=\; & U_g^{S*}(z_g; S_g, S_{-g}) \\ & +\beta(1-\delta)\{\int_Q \int_Z [\omega_g(S_g, S_{-g}) V_g^M(z_f, z_m, q; S_g, S_{-g}) \mathbb{1}_g(z_f, z_m, q; S_g, S_{-g}) \\ & +\omega_g(S_g, S_{-g})(1-\mathbb{1}_g(z_f, z_m, q; S_g, S_{-g})) V_g^S(z_g; S_g, S_{-g}) \\ & +(1-\omega_g(S_g, S_{-g})) V_g^S(z_g; S_g, S_{-g})] d\hat{S}_{-g}(z_{-g}; S_g, S_{-g}) dF(q)\}, \end{aligned} \tag{5}$$

where $\hat{S}_m(z_m)$ is the normalized distribution of single males, which is given by:

$$\hat{S}_m(z_m) = \frac{S_m(z_m)}{\int_{Z_m} S_m(z_m)}$$

The first term in the continuation value stands for the case that both spouses agree to get married. The second term represents the case that at least one of the potential spouses rejects the marriage. The last term is the case that the individual meets nobody. The sex ratio not only affects the meeting probability $\omega_g(S_g, S_{-g})$, but also the normalized distribution $\hat{S}_{-g}(z_{-g}; S_g, S_{-g})$.

2.6.2 Value of being married

The value function of being married is defined similarly to the value of being single. Consider a married individual of gender g. The couple's productivity type combination is given by z_f and z_m, and a realization of marriage quality shock to both spouses is q. The aggregate states of the single population distribution are S_g and S_{-g}. Given the optimal intra-household allocation obtained from the Nash bargaining problem defined in (2), the married individual obtains the indirect utility $U_g^{M*}(z_f, z_m; S_g, S_{-g})$. On top of it, there is an additional utility gain of q from marriage. In the next period, the marriage quality shock q updates to q' according to the distribution $G(q'|q)$, and two spouses decide whether to get divorced or not.

Divorce occurs if one of the two spouses wants to get divorced:

$$V_g^M(z_f, z_m, q; S_g, S_{-g}) < V_g^S(z_g; S_g, S_{-g}) \text{ } or \text{ } V_{-g}^M(z_f, z_m, q; S_g, S_{-g}) < V_{-g}^S(z_{-g}; S_g, S_{-g}) \quad (6)$$

Therefore, the indicator function $\mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g}) = 1$ if they want to stay in marriage, and takes the value of 0 if anyone wants to get divorced.

$$\mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g}) = \begin{cases} 1, & if \text{ (6) } holds, \\ 0, & otherwise \end{cases} \tag{7}$$

So for gender $g \in \{f, m\}$, the value function of being married is specified as follow:

$$\begin{aligned} V_g^M(z_f, z_m, q; S_g, S_{-g}) &= U_g^{M*}(z_f, z_m; S_g, S_{-g}) + q \\ &+\beta(1-\delta)\{\int_Q [V_g^M(z_f, z_m, q'; S_g, S_{-g})\mathbb{1}_g(z_f, z_m, q'; S_g, S_{-g}) \\ &+V_g^S(z_g; S_g, S_{-g})(1 - \mathbb{1}_g(z_f, z_m, q'; S_g, S_{-g}))]dG(q'|q)\}, \end{aligned} \tag{8}$$

where the first term in the continuation value gives the case of remaining in marriage, and the second term represents divorce.

2.7 The stationary equilibrium

The married household allocation problem is through a Nash bargaining process, where the outside option is the value of being single. As mentioned above, the value of being single is a function of the meeting probability. Since the meeting probability depends on the distribution of singles, to understand how the sex ratio affects the distribution of singles and thus the value functions and the intra-household allocation requires knowing the steady state population distributions.

The steady state distribution of the single female is defined by:

$$\begin{aligned}
S_f(z_f') &= (1-\delta)\omega_f(S_f,S_m)\int_Q\int_{Z_m}\int_{Z_f}^{z_f'}[1-\mathbb{1}_f(z_f,z_m,q;S_f,S_m)]dS_f(z_f)d\hat{S}_m(z_m)dF(q)\\
&+(1-\delta)\int_Q\int_Q\int_{Z_m}\int_{Z_f}^{z_f'}[1-\mathbb{1}_f(z_f,z_m,q;S_f,S_m)]dM_f(z_f,z_m,q_{-1})dG(q|q_{-1})\\
&+(1-\delta)(1-\omega_f(S_f,S_m))\int_{Z_f}^{z_f'}dS_f(z_f)\\
&+\delta, \qquad (9)
\end{aligned}$$

The first term represents single females who survived from last period, meet a male but does not get married. The second term captures females whose marriage breaks up. The third term gives single females who fail to meet any male. And the last term is the death replacement. Since the female population is normalized to be 1, and all females face the same mortality rate, the death replacement is simply δ.

