A "Reproductive Capital" Model of Marriage Market Matching

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We often consider the toll the ticking biological clock takes on women, but not the *economic* loss it causes via its impact on marriage market prospects. I document two new facts in Census data demonstrating the potential importance of fertility in marriage market outcomes. First, women who are older at first marriage are matched with poorer husbands, whereas men who marry when older marry richer spouses. Second, the returns to education on the marriage market have historically been non-monotonic, with graduate educated women marrying poorer spouses than college educated women. To study the economic effects of time-limited fertility, I provide a matching model of the marriage market where women's human capital investments impact a second dimension, "reproductive capital." The model predicts precisely the non-monotonic matching equilibrium observed in the data when the fertility loss from investment is large relative to the income gains. Moreover, the model predicts the more recent movement toward assortative matching as family sizes have fallen and the return to women's human capital has risen. To test the model's central mechanism, I use an incentive-compatible experiment to show that age has a causal negative impact on women's marriage market value. For every year a woman ages beyond 30, she must earn an extra $7,000 a year to remain equally attractive to potential partners.

JEL Codes: C78, D10, I26, J12, J13, J16

1 Introduction

Decades of research and popular press tell us the biological clock is a central driver of women's decision-making. However, it is usually treated as a matter of women optimizing according to their own conflicting preferences for both career and family. But, just as a woman's human capital is valued on the labor market, her fertility has value on the marriage market: men marry partly to have children, and marriages tend to improve the economic circumstances of women.[1] Therefore, women whose human capital investments limit their fertility may experience not just a personal loss, but an *economic* one. Using a combination of theory, historic Census data, and experimental

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[1] Marriage has economic value to both men and women, through the production of public goods and returns to scale, but may especially improve women's economic circumstances via hypergamy (Edlund and Pande, 2002).

evidence, this paper introduces the concept of fertility as "reproductive capital," and examines the economic impacts of its depreciation for women.

I first present stylized facts that suggest aging impacts women's marriage market success. Whereas men's reproductive systems age at the same rate as other bodily systems, women experience sharply declining fecundity beginning in their mid-thirties, and ending in menopause.[2] Mirroring this asymmetric biological pattern is a social one: Women who marry later (past their mid-twenties) marry poorer men with each passing year, whereas for men later marriage is associated with richer partners. If this is a causal relationship, it means men also hear the ticking of the biological clock. Seeking to marry and have children, they naturally prefer more fertile partners. Women thus face a tradeoff: human capital investments increase earnings, but take up crucial time during the reproductive years, as it is difficult to co-process career investments and family formation (Michael and Willis, 1976; Goldin and Katz, 2002; Bailey, 2006; Bailey, Hershbein and Miller, 2012; Adda, Dustmann and Stevens, 2017; Kleven, Landais and Sogaard, 2015). This loss of reproductive capital may cost them real economic returns on the marriage market, despite human capital itself being a positive trait. If additional years of investment might be linked to marrying a poorer partner, a woman could risk actually reducing her total household income, even as she raises her own earnings.

To look for suggestive evidence of this dual impact of human capital investments on the marriage market, I examine the marriage outcomes of women with graduate degrees compared to other educational levels. Women with graduate degrees make a useful stand-in for women who have made time-consuming human capital investments because they can be identified as a group in national data, versus women who have made on the job investments that cannot be observed. In general, matching in the US tends to be assortative by human capital—wealthier or better educated individuals marry wealthier partners. I document an important exception to this pattern: Until the 2000 Census, women with graduate degrees married significantly poorer spouses than women with college degrees, despite every other educational level yielding richer partners. In other words, there was a *non-monotonic* relationship between women's education and spousal income.

This fact cannot be explained by existing models of household formation. If households specialize between market and non-market work, then negative assortative matching is expected, with high earning men choosing lower human capital women who have lower opportunity costs of time. Similarly so if female education is somehow a "bad," because it yields additional bargaining power

[2] See discussion in Section 2.

for the female partner or creates greater intra-household conflict. Conversely, if marriage is taste-based, or there are consumption complementarities with one's spouse, or there are convex returns to investments in children, positive assortative matching is predicted. There is no current model that can simultaneously explain the positive relationship between education and spousal income for those with less than a college degree, and the negative relationship for those with college degrees and higher.[3]

In addition to matching with lower quality partners, popular press implies it is difficult for educated women to find mates at all. However, I show that women with only college degrees have actually always married at rates similar to women of other educational levels, while graduate educated women have married at substantially lower rates. Thus, a recently noted "reversal of fortune" for educated women on the marriage market (Fry, 2010) has actually been driven entirely by women with graduate degrees. This unique pattern and trend reversal for graduate educated women implies that the time cost of education is crucial in understanding its marriage market impact: age must be viewed as a factor in women's marriage market appeal.

To formally model the consequences of depreciating "reproductive capital," I present a bi-dimensional matching model capturing women's heterogeneity in both income and fertility. The model predicts exactly the non-monotonic matching pattern seen in historical data. Because the most skilled women, who would normally match with the most skilled men, are the ones most likely to make career investments, and thus be older at the time of marriage, the highest-earning men forgo matching with the highest-earning women in favor of lower-earning, more fertile partners. This marriage market response to skill investments in turn increases the cost of such investments to women. In addition to the personal utility cost of lower fertility, women face a real economic loss, both in terms of potentially marrying a poorer partner, and in ceding more of the marital surplus to that partner. The model allows a back-of-the-envelope calculation of the marriage market's contribution to the total welfare cost, suggesting that roughly one-third of the cost of declining reproductive capital for high-skilled women comes through equilibrium channels. This means that even if a woman placed no value on children personally, she would still experience a "tax" on her human capital investments via the marriage market equilibrium.

The model additionally explains the more recent changes in marriage prospects for women

[3] If tastes vary with income or education in such a way that optimal matching patterns change, it would be more likely for the pattern to flip in the opposite direction, with poorer individuals preferring more "traditional" households, while wealthier individuals matched more assortatively.

with graduate degrees. If the labor market return on investment rises or the fertility cost falls sufficiently (perhaps due to reduced family size desires by men and women),[4] the highest-earning women may be able to compensate their partners for forgone fertility, and thus match assortatively, producing the reversal in marriage outcomes for highly educated women. This partial elimination of the "graduate marriage-market penalty" may have amplified the effects of increasing labor market returns to education, helping to explain the dramatic rise in women pursuing higher education (Goldin, Katz and Kuziemko, 2006). That the rates of women attending higher education have increased to such an extent (despite labor market returns apparently being higher for men) has led to speculation that it must be the *marriage* market returns to education that are on the rise for women (Chiappori, Iyigun and Weiss, 2009; Ge, 2011), a phenomenon my model explains.

I then provide direct, causal evidence of the model's driving mechanism, that men value potential partners' fertility, and therefore age, through an incentive compatible experiment. The experiment isolates and quantifies the impact of age on women's marriage market value by randomly assigning age to dating profiles, holding attractiveness and other factors constant, and comparing its impact to that of of income. I find that for every year a woman ages beyond 30, she must earn an extra $7,000 a year to remain equally attractive to potential partners. This preference is driven by men who have no children, and those who have accurate knowledge of the age-fertility tradeoff, indicating that the causal effect of age on marriage market outcomes is likely due to fertility. When age is isolated from other factors, men who already have children do not penalize older partners, while men looking to become fathers do.

The experiment's findings reconcile with the stylized fact that today, graduate educated women marry richer spouses than college educated women. In the experiment, a woman who is one year older must earn $7,000 extra in order to compensate her husband. Graduate educated women marry on average one year later than college educated women in the 2010 ACS, and earn $16,000 more. Thus, their income premium overcomes their age penalty. Longer term investments (for example, the partnership track at a law firm) may certainly still be penalized overall on the marriage market, since each additional year depreciates reproductive capital further. This may help explain why women who earn advanced degrees "drop out" of high pressure careers down the line (Bertrand, Goldin and Katz, 2010), as well as why high-skilled women choose particular types of careers (e.g., pharmacology) where there is less convexity in wage returns to hours (Goldin and Katz, 2012;

[4] Family size desires transitioned rapidly from 4 to 2 during the 1970s (Livingston, Cohn and Taylor, 2010), shown in Appendix Figure A3. This change is obviously endogenous, but is outside the scope of my model.

Goldin, 2014). Simulations of the model suggest that policies that either decrease the fertility cost of such investments or increase their economic returns will lessen the reproductive capital tradeoff, providing substantial welfare gains to skilled women.[5]

This paper makes three key contributions to the existing literature. First, I document non-monotonic matching, and show that highly educated women experience marriage market outcomes very different than those of college educated women. This builds on literature noting improved marriage outcomes for college educated women (Rose, 2005; Isen and Stevenson, 2010; Bertrand et al., 2016; Fry, 2010), as well as increases in assortative mating (Chiappori, Salanié and Weiss, 2017; Hurder, 2013; Greenwood et al., 2016, 2014; Fernandez, Guner and Knowles, 2005; Schwartz and Mare, 2005),[6] by noting the unique pattern in graduate educated women's outcomes, which is not easily explained by existing theories. Second, using a bi-dimensional matching model, I demonstrate that the time-cost of large career investments can result in the non-monotonic matching patterns and the "reversal of fortune" evident in the data. In this setting, the bi-dimensional nature of the model is not just theoretically interesting, but crucial to matching empirical evidence, whereas in many other settings an "index" approach to characteristics is sufficient.[7] Additionally, this paper provides micro foundations of the mechanism through which age can affect women's marriage market outcomes, contributing to literature that starts from the premise that older women are less desirable on the marriage market (Siow, 1998; Dessy and Djebbari, 2010; Bronson and Mazzocco, 2012; Zhang, 2017*b*). Third, I provide the first well-identified evidence of men's preference for partner age, and thus the causal negative impact for women of aging on marriage market outcomes, something previously suggested in economics, but not shown conclusively empirically (Edlund, 2006; Edlund and Korn, 2002; Edlund, Engelberg and Parsons, 2009; Grossbard-Shechtman, 1986; Arunachalam and Naidu, 2006).[8] This paper also contributes a new, incentive compatible experi-

[5] Gershoni and Low (2017) provides empirical evidence aligning with this prediction, demonstrating that when the reproductive time horizon is lengthened via access to IVF technology, women complete more advanced degrees, earn more, and work in higher prestige occupations.

[6] Note that Gihleb and Lang (2016) find that there is not necessarily an increase in assortativeness, depending on measurement techniques, while Eika, Mogstad and Zafar (2014) look at assortativeness within education categories, and find the increase has been mostly among less educated groups.

[7] Matching models that look at two or more characteristics often reduce these characteristics to an index of overall desirability; however, if the value of either characteristic varies with the quantity of the other characteristic, the dimensions of the model cannot be collapsed. An example of this is Chiappori, Oreffice and Quintana-Domeque (2011), where smokers do not mind if their partners smoke, whereas non-smokers do, and thus no universal index of desirability can be found. Galichon and Salanié (2015) offer a multi-factor example. This type of model is an emerging strand of the literature, and equilibrium characteristics in this setting have only recently been explored. The model I develop builds on this literature by allowing the woman's two characteristics to be endogenously chosen, with one affecting the other (she can choose to improve her income only at the expense of reproductive capital), and showing that the unique matching patterns that emerge are predictive of real-world outcomes.

[8] It is taken as a given in other disciplines such as evolutionary biology (Trivers, 1972), anthropology (Bell and

mental methodology to the literature on what men and women value on the dating and marriage market (Fisman et al., 2006; Hitsch, Hortaçsu and Ariely, 2010; Bertrand, Kamenica and Pan, 2015), which would be broadly applicable to settings where one wants to measure preferences over individual characteristics without deception.

Combining these literatures allows a fuller picture of the impact of the age-fertility profile. While we may presume that women value fertility and consider it in their career decisions, demonstrating its effect on the marriage market allows us to measure its economic impact: women essentially experience a "tax" in terms of their husbands' earnings for their additional education. My model shows that men's micro preferences for more fertile partners, shown via experimental data, drive real matching patterns that result in the highest-educated women being paired with lower income spouses than women with less human capital, shown via decades of nationwide Census data. As "reproductive capital" happens to depreciate in value at a similar time in the life cycle to when human capital for high-skilled workers appreciates most rapidly, its marriage market impacts are likely to be extremely salient in human capital investment decisions, especially at the top of the distribution. The model's equilibrium responds to underlying factors, such as the labor market return to education, the ability to have children later in life, and flexibility in combining family and career. Thus, individuals, policymakers, and firms may be able to use a better understanding of this tradeoff to blunt the impact of reproductive capital's decline.

The remainder of the paper proceeds as follows: Section 2 documents empirical facts in the US Census that require a bi-dimensional matching model, Section 3 develops the model and its implications, Section 4 demonstrates the causal impact of aging for women through an experiment, Section 5 presents the result of a simulation, and Section 6 concludes.

2 Stylized facts on marriage outcomes for educated women

Aging impacts women's biological fecundity differently than men. Men experience a reproductive decline with age that is proportional to the decline in other bodily systems, whereas women experience a separate process—menopause—where reproductive capacity declines non-linearly to zero (Frank, Bianchi and Campana, 1994).[9] Because of this, delays in marriage may be expected to

Song, 1994), and sociology (Hakim, 2010)

[9]The exact date of this decline may be difficult to pinpoint, but a collage of evidence points to pregnancies being rarer (Menken, Trussell and Larsen, 1986), more likely to end in miscarriage (Andersen et al., 2000), and more likely to result in fetal abnormalities (Hook, Cross and Schreinemachers, 1983) later in life. Women lose 97% of eggs by age 40 (Kelsey and Wallace, 2010), while remaining egg quality declines (Toner, 2003). To help disentangle

have a differential impact on women's marriage outcomes. Figure 1 shows that this is indeed the case: women who are older at the time of first marriage (beyond age 30) tend to marry lower-income spouses.[10] In contrast, men's age is slightly positively correlated with spousal income.[11] This negative relationship between women's age and spousal income is non-causal, but given that individuals who marry later tend to be positively selected, it is suggestive of a negative impact of age on marriage market outcomes.[12] Additionally, the fact that the volume of marriages is highest in the early years, before the decline begins, shows that it is also not a matter of sorting the "lemons" from the "peaches" on the marriage market.

If indeed being older at the time of marriage causes one to marry lower-income spouses on average, then time-consuming human capital investments would represent a double-edged sword: on the one hand, human capital is generally considered a positive marriage market trait, and likely to help attract a high-income spouse. On the other hand, income-increasing investments take time, decreasing what could be another valuable asset on the marriage market, reproductive capital.

This suggests that not all human capital investments are created equal: those that take place later in life and take longer are more likely to carry potentially negative marriage market effects, whereas short investments before childbearing years, such as earning a college degree, would be unambiguously positive. To see if there is support for the marriage market reproductive capital - human capital tradeoff in the data, I examine how the marriage outcomes of women with graduate degrees compare to those of women with college degrees.[13]

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the co-movement of fecundity and fertility choice, such as the use of contraceptives, some literature uses couples in traditional societies that do not use birth control. Although these measures may suffer from downward bias due to potentially declining rates of intercourse with age, and lower overall health and access to medical care in societies without contraceptive use, more recent prospective studies also show an accelerating decline in fecundity by age 40 for women, whereas men's fertility is relatively stable. For example, Rothman et al. (2013), in a prospective study of 2,820 Danish women trying to conceive, find that women 35-40 years old will become pregnant 77% as frequently as women age 20-24, whereas for men this ratio is 95%.

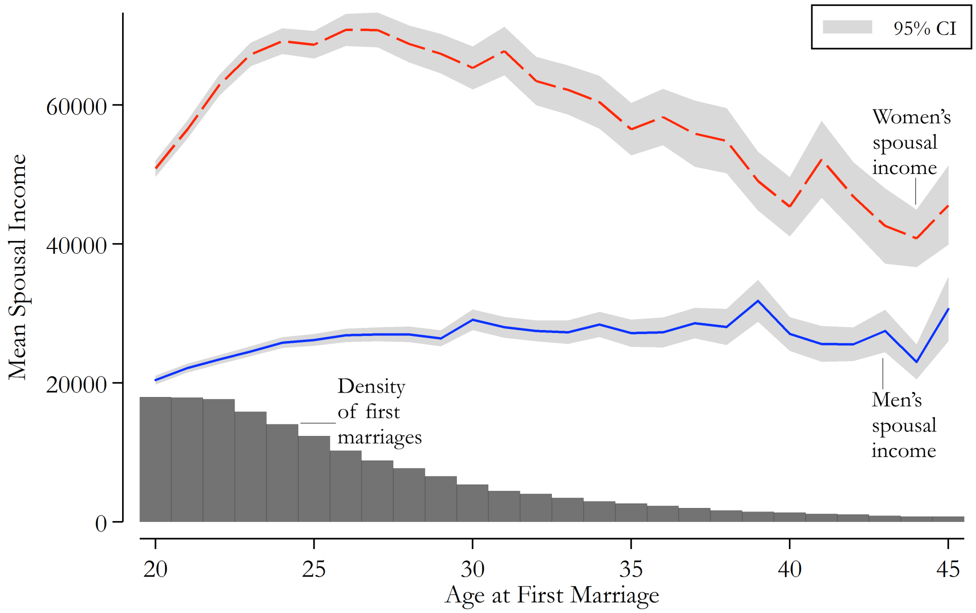
[10] For empirical results in this section: US Census and ACS sample is restricted to white individuals in their 40s, so that the vast majority of first marriage matching activity and educational investments have already taken place by the time they are observed. I analyze each ten-year cohort in a single Census year, rather than analyzing multiple groups retrospectively, which allows greater homogeneity of current life situation, since most variables, such as income, are reported for the present time only. I restrict to first marriages when showing results for only 1980 and 2010, but use all marriages when showing results across Census years, to allow for comparability with 1990 and 2000 data, which do not contain a variable for marriage number. Education used to indicate high human capital women rather than income because women's income is chosen endogenously after marriage, and therefore less representative of matching patterns.

[11] Appendix Figure A1 shows that in addition to the average pattern, within each income level marrying older is always linked to marrying a poorer spouse for women (but not for men).

[12] Zhang (2017*a*) notes that the selection of men who marry late tends to be negative, and so this pattern could be explained by available mates. However, the asymmetry in the pattern between men and women still relies on differential fecundity.

[13] Much empirical work categorizes all women with college degrees as "college plus." However, the "reproductive capital" hypothesis suggests women with college degrees and graduate degrees may have very different marriage

Figure 1: Spousal Income by Age at Marriage



Notes: Lines represent the average spousal income for first marriages by age at marriage for women versus men. Bars represent the portion of all women's marriages occurring at that age, to check whether selection is driving the effect. Source: 2010 American Community Survey (1 percent sample) marital histories for white men and women, 46-55 years old.

Women with graduate degrees (master's, MDs, JDs, PhDs, MBAs, etc.) make a good stand-in for women who have made time-costly human capital investments, because data shows they both earn more on the labor market and marry later and have fewer children.[14] Moreover, this group can be identified in the data clearly, whereas those who make time-intensive on-the-job investments are not clearly delineated.[15]

Figure 2 demonstrates that, until recently, graduate educated women married poorer spouses than college educated women. Although women who earned graduate degrees in the 60s and 70s may have indeed pursued less lucrative degrees and careers than women who earn them today, these women always earned substantially more than their college-educated counterparts (see Table 6), and yet their spouses earned substantially less. By separating the "college plus" category into college educated versus graduate educated, we can also see that the recently noted reversal

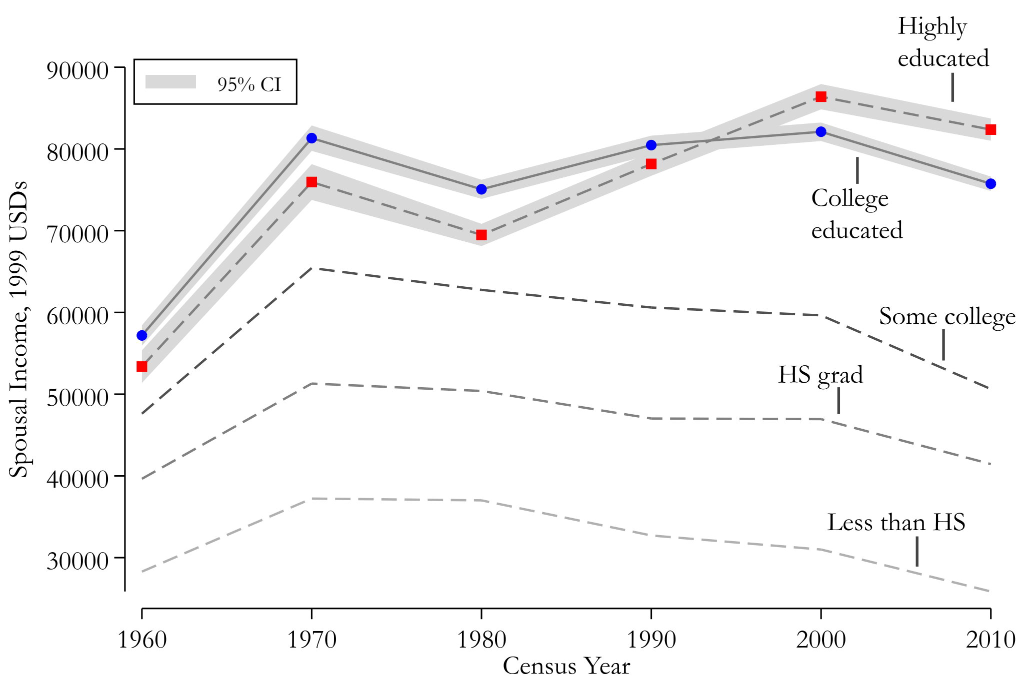
market outcomes, since women with college degrees only could still marry quite young and have large families.

[14] Data from the US Census shows that graduate educated women in the 2010 Census had married about one year later and earned $16,000 more than college educated women, shown in Table 6.

[15] And women with graduate degrees may be more likely to be in this group as well—for example, the natural course of action following law school is to become an associate at a law firm, after med school it is to become a resident, and after an MBA it is to pursue a corporate job.

in marriage outcomes for educated women on the marriage market was really driven by highly educated women, with graduate educated women finally matching with richer spouses by the 2000 Census.[16] Strikingly, the alignment of spousal income to every other educational level remained constant over this period. The spousal incomes of lower education levels are largely parallel. There is some growth in the incomes of college-educated women's spouses relative to other educational levels, consistent with increasing inequality and returns to skill during this period, but this cannot explain the crossing in college versus graduate women's marriage outcomes.

FIGURE 2: SPOUSAL INCOME BY WIFE'S EDUCATION LEVEL



Notes: Income of spouse based on wife's education level, with "highly educated" constituting all formal education beyond a college degree. Source: 1 percent Census data from 1960, 1970, 1980, and 1990, and American Community Survey from 2000 and 2010. Sample consists of white women, ages 41-50 years old.