Analogously, the distribution of single males is defined by:

$$\begin{aligned}
S_m(z_m') &= (1-\delta)\omega_m(S_f,S_m)\int_Q\int_{Z_f}\int_{Z_m}^{z_m'}[1-\mathbb{1}_m(z_f,z_m,q;S_g,S_{-g})]dS_m(z_m)d\hat{S}_f(z_f)dF(q)\\
&+(1-\delta)\int_Q\int_Q\int_{Z_f}\int_{Z_m}^{z_m'}[1-\mathbb{1}_m(z_f,z_m,q;S_f,S_m)]dM_m(z_f,z_m,q_{-1})dG(q|q_{-1})\\
&+(1-\delta)(1-\omega_m(S_f,S_m))\int_{Z_m}^{z_m'}dS_m(z_m)\\
&+\delta\boldsymbol{\phi} \qquad (10)
\end{aligned}$$

Compared with single females, the single male population is larger due to the imbalanced sex ratio. Since the male population is ϕ, given the same mortality rate, $\delta\phi$ amount of males are replaced by singles. Such difference is captured by the last term in the distribution function, the death replacement.[9]

[9] Even though each period there are more males being replaced by singles, the single male population is not explosive. A simple example to illustrate this is, suppose there are 100 females and 110 males in the

The steady state distribution of the married individual with gender g is given by:

$$M_g(z'_g, z'_{-g}, q') = (1-\delta)\omega_g(S_g, S_{-g})) \int_Q^{q'} \int_{Z_g}^{z'_g} \int_{Z_{-g}}^{z'_{-g}} \mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g}) d\hat{S}_{-g}(z_{-g}) dS_g(z_g) dF(q)$$
$$+(1-\delta) \int_Q^{q'} \int_Q \int_{Z_g}^{z'_g} \int_{Z_{-g}}^{z'_{-g}} \mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g}) dM_g(z_g, z_{-g}, q_{-1}) dG(q|q_{-1}) \quad (11)$$

The first term captures the survived individuals changing their marital status from single to married. The second term measures the survived marriages. The marriage market clears in the stationary equilibrium, which means $M_f = M_m$ and this defines a fixed-point problem.

Now the stationary equilibrium of the economy can be defined as following:

Definition 1: A stationary equilibrium is allocations for single and married households, c_g^S, ℓ_g^S, c^M, ℓ_f^M and ℓ_m^M; a set of value functions for singles and married, $V_g^S(z_g; S_g, S_{-g})$ and $V_g^M(z_g, z_{-g}, q; S_g, S_{-g})$; a matching rule for singles and married couples $\mathbb{1}_g(z_g, z_{-g}, q)$; and stationary distributions for singles and married couples $S_g(z_g)$ and $M_g(z_g, z_{-g}, q)$ for $g = m, f$, such that the following conditions hold:

- The allocations c_g^S, ℓ_g^S solve for the single household problem (1), and c^M, ℓ_f^M and ℓ_m^M solve for the married household Nash bargaining problem (2).
- The value function $V_g^S(z_g; S_g, S_{-g})$ defines the single's problem (5) given her/his indirect utility function $U_g^S(c*_g, \ell*_g)$ from problem (1); the value function for a married person $V_g^M(z_g, z_{-g}, q; S_g, S_{-g})$, the meeting probability $\omega_g(S_g, S_{-g})$, the matching rule for singles $\mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g})$, and the normalized distribution of singles $\hat{S}_g(z_g)$.

economy, and 80 of them are married. So the sex ratio is 1.1. Assume the mortality rate is 10%. Every period 10 females are dead, 8 of which are married and 2 are single; 11 males are dead, 8 of which are married and 3 are single. After the death replacement, there are 28 single females and 38 single males. Once 8 of them get married in the next period, it still guarantees a stationary equilibrium.

- The value function $V_g^M(z_g, z_{-g}, q; S_g, S_{-g})$ defines the a married person's problem (8) given her/his indirect utility function $U_g^M(c*_g, \ell*_g)$ obtained from problem (2); the value function for a single $V_g^S(z_g; S_g, S_{-g})$, and the matching rule for a married couple $\mathbb{1}_g(z_g, z_{-g}, q')$.

- The matching rule $\mathbb{1}(z_g, z_{-g}, q; S_g, S_{-g})$ is determined by equations (4) and (7), given the value functions $V_g^S(z_g; S_g, S_{-g})$ and $V_g^M(z_g, z_{-g}, q; S_g, S_{-g})$.

- The stationary distributions $S_g(z_g)$ and $M_g(z_g, z_{-g}, q)$ evolve according to equations (9), (10) and (11), given the matching rule $\mathbb{1}_g(z_g, z_{-g}, q; S_g, S_{-g})$.

2.8 Solving the Nash bargaining problem

It involves a complicated fixed point problem to solve the model. On the one hand, the allocations of married households require the knowledge of value functions which depends on the meeting probability and distributions. On the other hand, the distributions depend on optimal allocations and value functions.

The Nash problem itself is hard to solve directly given the interaction of value functions between the two counterparts. To solve the problem, since a Nash problem is Pareto optimal, I can transform the Nash problem in (2) into a weighted sum of utilities to pin down the allocation on the Pareto frontier:

$$\begin{aligned} \max_{c,\ell_f,\ell_m} \quad & \theta U_f(c, \ell_f) + (1-\theta) U_m(c, \ell_m) \\ s.t. \quad & c = w_m z_m \ell_m + w_f z_f \ell_f, \end{aligned} \tag{12}$$

where $\theta = \frac{V_m^M(z_g,z_{-g},q;S_g,S_{-g}) - V_m^S(z_m;S_g,S_{-g})}{(V_m^M(z_g,z_{-g},q;S_g,S_{-g}) - V_m^S(z_m;S_g,S_{-g})) + (V_f^M(z_g,z_{-g},q;S_g,S_{-g}) - V_f^S(z_f;S_g,S_{-g}))}$.

Lemma 1: When consumption is pure public, the Nash bargaining problem of a married household defined in (2) is equivalent to the problem of maximizing the weighted sum of utilities defined in (10).

Proof: see Appendix A1.

θ is the Pareto weight attached to the wife, which captures her bargaining power in the household. If the marriage is feasible, θ lies between 0 and 1. Upon a match, $\theta < 0$ if the male wants to stay single, and $\theta > 1$ if the female does not want to accept the marriage. Note that the weight is endogenous, which depends on the state $(z_g, z_{-g}, q; S_g, S_{-g})$. One can obtain a higher bargaining power if he/she has a higher productivity, a lower marriage quality shock, or a lower single population size, as it increases the value of being single and thus a better outside option.

The detailed algorithm to solve the structure model is illustrated in Appendix A2.

3 Calibration

To analyze the impact of changing sex ratios on marriage market and labor market outcomes, the model is solved numerically. The base year for calibration is set to be 1990. Even though after the implementation of the one-child policy in 1978 the sex ratio started to increase, I still use 1990 as the base year for two reasons. On the one hand, the sex ratio in 1990 is 1.072, which is still close the natural sex ratio of 1.05.[10] On the other hand, the labor supply data at the intensive margin is only available after 1989. Therefore, I choose 1990 as the base year where I have the information about individual marital status,

[10]The World Health Organization computes the statistics based on the UN World Population Prospect from 1990 to 2010, and suggests that the average number is 1.05. See http://www.searo.who.int/entity/health_situation_trends/data/chi/sex-ratio/en/.

education level and working hours. The key data targets are married individual working hours and the marital sorting of different education groups. Some parameters are assigned using a priori information, while the remaining parameters are calibrated to match some key data moments through the minimum distance estimation method.

Then in the next section, the model is simulated using the sex ratio in 2007 to access the role of the imbalanced sex ratio on marriage and labor market outcomes. The reason why the year 2007 is chosen as the reference year is because the sex ratio reached its peak of 1.2 in 2007. It is assumed that the economy is at steady state in both reference years.

3.1 Parameters based on a priori information

Some of the parameters are taken directly from the literature or from independent evidence. These parameters are summarized in Table 1.

The utility function parameters are taken from the literature, where the inverse of consumption elasticity is 2 as in Hall (1988) and the inverse of Frisch elasticity is 3 as estimated by Chetty, Guren, Manoli, and Weber (2011). The length of a model period is 1 year, so the discount factor takes the value of 0.96. Since I am interested in the marriage and labor market outcomes, the population I am focusing on in all the analysis is between 18 and 50 years old, which corresponds to an average lifespan of 33 years.[11] So the mortality rate being 1/33 gives an average live of 33 years. All data targets for the estimation are calculated for population within this age range. The gender wage gap in 1990 is taken from Zhang, Zhao, Park, and Song (2005), who control for the educational

[11] In China, the official female retirement age is between 50 and 55. Therefore I restrict the interested population to be below 50 years old to avoid the forced retirement.

Table 1: Pre-determined parameter values

Parameters	Value	Explanation
Utility		
γ	2	Inverse of consumption elasticity (Hall (1988))
μ	3	Inverse of Frisch elasticity (Chetty, Guren, Manoli, and Weber (2011))
β	0.96	Discount factor
δ	1/33	Mortality rate between age 18 and 50
κ	0.80	Gender wage gap (CHNS 1989)
ϕ	1.072	Sex ratio (1990 Census)
z_f, z_m	1	Educational premium
	1.075	(less than primary school=1, primary completed=1.075,
	1.191	secondary completed=1.191, university completed=1.375)
	1.375	(Liu (1998) and Zhang et al. (2005))
Z_f	0.3035	Distribution of female education (1990 Census)
	0.5782	
	0.1148	
	0.0035	
Z_m	0.1433	Distribution of male education (1990 Census)
	0.6821	
	0.166	
	0.0086	

level and experience. The sex ratio of the interested age group in 1990 is 1.072, as is calculated from the 1990 Census.

Lastly, the productivity distributions are directly taken from the educational attainment distribution from the 1990 Census. The productivity level is classified into 4 categories, which represents “Less than primary school”, “Primary school completed”, “Secondary school completed” and “University completed”. So the productivity z_g represents the education premium, and it is assumed to be identical between women and men with the same educational level. The productivity level of “Less than primary school” is normalized to 1. As estimated by Liu (1998) and Zhang, Zhao, Park, and Song (2005), the education premium of “Primary school completed”, “Secondary school completed” and “University completed” relative to “Less than primary school” on average is 7.5%, 19.1% and 37.5% respectively between 1989 and 1992. Therefore the productivity levels are 1, 1.075, 1.191 and 1.375 for the four categories respectively. The distributions of educational attainment Z_g are different between females and males. In 1990, males on average have a higher educational attainment than females. For males, 14.33% have less than primary school education, 68.21% complete primary school, 16.6% finish their secondary education and 0.86% have a college degree or above. For females, 30.35% of them do not complete primary education, 57.82% have their primary education done, 11.48% complete secondary school and only 0.35% finish higher education.