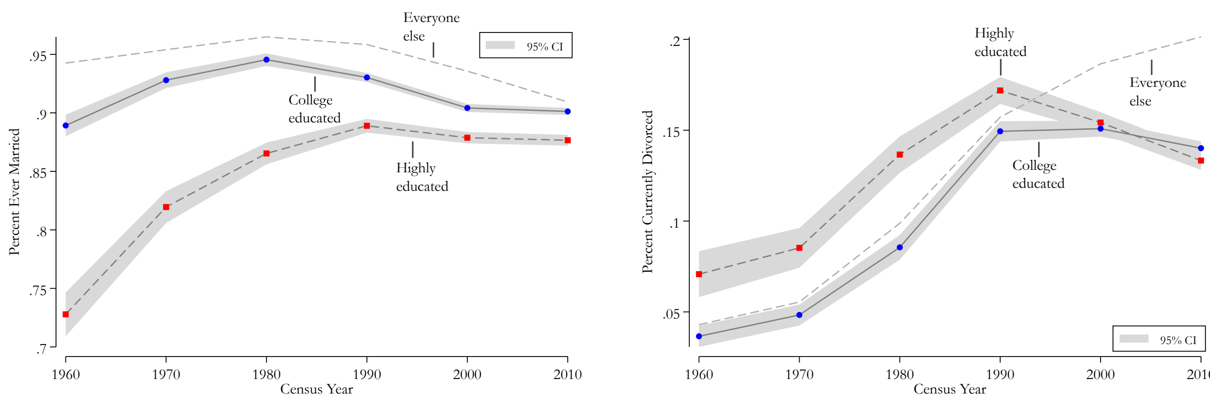
Over the same period, highly educated women's marriage rates rose precipitously, while divorce rates fell. Conventional wisdom holds that too-educated, too-high-earning women are punished on the marriage market. However, Figure 3 panel (A) demonstrates that *college* educated women actually always married at rates close to all other educational categories. It is only *highly* educated women who previously had comparatively low rates of marriage, and have now experienced sub-

[16] This is also evident in a regression with dummies for each cohort, as shown in Appendix Table A2.

stantial gains. Similarly, as shown in panel (B), college educated women's divorce rates were almost identical to all other educational categories, whereas highly educated women's were notably higher. College educated women's divorce rates have leveled off recently, while those of other educational categories have risen, but highly educated women's divorce rates have actually fallen.

Figure 3: Marriage and divorce rates by education level

(A) Marriage (B) Divorce

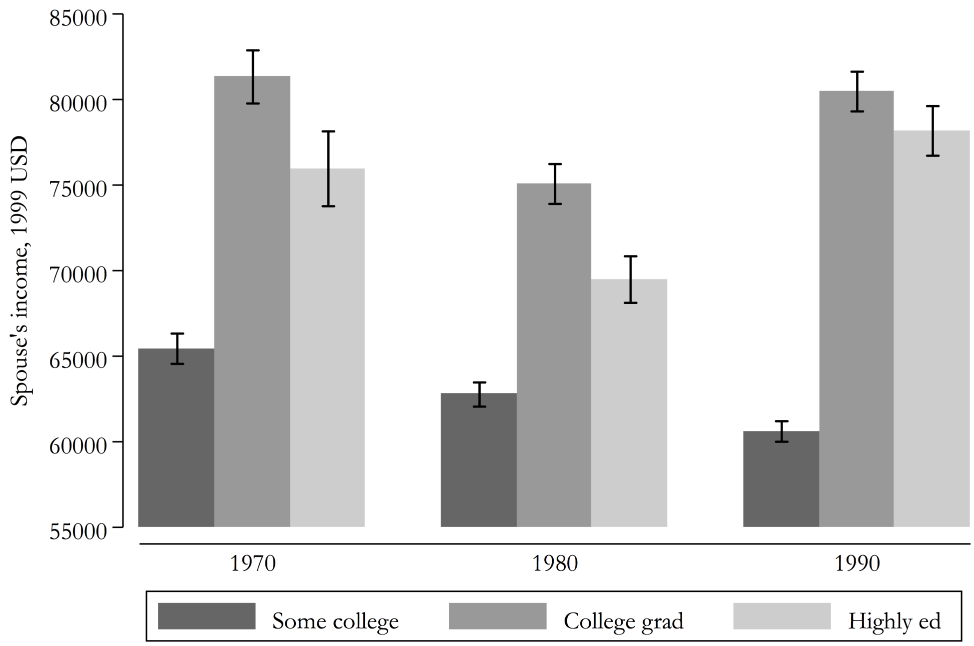


Notes: Ever married and currently divorced rates for women, based on education level, with "highly educated" constituting all formal education beyond a college degree. Ever divorced rates show a similar pattern, but are not available in all years. Source: 1 percent Census data from 1960, 1970, 1980, and 1990, and American Community Survey from 2000 and 2010. Sample consists of white women, ages 41-50 years old.

Several authors have noted an improvement in educated women's marriage outcomes (Rose, 2005; Isen and Stevenson, 2010; Bertrand et al., 2016; Fry, 2010), as well as increases in assortative mating over time (Chiappori, Salanié and Weiss, 2017; Hurder, 2013; Greenwood et al., 2016, 2014; Fernandez, Guner and Knowles, 2005; Schwartz and Mare, 2005). One could imagine this stems from a transition from Beckerian-style division of labor (Becker, 1973, 1974, 1981), and hence negative assortative matching, to a dual earner model. However, as Figure 4 highlights, matches during the period of Becker's work were not negative assortative, but rather non-monotonic. Women with some college married richer spouses than women with only high school degrees, and women with college degrees married richer spouses than women with some college. Yet, women with graduate degrees married poorer spouses than women with college degrees. This "penalty" to graduate education is significant in all three decades, although somewhat dissipating by 1990, and is also present in the 1960 data, although very few women received graduate degrees at the time.

Although there are many possible explanations for this phenomenom, which I discuss in Section 5, standard models fail to match this non-monotonicity. Division of labor, and thus substitutability

Figure 4: Non-monotonicity in spousal income by wife's education level



Notes: Income of spouse based on wife's education level, with "highly educated" constituting all formal education beyond a college degree. Difference between college and highly educated women's spousal earnings is significant in all three samples. 95% confidence interval shown by black lines. Source: 1 percent Census data from 1970, 1980, and 1990. Sample consists of white women, ages 41-50 years old.

between men's and women's incomes, could explain the negative relationship between education and spouse's income for college and graduate educated women, but not the positive relationship at other education levels. Complementarity in spouse's incomes, social class, or education (whether because of consumption complementarities, or convex returns to certain investments) could explain the positive relationship in most of the data, but not the apparent "penalty" to graduate education.

These stylized facts thus suggest that age matters on the marriage market, and thus time-costly human capital investments have a multi-dimensional effect. In the next section, I show that a matching model incorporating reproductive capital, in addition to human capital, predicts both the non-monotonicity present in the data and the more recent improved fortunes of highly educated women. Subsequently, I demonstrate a causal relationship between age and women's marriage market desirability experimentally.

3 A Theory of the Marriage Market with Reproductive Capital

If human capital investments affect two factors, earnings and age—which in turn impacts fecundity—and both have marriage market value, it will have consequences for marriage matching patterns and women's willingness to invest in human capital in the first place. This section studies these effects using a bi-dimensional transferable utility matching model. Importantly, the model's bi-dimensionality is necessary to replicate the patterns present in the stylized facts.[17] The model purposely abstracts away from gender-specific preferences over childbearing, instead examining whether an asymmetry between men and women in the age-fertility profile alone can produce patterns in human capital investment and marriage outcomes similar to those seen in observational data.

Transferable utility matching models derive matching patterns from the efficient creation and division of surplus (Shapley and Shubik, 1971; Becker, 1973). The equilibrium payoff of each individual in a marriage is set by the market as "offers" where both spouses are able to attract one another, essentially acting as prices based on the contribution of an individual's traits to the joint surplus and the scarcity of those traits on the market. Thus, instead of requiring the assumption that fertility impacts matching and household transfers, the model I present yields this as an equilibrium result, stemming from children entering men's utility functions.

I first illustrate the marriage market outcome of the bi-dimensional model, and show that it predicts non-monotonic matching. Then, I demonstrate that the marriage market penalty may cause women to limit their human capital investments. Finally, I demonstrate that the model can create transitions in matching and investment similar to those documented in the historical Census data.

3.1 Model set up

In this model, career investments yield earnings gains, but delay marriage and childbearing. This is intended to capture the impact for women of large, lumpy career investments such as medical

[17] The second factor could in principle represent multiple qualities: it could be that women who have higher market earnings have lower tastes for domesticity or lower productivity at it, it could be that the delay affects physical beauty through age, and it could be that women with high market earning potential *want* fewer children in addition to being able to produce fewer. It is necessary for the decision to invest in additional education to impact both the first factor, market earning potential, and the second factor, and for this latter factor to be relevant to marital surplus. It is also important that this second factor is non-linearly impacted by investments, since we do not see a parallel tradeoff in the stylized facts for college-educated women. Therefore, fecundity, which is impacted by educational decisions and deteriorates on a particular time schedule, is a good fit. In Section 4, I present evidence that fertility indeed has a causal negative impact on men's valuation of female partners, and thus is a likely candidate for this second factor.

school plus residency, pursuing partnership at a law firm, or a PhD and the academic tenure track. Women entering the marriage market therefore differ along two dimensions: their income—human capital—and their fertility—reproductive capital. In the model, women first choose whether to invest in their careers, then marriages form via transferable utility matching, then each couple has a child with probability π, and finally couples allocate income between children and private consumption.

Men and women are each endowed with skill. In the man's case, human capital investment is assumed to be costless, and he thus arrives on the marriage market with a single characteristic, income, y^h, distributed uniformly on $[1, Y]$.[18]

Women, starting with skill s distributed uniformly on $[0, S]$, can choose to improve their level of income, but doing so takes time, and this time is costly in terms of reproductive capital depreciation. As a result, if they choose to make investments, they will have a lower probability of becoming pregnant when they get married. Women are therefore characterized by a pair of characteristics, (y^w, π). This pair is equal to (s, P) if the woman marries without investing and $(\lambda s, p)$ if the woman marries after investing, where $\lambda > 1$ and $P > p$. Note that the "fertility penalty" of investment is the same for all women, whereas the wage difference from investment increases with skill. Thus, higher skilled women may have more to gain from investing.

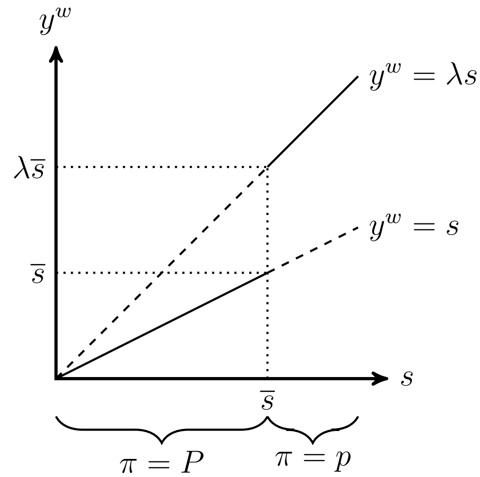
In this section, I assume an exogenous skill threshold, $\overline{s}$, above which women invest. After determining the equilibrium in the marriage market conditional on $\overline{s}$, I use this equilibrium to solve backwards for which women would optimally invest in the first stage. Thus, assume women with $s > \overline{s}$ invest, earn income of λs, and have fertility p, whereas women with $s < \overline{s}$ earn income s and have fertility P, as shown in Figure 5.

After couples match, each has a child with probability π, and allocates their income. This process determines the surplus created by a given marriage, and thus individuals' preferences over different matches. Thus, solving the model requires working backwards from consumption decisions if a child occurs, to the surplus function from marriage, to then determining the optimal match.

Married couples can spend income on private consumption given by q^h and q^w and a public

[18] Starting at 1 simplifies the model by ensuring all individuals want to marry, because marriage is only "profitable" if total income is greater than 1.

Figure 5: Women's income versus potential income: exogenous $\overline{s}$



Notes: Women are endowed with skill, s, shown on the x-axis. Their level of income, y^w, shown on the y axis, is determined by their investment decision. If women invest, they earn income λs, with $\lambda > 1$, but at the cost of reducing their fertility, π from P to $p < P$. In this section, we assume women with $s > \overline{s}$ invest.

good, investment in children, denoted by Q:

$$U^h(q^h, Q) = q^h(Q+1)$$

$$U^w(q^w, Q) = q^w(Q+1),$$

with budget constraint $q^h + q^w + Q = y^h + y^w$

These utilities satisfy the Bergstrom-Cornes property for transferable utility (Chiappori and Gugl, 2014; Bergstrom and Cornes, 1983), and thus consumption decisions can be found by maximizing the sum of utilities, subject to the budget constraint. Accordingly, the utility maximizing level of Q and the sum of private consumptions, q is given by:

$$q^* = \frac{y^h + y^w + 1}{2}$$

$$Q^* = \frac{y^h + y^w - 1}{2}.$$

(Corner solutions are avoided by restricting $y^h + y^w > 1$ based on the distributions of y and s).

The joint expected utility from marriage, T, is a weighted average between the optimal joint utility if a child is born and the fallback position of allocating all income to private consumption:

$$T(y^h, y^w, \pi) = \pi\frac{(y^h + y^w + 1)^2}{4} + (1-\pi)(y^h + y^w).$$

For simplicity, I normalize the utility of singles to be zero, so that T represents the surplus from marriage. The predictions of the model are unchanged if we assume each individual converts his or her income into private consumption when unmarried.

3.2 Matching equilibrium

A matching is defined as a set of probabilities that a given man is matched with a given woman, and value functions for each agent indicating their equilibrium surplus share from the resulting match. The value function is the indirect utility resulting after matching is complete and optimal consumption allocation decisions have been made. The value functions are based on individual characteristics only, since the stable matching pins down surplus shares along with partners based on each individual's characteristics:

$$\text{Husband's value function}: \quad u(y^h) \equiv U^h(q^{h*}, Q^*)$$

$$\text{Wife's value function}: \quad v(y^w(s), \pi(s)) \equiv U^w(q^{w*}, Q^*).$$

A matching is stable if no two individuals would both prefer being matched together to their current pairing, and no matched agent prefers singlehood. Thus, we require:

$$\forall y^h: \quad u(y^h) \geq 0$$

$$\forall s: \quad v(y^w(s), \pi(s)) \geq 0$$

$$\forall y^h, \forall s: \quad u(y^h) + v(y^w(s), \pi(s)) \geq T(y^h, y^w(s), \pi(s)),$$

where $u(y^h) + v(y^w(s), \pi(s)) = T(y^h, y^w(s), \pi(s))$ for individuals matched together.

With a broad set of utility functions where children are a public good, including the one specified here, the marital surplus is supermodular in incomes. Thus, conditional on fertility, we expect assortative matching, since the increase of the joint product in one partner's income is increasing in the other partner's income. However, when fertility *and* income change, the stable match need not adhere to this positive assortative form. I show that the stable match exhibits deviations from positive-assortative matching when the returns to career investment for women, λ, are low relative to the fertility costs, $\frac{P}{p}$.

Proposition 1. *The stable match will be positive assortative on income conditional on fertility. However, for any* $\lambda < (\frac{P}{p} - 1)\frac{(Y-1)}{S} + \frac{P}{p}$*, the stable match will not be overall positive assortative.*

Proof. As shown in Becker (1973), supermodularity of the surplus function yields positive assortativeness in a unidimensional setting. Thus, for any surplus function that is supermodular in incomes, for two women of the same fertility level, the woman with the higher income must be matched with a higher-income man. Here, since the two incomes enter additively, this is equivalent to convexity in income: $\frac{\partial^2 T}{\partial y^h \partial y^w} = \frac{\pi}{2} > 0$. Thus, for any two women of equal fertility, the woman with the higher income will be matched with the higher income man.

To prove that there will *not* be unconditional assortativeness for all parameter values, by contradiction, assume assortative matching everywhere. For this to be the stable equilibrium, total surplus must be maximized. However, whenever $\lambda < (\frac{P}{p} - 1)\frac{(Y-1)}{S} + \frac{P}{p}$, the gain in joint surplus from a richer husband is higher just below the investment threshold than just above, meaning surplus can be increased by switching husbands on either side of $\bar{s}$. For proof details, see Appendix B.2. $\square$

Thus, the stable match will not always be positive assortative across the investment threshold, where fertility levels change. Across this threshold, matching is determined by how the relative trade-off between fertility and income differs for couples with men of different incomes. If couples with richer men value fertility less relative to income, then there is no tradeoff in matching the richest, and least fertile, women with the richest men—matching would be positive assortative everywhere. However, if couples with richer men value fertility more, there is a fertility-income tradeoff that may make matching non-assortative across the investment threshold. It could be that the value of extra fertility, although increasing in income, never outweighs the value of extra income, which is *also* increasing in income due to supermodularity. Or, extra fertility could always outweigh the value of extra income. Finally, there could be a switching point, where as men increase in income, the value of fertility in the total surplus overtakes that of income.

To examine how the tradeoff between fertility and income varies with men's income, it is useful to explore how the marginal rate of substitution between the woman's two characteristics is changes with the husband's income:

$$MRS = -\frac{d\pi}{dy^w} = \frac{\frac{\partial T}{\partial y^w}}{\frac{\partial T}{\partial \pi}} = \frac{\pi\frac{y^h+y^w+1}{2} + (1-\pi)}{\frac{(y^h+y^w+1)^2}{4} - (y^h+y^w)}.$$

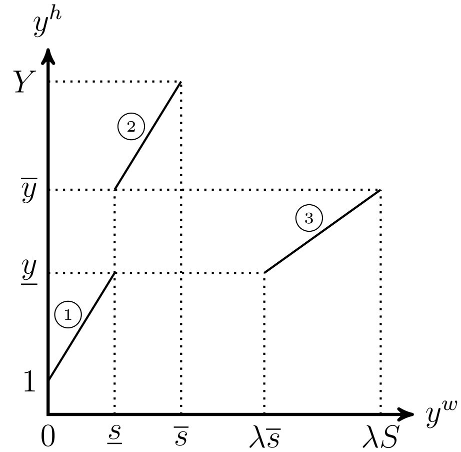
This is the relative change in surplus from an increase in y^w versus an increase in π. This ratio is decreasing in y^h:

$$\frac{\partial(MRS)}{\partial y^h} = -\frac{2(\pi(y^h+y^w-1)+4)}{(y^h+y^w-1)^3} < 0.$$

Therefore, the richer the husband is, the less improvement in fertility is required to compensate for income loss. In this sense, couples with richer husbands care more about fertility relative to income, and thus in equilibrium there may be a set of richer men who actually marry poorer, more fertile women than do poorer men.

An equilibrium displaying assortative matching for women with the same fertility, but non-assortative matching for women with different fertility levels, is shown in Figure 6. Recall $\bar{s}$ is the skill threshold for women becoming educated. Poor men, from 1 to $\underline{y}$, marry low-skill, fertile

Figure 6: Non-monotonic equilibrium match



Notes: Women's income, y^w is on the x-axis, and men's income, y^h on the y-axis. The diagonal lines represent matching between men and women. In this non-monotonic matching equilibrium, women with income between 0 and $\underline{s}$ match with men with income between 1 and $\underline{y}$. Women with incomes between $\underline{s}$ and $\overline{s}$ match with men with incomes between $\overline{y}$ and Y. Women who have invested, and thus have incomes between $\lambda\overline{s}$ and λS, match with men with incomes between $\underline{y}$ and $\overline{y}$.

women (matching assortatively). On the other side of the threshold, the richest group of women matches assortatively with the middle group of men, from $\underline{y}$ to $\overline{y}$. But the richest men, from $\overline{y}$ to Y, forego matching with the richest women and instead marry the "best of the rest"—the more high-skilled women among those who have not invested and are thus still fertile.[19]

The equilibrium value functions (solved for in the next section) can be used to show that this is indeed a stable match when λ, the income gain from investing, is high enough to overcome the fertility cost, $\frac{P}{p} - 1$, for some men, but not high enough that all men prefer women who have invested. In particular, when $\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p} - 1)\frac{Y-1}{S} < \lambda < (\frac{P}{p} - 1)\frac{Y-1}{S}$, the three-segment match is the unique stable match. For *any* value of S, $\overline{s}$, P, and p, such a λ exists, as $\frac{S-\overline{s}}{S+\overline{s}} < 1$. Thus, this model predicts the non-monotonic matching present in Census data.

Solving for equilibrium utilities At the stable match, for any two individuals of skill s and income y, the sum of their utilities in their current union must be greater than or equal to the utility they could obtain from marrying one another. That is, $u(y) + v(s) \geq T(y, s)$, because otherwise they would leave their current partners, and the match would not be stable. For married individuals, this holds with equality. We can imagine the matching process as each spouse choosing

[19]The matching functions in this uniform case are linear—in an arbitrary distribution, their form would be determined by the density of individuals, so that the number of women above any point exactly matches the number of men above that point.

the partner that maximizes his or her own share of the surplus conditional on keeping his or her spouse happy. Doing this maximization piecewise, and thus holding fertility constant, allows men and women to maximize over a single variable each: men over women's skill levels, and women over men's income. Thus, the subscript i below represents the section of the match, as shown in Figure 6:

$$u_i(y) = \max_s \{T_i(y, s) - v_i(s)\}$$

$$v_i(s) = \max_y \{T_i(y, s) - u_i(y)\}.$$

The first order condition of this problem dictates that the slope of the *other* partner's value function must equal the slope of that person's contribution to the surplus:

$$v_i'(s) = \frac{\partial T_i(y, s)}{\partial s}$$

$$u_i'(y) = \frac{\partial T_i(y, s)}{\partial y}.$$

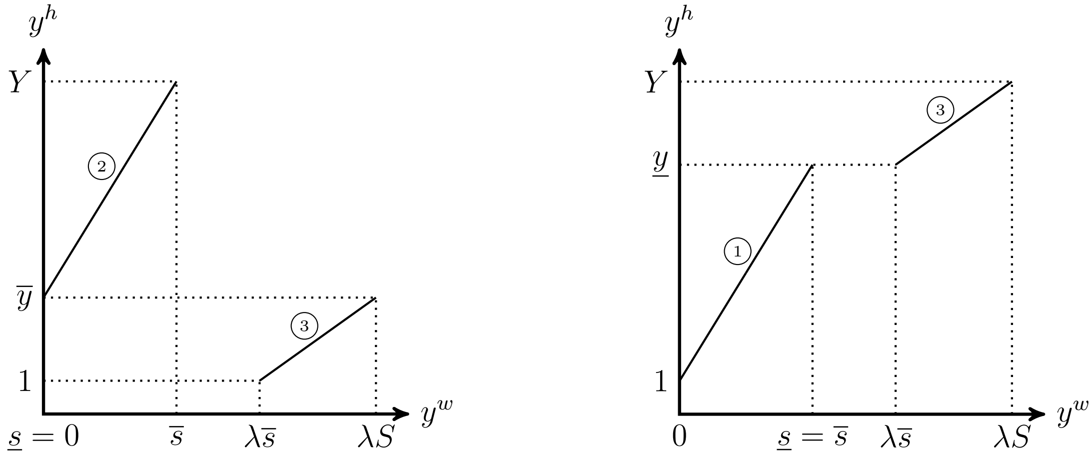
By using the matching function to plug in for y as a function of s and s as a function of y, we obtain a single-variable expression that can be integrated to pin down the value function to an additive constant. The boundary conditions that each man and woman agrees to marry, as well as the conditions that a man or woman at a "threshold" between segments must be indifferent, allow us to solve for the constants and obtain closed form solutions for the equilibrium value functions. Because of the piece-wise nature of the match, this differentiation and integration process must be completed for each "section" shown in Figure 6.[20]

Note that a woman's value function responds to fecundity loss through two channels. First, even if the woman's consumption level stayed constant, her utility would be reduced through the lower probability of conceiving, since children directly impact her utility. However, her consumption will also be reduced via the marriage market equilibrium, given that lower fecundity also lowers her husband's utility, and thus he requires a greater share of the available consumption in order to agree to the match. Section 3.4 demonstrates that approximately one-third of the welfare loss due to lower fertility for skilled women comes through this equilibrium channel.

Three benchmark equilibrium types The form of the stable match depends on the return to education, λ, relative to the fertility cost, $\frac{P}{p} - 1$, and other parameters. There are three main equilibrium types: non-monotonic matching, block-negative assortative matching, and positive assortative matching. When λ is between $\frac{S-\bar{s}}{S+\bar{s}}(\frac{P}{p} - 1)\frac{Y-1}{S}$ and $(\frac{P}{p} - 1)\frac{Y-1}{S}$, the stable match is

[20]Appendix B.1 shows the process of solving for the value functions in detail.