3.2 Parameters estimated by the minimum distance estimation method

The remaining 7 parameters, namely 2 disutility of working parameters $\{\psi_f,\ \psi_m\}$ and 5 marriage quality shock parameters $\{q_s,\ \sigma_s\ \ q_m,\ \sigma_m,\ \rho_m\}$, are estimated through the minimum distance estimation method.

The data targets are: *(i)* married female and male working hours, *(ii)* marriage market outcomes.

The married female and male working hours are calculated from the 1991 CHNS data. The data set reports the average hours individual works per day from last year (1990).The married population is defined by currently married or cohabiting couples, while the single population includes those who are never married, separated/divorced or widowed. Note that the fraction of population with a college degree or higher is very small in the Census data, and the number of college graduated observations in the 1991 CHNS data set is even smaller (less than 80). So instead of targeting the working hours data for each education group, I take the average married female and male working hours as the calibration target.[12]

The marriage market outcomes I am focusing on are: the fraction of females that are married in the female population and assortative mating patterns, which is defined by the number of marriages for all possible education combinations between two spouses. The marriage market outcome data targets are calculated with the 1990 Census data. Given

[12] According to my calculation, the average daily working hours between wives and husbands are almost the same. Zero working hour data are also included. The 1991 CHNS also reports the number of days individual works and the number of months individual works. I also calculate the average weekly working hours and the average yearly working hours. Even though the average weekly working hours and the average yearly working hours are slightly lower than the average daily working hours, there is very little difference between husbands and wives in all three measures. Therefore I directly use the average daily working hours as the data target.

the sex ratio and the fraction of females that are married, when the female population is normalized to 1, the fraction of males that are married can be directly obtained. So I do not take the fraction of males that are married as a data target. Since the population size with higher education is very small, the number of marriages with one partner being highly educated is also very low relative to other groups. Therefore, to eliminate the effect from the long tail of education distribution, the assortative mating targets are normalized by dividing the number of marriages in each education group combination by the number of married females for each female education level. As a result, for each female education level, the measure of assortative mating sums to 1.[13]

In total, I have 19 data targets. Two of them capture labor market outcomes, which are the married female working hours and the married male working hours. The rest 17 data targets are marriage market outcomes. One of them captures the overall married fraction of population, which is the fraction of females that are married. The other 16 represents the assortative mating pattern.

Let η be the vector of 7 parameters to be estimated, and $g(\eta)$ be the vector of moments generated by the model as a function of parameters. Denote g^{data} be the vector of 19 data moments obtained from 1989 CHNS and 1990 Census. So the vector of distances between data moments and model moments is given by $G(\eta) = g^{data} - g(\eta)$.[14] Then the minimum distance estimator is defined as:

$$\hat{\eta} = \arg\min_{\eta} G(\eta)' W G(\eta),$$

[13]The normalization by the population size of each education group is crucial in the calibration to capture marriage market outcomes. One example is, after the normalization, the share of marriages with the same education level is very prevailing in the college educated groups (as shown in Table 3). However, without normalization, it would only account for 0.2% of population.

[14]Here I use the absolute deviation instead of the percentage deviation, because the data targets are more or less in the same magnitude. The working hours are normalized to be between 0 and 1, and the other targets are measured by shares, which are also between 0 and 1.

where W is a positive semi-definite weighting matrix. [15]

Table 2 reports the calibrated parameter values.

Table 2: Calibrated parameters

Parameters	Explanation	Value
ψ_f	Disutility of working of female	16.136
ψ_m	Disutility of working of male	19.755
q_s	Mean of single's marriage quality distribution	-0.4964
q_m	Mean of married marriage quality distribution	0.399
ρ_m	Persistence of married marriage quality distribution	0.9654
σ_s	Variance of single's marriage quality distribution	2.0586
σ_m	Variance of married marriage quality distribution	1.0065

Females have a lower disutility of labor than males as expected. The lower disutility of labor leads to a higher female working hours, while the gender wage gap discourages her to work. Therefore the model is able to generate similar working hours between husbands and wives to match the data. The mean of the single's marriage quality shock is negative, as consumption in married household is public, which is already enough to motivate singles to get married, so the negative value of the love shock offsets some of the marital benefit to match the marriage data. The variance of the single's marriage quality shock is larger, which gives singles incentive to wait longer until they meet a better match. Once the two meeting partners get married, the marriage quality shock is very persistent ($\rho_m = 0.9654$).

The performance of the model is illustrated in Table 3, where the model moments and data moments are displayed. Overall, the model can replicate many of the data. The working hours can be matched well. The model can also mimic the overall marriage market as the fraction of females that are married is relatively well matched. Unfortunately some of the assortative mating moments are off the target especially in the high- educated groups,

[15] The weights are chosen such that I put equal weight on the labor market targets and on the marriage market targets. Since there are only 2 labor market moments and 17 marriage market moments, I set the weights on married female and male working hours to be 8.5 and the weights on each of the marriage market targets to be 1.

because not only the population of the high-educated group is too small which generates a very small meeting probability between high types, but also the model is over-identified that there are 19 data moments but only 7 parameters calibrated.

Table 3: Data and model moments

Statistics	Data moment				Model moment			
Working hours								
Wife	7.87				7.71			
Husband	7.93				7.96			
Marriage								
Fraction of Married Females	0.809				0.806			
Sorting (% of married females of each type)	husband				husband			
wife	<primary	primary	middle	uni	<primary	primary	middle	uni
<primary	0.3532	0.5915	0.0551	0.0002	0.1474	0.7024	0.1421	0.0077
primary	0.048	0.7937	0.1556	0.1331	0.1423	0.706	0.1441	0.0076
middle	0.0139	0.3757	0.5688	0.0416	0.135	0.6428	0.2152	0.007
uni	0.0004	0.0453	0.2284	0.726	0.1168	0.5556	0.3101	0.0175

Data source: 1989 CHNS and 1990 Census

4 The impact of the imbalanced sex ratio in 2007

In this section, I simulate the model for 2007 by only changing the sex ratio to 1.2 while holding the rest parameters constant to access the role of the imbalanced sex ratio on marriage and labor market outcomes. Then I compare the results of the two steady states. In particular, the interested outcomes are: (i) female and male working hours, (ii) female and male married fraction, their willingness of marriage and the degree of assortative mating, (iii) the welfare implication. The results are presented in Table 4.

4.1 Effects on labor market

The first part of Table 4 illustrates the impact of the increased sex ratio on labor market outcomes. On average, total female working hours drop by 5.1% from 8.08 to 7.67 hours per day, and total male working hours increase from 8.13 to 8.56 hours per day by 5.2% .

Table 4: The impact of the imbalanced sex ratio in 2007

	1990	2007
Labor market		
Total female working hours	8.08	7.67
Total male working hours	8.13	8.56
Married female working hours	7.71	7.38
Married male working hours	7.96	8.7
Marriage market		
Fraction of married females	0.8094	0.8354
Fraction of married males	0.7665	0.6962
Degree of assortative mating	0.456	0.4935
Other changes		
Married household consumption	0.9541	0.9675
Female bargaining power	0.4895	0.5485
Welfare (% change)		
Female		9.5%
Male		-14.42%

Two reasons explain the change: First, the higher female bargaining power enables wives to take more marital surplus in forms of leisure. Therefore, married female working hours fall. On the contrary, husbands need to work more as his bargaining position gets worse. Second, as will be shown later, there is a compositional effect from the change of share of married population. Given that married females work less than single females (8.08 vs. 9.47 hours), when there are more female getting married, total female labor supply falls. The opposite happens to males.

From 1990 to 2007, data suggest that married female working hours has dropped by 10.85% from 7.83 to 7.01 hours per day (2006 CHNS). The model prediction is in line with the data, where married female working hours decreased from 7.71 to 7.38. The decline of married female working hours can be mainly explained by the increased female bargaining power in the intra-household allocation problem. In order to stay in marriage, husbands are willing to shift more resources to wives. In the model setup, it is realized by allowing wives to work less and enjoy more leisure. As measured by the Pareto weight θ, female bargaining power has increased by 12.05% on average, and all types of females are able to obtain a higher bargaining power. Particularly, as illustrated in Table 5, the bargaining power of the lower female types increases most. [16]

Table 5: Change of female bargaining power by productivity type

	<primary	primary	secondary	college
Low	0.4761	0.4917	0.512	0.5456
High	0.5378	0.55	0.5676	0.5953
% Change	12.96%	11.85%	10.85%	9.12%

Unfortunately, the model predicts that husbands should increase their working hours as their bargaining power falls, which contradicts the data. In the data, married male working hours barely change. Two reasons can explain the difference. On the one hand, empirical studies find that the elasticity of female labor, especially married female labor is bigger than that of male labor (Triest (1990), Eissa (1995) and Keane (2011)), so married male labor reacts less to the change of the sex ratio. In my model the Fisch elasticity is assumed to be identical between females and males, which is common in most of the quantitative studies (Aiyagari, Greenwood, and Guner (2000), Greenwood, Guner, and Knowles (2003) and Greenwood, Guner, Kocharkov, and Santos (2016) etc.). On the other hand, the only interested variation in the model is the sex ratio from 1990 to 2007, and all the other parameters are fixed at the 1990 level. But during the period of interest, in the data the education distribution also shifts to the right, and females with higher education outnumber males with higher education. With a significant increased return to education, it could drive the male working hours down.[17]

[16] The income effect from marriage may play a less important role. Yao and Tan (2005) shows that the increased male income can only explain less than 10% of the drop in female labor supply.