Figure 7: Low (left) and High (right) λ equilibrium match



Notes: Women's income, y^w is on the x-axis, and men's income, y^h on the y-axis. The diagonal lines represent matching between men and women. In the left panel, λ is low enough relative to other parameters that all wealthy men prefer matching with women who have not invested, and so matching is "block negative assortative." In the right panel, λ is high enough that all men prefer women who have invested, and so matching is positive assortative.

of the three-segment form depicted in Figure 6. For smaller λs, segment 1 of the match collapses, and $\underline{s} = 0$, leaving a match with two segments that can roughly be described as "block negative" assortative matching, as shown on the left of Figure 7. When λ is very large, above $(\frac{P}{p} - 1)\frac{(Y-1)}{S} + \frac{P}{p}$, segment 2 of the match collapses, and $\underline{s} = \overline{s}$, creating positive assortative matching, as shown on the right of Figure 7. When λ is between that required for the non-monotonic match and that for the positive assortative match, the equilibrium transitions between the two with randomization, meaning some segment of men is indifferent between poorer, more fertile women, and richer, less fertile women.

The matching equilibrium implies that as λ grows relative to $\frac{P}{p}$, the world transitions from one where educated women are penalized for their investment, because the additional income they earn is insufficient to compensate wealthy male partners for their loss in fertility, to one where they are able to compensate, and thus match with, partners similarly high in the income distribution. This transition is simulated in Appendix Figure A4, showing the different equilibria types as p and λ change, for a given $\overline{s}$, P, Y, and S.

Proposition 2. *The unique stable match exhibits positive assortative matching on each side of the investment threshold, $\overline{s}$. Across the investment threshold:*

- *When $\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p} - 1)\frac{Y-1}{S} < \lambda < (\frac{P}{p} - 1)\frac{Y-1}{S}$, there will be non-monotonic matching. Men from 1 to $\underline{y}$ will match with uneducated women; men from $\underline{y}$ to $\overline{y}$ will match with educated women;*

and men from $\overline{y}$ to Y will match with uneducated women.

- *When* $\lambda < \frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p}-1)\frac{Y-1}{S}$*, there will be negative-assortative matching.*
- *When* $\lambda > (\frac{P}{p}-1)\frac{(Y-1)}{S}+\frac{P}{p}$*, there will be positive assortative matching.*

Proof. There is assortative matching on either side of the threshold because supermodularity of the surplus function is a sufficient condition for assortative matching in unidimensional settings, and, on either side of the threshold, women vary unidimensionally in income (and the surplus function is supermodular in incomes).

To verify that each of the specified match forms are stable, we need to check that no two individuals would rather match with each other than their current pairings, or remain single. We know that no one prefers to remain single, because each marriage generates strictly positive surplus, by the assumption that singles' utility is zero. We know that within segments (see illustrations), no re-matching could yield surplus gains, because otherwise there would not be positive assortative matching. So we need only check that no one could profitably deviate to match with someone from a *different* section.

To verify this, we check that the individuals with the most to gain from deviating from their current match (to pair with one another) do not get positive surplus improvements. For non-monotonic matching, this is true when $\lambda < (\frac{P}{p}-1)\frac{Y-1}{S}$. For $\lambda < \frac{Y-1}{S}\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p}-1)$, $\underline{s}$=0, and thus the negative-assortative match is the unique stable match. There is no profitable deviation from positive-assortative matching when $\lambda > (\frac{P}{p}-1)\frac{(Y-1)}{S}+\frac{P}{p}$.

For proof details, see Appendix B.3. □

These three equilibria show that matching outcomes for "top" educated women are expected to improve over time, as long as λ and p tend to rise, and thus that the non-monotonicity present in Census data will transition to positive assortative matching.

Transition equilibrium when $(\frac{P}{p}-1)\frac{Y-1}{S} < \lambda < (\frac{P}{p}-1)\frac{(Y-1)}{S}+\frac{P}{p}$, no pure stable matching exists, and the stable match exhibits randomization by men, some segment of whom will be indifferent between matching with poorer, more fertile women, and richer, less fertile women. In other words, the income gained (lost) exactly compensates for the fertility lost (gained) in switching from one type of partner to the other. The rate of randomization is endogenously determined in order to keep all men above the lower bound of the randomization segment, $\underline{y}$, indifferent between the women on the right or left of the threshold. When fewer men marry uneducated women, the change in women's skill with men's income will be steeper for men marrying uneducated women than for men marrying educated women, thus equalizing the change in surplus the man can expect between the two sides.[21]

In the real world, this could translate into the dating pool for richer, older women being limited, because men are also considering poorer, younger women as potential partners. Even as matching tends toward assortativeness, with this remaining randomization, spousal quality drops off faster for women at the top of the income distribution than it would in a purely assortative world, because only some portion of the men in a certain quality category are actually available, given that they are also willing to match with poorer, younger women.

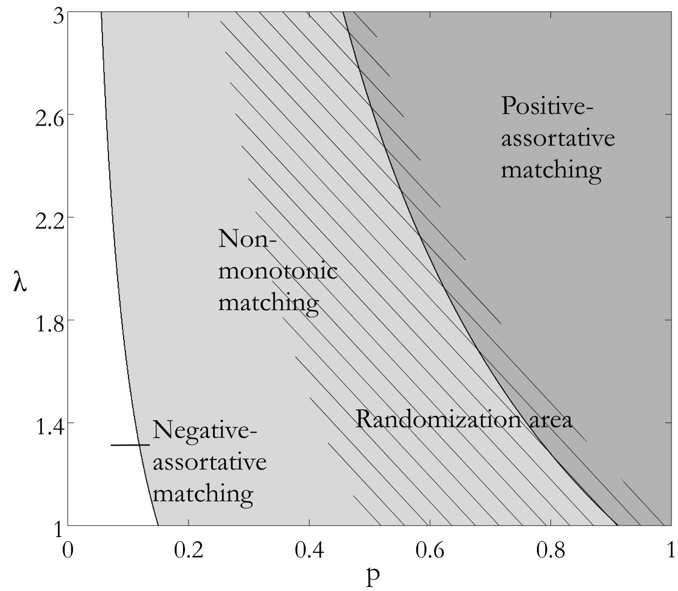
An illustration of the bounds of the different type of equilibria in terms of p and λ (for a given P, Y, and S), with an exogenous education threshold $\overline{s} = 0.7$ is shown in Figure 8. Negative assortative matching is the area in white, pure non-monotonic matching is shown in light grey with no shading, and pure positive-assortative matching is shown in dark grey with no shading. The non-pure transition equilibrium is shaded with diagonal lines.

Generalizability The existence of a potentially non-monotonic equilibrium arises from the surplus function exhibiting supermodularity in spouses' incomes and a marginal rate of substitution between income and fertility that decreases in income, something that is generalizable to a large number of functions and settings. The supermodularity assumption is fairly standard in marriage models with children acting as a public good, for example Lam (1988). It reflects the returns to income being multiplied by the ability to spend additional dollars on both private consumption and investments in children, where enjoyment from children is shared by both husband and wife. In a single dimensional model, this assumption is a good fit for aggregate data, where in general married partners are very similar to one another.

The marginal rate of substitution assumption has two intuitive explanations. First, it reflects diminishing marginal returns to income relative to other inputs in the surplus function. Although the surplus is supermodular in incomes, it is natural that if income is abundant, the value of

[21] This provides an appealing intuitive explanation of the λ cutoff at which randomization appears. When $\lambda = (\frac{P}{p} - 1)\frac{Y-1}{S}$, the return to education is exactly equal to the loss of fertility resulting from investment, weighted by the ratio of the income distributions, and thus the relative value of women's income compared to men. Thus, once that segment is passed, an interval of men (rather than just the men with exactly income $\underline{y}$ or $\overline{y}$) can be kept indifferent between the two types of women, by varying the matching "slope," which comes from the probability of matching with one type versus the other.

Figure 8: Matching equilibrium for varying λ and p
$Y = 2,\ S = 1,\ P = 1,\ \overline{s} = 0.7$



Notes: Figure depicts the matching equilibrium depending on the value of the return on investment, λ (y-axis) and the post-investment chance of fertility, p (x-axis). Negative assortative matching is the area in white, pure non-monotonic matching is shown in light grey with no shading, and pure positive-assortative matching is shown in dark grey with no shading. The non-pure transition equilibrium is shaded with diagonal lines.

additional income relative to fertility diminishes. Second, it is tied to the growing importance of additional surplus from the public good as the amount spent on that good rises. If a large amount of the value of additional income is coming from the ability to spend that income on a joint child, the couple's surplus will be very sensitive to the probability of being able to conceive in the first place.

The intuition for why these two characteristics are crucial is that supermodularity ensures there will be assortative matching conditional on fertility, while a decreasing marginal rate of substitution between income and fertility in terms of husband's income guarantees that there can exist a man rich enough that he values any fixed change in fertility more than the corresponding change in income that goes with it. Then, since every subsequent man values this fertility-income swap *more*, as long as the top man is "rich enough," there can be an efficiency improvement made by deviating from assortative matching to pair the richer man with the poorer, more fertile woman, and the poorer man with the richer, less fertile woman. The appendix provides more formal explication for why these two characteristics result in non-monotonic matching.[22]

[22]This may be applicable to other settings where one agent values two characteristics of another agent, but the

3.3 Endogenous human capital investments

Having now characterized the equilibrium in the matching stage, taking the number of women who invest as given, I now consider endogenous investment decisions. When women take the matching equilibrium into account when deciding whether to invest, then in addition to the commonly mentioned personal cost of lowered fertility, women would face a second cost: that of matching with a lower quality partner or compensating a higher quality partner to make up for forgone fertility.

Just as the matching problem has a dual surplus maximization problem, the educational investment and matching equilibrium can be solved for by *either* assuming women become educated first, and then marry, or by solving a complete surplus maximization problem encompassing both educational choices and matching. This is equivalent to finding the surplus-maximizing match, where couples then choose the best education level for the wife post-marriage—in this form, the match and educational investment equilibrium will be unique. In the case of matching with pre-marital investments, with some surplus forms there may be multiple equilibria (Cole, Mailath and Postlewaite, 2001; Mailath, Postlewaite and Samuelson, 2013). However, there is still a single *efficient* equilibrium, which always exists (Dizdar, 2013) and is the solution to the dual problem. I solve for this efficient equilibrium.

Lower skill bound for investment The lower bound on women's skill for them to be willing to invest, $\overline{s}$, can be found by using the payoff functions resulting from the matching equilibrium, and finding the point at which the investment payoff dominates the non-investment payoff. Adding a small fixed cost of education, c, provides a more realistic set of educational investment outcomes, and creates a broader range of parameter values that yield an interior solution (rather than all or no women getting educated). To simplify this section, let $Y = 2$, $S = 1$, and $P = 1$.

Using the equilibrium payoff functions, we seek the skill level at which $v_3(\overline{s}) = v_2(\overline{s})$, or $\overline{s}^*(\lambda, c, p)$. Although its functional form is complex, $\overline{s}^*$ varies with the parameters in expected

two are negatively correlated. For example, imagine a downstream manufacturer characterized by quality only, while the upstream supplier is characterized by both quality and speed. A higher quality downstream manufacturer could conceivably achieve greater profit gains from matching with a higher quality upstream firm than would a lower quality one. Nonetheless, there could also be diminishing marginal returns to quality *relative* to speed as the downstream firm's own quality increases. A higher quality downstream firm may be able to make up for some upstream quality deficiencies, but be less willing to absorb delays in timeline. Therefore, there could arise a matching equilibrium where high quality downstream firms are actually matched to low-quality, but speedy, upstream firms, while the highest quality upstream firms match with more mid-range downstream firms. In this setting, too, the two characteristics of the upstream firm could be endogenously determined—for example, if the firm knows it must sacrifice speed to develop quality, and does so even knowing it forgoes the opportunity to supply the best firms. As another example, Grossman, Helpman and Kircher (2017) provides a setting where a discontinuous matching pattern can emerge between workers and managers.

ways: it is increasing in c, decreasing in λ, and decreasing in p. In other words, the higher the fixed cost of investment, the higher the skill threshold for pursuing it; the higher the return to investment, the lower the skill threshold; and, the higher the chance of conceiving following investment, the lower the threshold. A higher threshold means fewer women making career investments. A lower threshold means more women making career investments, and this can be spurred by a lower fixed cost of investment, greater returns, or a higher chance of conceiving (e.g., through IVF technology).

If the returns to investment or fertility post-investment are very low, no woman will optimally choose to invest, i.e., the optimal $\overline{s}$ will be 1.[23] One could imagine that if these parameters are very high, all women will want to invest, but here the fixed cost of investment will keep some women from investing at all times, although an increasingly smaller share.

Note that the equilibrium payoff function internalizes not just the individual change in utility from a different fertility level, but also any change in the share of surplus received. This reflects the impact of traits on the overall surplus: someone with traits that yield a large surplus will in exchange receive a favorable match with a high surplus share. Someone with less desirable traits will face a less desirable match and a lower surplus share. Thus, when equalizing the payoff between investing and not investing to find the optimal threshold, both the personal cost of lower fertility and the cost to the marital surplus are considered. Accounting for these marriage market consequences yields a lower $\overline{s}$ for a given λ and p than if women only needed to consider their own preferences for fertility, or if matching were somehow irrespective of reproductive capital. Thus, the marriage market adds a "second cost" of investment to women's own valuation of forgone fertility. If this cost is reduced, women's investments will increase.[24]

Special case: upper bound for investment In the same way that increasingly richer men may value fertility higher relative to income, women who are at the very top of the skill distribution may possibly prefer to remain uneducated, due to the fertility loss. These women have an additional concern, which is that they may also forgo matching with the best men by becoming educated, despite increasing their own income. For some parameter values, there is an upper bound of skill-

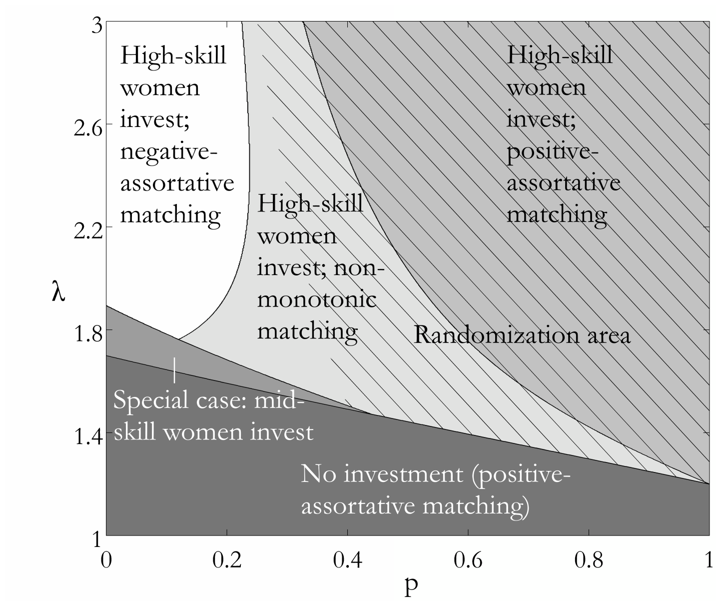
[23] Note, though, that there may be other reasons for investing in education that are not reflected in the utility function used in this model. In this case, some women may invest regardless, and the matching stage of the model can be informative about the marriage market consequences of this decision.

[24] My results are also consistent with Goldin's (2006) documentation of the "revolution" of women switching from marrying before solidifying their identities to now making pre-marriage investments: once seeking higher education is not penalized in the marriage market, women are more likely to invest before marriage.

level for investment, on the support of s. The results of the second-stage matching equilibrium are largely unchanged by this possibility, because any "top" women who choose to remain uneducated will simply match assortatively with the "top" men, allowing the other results to hold with a merely truncated distribution of available men and women who can be matched.

There may be an intuitive real-world corollary to this phenomenon: women whose matching outcome would be so appealing in the absence of investment that it is not worth forgoing these appealing marriages in order to better their own career outcome. (In the model, women's skill is uniformly distributed. In a more typical normal distribution, this would be women at the far right tail.) Thus, this simple model allows for non-monotonicity both in matching, and in women's own willingness to invest. This possibility only occurs when p is small but λ relatively large—i.e., the return on investment is large enough that some women want to invest, but the fertility cost is high enough that it deters the top women. As λ grows further, this "wedge" disappears, and the top women choose to invest.[25]

FIGURE 9: MATCHING AND EDUCATION EQUILIBRIUM FOR VARYING λ AND p
$Y = 2,\ S = 1,\ P = 1,\ c = 0.2$



Notes: Figure depicts optimal investment decisions and the resulting matching equilibrium depending on the value of the return on investment, λ (y-axis) and the post-investment chance of fertility, p (x-axis). In the lower dark grey region, λ and p are low enough that there is no investment. Lighter grey sliver shows where mid-skill women invest. Top three segments represent area where all women with s greater than $\bar{s}$ invest, with different matching patterns resulting.

[25]This transition is illustrated via simulation in Appendix Figure A7.

Figure 9 depicts the bounds on the three possibilities for education decisions and the resulting matching over a wide range of λ—from no return on investment, $\lambda = 1$, to a three-fold return—and p. The lower dark grey region represents the area for which λ and p are low enough that there is no investment (note that below a λ of 1.2 there is no investment no matter the p, due to the fixed cost of 0.2). The lighter grey sliver represents the region for which there is an upper and lower bound on investment—some woman wants to invest, but not the top-skilled women. And, the top three segments represent the region for where all women with s greater than some $\overline{s}$ invest. These women then match negative-assortatively, non-monotonically, or positive-assortatively, depending on λ relative to p.

3.4 Theory Summary and Extensions

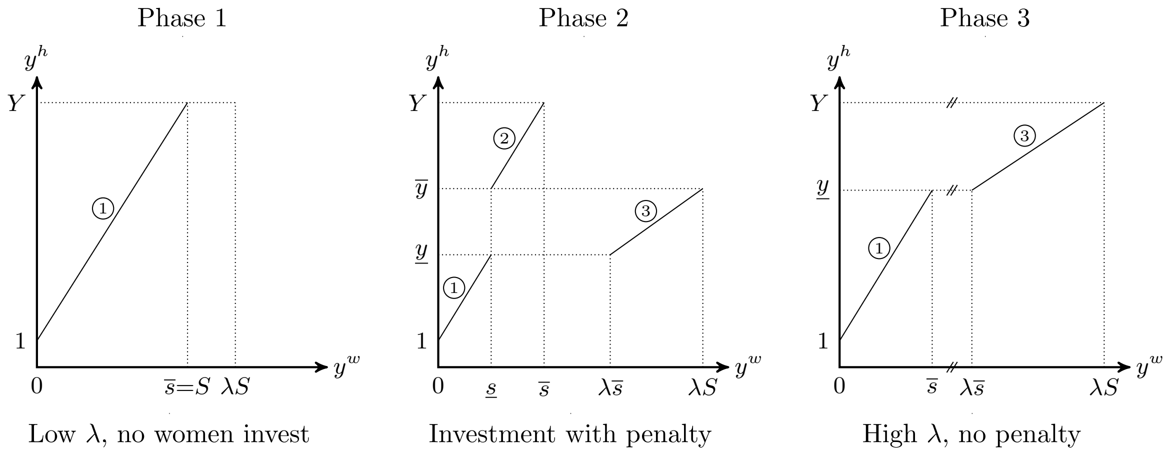
Historical transitions The matching model demonstrates that introducing a second factor that endogenously trades off with human capital can produce the non-monotonic matching patterns exhibited by Census data, as well as the historical transition toward more assortative matching, driven only by the top-skilled women. Figure 10 shows the potential transitions one can derive as λ and $\frac{P}{p}$ increase. These predictions mirror the transitions exhibited in Census data, from very few women investing in higher education, to women investing with a marriage market penalty, to the penalty dissipating and assortative matching returning.[26]

Welfare Crucially, the model provides a mechanism through which the biological clock impacts women's welfare through a channel other than her own desire for children. That is, even if a woman did not care at all about having a family, she would still be negatively impacted by her fertility loss through her loss of status on the marriage market.

In fact, a back of the envelope calculation using the model suggests that approximately one-third of the utility cost from the post-investment fertility loss comes through the marriage market, rather than directly through women's utility over children. Figure 11 compares the utility loss of lower fertility from the marriage market alone to the loss including women's personal valuation of fertility. The portion of the welfare loss stemming from being matched with a lower quality spouse and needing to cede more of the marital surplus to that spouse ranges from 20-40% of the total

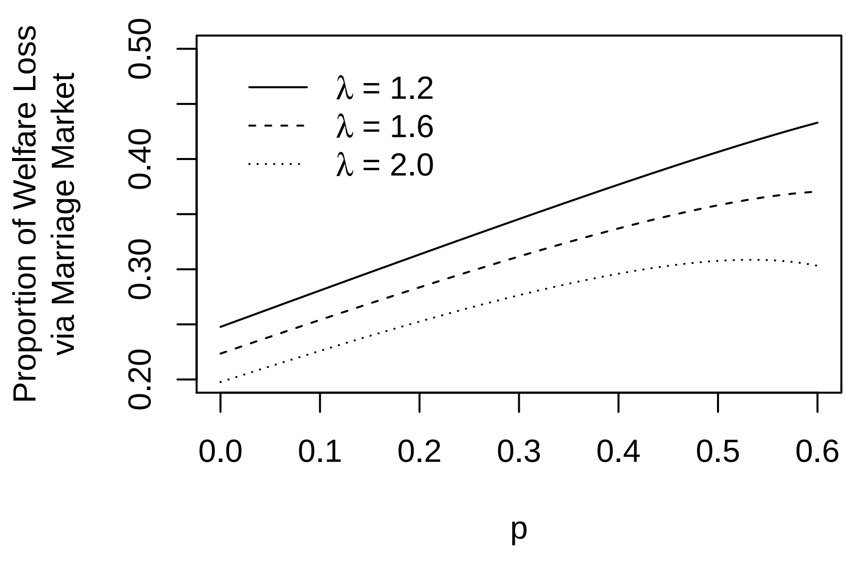
[26]Note, this transition would be consistent with the predictions of Bertrand et al. (2016), who find that skilled women's marriage rates should first fall as the skill premium rises, then fall. If skilled women at first choose not to make time-consuming career investments, their marriage outcomes should be similar to other women. Once they begin making time consuming investments, their marriage outcomes fall, and then when the skill premium rises sufficiently for them to compensate their partners, their marriage outcomes rise.

Figure 10: Potential historical transitions



Notes: Figure depicts the model's prediction depending on the value of the return on investment, λ relative to the post-investment chance of fertility, p. At first, the returns are low enough—and the potential marriage market cost high enough—that no women invest (and thus matching is assortative). As λ rises relative to other parameters, some women invest, but these top-skilled women are penalized on the marriage market. Finally, λ is sufficiently high that the top-skilled women can compensate their potential partners, and assortative matching, with female investment, returns.

Figure 11: Proportion of Welfare Loss from Time-Limited Fertility due to Marriage Market



Notes: Figure depicts the portion of the welfare loss (y-axis) from lower fertility that comes through the marriage market compared to the total welfare loss, for varying values of λ, across a range of values for p (x-axis). This is shown for the most-skilled woman, with skill-level S, across the range of ps where non-monotonic matching results, for parameter values $Y = 2$, $S = 1$, and $P = 1$, with an exogenous t, investment threshold, of 0.7. The graph is produced by calculating the change in woman's indirect utility between a scenario with zero fertility cost of investment, to one where post-investment fertility equals p, and comparing that to the same effect if her partner and share of the marital surplus were held constant.

utility cost.[27] This simple calculation highlights that the loss of reproductive capital is an economic loss, just as worker disability is. Because the marriage market creates value for women, the loss of a valuable asset on that market creates real economic impacts.