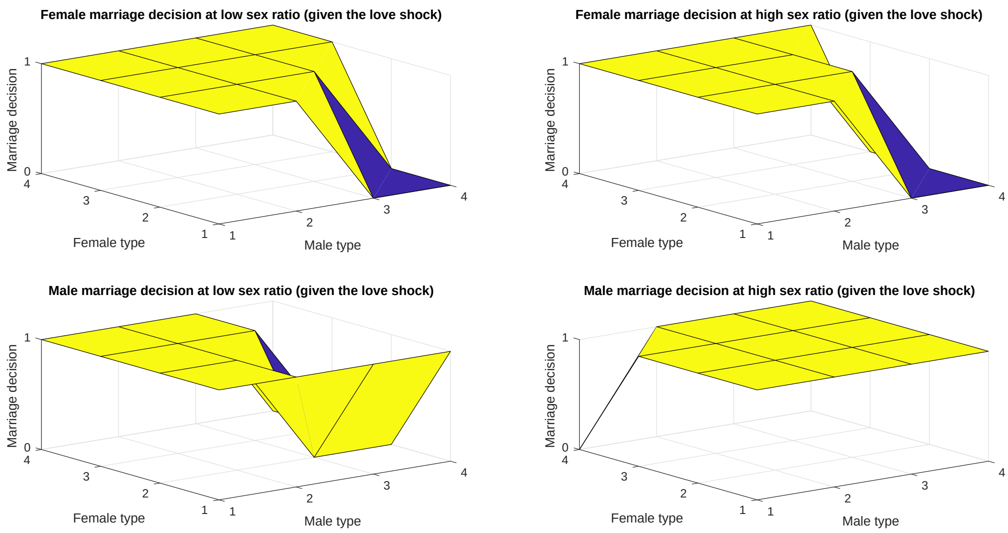
[17] During this period, the gender wage gap is almost constant, which increased to 0.76 from 1990 to 1997, but falls to 0.78 in 2007.

4.2 Effect on marriage market

In 2007, marriage became more attractive to females but less attractive to males. In particular, in the data, the fraction of females that are married increased from 80.47% to 81.38%, and the fraction of males that are married dropped from 74.44% to 71.35% (2010 Census). The model can capture the trend in the data. In the model, the fraction of females that are married increased from 80.94% in 1990 to 83.54% in 2007, while the fraction of males that are married dropped from 76.65% to 69.62%. For females, the increased fraction comes from: (i) The increased bargaining power makes marriage more attractive, and (ii) There are more males available on the marriage market, which gives females a wider choice of husbands. But on the other hand, giving a wider choice of husbands, females can wait longer for a better husband. They are more picky in finding a partner. This can be illustrated from the top two graphs in Figure 3, which show the marriage policy function of females for a given level of the marriage quality shock. When the sex ratio is higher, fewer females would like to accept the marriage given the type of the potential husband. The fraction of males that are married decreased, however, is due to the increased pickiness of females. For males, they become less picky, as shown in the bottom two graphs in Figure 3. Instead, they were more willing to marry lower type of females than before.

In addition, the degree of assortative mating increases, which is defined by the fraction of married couple with the same education level. In 1990 when the sex ratio was low, 45.6% of couples had the same education level, but this number dropped to 49.35% in 2007. Since males are less picky and females are more picky, more low-educated women were able to find a high-educated husband, which reduced the degree of assortative mating. This is in line with the empirical literature (Edlund, Li, Yi, and Zhang (2013)).

Figure 3: Marriage policy functions before and after the sex ratio changes



4.3 Effect on welfare

Since the population size in the two stationary distributions is different, some of the males do not present in the low sex ratio economy. Therefore, to measure the welfare change, instead of looking at the single and married population separately, I focus on a new-born agent who just enters the marriage market.

Consider a new-born agent of gender $g \in \{f, m\}$ with productivity type z_g, who just enters the economy as a single individual. The value function for the new-born individual is given by:

$$\begin{aligned}\tilde{V}_g(z_g; S_g, S_{-g}) \quad = \quad & \int_Q \int_Z (\omega_g(S_g, S_{-g}) \max\{V_g^{S*}(z_g; S_g, S_{-g}), V_g^{M*}(z_g, z_{-g}, q; S_g, S_{-g})\} + \\ & (1 - \omega_g(S_g, S_{-g}))V_g^{S*}(z_g; S_g, S_{-g}))d\hat{S}_{-g}(z_{-g})dF(q),\end{aligned}$$

As being single, the new-born agent meets a spouse of type z_{-g} with probability ω_g and draws the marriage quality shock q. Given the optimal allocations of being single and being married, the individual decides whether to get married by comparing the value of being married and the value of being single. If the agent meets no one with probability $1 - \omega_g$, he/she has to stay single.

Then the change of welfare is measured in consumption equivalent units for the new-born female and male. On average, females benefit from the more imbalanced sex ratio by gaining 9.5%. Three channels contribute to the change. Firstly, the higher bargaining power enables married females to enjoy more leisure.[18]. Secondly, more females can find a husband due to a higher meeting probability. Given the beneficiary of marriage (economies of scale and the love shock), females gained even more. Thirdly some of the females can marry a better husbands, and they benefit from the income effect. In contrast, male welfare dropped by 14.42%. It comes not only from the additional working hours, but more importantly from the lower likelihood of finding a wife. In the next section, a decomposition exercise is done to show the importance of each channel.

5 Decomposition exercises

The effect of a higher sex ratio on female and male welfare are through 3 channels: the bargaining channel, the channel of females facing a wider choice of husbands, and the channel of females finding a better husband. The bargaining channel is captured by the bargaining weight θ, the wider choice of husbands channel is represented by the meeting probability ω_g and the better husband is defined by the marriage policy function $\mathbb{1}_g$. To pin down the effect from each of the three channels, I use 1990 as the benchmark year,

[18]Home public consumption plays little role, which is almost constant (from 0.9541 to 0.9675)

then change the sex ratio to the level in 2007, and turn on each channel separately while holding the other two channels fixed at the 1990 level. Then the effect of the interested channel can be captured by comparing each variable value with its 2007 counterpart in column (5). The closer the values are, the more important the channel is in explaining the change. The results are shown in Table 6.

Table 6: Decomposition exercises

	1990 (1)	Higher bargaining power (2)	Higher meeting prob. (3)	Better husband (4)	2007 (5)
Welfare (% change)					
Female		3.48%	5.93%	0.12%	9.5%
Male		-5.43%	-8.87%	-0.02%	-14.42%
Labor market					
Working hours (wife)	7.71	7.45	7.71	7.64	7.38
Working hours (husband)	7.96	8.54	7.96	8.03	8.7
Marriage market					
Fraction of females, married	80.94%	81.26%	84.62%	80.78%	83.54%
Fraction of males, married	76.65%	75.87%	70.51%	76.69%	69.62%

5.1 A higher female bargaining power only

The bargaining channel effect can be captured by the change of the endogenous bargaining weight θ. To pin down the effect from this channel, I allow the weight to change with the value functions by $\theta = \frac{V_m^M - V_m^S}{(V_m^M - V_m^S) + (V_f^M - V_f^S)}$, while holding the meeting probability ω_g and marriage policy functions constant at the 1990 level.[19] Then the model is simulated with the 2007 sex ratio. The results are listed in the second column of Table 6.

[19] Note that even when the meeting probability is fixed, the relative single population size still changes, which affects the normalized single distribution in the value function. So the bargaining weight still changes.

The result suggests that the bargaining channel can explain about one third of the increased female welfare. Females are 3.48% better off, and males are -5.43% worse off due to the variation of the bargaining power. The change is resulted from a higher married fraction and lower working hours of females. 78% change of the lower married female labor supply can be explained by the higher bargaining power. For husbands, they have to work 0.58 hours. Regarding the marriage market, the fraction of females that are married increases by 0.32 percentage points, as the higher bargaining power makes marriage more attractive for females. Males marry 0.78 percentage point less as the marriage is less beneficial due to the lower bargaining power.

5.2 A wider choice of husbands only

The wider choice of husbands channel is measured by the change of the meeting probability ω_g. In particular, the meeting probability evolves according to $\omega_g = \frac{S_{-g}}{S_g + S_{-g}}$, while the bargaining weight and the marriage decision are fixed at 1990 level. Column 3 in Table 6 presents the simulated outcomes.

The wider choice of husbands is the most crucial channel in explaining the change of welfare. It makes females 5.93% better off and males 8.87% worse off. The effect comes from the sex ratio impact on the marriage market. While married individual working hours barely react to the increased sex ratio, the increased meeting probability for females is essential in explaining the increased fraction of females that are married. It accounts for 3.68 percentage points of the increase. On the contrary, males reduce the married fraction by 6.14 percentage points.

5.3 A better husband only

The better husbands channel is captured by allowing the marriage decision to be endogenous while holding the bargaining weight and meeting probabilities constant. The results are illustrated in column 4 of Table 6.

Overall, the better husbands channel only plays a marginal role in the change of welfare. Females are 0.12% better off and males are -0.02% worse off. When agents are free to choice their partner, the married female labor supply falls by 0.07 hours. A better husband enables the wife to work less due to the income effect. Married males work 0.07% more. The impact on the fraction of females and males that are married is small, where there are 0.16 percentage points decrease in married females and 0.04 percentage points more of married males, because females are more picky in choosing husbands and males are less picky.

6 Conclusion

The sex ratio in China has been growing significantly during the last 20 years. The imbalanced sex ratio generates more than 30 million additional males than females, which has a large impact on China's labor and marriage market.

To investigate its effects, I develop a unified model of marriage and divorce, imbalanced sex ratios and an intra-household bargaining process. The intra-household allocation problem of married households is through a Nash bargaining with being single as the outside option. Marriage is attractive for two reasons, the love shock and economies of scale from household public consumption. The meeting probability in the marriage market depends on the population size of singles with the opposite sex. Therefore, the scarcer gender has a higher probability of finding a spouse. The sex ratio effects females and males through the bargaining channel and the marriage channel. The marriage channel is further consisted of a wider choice of husbands and chances of finding a better husband. When the sex ratio rises, it increases the meeting probability for females, and thus leads to a better outside option, which gives females a higher bargaining power. The higher bargaining power enables females to enjoy more leisure, and makes marriage more attractive. In addition, the additional males in the marriage market gives females a wider choice of husbands which makes them easier to find a husband, and the chance of finding a better husband which further generates a stronger income effect on wives. All of them contribute to the change of welfare.

The model is calibrated to match China data in 1990 when the sex ratio was low. The data targets are chosen to capture married households working hours and marriage market patterns. The calibrated parameter values are reasonable and the model overall fits the data well. Then the sex ratio increases to 1.2 exogenously to simulate 2007 outcomes. Results suggest that the increased sex ratio leads to a lower female labor supply and a higher male labor supply. On the one hand, married females work less and married males work more due to the change of the relative bargaining power. On the other hand, the fraction of females that are married rises, and the fraction of males that are married falls. Since single females work more than married females, total female working hours fall by 5.1%. The degree of assortative mating drops with the higher sex ratio, since males are less picky in choosing their partners. The model predictions are in line with the data trends broadly. Overall, females are 9.5% better off from the change in labor and marriage market outcomes while males are 14.42% worse off.