Marriage rates Although the model has no formal predictions regarding marriage or divorce rates, if we imagine introducing shocks that make marriage less attractive, either before or during marriage, women in lower surplus matches will be more affected by these shocks, and less likely to marry and more likely to divorce. When matching is non-assortative, highly educated women may be in lower surplus matches than their less educated counterparts. As marriage becomes more assortative, high-skilled women are likely to be in the highest surplus matches, and thus are predicted to be the most likely to marry and least likely to divorce, again mirroring the patterns in Census data.

Mechanism Would any two-factor model be able to provide similar predictions? First, the second factor cannot be randomly distributed throughout the population, but rather must systematically relate to either innate talent or human capital acquisition (which creates a correlation with skill, as higher skilled women seek more education). Thus something related to age or the time required to seek human capital is especially likely to take this form. Additionally, this second factor must be something that high income men value more than low income men, thus outweighing super modularity in incomes. Without super-modularity in incomes, it would be difficult to produce the assortative mating seen in Census data across all educational levels before graduate education. But, once we postulate that the marital surplus is super-modular in incomes (or at least underlying skill/earning potential) the only way to *reverse* the assortativeness in mating for the far right tail of women is for high-income men to value the second factor, of which they have a lower stock, more than low income men. When children are a public good within the household, and there is a standard child quality effect where high-income parents wish to invest more in children, fertility as the second factor naturally has this feature.

Yet, there is no well-identified empirical evidence that men do in fact value their wives' potential fertility on the marriage market, and would change their valuations of mates based on this factor.

[27]The calculation is $1 - \frac{v(S|p=P)-v(S|MM)}{v(S|p=P)-v(S)}$ where $v(S|p = P)$ is the woman with skill S's indirect utility if she invests but fertility is unaffected, $v(S)$ is her actual indirect utility, and $v(S|MM)$ is her indirect utility if there is indeed a fertility penalty, but she were to still match with man Y and receive the same *share* of the surplus as if there were no fertility loss.

Although it is logical from an evolutionary standpoint, and consistent with anecdotal evidence, strong evidence of valuation of fertility would help bolster the case that reproductive capital is indeed the "second factor" underlying non-assortative matching patterns, and their recent reversal. I thus turn to an incentive compatible experiment that tests, for the first time, the monetary value that men place on women's age, and thus fertility.

4 Experiment

A large amount of anecdotal evidence suggests that men prefer younger women on the dating market. For instance, dating website OK Cupid has published data showing that men list their preferred age ranges for women as much younger than themselves, and target their messaging at the younger end of that range.[28] This pattern has also been documented by sociologists England and McClintock (2009), who find that the age gap between spouses is increasing in the man's age at marriage. A 30-year-old man may marry a woman only a couple years younger than himself, whereas a 50-year-old man will, on average, marry a woman ten years younger.

However, there is no well-identified *causal* evidence of the impact of age on marriage market value. This is inherently hard to measure, since individuals' incomes, lifestyles, and appearances change with age, and thus a simple association between women's age and worse marriage outcomes is not conclusive. Men may simply have tastes for women who *look* younger, which is then correlated with actual age. This preference may nonetheless be rooted in an evolutionary-driven desire for fertility, but the policy implications for a conscious preference for fertility, versus an instinctive one, differ. If a true preference for fertility underlies the preference for younger women, then policies promoting access to assisted reproductive technology could help alleviate the marriage-market penalty to delayed marriage. If the preference for youth is exclusively a preference for younger looks, though, such policies would be ineffective (and the government may want to consider subsidies for Botox instead). The age "penalty" could also result from social norms or meeting opportunities, rather than men's preferences.

To determine whether there is indeed a causal impact of age on women's marriage market value, and quantify its impact, I implement an incentive compatible online experiment in which single men and women rate profiles of hypothetical partners, with age randomly assigned, while other characteristics, such as beauty, remained fixed. Income is also randomly assigned to the profiles,

[28] OK Trends, "The Case for an Older Woman," February 16th, 2010.

providing a "numeraire" by which to quantify the preference for age. I then connect these results to an underlying preference for fertility by examining the heterogeneity in male raters' tastes.

4.1 Methodology

The methodology I use isolates age from other factors, while incentivizing participants to give honest responses. Respondents were recruited online to rate dating profiles, with each respondent rating 40 profiles. All characteristics on these (hypothetical) profiles were fixed, except for age and income, which were randomly assigned as the profile was viewed. In order for the online experiment data to be valid, subjects must rate the profiles according to their own preferences. Yet, unlike in most traditional economics experiments, there is no clear way to incentivize truthful reporting in rating dating profiles. If the profiles were presented as real, in the context of a dating site or speed dating exercise, deception would be involved (since at least some portion of the profile, the exogenously assigned age and income, must be fake), which makes the experiment not truly incentive compatible. In order to elicit honest responses without deception, I introduce a novel strategy of providing non-monetary compensation whose value is tied to honest ratings in the study. Participants were offered free customized advice on their own online dating profiles to attract the type of people they had indicated interest in *based on their answers to the experimental questions.* The customized advice was provided by a dating coach hired for this purpose. Because participants are recruited using advertising for this compensation, and spend their time on the study in expectation of receiving it, I can assume participants place value on the advice and therefore want to increase its quality. The way to maximize the value of the compensation used here is to respond truthfully in the study, just as in a traditional experiment the way to maximize one's payoff is by exerting effort in the game.

For the initial sample, subjects were recruited using online ads, placed on dating sites or linked to searches of dating-related keywords. A sample Google ad is shown below:

A Better Dating Profile

Single & 30-40? Take this survey &

get expert dating profile advice!

www.columbiadatingstudy.com

Following the implementation of this initial experiment, I conducted a second, similar experiment in order to test for heterogeneity in men's preferences for age. Because this required a larger sample, I enlisted a survey firm, Qualtrics, to recruit respondents. These respondents were recruited through Qualtrics' relationship with marketing partners, which offer survey opportunities to their mailing lists in exchange for incentives (e.g., frequent flyer miles, gift certificates). As this second population was provided with other incentives in addition to the date coaching (which was still provided), the date coaching incentive could have been less powerful in eliciting truthful responses—nonetheless, the parameter estimates for valuation of age are nearly identical between the two groups.

To generate the hypothetical dating profiles, a stock photo was randomly combined with a user name, height, and interests, which remained fixed to the profile. Then, as each participant viewed the profiles, an age (between 30 and 40) and income would be randomly generated. For additional details on the experimental methodology, as well as data summary statistics, see Appendix C.1.

4.2 Men's Preferences Over Age

I identify the effect of randomly assigned ages on ratings for men-rating-women and women-rating-men, using the specification:

$$Rating_{ij} = \beta_0 + \beta_1 age_{ij} + \beta_2 income_{ij} + \alpha_i + \theta_j + u_{ij}$$

Because each individual rates 40 profiles, and each profile is seen by multiple individuals, I can include both rater, α_i, and profile, θ_j, fixed effects.[29]

Table 1 shows results for those who meet my sample requirements (of being between 30 and 40 and white), as well as all data collected (including incomplete responses).[30]

These results show that men rate women lower when the profile is presented with a higher age, whereas women rate men more highly when a higher age is shown. This lower rating is stronger for the targeted group of white men between the ages of 30 and 40, potentially because restricting in this way excludes individuals who were much older than the targeted age range, and may have less intense age preferences. The reduction in rating for an additional year of age is 0.044 points, on a scale from 1 to 10. Thus, if a woman is 10 years older, she will be rated 0.4 points lower on average. A woman who is $10,000 poorer is rated 0.06 points lower; thus to make up for each

[29] In this section, I present heteroskedasticity-robust standard errors. Although errors may be correlated within an individual's responses, the "group" status, the individual, is not correlated with the x variable of interest, age, since it is orthogonally assigned within subject's rankings, and thus the criterion for requiring a cluster correction is not met. When examining heterogeneity among respondents in subsequent sections, I cluster results at the respondent level. See: Angrist and Pischke (2008), page 311.

[30] The considerable difference in observations between those specifications is because the complete dataset includes some individuals who did not complete the entire survey, and thus I lack information on their race or ethnicity.

Table 1: Age-Rating Relationship for Men vs. Women

	Dependent variable: Profile rating			
	Men in Sample	Men All	Women in Sample	Women All
	(1)	(2)	(3)	(4)
Age	-0.044***	-0.024**	0.131***	0.079***
	(0.015)	(0.010)	(0.015)	(0.010)
Income ($0,000s)	0.061***	0.023**	0.134***	0.147***
	(0.016)	(0.011)	(0.016)	(0.011)
Constant	6.252***	5.811***	-0.160	4.493***
	(0.662)	(0.467)	(0.692)	(0.457)
Observations	1440	3752	1800	4220
R-Squared	0.471	0.487	0.394	0.452

Notes: Regression of profile rating on randomly assigned age and income, for men-rating-women in columns 1 and 2 and women-rating men in columns 3 and 4. Columns 1 and 3 are restricted to white individuals between 30 and 40. Columns 2 and 4 includes all data collected, including incomplete responses where not all profiles were rated. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

additional year of age, a woman must earn $7,000 more.

The contrasting results for men versus women demonstrate that the negative relationship between a female profile's listed age and the rating cannot only be a "lemons" effect, where older women still on the market are judged to be less appealing. If this were entirely the channel of this negative preference, women rating men should show a similar aversion to age, although potentially less intense because men marry later. Instead, women show the opposite reaction to age.

Table 2 shows the results for men with several robustness checks. First, I restrict the analysis to only those who completed and submitted the survey, as those who did not may not have been incentivized to provide accurate data, since they did not claim the compensation. Then, I exclude subjects who opted out of the compensation, which happened in a small number of cases.[31] I next exclude individuals who have a low correlation between their "rate" responses and their "rank" responses, since this may indicate low attention. Next, I exclude the small number of individuals who took the survey prior to the implementation of a one-second load delay on the photographs, so that individuals would read the profile information more carefully before responding to the photo alone. None of these changes significantly alter the results.

Finally, I check whether photographic appearance versus reported age may be influencing the results. Photos likely *look* a certain age, and so when these photos are paired with higher ages, the person looks "good for their age," whereas when paired with lower ages the person looks "bad for their age." When the interaction between visual age (calculated by having 120 undergraduates

[31] As the compensation involved the sharing of individual data with a third party, human subjects considerations required I provide the option to opt out.

guess the age of the person in each photo) and age is controlled for, the penalty for age, if anything, gets larger, although neither coefficient is significant.

Table 2: Robustness Checks

	Dependent variable: Profile rating (Male subjects)				
	Finished	No Opt Out	High Corr	Load Delay	Visual Age Control
	(1)	(2)	(3)	(4)	(5)
Age	-0.040**	-0.049***	-0.045***	-0.044***	-0.119
	(0.018)	(0.016)	(0.016)	(0.016)	(0.183)
Income ($0,000s)	0.066***	0.069***	0.062***	0.051***	0.061***
	(0.018)	(0.018)	(0.017)	(0.017)	(0.016)
vis_ageXage					0.002
					(0.005)
Observations	1120	1280	1360	1320	1440
R-Squared	0.435	0.460	0.465	0.465	0.471

Notes: Regression of profile rating on randomly assigned age and income, for men-rating-women. Column 1 restricts to individuals who finished and submitted the survey, column 2 restricts to those who did not opt out of compensation, column 3 restricts to those with a high correlation in the two rating measures (to assure attention), column 4 excludes individuals taking the survey before a load delay in photos was implemented, and column 5 controls for the visual age of the photo (guessed by 120 undergraduates) interacted with actual age. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

The experimental results show that men have a robust preference for younger partners, even when beauty is controlled for by exogenously assigning age to fixed profiles of potential partners. The remaining question is whether this preference is actually driven by a preference for fertility, and whether some of this preference operates on a conscious level.

4.3 Drivers of Preference

To understand the drivers of men's preferences for younger partners, I conducted a second experiment to examine how raters' characteristics interact with profile age. Because this requires looking at heterogeneity among respondents, rater-profile observations cannot be treated as independent, and thus a larger sample size is required (in this section, results are clustered at the rater level when rater characteristics are interacted with profile characteristics). For this experiment, I gathered a sample of 200 men through the survey firm Qualtrics, over-sampling high-income men in order to better match the distribution of respondents recruited through Google ads. I also collected data on 100 women to help confirm that the earlier results hold in this sample (and thus that the different recruitment mode did not alter the experiment's validity).

The basic age-rating analysis performed with Qualtrics data is shown in Table 3, confirming that this sample exhibits the same tradeoff between age and rating for male respondents, despite the different recruitment technique. In fact, the coefficient on age as a factor in male preferences has a remarkably similar coefficient between the two samples. The contrasting positive coefficient for women is also present in this sample.

Table 3: Age-Rating Relationship for Men vs. Women: Qualtrics Sample

	Dependent variable: Profile rating		
	Men + oversample	Men	Women
	(1)	(2)	(3)
Age	-0.043***	-0.062***	0.028***
	(0.006)	(0.009)	(0.010)
Income ($0,000s)	0.032***	0.007	0.036***
	(0.007)	(0.009)	(0.010)
Constant	9.768***	7.475***	3.340***
	(0.271)	(0.426)	(0.552)
Observations	8080	4040	4040
R-Squared	0.490	0.479	0.463

Notes: Regression of profile rating on randomly assigned age and income, for men-rating-women (columns 1 and 2) and women-rating-men (column 3), from the second sample collected via Qualtrics. High-income men were oversampled and are included in column 1, whereas column 2 restricts to the "natural" sample only. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.1

As a first check on mechanisms driving preferences, I examine whether taste for similarly-aged partners may affect the relationship between age and ratings, in Table 4. Is it possible men prefer not younger women, but rather similarly aged women, or women who are *slightly* younger only? To test for this, I control for the age difference squared, or the taste for similarity, as well as similarity with the male partner being slightly (two years) older.[32] Neither of these additions eliminates men's preference for younger partners.

One of the surprising results in the experimental data is that women appear to have a *positive* preference for age. This positive preference indicates something residual besides fertility being captured by the age variable. Yet, when the distance from the two year "ideal" age difference is controlled for in the case of female raters, this apparent preference over age disappears (column 4). Thus, this apparent preference for older men is really a preference for men two years older than oneself. This is in contrast to the results for men, where a significant secular age preference is still present even after controlling for the two year age difference.

I now turn to establishing the true drivers behind men's preferences for age, by interacting age with rater characteristics that may make men care more or less about fertility, in Table 5 . In each case, men's characteristics that indicate a preference for fertility result in a bigger rating penalty

[32]There is some evidence of tastes for partner age taking this form, e.g., Hitsch, Hortaçsu and Ariely (2010); Choo and Siow (2006); Buss, Shackelford and LeBlanc (2000).

Table 4: Controlling for Age Difference as Preference Channel: Qualtrics Sample

	Dependent variable: Profile rating			
	Male Raters		Female Raters	
	Age Diff Control	"Ideal" Age Diff	Age Diff Control	"Ideal" Age Diff
	(1)	(2)	(3)	(4)
Age	-0.040***	-0.024**	0.036**	0.004
	(0.009)	(0.010)	(0.015)	(0.012)
Income ($0,000s)	0.032***	0.032***	0.035**	0.035**
	(0.009)	(0.009)	(0.014)	(0.014)
(Age diff)^2	-0.004***		-0.008***	
	(0.001)		(0.002)	
(Age diff − "ideal")^2		-0.004***		-0.008***
		(0.001)		(0.002)
Observations	8080	8080	4040	4040
R-Squared	0.491	0.491	0.468	0.468

Notes: Regression of profile rating on randomly assigned age and income for men-rating-women (columns 1 and 2) and women-rating-men (columns 3 and 4), from the second sample collected via Qualtrics. Columns 1 and 3 controls for the age difference between the rater and the profile squared. Columns 2 and 4 control for the "ideal" age difference of the rater being two years older than the profile for male raters and the rater being two years younger for female raters. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses, clustered at the rater level.

*** p<0.01, ** p<0.05, * p<0.1

to women's age. The characteristics I look at are wanting to get married, *Want marr*, wanting kids, *Want kids*, having no children, *No kids*, and having accurate knowledge of the age - fertility tradeoff, *Knowledge*.[33]

When women's age is interacted with the rater wanting to get married, the main effect of age becomes smaller, while the interaction is negative and significant (although only at the 10% level). This means that men who want to get married dislike age *more* than men who may be looking for more casual relationships. This provides our first evidence that fertility may be driving the preference, because if it were a preference for the amenity value of younger women, we may expect men who *do not* want to get married to value it more. In the second column, we can see that men who want children soon also demonstrate a stronger preference for younger partners.

In the third column, I look at whether men already have children, as having no children currently may be a stronger indicator of seeking the option value to have kids than stated preferences. As expected, men with no kids have a very strong preference for younger women. In fact, with this interaction term inserted, the main effect of age becomes zero, indicating that men who already have children have *no preference* over randomly assigned age. This provides strong evidence that the preference being identified is really a preference over fertility, as other possible residual factors

[33]Measured as an indicator for awareness that fertility decreases for women before age 45.

would be unlikely to diverge so strongly between men who have children and those who do not.

The final column interacts age with knowledge about fertility. The variable "Knowledge" represents the rater being aware that women's fertility begins to decline by age 45 (asked in the post-survey as "at what age does it become biologically difficult for a woman to conceive?"). For men who lack such knowledge, there is again *no preference* over age—the main effect is statistically zero—whereas for the knowledgeable men the negative perception of age is much stronger. Taken together, this table shows that the age preference found by this experiment is driven by men who have reason to care about fertility and have the knowledge to connect age to fertility.[34]

Table 5: Fertility Mediators: Qualtrics Sample

	Dependent variable: Profile rating (Male subjects)				
	Base	Marriage	Want kids	Current kids	Knowledge
	(1)	(2)	(3)	(4)	(5)
Age	-0.043***	-0.028**	-0.033***	0.002	-0.007
	(0.006)	(0.011)	(0.009)	(0.019)	(0.010)
Income ($0,000s)	0.032***	0.032***	0.032***	0.032***	0.032***
	(0.007)	(0.009)	(0.009)	(0.009)	(0.010)
Want marr × age		-0.032*			
		(0.019)			
Want kids × age			-0.055*		
			(0.032)		
No kids × age				-0.055**	
				(0.021)	
Knowledge × age					-0.057***
					(0.017)
Observations	8080	8080	8080	8080	7800
R-Squared	0.490	0.490	0.491	0.491	0.488

Notes: Regression of profile rating on randomly assigned age and income for men-rating-women, from the second sample collected via Qualtrics. Column 2 interacts profile age with whether the rater wants to get married. Column 3 interacts age with whether the rater wants kids. Column 4 interacts age with whether the rater currently has no children currently. Column 5 interacts age with whether the rater is aware that fertility declines before for women before age 45. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses, clustered at the rater level in columns 2 – 5
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

These results suggest that at least some of the observed preference for younger partners stems from preferences for fertility. If the negative coefficient on age instead captured a latent preference for attractiveness or other youthful qualities, whether or not the man wants to have children or knows about the age-fertility relationship should have no bearing on the strength of his preference.

[34] Appendix C.3 further exploits the individual beliefs about when female fertility starts to decline to look for non-linearity in preferences over age as it relates to fertility. If the preference for age is really a preference for fertility, not all years should be the same: years closer to the fertility decline should affect dating market appeal much more than additional years very far from the fertility decline, or after the fertility decline, when there will be little marginal change to fertility. Table A6 shows that preferences indeed take this shape: additional years close to a rater's perceived fertility cutoff have a much greater impact on rating than age changes more than 10 years before the perceived cutoff or after the cutoff.

Moreover, these findings are consistent with a model of agents with heterogenous preferences for fertility rationally maximizing utility: those with stronger preferences for children and more knowledge to act on these preferences penalize older partners more. Thus, instinctive forces connecting age to beauty are not all that are at play, and policies that impact older-age fertility may very well change the costs to aging on the dating market.

5 Simulation and Discussion

From Hollywood films to online dating sites, it seems evident that men prefer younger women, yet the *cost* of this preference to women has not been modeled or measured. If, indeed, younger partners are preferred by men, then time-consuming career investments carry an additional price to women. By bringing together theory, national data, and micro experimental evidence, this paper demonstrate the dollars-and-cents cost of depreciating reproductive capital to women's economic well-being, and shows how understanding this force sheds light on both historical patterns and current gender inequity. This section uses the model to simulate historical transitions, and then considers the reproductive capital hypothesis alongside alternative explanations for the patterns in historical data.

Figure 12 simulates the model in the presence of growing returns to women's education and falling fertility costs. Market opportunities for women have risen dramatically in the past 50 years (Hsieh et al., 2013). Meanwhile, average family size has fallen, with a rapid transition from "four or more" as the modal answer for ideal family size to "two" between 1965 and 1975 (Livingston, Cohn and Taylor, 2010).[35] If couples wish to have four children, graduate education may significantly interfere with the probability of reaching this desired family size. With a desire for fewer children, longer delays are possible with less of an impact on reproductive success.

The first row of images in Figure 12 show that at first, no woman is willing to risk the marriage market costs of investing, so human capital accumulation by women is limited, and matching is assortative. As λ, the gain from investing, slowly increases while the fertility cost falls (via increasing the success of post-investment conception, p), the education and marriage market transforms. The first women to invest, shown by dark blue dots, are penalized through worse marriage matches, creating the non-monotonic equilibrium exhibited in the early Census data. Over time, as labor

[35] Shown in Appendix Figure A3.This change was most likely brought on from the substitution from child quantity to child quality as overall wealth increases, and the rise of women in the workforce, increasing the opportunity cost of childbearing, but is treated as exogenous here.

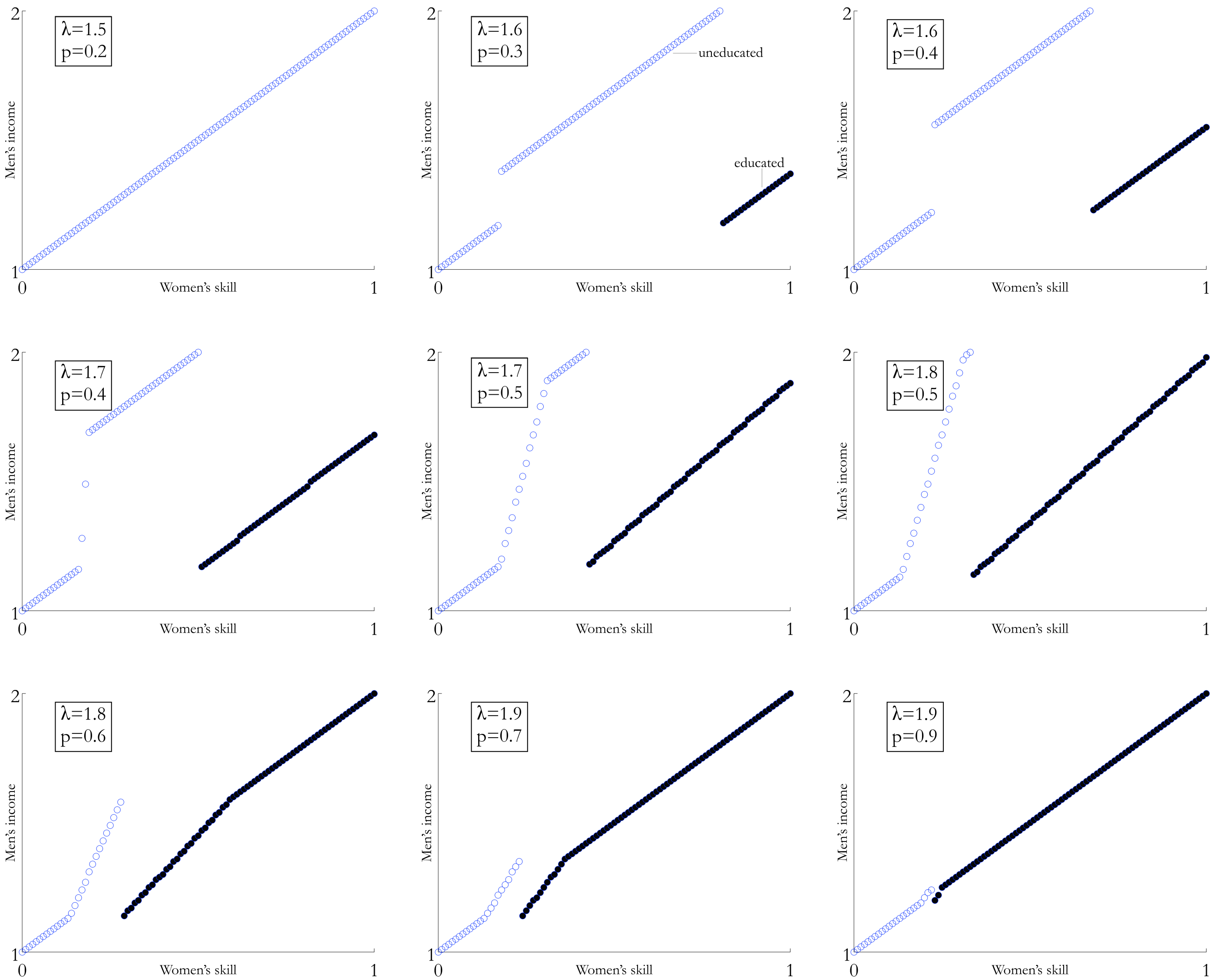
market returns to investment rise and the fertility cost falls, the marriage matches of these women gradually improve, as seen in the second row of images. This, in turn, creates a feedback loop, with more women being willing to invest (which also matches the dramatic rise in US women pursuing higher education). Finally, the market becomes essentially assortative, with some "randomization" by the highest earning men: some marry the very richest women, while others still choose the best among the women who have not invested.

This simulation demonstrates that, in addition to producing the non-monotonic matching present in the data, the model also predicts the realignment that eliminated the marriage market penalty for graduate educated women. The rapid transition in spousal income for graduate women shown in Figure 2 is consistent with the timing of the shift in desired family size. Thus, by framing fertility as an economic asset, and evaluating the tradeoffs its depreciation creates for women, this paper explains previously unexplored historical patterns in women's marriage outcomes and human capital investments. Policies that further eliminate the reproductive cost of career investments, or increase returns via career opportunities, could bring about the possible future depicted in the third row of images: heavy human capital investments by women, and a complete erasure of the associated marriage market penalty.

The rise in spousal income, rise in marriage rates, and fall in divorce rates for highly educated women are consistent with a body of literature noting an increase in marriage success for educated women as well as an increase in assortativeness in mating. However, I distinguish between women with bachelor's degrees and women with post-bachelor's education, which makes clear that these trends have been driven by the highly educated women. Thus, existing explanations for the overall increasing marital assortativeness fail to explain the patterns documented here.

The increase in the skill premium noted as a driver of increasing assortativeness in Fernandez, Guner and Knowles (2005) would be expected to increase marital outcomes, and marriage rates, for college educated women as well as graduate-educated women. Stevenson and Wolfers (2007) note that consumption complementarities are likely to overtake the traditional Beckerian division of labor household model (Becker, 1973, 1974, 1981) as women enter the labor force and household work becomes more easily substituted by technology. The "Engines of Liberation" (Greenwood, Seshadri and Yorukoglu, 2005) such as dishwashers (or infant formula, as in Albanesi and Olivetti (2016)) have also been noted as a possible driver of increased assortativeness by Greenwood et al. (2016), in addition to the skill premium. Relatedly, Chiappori, Salanié and Weiss (2017) present a model where a transition from household production focused on chores to one focused on child

FIGURE 12: FULL TWO-STAGE OPTIMIZATION SIMULATION



Notes: Figure depicts the results of a simulation of the investment and matching equilibrium as the value of the return on investment, λ, and post-investment chance of fertility, p, increases. Women's skill is shown on the x-axis and men's income on the y-axis, with dots depicting marriage matches. At first, the returns are low enough—and the potential marriage market cost high enough—that no women invest (and thus matching is assortative). As λ and p rise, some women invest (shown by dark blue dots, but these top-skilled women are penalized on the marriage market, and matching is non-monotonic. As λ and p continue to grow, matching becomes more assortative. Simulation shown for Y=2, S=1, P=1, and c=0.2.

human capital can increase the marriage market returns to women's education. However, these explanations all predict a transition from negative assortative matching to positive assortative matching. No existing model matches the non-monotonicity in the data. And, starting from such a pattern, technological change would have to be extremely non-linear in its impacts to match the particular divergence of graduate educated women's outcomes from college-educated.

Another possible explanation for these patterns is that the selection of women into post-bachelor's education has changed in a way that could align with the observed matching patterns. For example, if women previously selected into post-bachelor's education after receiving a signal that they had a low chance of success on the marriage market, whereas in later years women have sought further education due to having higher marginal career returns.[36] Two facts appear to counter this story, however. First, the same selection forces may have also applied to college-educated women in earlier cohorts, since college education was still somewhat rare at that time (in the 1960 Census, 6.5% of women had a college-or-greater degree). Second, as shown in Figure 13, the spousal income gap does not respond to the percentage of women earning graduate degrees. Between the 1970 and 1980 Census, the number of women who achieved post-bachelor's education approximately doubled (thus drastically affecting selection effects, if they were present), while the "penalty" in spousal income compared to college education remained unchanged. From 1990 to 2000 the increase as a percent of the base is much slower, and thus the "pool" of women with graduate degrees cannot have changed substantially, whereas the spousal income gap showed a rapid reversal.[37] Rather, Figure 13 is more consistent with the predictions of the model, where as the reproductive costs of investment fall and career returns rise, women first become more willing to pursue graduate education despite the marriage market costs, and then the marriage gap closes, reinforcing this trend.

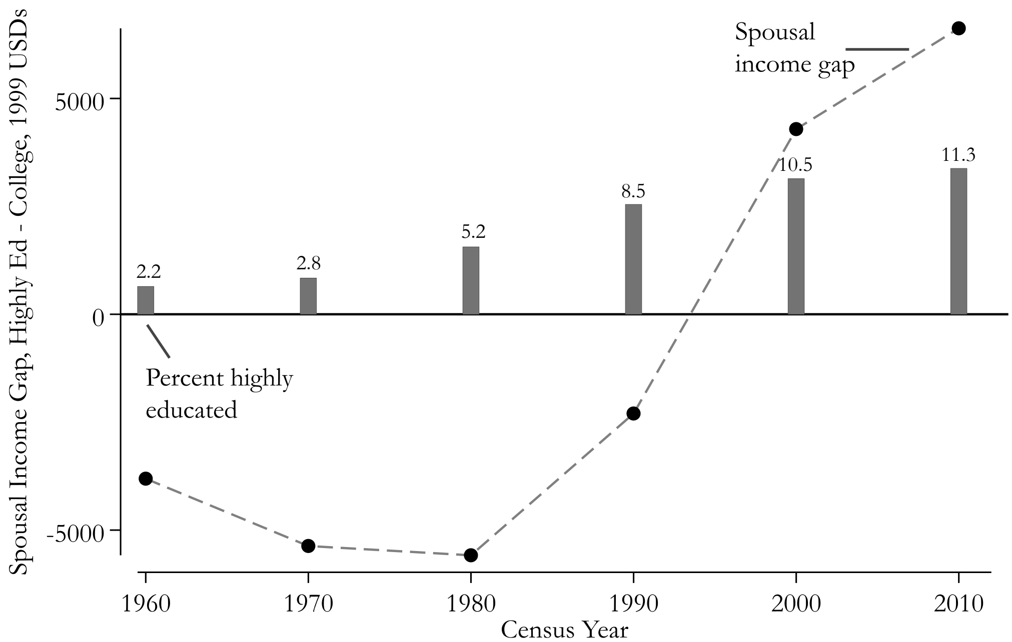
These patterns are also unlikely to be driven by high-earning women having different tastes for partners. Matching could potentially be non-assortative if high-earning women prefer lower-earning partners either because of income effects or a preference for partners who are more likely to be able to spend time at home (although, such preferences in traditional models would tend to predict negative assortative matching, since partner income is complementary, rather than non-monotonic matching). However, both of these forces would strengthen, rather than weaken, as

[36]It also could be that highly educated women are unobservably better along some dimension than college educated women, especially those highly educated women who managed to pursue such education at a time when it was rare for women, making the result of college educated women matching with "better" men at some point all the more striking.

[37]I perform an additional check using data from the National Longitudinal Surveys (NLS) to explicitly examine whether there has been an increasing skill premium among women who attain post-bachelor's education. If women were previously selecting into post-bachelor's education due to negative selection in other areas, they may be expected to be less positively selected on intelligence and academic potential. Appendix table A1 examines this, using data from aptitude scores and educational attainment of three NLS cohorts—the 1968 Young Women panel, the 1979 Youth panel, and the 1997 Youth panel. (Unfortunately, the earliest cohort with the necessary test score information only captures the tail end of the non-assortative matching group, but nonetheless, the trend should be informative about whether there are large shifts occurring in underlying selection factors.) The data shows that highly educated women indeed had substantially higher aptitude scores on average than college educated women even in the earliest cohort, and that the two numbers are not systematically diverging (which would indicate greater skill-driven selection).

female earning power at the top grows, failing to predict the reversal in marriage market outcomes for the "top" women in recent years.[38] Bertrand et al. (2016) offers gender norms against career women as a possible explanation, suggesting these norms may have dissipated in recent years. My model demonstrates, though, that such a norm shift could at least partly be driven by economic fundamentals.

Figure 13: Rates of women's graduate education versus the spousal income gap



Notes: "Highly educated" constitutes all formal education beyond a college degree. "Spousal income gap" is defined as the average spousal income for highly educated women minus the average spousal income for college educated women. Source: 1 percent Census data from 1970, 1980, and 1990. Sample consists of white women, ages 41-50 years old.

Table 6 examines the differences between women with college and graduate degrees in 1980, when there was a "penalty" to being highly educated, and 2010, when there was a premium. The shift in spousal matching might be expected if graduate education previously offered little earnings benefit. However, table 6 shows that even in the 1980 Census, highly educated women earned substantially more than college educated women, with annual income almost 50% higher.[39] This

[38]Moreover, in appendix C.2, I use data from the dating experiment to test for whether male income is less important for high-income women in evaluating potential partners, and find that high-income women actually care *more* about income.

[39]Women's own income in all years, with no crossing, is shown in Appendix Figure A2. This means that the results are also unlikely to be driven by measurement error in classifying women as highly educated, as noted by Kominski

Table 6: Income, age at marriage, and children by education

	1980			2010		
	College Ed.	Highly Ed.	Difference	College Ed.	Highly Ed.	Difference
Income	$18,462	$28,653	$10,190***	$32,326	$48,030	$15,703***
AFM	23.01	23.75	0.73***	26.28	27.23	0.95***
Children	1.98	1.46	-0.52***	1.64	1.52	-0.12***

Notes: "Highly Educated" constitutes all formal education beyond a college degree. Income in 1999 USD. Children measured as children in household (this may be downward biased by older children leaving the household, but this bias will be stronger for college educated women, who have children younger. Children ever born is available for 1980 only, and shows the same pattern, with a difference of 0.50 between college and highly educated). Source: 1 percent samples of 1980 US Census and 2010 American Community Survey. Sample consists of white women, age 36-45, with children in household measured for age 36-40 (to avoid children leaving) and age at first marriage measured for ages 41-45, to ensure most marriages are complete.
*** p<0.01, ** p<0.05, * p<0.1

table also shows the impact of graduate education on delaying marriage and decreasing the number of children. In the 1980 as well as 2010 data, women with graduate degrees married almost a year later than women with college degrees on average, meaning some women may have delayed substantially.

This data further suggests that falling family sizes may indeed help explain the transition to more assortative matching. In 1980, women with college degrees had households with about one half fewer children than women with graduate degrees. This difference dissipated substantially in 2010, but driven entirely by falling family sizes among the college educated, while graduate educated women's family sizes stayed largely constant.[40] This is consistent with the idea that while graduate educated women continued to delay marriage, falling family size desires meant this delay was less costly in terms of realized fertility.

Moreover, the income-age tradeoff between college and graduate educated women aligns well with the penalty to aging found in the experiment. Today, women with graduate degrees marry richer spouses than women with college degrees, but they also earn $16,000 more while only marrying on average 1 year later, as shown in Table 6. The experiment suggests that each year older requires an additional $7,000 of annual income, so graduate educated women's extra earning more than makes up for their later marriage.

Despite this elimination of the penalty to graduate education, the experiment highlights that the *age* penalty still exists, it is just that women with sufficiently high incomes can compensate partners for their higher age. As previously mentioned, falling family sizes have likely reduced the magnitude of this penalty in the last two decades. But this means that while women may be able

and Siegel (1993), since the income gap between "highly educated" and college educated has remained relatively constant over time.

[40]Consistent with the finding of Hazan and Zoabi (2015) that highly educated women have had larger families recently.

to delay starting families until after their education, they could still anticipate penalties if they continue to delay into their late thirties, when even small family size ambitions might be impacted by declining fecundity. Thus, reproductive capital might help explain the lack of women in top executive positions (women make up just 4% of Fortune 500 CEOs), or certain fields with rapid human capital depreciation (tech) or heavy on-the-job training requirements (surgery). Future research may want to examine investments of different lengths, with accordingly differing career payoffs, rather than the binary investment modeled here.

The reproductive capital framework may also provide useful insights to firms interested in attracting and retaining top female talent. Optimal contracts for women may be very different from those that have evolved in a historically male-dominated workforce. For example, firms may wish to adjust women's compensation nonlinearly to reflect the ever-increasing opportunity cost of career investment as reproductive capital depreciates. Creating greater flexibility to allow women to marry and start families while still making career investments, or easing re-entry into the workforce once they have already had children, would also lower the reproductive cost of "on the job" investments. Policy-makers could utilize a better understanding of reproductive capital to inform efforts to promote women's human capital accumulation, such as parental leave policies and workforce re-entry programs, and calculate the welfare effects of such policies. Moreover, government policies that ease access to infertility treatments may have spillover impacts on human capital decisions (see Gershoni and Low (2017) for example). When viewed through this framework, insurance coverage of infertility treatments becomes a question of not just health policy, but also labor and economic policy.

Moreover, the concept of reproductive capital suggests substantial welfare effects of aging for women, or to the premature loss of fertility (as may be experienced by women in developing countries facing illness or childbirth-related infertility). The lower are the returns to female skill, e.g., due to labor market discrimination, the more such losses of reproductive capital will limit a woman's overall well-being; the higher the reproductive costs of career investment, the more high human capital women will be penalized on the marriage market. Thus, evaluating women's labor market opportunities together with the reproductive costs of capitalizing on such opportunities provides a more complete picture of women's economic well-being.

6 Conclusion

This paper treats women's decisions as a tradeoff between two assets: human capital, which grows based on investment, and reproductive capital, which depreciates with time. The theoretical and empirical work presented here indicates that reproductive capital may be a crucial factor in understanding women's human capital investments. Without acknowledgement of the very real economic costs to women of delaying marriage, differing labor market outcomes for women can easily be dismissed as resulting from differing preferences or abilities. The model presented here demonstrates that any agent endowed with "depreciating reproductive capital" would make more limited human capital investments. This paper additionally shows that reproductive capital is a parsimonious explanation for recent historical trends in marriage and education.

I document that later age at first marriage is linked to lower income spouses for women, but not men. Moreover, in recent US history, the returns to education on the marriage market were non-monotonic: women with college degrees matched with richer spouses than those without, but women with even more education, graduate degrees, matched with poorer spouses than those with college degrees. Multiple authors have noted an improvement in marriage outcomes for educated women, but I show that these changes have been driven by the *most* educated only. College-educated women always enjoyed high marriage market outcomes, whereas graduate educated women previously married less and divorced more, in addition to matching with lower-income spouses. These facts indicate age affects women's marriage market success, and thus there are multiple dimensions to education on the marriage market: on the one hand, it increases human capital, on the other hand, it requires time, which reduces reproductive capital.

To explore the consequences of this human capital - reproductive capital tradeoff, I develop a bi-dimensional marriage matching model where women first choose whether to invest in their career, at the cost of fertility. Matching is predicted to be non-monotonic when the fertility cost of career investments are large relative to the income gains. In this case, since skilled women are most likely to make career investments, they are passed over as spouses by the highest earning men, who prefer poorer, more fertile women. This adds a second cost to women considering time-consuming career investments—not only do they themselves potentially lose out on fertility, but they experience a "tax" on the marriage market as well. This equilibrium effect means that even if a woman herself wants no children, her human capital investments will still carry additional costs versus men's. In other words, the inequality is not just a matter of differing preferences. This fact is essential to understand why women may make time-consuming career investments at lower rates than men, and also which policies are likely to support greater investments by women.

Evidence for the model's driving mechanism, men optimizing over partners' expected fertilities, is shown through a novel online dating experiment. The experiment aims to separately identify the causal impact of age from other factors, such as beauty, by randomly assigning age to dating profiles, and using a new incentive mechanism to elicit truthful assessments from male raters. I show that men, but not women, have preferences over partner age, particularly when they have no children currently and are aware of the age-fertility tradeoff.

This paper shows that incorporating reproductive capital into models of women's decisions may be crucial to understanding not just historical marriage patterns, but also women's current human capital investment decisions. This fundamental reproductive capital - human capital tradeoff shows the unique costs to women of large human capital investments within the framework of a rational economic model. Crucially, this takes the form of a financial penalty—even if women themselves do not desire children, they will experience a material loss from lower fertility via the marriage market. This casts a new light on policies that alleviate the work-family tradeoff or extend reproductive time horizons.

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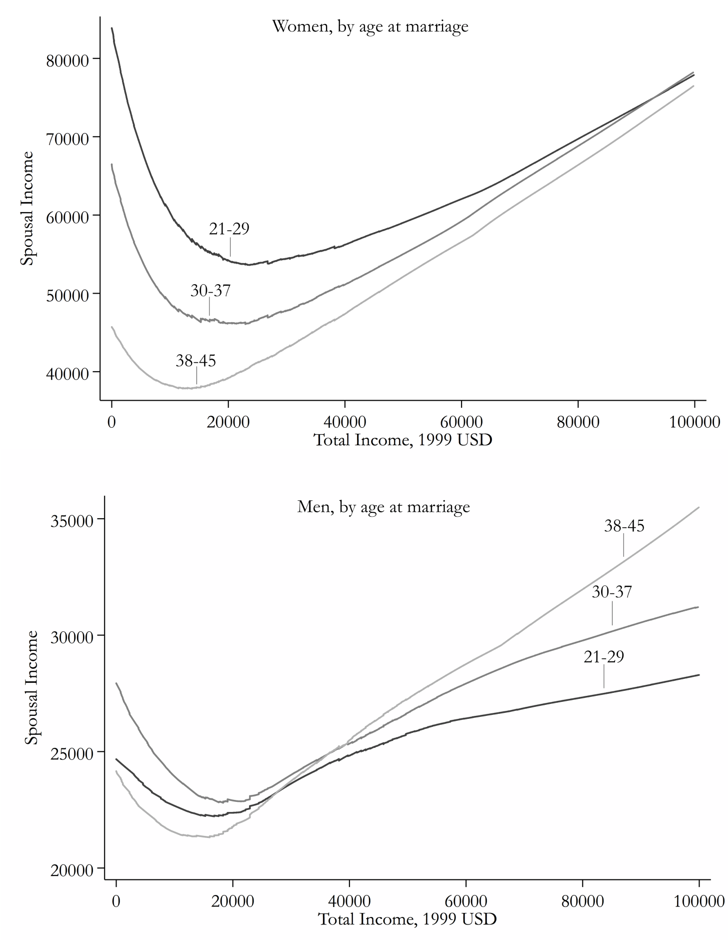
For Online Publication

A Appendix: Census Data

A.1 Establishing stylized facts

Figure A1 shows that conditional on income, marrying older is always worse for women, but not for men. For women, no matter their own income, women who marry at an older age have a lower-earning spouse. For men, on the other hand, marrying at an older age is linked to a higher-earning spouse when they themselves are high-earning.

FIGURE A1: LOWESS-SMOOTHED SPOUSAL INCOM BY AGE AT MARRIAGE



Notes: Figure graphs own income versus spousal income for three different age groups. Women who marry older marry poorer men no matter their own income, whereas for wealthy men, those marrying older are matched with higher-earning spouses.

A.2 Changes over time

Table A1 examines whether there has been an increasing skill premium among women who attain post-bachelor's education, using data from aptitude scores and educational attainment of three National Longitudinal Surveys NLS cohorts—the 1968 Young Women panel, the 1979 Youth panel, and the 1997 Youth panel. (Unfortunately, the earliest cohort with the necessary test score information only captures the tail end of the non-assortative matching group, but nonetheless, the trend should be informative about whether there are large shifts occurring in underlying selection factors.) If women were previously selecting into post-bachelor's education due to negative selection in other areas, they may be expected to be less positively selected on intelligence and academic potential. The data shows, to the contrary, that highly educated women indeed had substantially higher aptitude scores on average than college educated women even in the earliest cohort, and that the two numbers are not systematically diverging (which would indicate greater skill-driven selection).

Table A1: Relative college and post-bachelor's average test score percentiles of three NLS cohorts

	NLS Young Women 1944-1954 birth cohort	NLS Youth '79 1957-1964 birth cohort	NLS Youth '97 1980-1984 birth cohort
College graduate	66.5	70.3	63.6
Highly educated	72.0	74.9	69.3

Notes: Numbers represent percentiles for test scores by education group, compared to other women of all education levels with test score information available, in three different National Longitudinal Study cohorts. Young Women test score data is from the SAT converted into an IQ measurement. 1979 and 1997 data is from the Armed Forces Qualification Test. Gap in scores between college and graduate-educated women is large and relatively stable. The difference in percentile at both educational levels between the different years may be attributable to score data being available for a different selection of individuals in different survey rounds (e.g., for the Young Women sample, it was only available for individuals who reached the later years of high school).

Table A2 demonstrates the change in spousal income based on educational status over time in a regression format.

Figure A2 shows that women who were highly educated always made higher wages than women who were only college educated, and thus that own income does not show a similar "crossing" as does husband's income.