A decomposition exercise is further conducted to pin down the effects from each of the 3 channels on welfare changes. The wider choice of husbands channel is the most important channel in explaining the welfare change , which affects the share of married population most. The bargaining channel is also essential due to the higher leisure enjoyment of females. It also makes marriage more attractive for females. The better husband channel is relatively less important, which affects the labor choice through the income effect and the marriage decision since females are more picky.

7 Appendix

7.1 A1

Proof of Lemma 1:

In order to prove that the two questions defined in (2) and (11) are equivalent, I will show that the first order conditions in the two problems when the solutions are interior are exactly the same.

The first order conditions of problem (2) are given by:

$$\begin{aligned} c: \; c^{-\gamma}(V_f^M - V_f^S) + c^{-\gamma}(V_m^M - V_m^S) &= \lambda \\ \ell_f: \; -\psi \ell_f^{\mu}(V_m^M - V_m^S) &= \lambda w_f z_f \\ \ell_m: \; -\psi \ell_m^{\mu}(V_f^M - V_f^S) &= \lambda w_m z_m \\ B.C.: \; c &= w_m z_m \ell_m + w_f z_f \ell_f, \end{aligned}$$

where λ is the Lagrange multiplier.

Collecting terms to eliminate λ gives:

$$\begin{aligned} -\psi \ell_f^{\mu}(V_m^M - V_m^S) &= c^{-\gamma}[(V_f^M - V_f^S) + (V_m^M - V_m^S)] w_f z_f \\ -\psi \ell_m^{\mu}(V_f^M - V_f^S) &= c^{-\gamma}[(V_f^M - V_f^S) + (V_m^M - V_m^S)] w_m z_m \\ c &= w_m z_m \ell_m + w_f z_f \ell_f, \end{aligned}$$

The first order conditions of problem (11) are:

$$
\begin{aligned}
c:\ & c^{-\gamma}\theta + c^{-\gamma}(1-\theta) = \lambda \\
\ell_f:\ & -\psi \ell_f^{\mu}\theta = \lambda w_f z_f \\
\ell_m:\ & -\psi \ell_m^{\mu}(1-\theta) = \lambda w_m z_m \\
B.C.:\ & c = w_m z_m \ell_m + w_f z_f \ell_f,
\end{aligned}
$$

where λ is the Lagrange multiplier and $\theta = \frac{V_m^M - V_m^S}{(V_m^M - V_m^S) + (V_f^M - V_f^S)}$.

Eliminating λ and replacing θ:

$$
\begin{aligned}
-\psi \ell_f^{\mu} \frac{V_m^M - V_m^S}{(V_m^M - V_m^S) + (V_f^M - V_f^S)} &= c^{-\gamma} w_f z_f \\
-\psi \ell_m^{\mu} \frac{V_f^M - V_f^S}{(V_m^M - V_m^S) + (V_f^M - V_f^S)} &= c^{-\gamma} w_m z_m \\
c &= w_m z_m \ell_m + w_f z_f \ell_f,
\end{aligned}
$$

which are the same as the FOCs of problem (2).

Therefore, the two problems are equivalent to each other.

7.2 A2

The algorithm to solve the structure model:

It involves a complicated fixed point problem to solve the model. The following steps illustrate the detailed algorithm.

1. Solve for the single household allocation problem and obtain the indirect utilities of being single.

2. Initialize guesses of distributions $S_{f,0}$, $S_{m,0}$, $M_{f,0}$ and $M_{m,0}$; value functions $V^S_{f,0}$, $V^S_{m,0}$, $V^M_{f,0}$ and $V^M_{m,0}$, continuation values $cont_f$ and $cont_m$ of married households and the initial bargaining weights θ_0.

3. Solve for married household allocations with bargaining weights θ_0 by solving the problem (12).

4. Update indirect utilities U^M_m and U^M_f.

5. Then the bargaining weight is updated by:

$$\theta = \min\{\max\{\frac{U^M_m + cont_m - V^S_{m,0}}{(U^M_m + cont_m - V^S_{m,0}) + (U^M_f + cont_f - V^S_{f,0})}, \lambda\}, 1 - \lambda\},$$

   where λ is a small number. This guarantees the weight is between 0 and 1.

6. Update value functions of being married given indirect utilities.

7. Given the updated value functions, determine marriage decisions and divorce decisions by comparing value functions of singles and married as equations (4) and (7).

8. Update meeting probabilities given distributions.

9. Given marriage and divorce decisions and meeting probabilities, update continuation values, value functions, and distributions according to equations (5), (8), (9), (10) and (11).

10. Calculate errors between V_m^S and $V_{m,0}^S$, between V_f^S and $V_{f,0}^S$, between V_m^m and $V_{m,0}^m$, between V_f^M and $V_{f,0}^M$, between S_m and $S_{m,0}$, between S_f and $S_{f,0}$, between M_m and $M_{m,0}$, and between M_f and $M_{f,0}$.

11. Update guesses of value functions and distributions by a convex combination of their own values and new values in value function and distribution iterations, and update the bargaining weight θ_0 by θ.

12. Check if the error is small enough. If not, go back to step (3).

Note that the bargaining weight is between 0 and 1. If it reaches the corner, it means at least one of the spouses rejects the marriage. This is consistent with marriage policy functions.

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