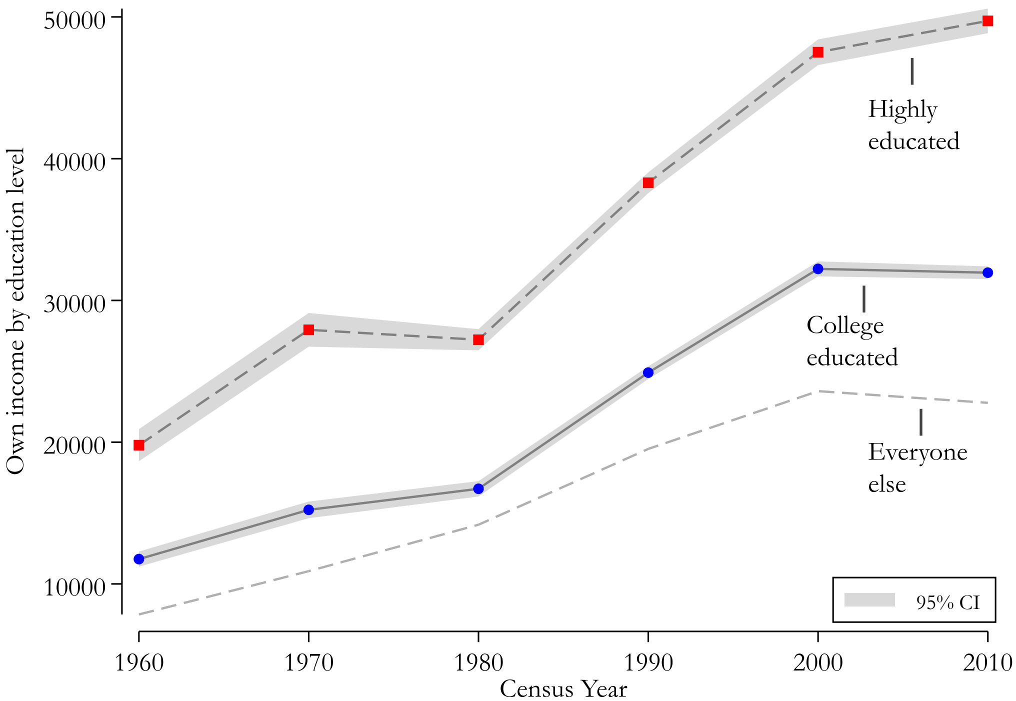
Table A2: Spousal income by wife's education level

	Dependent variable: Spousal income, 1999 USD		
	(1)	(2)	(3)
1960 × highly ed	-3,809	-3,809	-3,833
	(2,355)	(2,355)	(2,355)
1970 × highly ed	-5,368***	-5,368***	-5,386***
	(1,926)	(1,926)	(1,926)
1980 × highly ed	-5,584***	-5,584***	-5,580***
	(1,554)	(1,554)	(1,554)
1990 × highly ed	-2,300**	-2,300**	-2,300**
	(1,055)	(1,055)	(1,055)
2000 × highly ed	4,290***	4,290***	4,268***
	(828.7)	(828.7)	(829.1)
2010 × highly ed	6,625***	6,625***	6,623***
	(758.9)	(758.9)	(758.9)
Year FEs	Y	Y	Y
YOB FEs		Y	Y
Spouse age			Y
Observations	115,223	115,223	115,223
R-squared	0.540	0.540	0.540

Notes: Regressions of spousal income on wife's education level interacted with year for women with at least a college degree, with "highly educated" constituting all formal education beyond a college degree. No constant or "highly" dummy is included, so coefficients can be interpreted as the additional spousal income for those in the highly educated category in each sample. Source: 1 percent samples of US Census data from 1960, 1970, 1980, 1990 Census, and 2000 and 2010 American Community Survey. Sample consists of white women, ages 41-50. Standard errors in parentheses.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

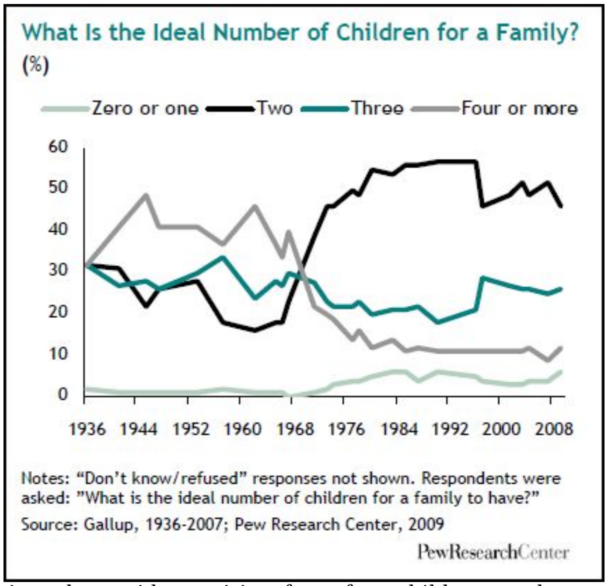
Figure A2: Own income by education level



Notes: Figure shows own income for women by education level, with "highly educated" constituting all formal education beyond a college degree. 1 percent samples of US Census data from 1960, 1970, 1980, 1990 Census, and 2000 and 2010 American Community Survey. Sample consists of white women, ages 41-50. Income in 1999 USDs.

A.3 Change in desired family size

Figure A3: Desired family size transition



Notes: Figure depicts the rapid transition from four children as the modal desired family size to two children, as evidenced by Gallup polls of men and women. As published in: Pew Center, The New Demography of American Motherhood, August 2010

B Appendix: Theory

B.1 Finding the payoff functions

This section shows in detail how to solve for the payoff functions, using the three-segment, non-monotonic match as an example.

We first need to determine what y is matched with what s in any equilibrium. Because matching must be assortative on either side of $\overline{s}$, the matching function is defined as the function that ensures an exactly equal number of women with income less than some level are matched to the number of men with income less than some level. Along the first segment, the man who has income 1 will be matched with the woman who has income 0, and similarly, the man with income $\underline{y}$ will be matched with the woman of skill $\underline{s}$, and we can use the fact that the density of $\underline{s} - 0$ must equal $\underline{y} - 1$ to solve for s. In the uniform case, this yields a linear matching function between s and y. For example, for segment 1:

$$\begin{aligned}\frac{s}{S} &= \frac{y-1}{Y-1}\\ s &= \frac{y-1}{Y-1}S\\ s &= \tfrac{S}{Y-1}(y-1)\end{aligned}$$

For notational simplicity, let's define $\theta \equiv \frac{S}{Y-1}$.

Thus we have the matching function for the first segment:

$$y = \frac{1}{\theta}s + 1$$

$$s = \theta(y-1)$$

And the surplus function:

$$T_1(y,s) = \left(\frac{y+s+1}{2}\right)^2 P + (y+s)(1-P)$$

Through the maximization procedure outlined in the body of the paper, we can then determine the value function. Recall:

$$u_i(y) = \max_s\{T_i(y,s) - v_i(s)\}$$

$$v_i(s) = \max_y\{T_i(y,s) - u_i(y)\}$$

Yielding FOCs:

$$\begin{aligned}v_i'(s) &= \frac{\partial T_i(y,s)}{\partial s}\\ u_i'(y) &= \frac{\partial T_i(y,s)}{\partial y}\end{aligned}$$

This equation can then be integrated, plugging in y as a function of s, and s as a function of y.

Following these steps for segment 1:

$$
\begin{aligned}
u_1\prime\,(y) &= (y+s+1)\,\frac{P}{2} + (1-P) \\
&= (y+\theta(y-1)+1)\,\frac{P}{2} + (1-P) \\
u_1\,(y) &= \int (y+\theta(y-1)+1)\,\frac{P}{2} + (1-P)\,dy \\
u_1\,(y) &= \left(\frac{y^2}{2}\,(1+\theta) + y\,(1-\theta)\right)\frac{P}{2} + y\,(1-P) + C_1
\end{aligned}
$$

$$
\begin{aligned}
v_1\prime\,(s) &= (y+s+1)\,\frac{P}{2} + (1-P) \\
&= \left(\frac{1}{\theta}s+1+s+1\right)\frac{P}{2} + (1-P) \\
v_1\,(s) &= \int \left(\frac{1}{\theta}s+1+s+1\right)\frac{P}{2} + (1-P)\,ds \\
v_1\,(s) &= \left(\frac{s^2}{2}\left(\frac{1}{\theta}+1\right) + 2s\right)\frac{P}{2} + s\,(1-P) + K_1
\end{aligned}
$$

Note that for two matched individuals, $u(y) + v(s) = T(y, s)$. Thus:

$$
\begin{aligned}
&\left(\frac{y^2}{2}\,(1+\theta) + y\,(1-\theta)\right)\frac{P}{2} + y\,(1-P) + C_1 \\
&+ \left(\frac{s^2}{2}\left(\frac{1}{\theta}+1\right) + 2s\right)\frac{P}{2} + s\,(1-P) + K_1 \\
&= \left(\frac{y+s+1}{2}\right)^2 P + (y+s)\,(1-P)
\end{aligned}
$$

Plugging in for y:

$$
\begin{aligned}
u_1(\frac{1}{\theta}s+1) + v_1(s) &= T_1(\frac{1}{\theta}s+1, s) \\
\Rightarrow C_1 + K_1 &= \frac{1}{4}P\,(\theta+1)
\end{aligned}
$$

For segment two, the matching function is:

$$y = \overline{y} + \frac{1}{\theta}(s - \underline{s})$$
$$s = \underline{s} + \theta(y - \overline{y})$$

Following the same maximization and integration procedure, plugging in for y as a function of s, and s as a function of y, and noting that $T_2(y,s) = T_1(y,s)$, we find:

$$\begin{aligned}
u_2\prime\,(y) &= (y+s+1)\,\frac{P}{2} + (1-P)\\
&= (y+\underline{s}+\theta(y-\overline{y})+1)\,\frac{P}{2} + (1-P)\\
u_2\,(y) &= \int (y+\underline{s}+\theta(y-\overline{y})+1)\,\frac{P}{2} + (1-P)\,dy\\
u_2\,(y) &= \left(\frac{y^2}{2}\,(1+\theta) + y\,(1+\underline{s}-\theta\overline{y})\right)\frac{P}{2} + y\,(1-P) + C_2
\end{aligned}$$

$$\begin{aligned}
v_2\prime\,(s) &= (y+s+1)\,\frac{P}{2} + (1-P)\\
&= \left(\overline{y} + \frac{1}{\theta}(s-\underline{s}) + s + 1\right)\frac{P}{2} + (1-P)\\
v_2\,(s) &= \int \left(\overline{y} + \frac{1}{\theta}(s-\underline{s}) + s + 1\right)\frac{P}{2} + (1-P)\,ds\\
v_2\,(s) &= \left(\frac{s^2}{2}\left(\frac{1}{\theta}+1\right) + s(1+\overline{y}-\frac{1}{\theta}\underline{s})\right)\frac{P}{2} + s\,(1-P) + K_2
\end{aligned}$$

Again we have the restriction that $u(y) + v(s) = T(y,s)$, yielding:

$$\begin{aligned}
&\left(\frac{y^2}{2}\,(1+\theta) + y\,(1+\underline{s}-\theta\overline{y})\right)\frac{P}{2} + y\,(1-P) + C_2\\
+&\left(\frac{s^2}{2}\left(\frac{1}{\theta}+1\right) + s(1+\overline{y}-\frac{1}{\theta}\underline{s})\right)\frac{P}{2} + s\,(1-P) + K_2\\
=&\left(\frac{y+s+1}{2}\right)^2 P + (y+s)\,(1-P)
\end{aligned}$$

Plugging in for y:

$$
\begin{aligned}
&u_2(\overline{y} + \frac{1}{\theta}(s - \underline{s})) + v_2(s) = T_2(\overline{y} + \frac{1}{\theta}(s - \underline{s}), s) \\
\Rightarrow &C_2 + K_2 = \frac{1}{4\theta}\left(P\underline{s}^2 - 2P\underline{s}\overline{y}\theta + P\overline{y}^2\theta^2 + P\theta\right)
\end{aligned}
$$

Plug in for $\overline{y}$:

$$
C_2 + K_2 = \frac{1}{4\theta}\left(PY^2\theta^2 - 2PY\overline{s}\theta + P\overline{s}^2 + P\theta\right)
$$

In the final segment, the matching function is:

$$
\begin{aligned}
y &= \underline{y} + \frac{1}{\theta}(s - \overline{s}) \\
s &= \overline{s} + \theta(y - \underline{y})
\end{aligned}
$$

In this case, the joint product, T_3, has a different form:

$$
T_3(y, s) = \left(\frac{y + \lambda s + 1}{2}\right)^2 p + (y + \lambda s)(1 - p)
$$

Again maximizing and integrating gives:

$$\begin{aligned}
u_3\prime(y) &= (y+\lambda s+1)\,\frac{p}{2}+(1-p)\\
&= \left(y+\lambda(\overline{s}+\theta(y-\underline{y}))+1\right)\frac{p}{2}+(1-p)\\
u_3(y) &= \int \left(y+\lambda(\overline{s}+\theta(y-\underline{y}))+1\right)\frac{p}{2}+(1-p)\,dy\\
u_3(y) &= \left(\frac{y^2}{2}(1+\lambda\theta)+y\left(1+\lambda(\overline{s}-\theta\underline{y})\right)\right)\frac{p}{2}+y\,(1-p)+C_3
\end{aligned}$$

$$\begin{aligned}
v_3\prime(s) &= (y+\lambda s+1)\,\frac{\lambda p}{2}+\lambda\,(1-p)\\
&= \left(\underline{y}+\frac{1}{\theta}(s-\overline{s})+\lambda s+1\right)\frac{\lambda p}{2}+\lambda\,(1-p)\\
v_3(s) &= \int \left(\underline{y}+\frac{1}{\theta}(s-\overline{s})+\lambda s+1\right)\frac{\lambda p}{2}+\lambda\,(1-p)\,ds\\
v_3(s) &= \left(\frac{s^2}{2}\left(\frac{1}{\theta}+\lambda\right)+s(1+(\underline{y}-\frac{1}{\theta}\overline{s}))\right)\frac{\lambda p}{2}+s\lambda\,(1-p)+K_3
\end{aligned}$$

The restriction that $u(y)+v(s)=T(y,s)$ gives:

$$\begin{aligned}
&\left(\frac{y^2}{2}(1+\lambda\theta)+y\left(1+\lambda(\overline{s}-\theta\underline{y})\right)\right)\frac{p}{2}+y\,(1-p)+C_3\\
+&\left(\frac{s^2}{2}\left(\frac{1}{\theta}+\lambda\right)+s(1+(\underline{y}-\frac{1}{\theta}\overline{s})\right)\frac{\lambda p}{2}+s\lambda\,(1-p)+K_3\\
=&\left(\frac{y+\lambda s+1}{2}\right)^2 p+(y+\lambda s)\,(1-p)
\end{aligned}$$

Plugging in for y:

$$\begin{aligned}
u_3(\underline{y}+\frac{1}{\theta}(s-\overline{s}))+v_3(s) &= T_3(\underline{y}+\frac{1}{\theta}(s-\overline{s}),s)\\
\Rightarrow C_3+K_3 &= \frac{1}{4\theta}\left(p\lambda\overline{s}^2-2p\lambda\overline{s}\underline{y}\theta+p\lambda\underline{y}^2\theta^2+p\theta\right)
\end{aligned}$$

Plugging in for $\underline{y}$:

$$C_3+K_3=\frac{p}{4\theta}\left(\lambda\underline{s}^2-2\lambda\underline{s}\overline{s}+2\lambda\underline{s}\theta+\lambda\overline{s}^2-2\lambda\overline{s}\theta+\lambda\theta^2+\theta\right)$$

The final payoff functions are:

$u_1(y) = \left(\frac{y^2}{2}\left(1+\frac{S}{Y-1}\right)+y\left(1-\frac{S}{Y-1}\right)\right)\frac{P}{2}+y(1-P)+C_1$

$v_1(s) = \left(\frac{s^2}{2}\left(\frac{Y-1}{S}+1\right)+2s\right)\frac{P}{2}+s(1-P)+K_1$

$u_2(y) = \left(\frac{y^2}{2}\left(1+\frac{S}{Y-1}\right)+y\left(1+\underline{s}-\frac{S}{Y-1}\overline{y}\right)\right)\frac{P}{2}+y(1-P)+C_2$

$v_2(s) = \left(\frac{s^2}{2}\left(\frac{Y-1}{S}+1\right)+s(1+\overline{y}-\frac{Y-1}{S}\underline{s})\right)\frac{P}{2}+s(1-P)+K_2$

$u_3(y) = \left(\frac{y^2}{2}\left(1+\lambda\frac{S}{Y-1}\right)+y\left(1+\lambda(\overline{s}-\frac{S}{Y-1}\underline{y})\right)\right)\frac{p}{2}+y(1-p)+C_3$

$v_3(s) = \left(\frac{s^2}{2}\left(\frac{Y-1}{S}+\lambda\right)+s(1+(\underline{y}-\frac{Y-1}{S}\overline{s}))\right)\frac{\lambda p}{2}+s\lambda(1-p)+K_3$

The constants can then be solved for using the constraints that, in order for the match to be stable, two men with the same income cannot receive different utilities. Thus, the men at all "break points," between two segments, must be indifferent. Additionally, a woman of the same income level must always receive a unique payoff. (For now, we do not restrict that all women of the same skill level must receive the same payoff, since the educational decision was undertaken before entering the marriage market, and cannot be changed).

In particular, $v_1(\underline{s}) = v_2(\underline{s})$ yields a relationship between K_1 and K_2. But, given $K_1 = 0$, this allows us to solve for $K_2 = \frac{1}{2\theta}(P\underline{s}\theta + P\underline{s}\overline{s} - PY\underline{s}\theta)$.

From segment 2, we have $C_2+K_2 = \frac{1}{4\theta}\left(PY^2\theta^2 - 2PY\overline{s}\theta + P\overline{s}^2 + P\theta\right)$, which allows us to solve for $C_2 = \frac{1}{4}\frac{P}{\theta}\left(\theta + Y^2\theta^2 - 2\underline{s}\theta - 2\underline{s}\overline{s} + \overline{s}^2 + 2Y\underline{s}\theta - 2Y\overline{s}\theta\right)$.

Then $u_2(\overline{y}) = u_3(\overline{y})$ gives us a relationship between C_2 and C_3, which allows us to solve for $C_3 + \frac{1}{2}p\left(\left(\lambda\left(\overline{s}-\theta\left(\frac{s}{\theta}+1\right)\right)+1\right)\left(Y+\frac{1}{\theta}(\underline{s}-\overline{s})\right)+\frac{1}{2}(\theta\lambda+1)\left(Y+\frac{1}{\theta}(\underline{s}-\overline{s})\right)^2\right) - \left(Y+\frac{1}{\theta}(\underline{s}-\overline{s})\right)(p-1)$.

Then, using the relationship from segment 3 between the two constants, we can solve for $K_3 = \frac{p}{4\theta}\left(\lambda\underline{s}^2 - 2\lambda\underline{s}\overline{s} + 2\lambda\underline{s}\theta + \lambda\overline{s}^2 - 2\lambda\overline{s}\theta + \lambda\theta^2 + \theta\right) - C_3$.

We then can use $u_1(\underline{y}) = u_3(\underline{y})$ to solve for C_1, which gives us two equations for C_1, which can be used to find $\underline{s}$, giving us:

$$\underline{s} = \frac{(P-p)(\overline{s}-\theta(Y-1)) + \theta\lambda(\overline{s}+\theta(Y-1))\,p}{2(P-p)+2P\theta}$$

which is the same as the equation found through the surplus maximization method. Together with the equations $\underline{y} = \frac{1}{\theta}\underline{s}+1$ and $\overline{y} = Y - \frac{1}{\theta}(\overline{s}-\underline{s})$, we now have eliminated the unknowns.

For the negative assortative case, $\underline{s}$ simply equals zero, and the payoff functions are exactly as above for segments 2 and 3. For the positive assortative case, $\underline{s} = \overline{s}$, and the payoff functions are those for segments 1 and 3. For the case with randomization, the payoff functions on segments 1 and 2 are the same, except for the constants and there being an $\underline{s}_1$ and $\underline{s}_2$ instead of a single break, while the payoff function on segment 3 has a different form, and there are some individuals matching on a new "randomization segment" between segments 1 and 2. The payoff function on segment 3 is very similar to the current form, but the matching equation, and thus slope of the payoff function, is weighted by the randomization probability, ξ. On the new randomization segment, the payoff function has a very similar form to that of 1 and 2, with a matching function weighted by $1 - \xi$.

B.2 Detailed proof of non-assortative matching

Proposition 3. *The stable match will be positive assortative on income conditional on fertility. However, for any $\lambda < (\frac{P}{p} - 1)\frac{(Y-1)}{S} + \frac{P}{p}$, the stable match will not be overall positive assortative.*

Proof. As shown in Becker (1973), supermodularity of the surplus function yields positive assortativeness in a unidimensional setting. Thus, for any surplus function that is supermodular in incomes, for two women of the same fertility level, the woman with the higher income must be matched with a higher-income man. Here, since the two incomes enter additively, this is equivalent to convexity in income: $\frac{\partial^2 T}{\partial y^h \partial y^w} = \frac{\pi}{2} > 0$. Thus, for any two women of equal fertility, the woman with the higher income will be matched with the higher income man.

To prove that there will *not* be unconditional assortativeness for all parameter values, by contradiction, assume assortative matching everywhere. Let $y(s)$ represent the income of the man matched with a woman of skill s. With positive assortative matching, the lowest skill individuals are matched together, $y(0) = 1$, and the highest skill individuals are matched together, $y(S) = Y$. In between, the matching is given by $y(s) = 1 + \frac{Y-1}{S}s$. The income of the man matched with woman of skill $\bar{s}$ is $y(\bar{s})$. For assortative matching to be stable, it must maximize the total surplus.

For simplicity, define two forms of the surplus function, depending on whether the wife has invested, representing the husband's income simply by y:

$$T(y^h, y^w(s), \pi(s)) \equiv \begin{cases} T^P(y,s) = P\frac{(y+s+1)^2}{4} + (1-P)(y+s) & \text{for } s < \bar{s} \\ T^p(y,s) = p\frac{(y+\lambda s+1)^2}{4} + (1-p)(y+\lambda s) & \text{for } s > \bar{s}. \end{cases}$$

Then, it must be the case that for a small dy: $T^p(y(\bar{s}) + dy, \bar{s} + \frac{S}{Y-1}dy) + T^P(y(\bar{s}) - dy, \bar{s} - \frac{S}{Y-1}dy) \geq T^p(y(\bar{s}) - dy, \bar{s} + \frac{S}{Y-1}dy) + T^P(y(\bar{s}) + dy, \bar{s} - \frac{S}{Y-1}dy)$. Rearranging, and dividing by dy yeilds: $\frac{T^p(y(\bar{s})+dy,\bar{s}+\frac{S}{Y-1}dy)-T^p(y(\bar{s})-dy,\bar{s}+\frac{S}{Y-1}dy)}{dy} \geq \frac{T^P(y(\bar{s})+dy,\bar{s}-\frac{S}{Y-1}dy)-T^P(y(\bar{s})-dy,\bar{s}-\frac{S}{Y-1}dy)}{dy}$. As dy goes to 0, this implies that the return to husband quality is higher for women just above the threshold than just below: $\frac{\partial}{\partial y}(T^p) \geq \frac{\partial}{\partial y}(T^P)$. Calculating these derivatives and plugging in $y(s)$

yields: $\lambda \geq (\frac{P}{p} - 1)\frac{(Y-1)}{S} + \frac{P}{p}$. Therefore, whenever $\lambda < (\frac{P}{p} - 1)\frac{(Y-1)}{S} + \frac{P}{p}$, there will be some deviation from assortative matching. $\square$

B.3 Detailed proof of stable match

Proposition 4. *The unique stable match exhibits positive assortative matching on each side of the investment threshold, $\overline{s}$. Across the investment threshold:*

- *When $\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p} - 1)\frac{Y-1}{S} < \lambda < (\frac{P}{p} - 1)\frac{Y-1}{S}$, there will be non-monotonic matching. Men from 1 to $\underline{y}$ will match with uneducated women; men from $\underline{y}$ to $\overline{y}$ will match with educated women; and men from $\overline{y}$ to Y will match with uneducated women.*
- *When $\lambda < \frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p} - 1)\frac{Y-1}{S}$, there will be negative-assortative matching.*
- *When $\lambda > (\frac{P}{p} - 1)\frac{(Y-1)}{S} + \frac{P}{p}$, there will be positive assortative matching.*

Proof. There is assortative matching on either side of the threshold because supermodularity of the surplus function is a sufficient condition for assortative matching in unidimensional settings, and, on either side of the threshold, women vary unidimensionally in income (and the surplus function is supermodular in incomes).

To verify that each of the specified match forms are stable, we need to check that no two individuals would rather match with each other than their current pairings, or remain single. We know that no one prefers to remain single, because each marriage generates strictly positive surplus, by the assumption that singles' utility is zero. We know that within segments (see illustrations), no re-matching could yield surplus gains, because otherwise there would not be positive assortative matching. So we need only check that no one could profitably deviate to match with someone from a *different* section.

For non-monotonic matching, this requires six conditions (where the subscripts indicate the section, and $T_1 = T_2 = T(y, s, P)$, and $T_3 = T(y, \lambda s, p)$):

1. $u_1(y) + v_2(s) \geq T_2(y, s)$ for $y \in [1, \underline{y}]$, $s \in [r, \overline{s}]$
2. $u_1(y) + v_3(s) \geq T_3(y, s)$ for $y \in [1, \underline{y}]$, $s \in [\overline{s}, S]$
3. $u_2(y) + v_1(s) \geq T_1(y, s)$ for $y \in [\overline{y}, Y]$, $s \in [0, \underline{s}]$
4. $u_2(y) + v_3(s) \geq T_3(y, s)$ for $y \in [\overline{y}, Y]$, $s \in [\overline{s}, S]$
5. $u_3(y) + v_1(s) \geq T_1(y, s)$ for $y \in [\underline{y}, \overline{y}]$, $s \in [0, \underline{s}]$
6. $u_3(y) + v_2(s) \geq T_2(y, s)$ for $y \in [\underline{y}, \overline{y}]$, $s \in [\underline{s}, \overline{s}]$

Conditions 1 and 3 hold for all values of λ, because if not, there would not be assortative matching conditional on fertility, since fertility does not vary between segments 1 and 2. Conditions 2, 4, 5, and 6 each involve segment 3, where both income and fertility change, and thus only hold for some values of λ, in particular, when $\lambda < (\frac{P}{p}-1)\frac{Y-1}{S}$. I demonstrate the procedure for verifying the stability when $\lambda < (\frac{P}{p}-1)\frac{Y-1}{S}$ in detail for condition 2—the same procedure can be followed for the other conditions.

First, find the s that minimizes $u_1(y) + v_3(s) - T_3(y,s)$: $\frac{d}{ds}(u_1(y) + v_3(s) - T_3(y,s)) = \frac{p}{2}\frac{Y-1}{S}\lambda\left(s - \overline{s} + \underline{y}\frac{S}{Y-1} - y\frac{S}{Y-1}\right) \to \widehat{s} = \overline{s} + \frac{S}{Y-1}(y-\underline{y})$ which is always less than $\overline{s}$, given the range of y for this condition, meaning the minimizer is $\overline{s}$. $(\frac{d^2}{ds^2}(u_1(y) + v_3(s) - T_3(y,s) = \frac{p}{2}\frac{Y-1}{S}\lambda > 0.)$

Next, find the y that minimizes $u_1(y)+v_3(s)-T_3(y,s)$: $\frac{d}{dy}(u_1(y)+v_3(s)-T_3(y,s)) = \frac{1}{2}p - \frac{1}{2}P - \frac{1}{2}P\frac{S}{Y-1} + \frac{1}{2}Py - \frac{1}{2}py + \frac{1}{2}Py\frac{S}{Y-1} - \frac{1}{2}ps\lambda \to \widehat{y} = \frac{1}{P-p+P\frac{S}{Y-1}}\left(P - p + P\frac{S}{Y-1} + ps\lambda\right)$. $(\frac{d^2}{dy^2}(u_1(y) + v_3(s) - T_3(y,s)) = \frac{1}{2}(P-p) + \frac{1}{2}P\frac{S}{Y-1} > 0.)$

At $\widehat{s} = \overline{s}$, $\widehat{y} = \frac{\left(P-p+P\frac{S}{Y-1}+p\overline{s}\lambda\right)}{P-p+P\frac{S}{Y-1}} = \underline{y} + \frac{\left(p-P+p\frac{S}{Y-1}\lambda\right)(\overline{s}-S)}{2\frac{S}{Y-1}\left(P-p+P\frac{S}{Y-1}\right)}$, which is less than $\underline{y}$, and thus interior, when $p - P + p\frac{S}{Y-1}\lambda > 0$ or when $\lambda > (\frac{P}{p}-1)\frac{Y-1}{S}$.

Meaning, when $\lambda < (\frac{P}{p}-1)\frac{Y-1}{S}$, $\underline{y}$ is the minimizer. $u_1(\underline{y}) + v_3(\overline{s}) - T_3(\underline{y},\overline{s}) = 0$, and the match is stable. But when $\lambda > (\frac{P}{p}-1)\frac{Y-1}{S}$, there is an interior minimizer, $\widehat{y}$. $u_1(\widehat{y}) + v_3(\overline{s}) - T_3(\widehat{y},\overline{s}) < 0$, meaning this condition is not met, and the match is unstable.

But, when $\lambda = \frac{Y-1}{S}\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p}-1)$, the cutoff between the uneducated women who match with (poor) men from 1 to $\underline{y}$ and the uneducated women who match with (rich) men from $\overline{y}$ to Y, $r = \frac{(P-p)(\overline{s}-S)+\frac{S}{Y-1}\lambda(\overline{s}+S)p}{2(P-p)+2P\frac{S}{Y-1}}$, is equal to zero, and thus $\underline{y} = 1$, and there are no uneducated women matching with poor men. Rather, the least-skilled educated woman matches with the man with income 1, the poorest man, and thus the three-segment non-monotonic match collapses to "block" negative-assortative matching for any $\lambda < \frac{Y-1}{S}\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p}-1)$.

Thus, for $\frac{Y-1}{S}\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p}-1) < \lambda < (\frac{P}{p}-1)\frac{Y-1}{S}$, the three-segment match is the unique stable match. For $\lambda < \frac{Y-1}{S}\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p}-1)$, the negative-assortative match is the unique stable match (the same conditions verify its stability, as the utilities remain unchanged).

When there is positive-assortative matching, there are only two segments—these segments are identical to segments 1 and 3 of the three segment match, with $\underline{s} = \overline{s}$. We thus need to check conditions:

$u_1(y) + v_3(s) \geq T_3(y,s)$ for $y \in [1,\underline{y}]$, $s \in [\overline{s}, S]$

$u_3(y) + v_1(s) \geq T_1(y,s)$ for $y \in [\underline{y}, Y]$, $s \in [0,\overline{s}]$

Recall:

$T_1(y,s) = \left(\frac{y+s+1}{2}\right)^2 P + (y+s)(1-P)$

$u_1(y) = \left(\frac{y^2}{2}\left(1+\frac{S}{Y-1}\right) + y\left(1-\frac{S}{Y-1}\right)\right)\frac{P}{2} + y(1-P) + C_1$

$v_1(s) = \left(\frac{s^2}{2}\left(\frac{Y-1}{S}+1\right)+2s\right)\frac{P}{2} + s(1-P) + K_1$

$T_3(y,s) = \left(\frac{y+\lambda s+1}{2}\right)^2 p + (y+\lambda s)(1-p)$

$u_3(y) = \left(\frac{y^2}{2}\left(1+\lambda\frac{S}{Y-1}\right) + y\left(1+\lambda(\overline{s}-\frac{S}{Y-1}\underline{y})\right)\right)\frac{p}{2} + y(1-p) + C_3$

$v_3(s) = \left(\frac{s^2}{2}\left(\frac{Y-1}{S}+\lambda\right) + s(1+(\underline{y}-\frac{Y-1}{S}t))\right)\frac{\lambda p}{2} + s\lambda(1-p) + K_3$

For condition 1, $\frac{d}{ds}(u_1(y)+v_3(s)-T_3(y,s))$ $= \frac{1}{2S^2}p\lambda(Y-1)\left(\frac{S^2}{Y-1}+S\overline{s}-S^2\frac{y}{Y-1}-S\frac{s}{Y-1}+S\frac{\overline{s}}{Y-1}+SY\frac{s}{Y-1}-SY\frac{\overline{s}}{Y-1}\right) \to \widehat{s} = \frac{S(y-1)}{Y-1}$, which is always less than $\overline{s}$, given the range of y for this condition, implying $\overline{s}$ is the minimizer ($\frac{d^2}{ds^2}(u_1(y)+v_3(s)-T_3(y,s)) = \frac{1}{2}\frac{p}{\frac{S}{Y-1}}\lambda > 0$).

$\frac{d}{dy}(u_1(y)+v_3(s)-T_3(y,s))$ $= \frac{1}{2(Y-1)}(P-p-PS-PY-Py+Yp+py+ps\lambda+PSy+PYy-Ypy-Yps\lambda)$ $\to \widehat{y} = \frac{1}{-P+p+PS+PY-Yp}(-P+p+PS+PY-Yp-ps\lambda+Yps\lambda)$.

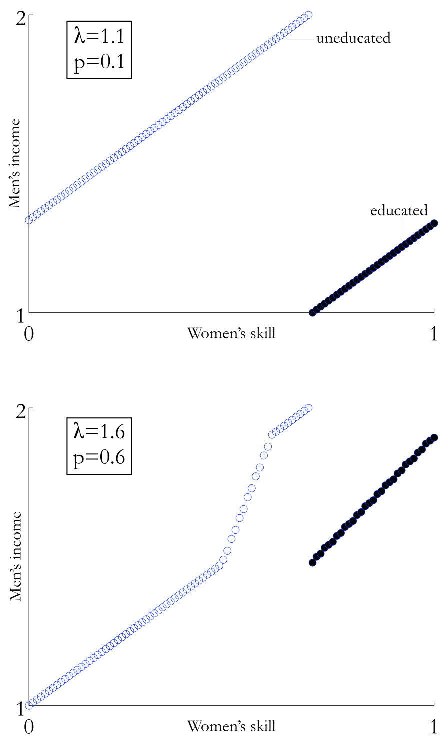
At $\widehat{s}=\overline{s}$, $\widehat{y} = -\frac{1}{S}\frac{Y-1}{p-P+PS+PY-Yp}(p\overline{s}-P\overline{s}+PS\overline{s}+PY\overline{s}-Yp\overline{s}-Sp\overline{s}\lambda)$. Note that $-\frac{1}{S}\frac{Y-1}{p-P+PS+PY-Yp}(p\overline{s}-P\overline{s}+PS\overline{s}+PY\overline{s}-Yp\overline{s}-Sp\overline{s}\lambda)-\underline{y}$ $= -\frac{Y-1}{S}\frac{1}{(P-p)(Y-1)+PS}\overline{s}(P-p)(Y-1)+S\overline{s}(P-p\lambda))$, which is less than 0 when $\overline{s}(P-p)(Y-1)+S\overline{s}(P-p\lambda))>0$, or $\frac{P-p}{p}\frac{Y-1}{S}+\frac{P}{p}>\lambda$. In that case, the minimizer is the interior minimizer $\widehat{y}$ ($\frac{d^2}{ds^2}(u_1(y)+v_3(s)-T_3(y,s)) = \frac{1}{2(Y-1)}((P-p)(Y-1)+PS)>0$).

$u_1(\widehat{y})+v_3(\overline{s})-T_3(\widehat{y},\overline{s})<0$, meaning the condition is not met, and the match is unstable. When $\lambda > \frac{P-p}{p}\frac{Y-1}{S}+\frac{P}{p}$, though, the minimzer is $\underline{y}$, and $u_1(\underline{y})+v_3(\overline{s})-T_3(\underline{y},\overline{s})=0$.

The second condition also holds when $\lambda > \frac{P-p}{p}\frac{Y-1}{S}+\frac{P}{p}$(shown through a similar process). Thus, the positive-assortative match is the unique stable match when $\lambda > \frac{P-p}{p}\frac{Y-1}{S}+\frac{P}{p}$. □

B.4 Illustration of matching equilibria

Figure A4: Matching Equilibria Simulation



B.5 General form of the matching equilibrium

This section formalizes the intuition for the two key characteristics of the surplus function, supermodularity and a marginal rate of substitution between income and fertility that decreases in income, leading to a potentially non-monotonic match.

Assume a population of men, characterized by income $y^h \in (0, Y)$, and a population of women endowed with skill $s \in (0, S)$, characterized by income and fecundity (y^w, π). Due to time-consuming career investments by high-skill women,

$$(y^w, \pi) = \begin{cases} (s, P), & \text{if } s < \overline{s} \\ (\lambda s, p), & \text{if } s \geq \overline{s} \end{cases}$$

For simplicity, assume the populations of men and women are equal, atomless and continuous in y and s, and have outside options such that all prefer to marry. When the surplus function $T(y, \pi)$, where $y = y^w + y^h$, increasing in both arguments, exhibits the following properties:

1. $\frac{\partial^2 T}{\partial y^w \partial y^h} = \frac{\partial^2 T}{\partial y^2} > 0$ (supermodularity in both spouses' income, equivalent here to convexity in income)
2. $\frac{\partial MRS}{\partial y} < 0$ where $MRS \equiv \frac{\frac{\partial T}{\partial y}}{\frac{\partial T}{\partial \pi}}$ (The marginal rate of substitution between fertility and income in the surplus function is decreasing in income, meaning higher-income couples value fertility more relative to income),

then the stable match has three characteristics:

- Women with $s < \overline{s}$ will match positive-assortatively with men with regard to income: if $s < s' < \overline{s}$, and s is matched with y and s' with y', then $y < y'$. Similarly, women with $s > \overline{s}$ will match positive-assortatively with men with regard to income

- There exist parameter configurations for which some high-earning men can marry a woman with $s < \overline{s}$, while some lower-earning men marry women with $s > \overline{s}$, thus matching negative-assortatively with regard to income across $\overline{s}$.

- If some man who marries a woman with $s < \overline{s}$ with certainty is richer than another who marries a woman with $s > \overline{s}$ with certainty, then every man richer than the first also marries a woman with $s < \overline{s}$.

I will examine the logic for each feature of the match in turn. First, that there is positive assortative matching between men and women on the sets $(\overline{s}, S) \times (0, Y)$ and $(0, \overline{s}) \times (0, Y)$.

Define $\phi(s) = \{y\}$ such that the probability that y is matched with s is greater than 0. For $(\overline{s}, S) \times (0, Y)$: Suppose that for two women with skills s and s', $s' > s$, each having fertility level π, that $y \in \phi(s)$ and $y' \in \phi(s')$, with $y > y'$. Because T is convex in total income, $T(\lambda s' + y, \pi) + T(\lambda s + y', \pi) > T(\lambda s' + y', \pi) + T(\lambda s + y, \pi) = u(y) + u(y') + v(s) + v(s')$, given the current matching.

This violates the constraints that $u(y) + v(s') \geq T(\lambda s' + y, \pi)$ and $u(y') + v(s) \geq T(\lambda s + y', p)$. Therefore, $y' \geq y$, and μ exhibits positive assortative matching for all women with the same fertility level, and thus on $(\overline{s}, S) \times (0, Y)$ and $(0, \overline{s}) \times (0, Y)$.

I will now demonstrate that the marginal rate of substitution condition is sufficient for non-assortative matching under some parameter values. To do this, I first establish that this assumption implies increasing differences in the husband's income of the surplus gain from swapping a high-fertility, low-income wife for a low-fertility, high-income wife: $\frac{\frac{\partial T}{\partial y}}{\frac{\partial T}{\partial \pi}} \equiv MRS$ decreasing in income implies that $T(y + \delta, P) - T(y' + \delta, p)$, $y' > y$, is an increasing function of δ.

An illustration of this implication of the decreasing MRS is shown in Figure A5.

Define $z(\delta)$ as the level of income that makes $T(z(\delta) + \delta, p) = T(x + \delta, P) \equiv T_\delta$.

Since $T(z(\delta) + \delta, p) = T(x + \delta, P)$, to show $T(x + \delta, P) - T(x' + \delta, p)$, we only need show that $T(z(\delta) + \delta, p) - T(x' + \delta, p)$ increasing in δ.

To show that $T(z(\delta) + \delta, p) - T(x' + \delta, p)$ is increasing in δ, it is sufficient to show $z(\delta)$ is weakly increasing in δ, since convexity of the surplus in income means that a given loss of y decreases the surplus more for higher y couples:

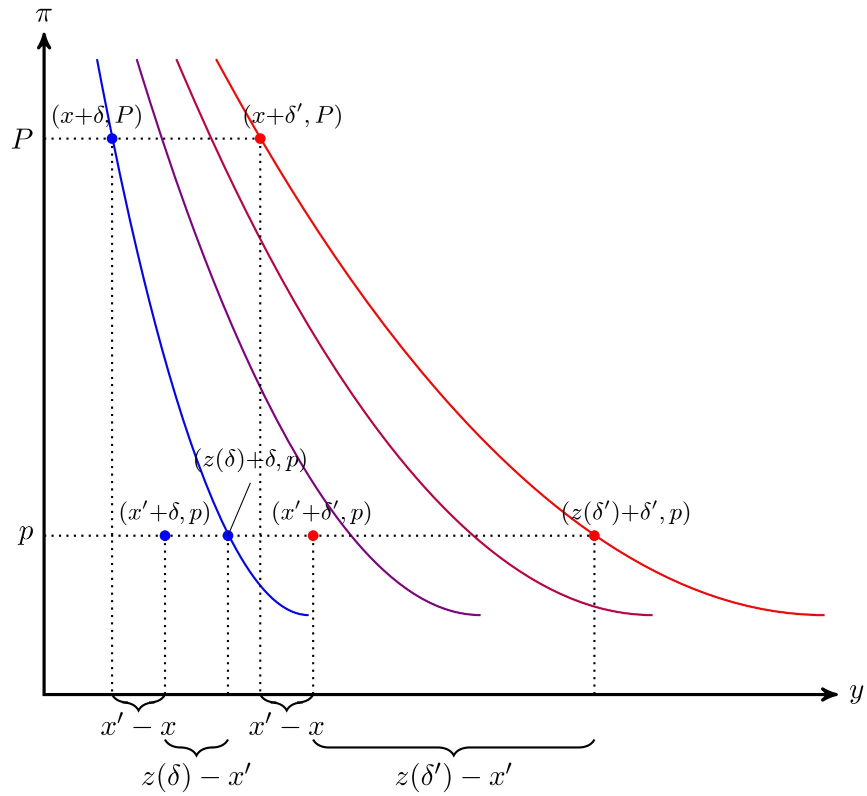
$$\begin{aligned} T(z(\delta) + \delta, p) - T(x' + \delta, p) &= \int_{x'}^{z(\delta)} \frac{\partial T}{\partial y}(y + \delta, p) dy \\ &> (z(\delta) - x') \frac{\partial T}{\partial y}(y + \delta, p) \end{aligned}$$

because $\frac{\partial^2 T}{\partial y^2} > 0$. (Note, this is where the assumption that $T(x + d, P) > T(x' + d, p)$, and hence $T(z(\delta) + \delta, P) > T(x' + \delta, p)$, is required.)

Thus, we need to show that $z(\delta)$ is weakly increasing in δ.

Let $g \in [0, 1]$ define the distance traveled from P to p along an iso-surplus curve at level T_δ starting from $(x + \delta, P)$, such that $\pi(g) = P - g(P - p)$, and finishing at $(z(\delta) + \delta, p)$. Note that $\pi(g)$ is independent of δ. At each g, we define $y_\delta(g)$ as the value of y such that $T(y_\delta(g), \pi(g)) = T_\delta$.

Figure A5: Illustration showing $z(\delta) - x'$ increasing in δ for MRS decreasing in y
Iso-surplus curves in fertility, π, and income, y



Notes: Figure depicts income on the x-axis and fertility on the y-axis, and graphs hypothetical iso-surplus curves that exhibit a decreasing marginal rate of substitution between fertility and income as income increases. $z(\delta)$ is the amount of income required to keep the surplus constant if P decreases to p. The picture demonstrates that $z(\delta)$ is increasing in δ, and thus the surplus gain from trading a higher-income woman for a higher-fertility woman is increasing in income.

Then:

$$\frac{\partial T}{\partial y}\frac{\partial y_\delta}{\partial g} + \frac{\partial T}{\partial \pi}\frac{\partial \pi}{\partial g} = 0$$

as we "walk" along the iso-surplus curve.

This implies

$$\frac{\partial y_\delta}{\partial g} = -\frac{\frac{\partial T}{\partial \pi}}{\frac{\partial T}{\partial y}}\frac{\partial \pi}{\partial g}$$

$$\int_0^1 \frac{\partial y_\delta}{\partial g} dg = \int_0^1 -\frac{1}{MRS(y_\delta(g), \pi(g))}\frac{\partial \pi}{\partial g} dg$$

$y_\delta(0) = x + \delta$ at the starting point of the T_δ iso-surplus curve. $\frac{\partial \pi}{\partial g}$ can be replaced with the linear function $-(P-p)$. This yields:

$$\begin{aligned}\int_0^1 \frac{\partial y_\delta}{\partial g}dg &= \int_0^1 -\frac{1}{MRS(y_\delta(g),\pi(g))}\frac{\partial \pi}{\partial g}dg \\ \Rightarrow z(\delta)+\delta-(x+\delta) &= \int_0^1 \frac{1}{MRS(y_\delta(g),\pi(g))}(P-p)dg \\ \Rightarrow z(\delta) &= \int_0^1 \frac{1}{MRS(y_\delta(g),\pi(g))}(P-p)dg + x\end{aligned}$$

I will now show that the righthand side expression is increasing in δ. $\pi(g)$ is constant in δ, by definition. T_δ is strictly increasing in δ, because $T(x+\delta,P)$ is strictly increasing in δ, and $T_\delta \equiv T(x+\delta,P$. Thus, since $\pi(g)$ is constant in δ, $y_\delta(g)$ must be increasing in δ.

The MRS is decreasing in y, by assumption. Therefore, $\frac{1}{MRS}$ is increasing in y. As $y_\delta(g)$ is increasing in δ, and $\pi(g)$ is constant, $\frac{1}{MRS}$ is increasing in δ. Because the expression inside the integral is increasing in δ for each g, the integral must also be increasing in δ, and thus the righthand side expression is increasing in δ.

Then, the lefthand side must also be increasing, meaning $z(\delta)$ is increasing in δ. $z(\delta)$ increasing in δ implies:

$$\begin{aligned}&T(z(\delta)+\delta,p)-T(x'+\delta,p) \text{ increasing in } \delta \\ \Rightarrow &T(x+\delta,P)-T(x'+\delta,p) \text{ increasing in } \delta\end{aligned}$$

Now I discuss how this increasing difference implies non-assortative matching for Y big enough (i.e., that if $\overline{s}$, Y, $\frac{P}{p}$, S and λ are such that $T(\overline{s}+Y,P) > T(\lambda S+Y,p)$, there exists y and y', $y<y'$, such that $\psi(y')<\psi(y)$).

Because $T(\overline{s}+Y,P) > T(\lambda S+Y,p)$, by continuity there exists $s<\overline{s}$ and $s'>\overline{s}$ such that $T(s+Y,P) > T(\lambda s'+Y,p)$. Because $\frac{\frac{\partial T}{\partial y}}{\frac{\partial T}{\partial \pi}}$ is monotonically decreasing in income, if $T(s+y',P) > T(\lambda s'+y',p)$, then $T(s+y',P)-T(\lambda s'+y',p) > T(s+y,P)-T(\lambda s'+y,p)$ for $y<y'$.

Now suppose that $\psi(Y) > \overline{s} > \psi(y)$ for all $y<Y$. $T(\overline{s}+Y,P) > T(\lambda S+Y,p)$ and $y<Y \Rightarrow T(s+Y,P)-T(\lambda s'+Y,p) > T(s+y,P)-T(\lambda s'+y,p)$, which can be rearranged to yield $T(s+Y,P)+T(\lambda s'+y,p) > T(s+y,P)+T(\lambda s'+Y,p)$, and thus the total surplus can be increased by exchanging the partners of Y and y, which is a contradiction. Thus $\psi(Y)<\psi(y)$ for some $y<Y$.

A stronger condition on the marginal rate of substitution, that it goes to zero as y goes to infinity, is sufficient to guarantee that for Y large enough $T(\overline{s}+Y,P) > T(\lambda S+Y,p)$. But, note that this region will still not always exist, because Y may not be large enough relative to λS and the fertility loss, $\frac{P}{p}-1$.

Finally, I show that if there is non-assortative matching, there is a single "break" from the assortative match: If there exists some $\hat{y}$ with $\psi(\hat{y}) < t < \psi(y)$ with certainty, for $y < \hat{y}$, then for all $y' > \hat{y}$, $\psi(y') < \overline{s}$.

Suppose, to the contrary, that for $y' > \hat{y} > y$, $\psi(\hat{y}) < \overline{s} < \psi(y)$ but $\psi(y') > \overline{s}$. Denote $s' = \psi(y')$, $\hat{s} = \psi(\hat{y})$, and $s = \psi(y)$. In order for this match to be surplus maximizing, $T(\hat{s}+\hat{y},P) + T(\lambda s' + y',p) > T(\lambda s' + \hat{y},p) + T(\hat{s}+y',P)$.

However, because $\frac{\frac{\partial T}{\partial y}}{\frac{\partial T}{\partial \pi}}$ is decreasing in income, for $y' > \hat{y}$, $T(\hat{s}+\hat{y},P) - T(\lambda s' + \hat{y},p) < T(\hat{s}+y',P) - T(\lambda s' + y',p)$. But then $T(\hat{s}+\hat{y},P) + T(\lambda s' + y',p) < T(\lambda s' + \hat{y},p) + T(\hat{s}+y',P)$, which is a contradiction. Therefore, if any $\hat{y}$ has $\psi(\hat{y}) < \overline{s} < \psi(y)$ with certainty, where $\hat{y} > y$, so must every $y' > \hat{y}$.

This result provides insight into how the marginal rate of substitution between two characteristics can impact matching in bi-dimensional settings, and is thus applicable to any matching problem where one side of the market is characterized by a single characteristic and the other side is characterized by two negatively correlated characteristics that cannot be summarized by an index.

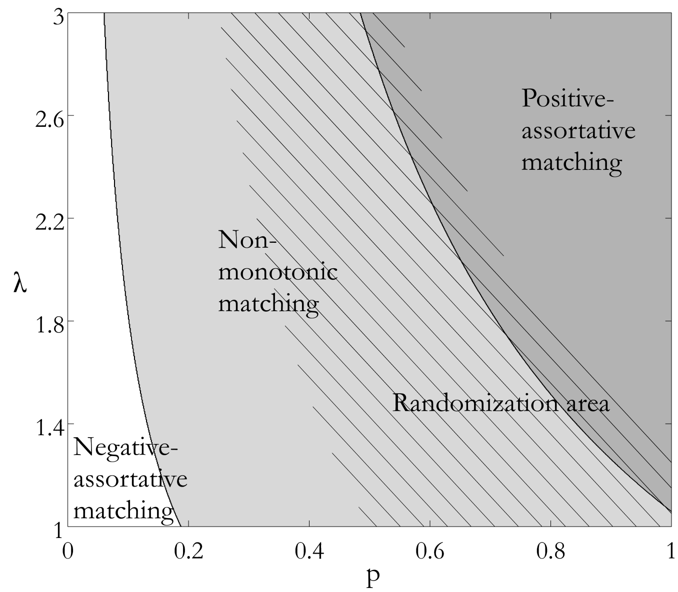
B.6 Matching equilibria with fixed cost

Adding a fixed cost of education in the matching stage, as is required to arrive at an endogenous human capital investment equilibrium, only slightly shifts the bounds of each type of equilibria, without substantively changing the predictions of the model, as shown in Figure A6.

The cutoffs for each section are also slightly different (except the boundary between non-monotonic matching and randomization, which is unchanged by the size of the fixed cost):

- Cutoff from negative-assortative to non-monotonic: $\lambda > \frac{(S-\overline{s})}{S+\overline{s}}\frac{(P-p)}{p}\frac{Y-1}{S} + \frac{2c}{S+\overline{s}}$
- Cutoff from non-monotonic to randomization transition: $\lambda > \frac{(P-p)}{p}\frac{(Y-1)}{S}$
- Cutoff from randomization to positive assortative matching: $\lambda > \frac{(P-p)}{p}\frac{(Y-1)}{S} + \frac{P}{p} + \frac{c}{\overline{s}}$

FIGURE A6: MATCHING EQUILIBRIUM FOR VARYING λ AND p
$Y = 2,\ S = 1,\ P = 1,\ \bar{s} = 0.7,\ c = 0.2$

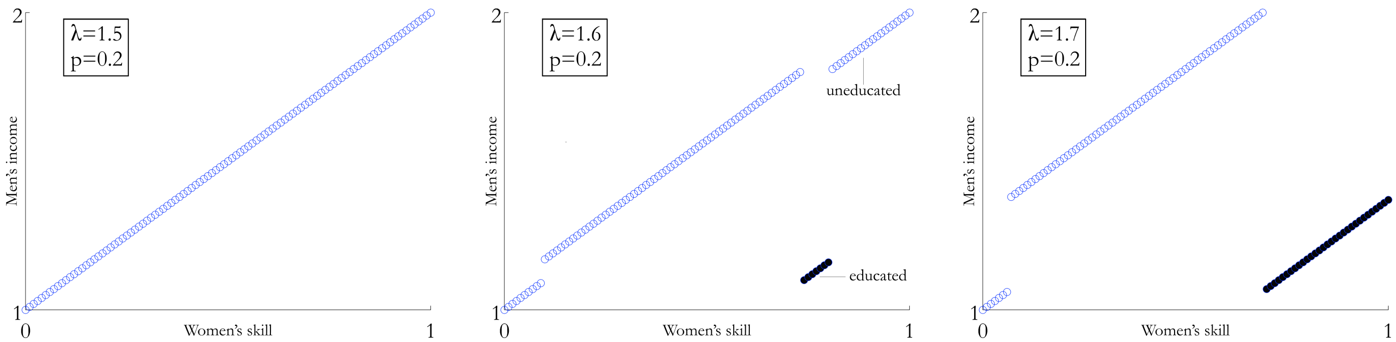


Notes: Figure depicts the matching equilibrium depending on the value of the return on investment, λ (y-axis) and the post-investment chance of fertility, p (x-axis), in the presence of a fixed cost of education. Negative assortative matching is the area in white, pure non-monotonic matching is shown in light grey with no shading, and pure positive-assortative matching is shown in dark grey with no shading. The non-pure transition equilibrium is shaded with diagonal lines.

B.7 Educational investments

Figure A7 depicts the special case where higher skilled women invest, but the highest skilled women forgo investment. These images come from the same optimization program displayed in Figure 12, but for parameter values that highlight this special case.

Figure A7: Example of Lower and Upper Investment Bounds



Notes: Figure depicts the results of a simulation of the investment and matching equilibrium as the value of the return on investment, λ, increases. Women's skill is shown on the x-axis and men's income on the y-axis, with dots depicting marriage matches. At a p of 0.2, for a low λ no women invest, at a slightly higher λ the mid-skill women invest (as the top-skill women have too much to lose), and at a higher λ the top-skill women invest. Simulation shown for Y=2, S=1, P=1, and c=0.2.

C Appendix: Experiment

C.1 Methodology

To generate the hypothetical dating profiles, I purchased stock photos that were similar in appearance to photos on dating websites, then randomly assigned characteristics. I started with 50 photos of men and 50 photos of women, depicting caucasian individuals of "ambiguous age," meaning no balding or gray hair, no obvious facial wrinkles, and no overly youthful hairstyles or clothing. I then had 120 undergraduate students rate each photo's physical attractiveness and guess the age of the individual in the photo. Average attractiveness and average "visual age" was then balanced between the men and women, and photos with an average guessed age outside the ages being used for the study were removed.

Using the selected photos, 40 male and 40 female dating profiles were created. The following characteristics were randomly assigned to each dating profile: a username, a height, and three interests. The usernames were assigned by using the top 40 names for men and women from the decade of birth for 30-40-year-olds, then assigning a random three-digit number. The heights were assigned randomly from a normal distribution using the mean and standard deviation of heights for caucasian men and women. Gender-neutral interests were assigned from a list of top hobbies, with more popular interests being assigned more frequently. All profiles listed the person as "looking for: serious relationship," in order to signal that the rater should consider this person as a potential long-term partner. Each of these characteristics were assigned to the profile and remain fixed throughout the experiment. Then, as each profile was shown, age and income were randomly assigned: age between 30 and 40 (inclusive), and an income range from roughly the 25th to 95th percentile for single individuals with at least an associate's degree in the 2010 Census. Each respondent who completed the survey viewed all 40 profiles.[41]

Summary statistics from the data are presented in Table A3, for my target sample of white individuals between 30 and 40.[42] Without these restrictions, in the initial sample 77% of male and 78% of female participants are white, and 74% fall within the targeted age range. In the Qualtrics sample all individuals are white and within the specified age range, due to pre-screening by Qualtrics.

Table A3: Summary Statistics

	Initial Sample				**Qualtrics Sample**			
	Men N=35		**Women N=44**		**Men N=207**		**Women N=104**	
Variable	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
Age	35.216	3.637	35.933	3.512	34.647	3.049	34.375	3.206
High Income	0.486	0.507	0.356	0.484	0.386	0.488	0.154	0.363
College Grad	0.676	0.475	0.689	0.468	0.493	0.501	0.462	0.501
Has kids	0.351	0.484	0.432	0.501	0.203	0.403	0.423	0.496
Wants (more) kids now	0.243	0.435	0.159	0.370	0.184	0.388	0.183	0.388
Wants marriage	0.459	0.505	0.432	0.501	0.469	0.500	0.442	0.499
Date lowest age	25.838	3.571	32.955	3.929	24.865	4.330	29.971	4.130
Date highest age	40.838	5.419	46.864	6.920	41.575	6.094	44.212	7.384
Preferred low	28.486	3.731	35.295	4.322	27.029	4.702	32.519	4.384
Preferred high	37.216	4.547	44.205	6.341	37.432	5.550	41.337	6.662
Fem Fert cutoff?	1.000	0.000	1.000	0.000	0.975	0.157	0.990	0.099
Fem cutoffage	41.189	6.368	39.674	4.719	43.108	7.113	41.098	6.231
Male fert cutoff?	0.892	0.315	0.767	0.427	0.835	0.372	0.796	0.405
Male cutoff age	53.667	8.912	55.455	8.460	51.946	9.091	56.549	9.077

Notes: Summary statistics for in-sample men and women from online dating experiment. High income is classified as earning over $60,000 annually. The fertility variables ask if there is an age at which it becomes biologically difficult for women or men to conceive, and then what that age is.

Because the recruitment of additional respondents was motivated by testing for heterogeneity

[41] After agreeing to the consent form, respondents were asked to rate profiles on a scale from 1 to 10. After 10 profiles, the respondents ordered the profiles from most preferred to least preferred, both to break up the monotony of the rating, and to provide a check for participants randomly entering answers without thinking about them (in which case there would be a low correlation between their ratings and rankings). Following this, they completed a brief post-survey including demographic information, dating preferences, and, finally, their knowledge of age-fertility limits for men and women.

[42] The consent form required respondents to certify that "I am between 30 and 40 years old, currently single, and seeking a partner of the opposite gender." However, in the post survey, some initial-sample respondents listed birth years outside the 30-40-year-old range. In my main specification, I exclude these responses. Also, although the profiles feature only white men and women, I did not restrict the race of respondents, so I also exclude non-white respondents during the analysis phase, since cross-racial rankings may be driven by different factors. For the Qualtrics sample, respondents were pre-screened based on race, relationship status, and age.

in male responses, male respondents in the Qualtrics sample were enrolled at a 2:1 ratio to female respondents. The oversampled males were also drawn from the higher end of the income distribution, in order to have an income distribution that better mirrors the general population (Qualtrics respondents, in absence of this sampling concentration, tended to be lower-income, which would not allow for a test of income heterogeneity).

Table A3 shows that men and women taking the survey display similar characteristics, although the men are more likely to be high-income, defined as income over $65,000 per year, in the initial sample. Where men and women differ substantially is their stated preferences for the age of their partner. In the initial sample, men state on average that the youngest they would date is a 26-year-old and the oldest is a 41-year-old, whereas women state averages of 33 and 47. When it comes to their preferred dating range, men look for women aged 29 to 37, whereas women seek partners between the ages of 35 and 44. This pattern provides some preliminary evidence that men have differential preferences over their partner's age, compared to women.

The final questions on the survey ask men and women at what age they believe it becomes biologically difficult for members of each gender to conceive a child. 100% of initial-sample respondents believe there is a cutoff for women (97% of men and 99% of women in the Qualtrics sample), indicating that there is some knowledge of differential fertility decline, whereas 89.2% of men and 76.7% of women believe that such a cutoff exists for men. Female respondents put the start of the fertility decline for women somewhat earlier than male respondents, at 39.7 years, as compred to 41.2 for men. Both male and female respondents, conditional on thinking there *is* a cutoff, believe the cutoff to be higher for men.

C.2 Alternative hypotheses: heterogeneous male or female tastes for income

I now examine evidence of possible alternative hypotheses that explain negative assortative matching at the top of the income distribution, using the experimental data. The first alternative hypothesis that I test for is that *women* who are very high-earning may exhibit a less strong preference for income than lower-earning women, and thus the observed non-assortative matching could really be driven by women's tastes. The question, essentially, is whether women who are very high-earning have a lower marginal utility of additional income. Table A4 interacts the rater's income with the profile's income for both men rating women (column 1) and women rating men (column 2)—the resulting coefficients are positive, although only significant for male raters. As mentioned, this indicates that tastes over income appear to take the supermodular form assumed by the model: those with more income value additional partner income more. Columns 4 and 5 show that "high-income" raters, both male and female, have a greater taste for additional income, by interacting a dummy for having annual income over $65,000 with the income in the profile. Thus, I find no evidence of a decreasing marginal utility of income for women.

Table A4: Preferences over partner income, men and women: Qualtrics sample

	Dependent variable: Profile rating			
	Men	Women	Men	Women
	(1)	(2)	(3)	(4)
Age	-0.043***	0.028*	-0.043***	0.028*
	(0.010)	(0.015)	(0.010)	(0.015)
Income ($0,000s)	-0.008	0.001	0.016	0.024*
	(0.018)	(0.025)	(0.012)	(0.014)
Inc × rater inc	0.007***	0.008		
	(0.002)	(0.005)		
Inc × rater high inc			0.040**	0.083*
			(0.018)	(0.046)
Observations	8080	4040	8080	4040
R-Squared	0.491	0.464	0.491	0.464

Notes: Regression of profile rating on randomly assigned age and income for men-rating-women and women-rating-men, from the second sample collected via Qualtrics. Columns 1 and 2 interact profile income with rater income. Columns 3 and 4 interact profile income with whether the rater is "high income," earning above $60,000 annually. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses, clustered at the rater level.
*** p<0.01, ** p<0.05, * p<0.1

It is also possible that men dislike income itself in potential mates, perhaps due to gender norms, which could lead to the non-assortative matching at the top without reproductive capital. Men may not dislike all income equally, but may dislike it when women earn more than they do (e.g., Bertrand, Kamenica and Pan (2015)), or may dislike *very* high-earning women. Table A5, column 1, regresses men's ratings on a dummy for whether the profile's listed income is higher than the rater's own income. The coefficient on "profile earns more" is positive, indicating men in this sample do not exhibit distaste for women earning more than them. The second column interacts "profile earns more" with profile income, to see if the slope of additional income turns negative, or is much smaller, for marginal dollars after the rater's own income. The coefficient is negative, but non-significant, and it is much smaller than the main effect. Thus, marginal dollars of income still contribute positively to rating. The last column examines whether very high-income women are viewed less positively. Using a dummy for each income level, with the lowest income level, $20-34,999, as a baseline, we see that the coefficients on income level rise monotonically: the highest income level has a higher coefficient than all income levels before it.

This demonstrates that there does not appear to be a penalty to being high earning, or earning more than one's partner, in contemporary data. It is possible the dissipation of these gender norms have driven the changes over time, as suggested by Bertrand et al. (2016) but my model demonstrates that a portion of what appears as a "norm shift" may be driven by economic fundamentals.

Table A5: Male preferences over partner income: Qualtrics sample

	Dependent variable: Profile rating (Male subjects)		
	Binary	Interaction	By income level
	(1)	(2)	(3)
Age	-0.043***	-0.043***	-0.043***
	(0.009)	(0.009)	(0.010)
Income ($0,000s)	0.027***	0.039***	
	(0.010)	(0.014)	
Profile earns more	0.053		
	(0.082)		
Earns more × inc		-0.006	
		(0.010)	
$35-49,999			0.134
			(0.083)
$50-64,999			0.151*
			(0.087)
$65-79,999			0.205**
			(0.085)
$80-94,999			0.213**
			(0.093)
$95-109,999			0.264***
			(0.095)
$110-124,999			0.343***
			(0.099)
Observations	8080	8080	8080
R-Squared	0.490	0.490	0.490

Notes: Regression of profile rating on randomly assigned age and income for men-rating-women, from the second sample collected via Qualtrics. Columns 1 controls for whether the profile has a higher income than the respondent. Column 2 interact profile income with the profile having a higher income than the respondent. Column 3 shows the shape of the rating - profile income relationship by including dummies for different profile income levels (lowest income level omitted). Rater and profile fixed effects included in all columns. Robust standard errors in parentheses, clustered at the rater level.

*** p<0.01, ** p<0.05, * p<0.1

C.3 Non-linearity in preferences over age

Table A6 checks for non-linearity in men's preferences over their partners' ages. If the preference for younger women displayed in the experiment is really a preference for fertility, then all years should not have equal weight in this calculation. Aging that takes place closer to the time when a woman may begin to have difficulty conceiving should be viewed more negatively than aging that is far before or far after this “infertility threshold.” The age range that was presented to participants, from 30 to 40 years old, was too narrow to detect any non-linearity in the response to age. However, this non-linearity should most naturally occur in relation to the *perceived* infertility threshold of each respondent. Thus, by creating a new variable of profile age minus each respondent’s individual belief regarding the infertility age, I effectively recover an expanded range of ages: from 20 years before infertility to 4 years after, restricting to cells with more than 100 data points. For example, if someone says that it becomes biologically difficult for a woman to conceive at age 36, and the profile age shown is 40, that data point becomes four years past infertility. If the respondent believes the age is 50, and the profile age shown is 40, that would be ten years prior to infertility, or -10.

For the analysis in table A6, errors are clustered at the profile level, because the “treatment” will be correlated with the raters’ underlying characteristics, since only individuals who list very high infertility ages can have very negative values for “years past infertility,” and only those who list very low infertility ages can have the upper range of “years past infertility” values. This also means that these results should be taken as suggestive only, as individual factors that may bias the response to age may be connected to those factors that cause one to list a higher or lower age at infertility. As in the other analysis that relies on heterogeneity across male respondents, these results are most reliably interpreted in Panel B, with the larger Qualtrics sample.

Column 1 substitutes the constructed years past the individual rater’s “infertility cutoff” variable for profile age, showing a coefficient with similar magnitude and significance to the age analysis. Column 2 shows that when a squared term is added, this term is also negative and significant, indicating that distaste for additional years intensifies as age approaches and crosses the perceived infertility cutoff. Finally, column 3 demonstrates that the negative relationship between age and rating follows a backwards “s-curve”: shallow, then steep, then shallow. The coefficient grows stronger as age approaches the respondent’s perceived cutoff, with a negative and significant slope interaction for being between 6 and 10 years from the cutoff, and a stronger negative and significant effect for additional years within 5 years of the cutoff. Then, once the cutoff has been passed, the coefficient on additional years reverts back to its baseline level (with the interaction being statistically zero), the same as additional years more than 10 years from the cutoff. In the initial sample, these effects are not significant, but follow the same pattern.

Table A6: Non-linearity in aging using rater-specific fertility cutoffs: Qualtrics sample

	Dependent variable: Profile rating (Male subjects)		
	Ind cutoff	Cutoff^2	By phase
	(1)	(2)	(3)
Years past	-0.038***	-0.083***	-0.031***
	(0.010)	(0.019)	(0.010)
Years past^2		-0.003***	
		(0.001)	
Yrs past × 10-6 yrs pre			-0.031***
			(0.009)
Yrs past × 5-1 yrs pre			-0.046**
			(0.019)
Yrs past × 0-4 yrs post			-0.001
			(0.048)
Income ($0,000s)	0.032***	0.032***	0.032***
	(0.011)	(0.011)	(0.011)
Observations	6833	6833	6833
R-Squared	0.465	0.467	0.467

Notes: Regression of profile rating on randomly assigned age and income for men-rating-women, from the second sample collected via Qualtrics. The respondent's estimate as to when women become less fertile is subtracted from the profile age to form a measure of how many "years past" this fertility cutoff the profile is. Column 1 shows that this measure can be used as an alternate measure of age, based on the subjective view of fertility by the respondent, and yield similar effect sizes. Column 2 shows the non-linear relationship between rating and this variable. Column 3 shows the value of the coefficient on "years past" at different stages: far from the fertility cutoff, close to the fertility cutoff, and after the fertility cutoff. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses, clustered at the rater level.
*** p<0.01, ** p<0.05, * p<0.1

C.4 Test for plausibility of surplus function properties

The theoretical model derives predictions from two crucial assumptions. First, the surplus function is supermodular in the two spouses' incomes. Second, the surplus function exhibits a marginal rate of substitution between income and fertility that declines with income. This section uses the experimental data to test the plausibility of these assumptions. Although I cannot test the effect on the surplus function as a whole, which involves the men's and women's utilities added together, I can derive an understanding of the shape of the surplus function from individual preferences.

For the first property, supermodularity in incomes, I look at the effect of the interaction between own income and profile income on overall rating, as discussed in the context of women's preferences for income. Table A4 shows that taste for partner income is indeed an increasing function of own income. In columns 1 and 2, the rater's own income interacted with the profile's income has a positive and significant coefficient for regressions of each male and female ratings on profile characteristics, providing evidence for the supermodularity assumption. This table is discussed in more detail in the next section.

Table A7 tests for the second assumption, decreasing marginal valuation of income relative to fertility as income increases. The relationship between men's ratings of profiles and women's ages shown in the profile is indeed heterogeneous across income groups. This justifies the non-index approach to solving the matching model, since not all men value partner characteristics alike. However, rather than merely increasing in income, the age penalty appears to be U-shaped, with the poorest men having the greatest preference for young partners, middle income men having the lowest preferences, and the highest income men having higher preferences than the middle-income. This may be due to cultural norms acting on the lowest income men, while the model's mechanism of decreasing marginal valuation of income relative to fertility (due, in part, to the growing importance of investments in children in the overall surplus produced by marriage) may be causing the heightened valuation of age among the higher-income men. The increasing side of the "U," though, is the one most likely to impact individuals considering post-bachelor's educational investments, and thus the relevant section for the model presented here. Additionally, because in both the three-segment and the positive assortative equilibrium the very poorest men *do* match with fertile women in the model, these equilibria would be robust to the very poorest men, in addition to the richest men, having heightened sensitivity to age. The negative assortative matching equilibrium may be ruled out by these preferences, however (in addition to being unlikely to appear due to typically assortative matching on social class).

Table A7: Income heterogeneity: Qualtrics Sample

	Dependent variable: Profile rating		
	Age interaction	Income and age	Control for knowledge
	(1)	(2)	(3)
Age	-0.001	-0.001	-0.026
	(0.015)	(0.015)	(0.018)
Income ($0,000s)	0.032***	0.034**	0.032*
	(0.009)	(0.016)	(0.017)
High income × age	-0.038*	-0.038*	-0.037*
	(0.022)	(0.022)	(0.022)
Low income × age	-0.070***	-0.070***	-0.063***
	(0.022)	(0.022)	(0.021)
High income × inc		0.022	0.025
		(0.021)	(0.022)
Low income × inc		-0.029	-0.025
		(0.024)	(0.024)
No knowledge × age			0.057***
			(0.017)
Observations	8080	8080	7800
R-Squared	0.491	0.492	0.490

Notes: Regression of profile rating on randomly assigned age and income for men-rating-women and women-rating-men, from the second sample collected via Qualtrics. Column 1 interacts profile age with the rater being high income or low income. Column 2 adds interactions with profile income. Column 3 controls for rater fertility knowledge interacted with profile age. Rater and profile fixed effects included in all columns. Robust standard errors in parentheses, clustered at the rater level.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$