Quantifying the Impact of Childcare Subsidies on Social Security

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Abstract

Female labour force participation and fertility levels directly impact social security, especially when it relies on a pay-as-you-go scheme. In this paper, we quantify the impact of childcare subsidisation policies on a PAYG social security system. We build an overlapping generations model in which women decide how many children to have, the allocation of childcare time among different alternatives and their labour force participation along their life cycle. We calibrate the model to Spanish data and use it to experiment with different childcare subsidisation policies. We find that childcare subsidies increase mother's labour force participation, although they also induce some shift from full to part-time work. They also provide a small boost to fertility. However, the magnitude of the effects is insufficient to generate a positive effect on the present value of the social security's budget balance.

Keywords: childcare subsidies, fertility, female labour force participation, Social Security

JEL Codes: J11, J13, J22

1 Introduction

Higher female labour force participation and decreased fertility levels are two of the most significant changes in women's socio-economic behaviour in developed countries in recent decades. Social security systems are directly affected by them, especially those that rely on pay-as-you-go schemes. The increase of female labour force participation brings in more contributions to Social Security in the short run. In the future, this implies additional pension entitlements once these women retire. Moreover, a drop in fertility levels causes the old-age dependency ratio to increase eventually (absent migration). As the share of pensioners over contributors rise, pressure on the system's finances builds.

Naturally, changes in female labour force participation and fertility are not independent of each other. They are jointly determined. Any policy that affects this joint decision affects the finances of social security.

Using a variety of data sources, including administrative, household panel and time use surveys, we provide evidence suggesting that childcare policies may boost fertility and strengthen mother's labour market attachment in Spain. A number of facts underlie this claim. First, like in most developed countries, there is a family gap in female labour force participation. In other words, women with children participate less in the labour market than childless women, and are more likely to be part-time workers if they do. Second, there is government provided universal coverage of preschool for children 3-6, and families use it extensively. In contrast, preschool usage for children 0-3, which is paid mostly out of pocket by families, is much lower. Moreover, access to informal unpaid childcare provided by other family members[1] is limited as well. Taken together, this suggests families with young children face important constraints when deciding how to cover their childcare needs. Finally, there is a positive gap between desired and realised fertility. That is, women have fewer children than they would like to have. Moreover, the number one reason women say prevents them from attaining their desired fertility is lack of compatibility between career and family. We hypothesise then that the introduction of a policy that subsidises childcare for children aged 0-3 could improve the social security's budget balance by jointly stimulating fertility and female labour force participation.

To quantify the effect of the introduction of such a policy, we build an overlapping generations model in which women choose fertility levels at the beginning of life. In subsequent periods they decide on labour force participation, and if they have young children, childcare allocation among maternal, informal and out-of-pocket alternatives. Simultaneously, the government runs a Social Security PAYG system financed with a payroll tax and pays

[1] For instance, grandparents.

retirement benefits.

We calibrate the model to Spanish data and social security regulations, and perform a series of policy experiments. First, we test in the model two policies that have actually been implemented by the Spanish government in the past and have been evaluated in the literature, namely universal childcare for children 3-6 and cash transfers upon birth. Then, we experiment with the introduction of partial (50%) and full (100%) childcare subsidies for women with children aged 0 to 3, a new policy.

For the first two policies, the findings are similar qualitatively with respect to previous literature. The newly proposed policies have small effects on female labour force participation and fertility decisions, which translates into a negative impact on the present value of the budget balance of Social Security.

In particular, the subsidies on childcare for children aged 0-3 induce an increase in part-time and a decrease in full-time work among women aged 33-39 with children in this ages, with a small net positive effect on total participation. The total fertility rate rises marginally, from 1.25 in the baseline economy to 1.26 under partial subsidies and 1.27 under full subsidies. These results are mainly driven by couples who did not have any children in the baseline economy, and under the new scenario, they choose to have one. Therefore, it suggests that introducing childcare subsidisation in Spain has a small positive effect on the number of children in the extensive margin for couples without any in the baseline economy.

The combined effects of the subsidies on female labour force participation and fertility are to worsen the Social Security budget balance in all future periods, which lowers its present value. The effect of fertility is too small to compensate for the immediate cost that the subsidies entail. Therefore, the policy fails to deliver the benefits hypothesised above.

This paper contributes to different strands of the literature. It is broadly related to the extensive body of research on the relationship between fertility, women's labour market outcomes, and policies relevant to it. Among the papers that rely on structural, dynamic models to study the issues at hand are Erosa et al. (2002), Da Rocha and Fuster (2006) and Adda et al. (2017). Unlike these, the labour market is not modelled here in detail because there are no search frictions or occupational choices. In contrast, a detailed social security system is included. The model economy is similar to the one in Bick (2016), Hannusch (2019) and Laun and Wallenius (2021). Women's labour force and childcare utilisation decisions during their child-rearing years are heavily inspired by Bick (2016). The contributions consist on modelling non-child rearing years, retirement benefits, and changing the way decisions regarding the number of children are made, by relying on desired fertility data. Differently from Hannusch (2019), women are allowed to decide to use less than their endowment of informal childcare. It may be that some mothers who value maternal childcare time highly do not use all the informal childcare endowment. The arrival of children to the household conditional on the fertility level follows a deterministic schedule, as in Laun and Wallenius (2021). That is, the model does not study the timing and spacing of multiple children, only total fertility. Even though the last two papers introduce pensions, they do not quantify family policies' effect on the system's finances.

This paper relates as well to the literature on Spanish child-related policies. Sánchez-Mangas and Sánchez-Marcos (2008) examine the introduction of a reform of Spain's income tax in 2003. The reform introduced two main changes. First, it increased children's taxable income deduction [2]. Second, it raised the additional monthly cash benefit/tax deduction to working mothers with children under three years[3]. Through a difference-in-differences-in-differences (DDD) estimator, they find that this policy increased labour market participation rate of eligible mothers by three percentage points compared to non-policy-eligible ones. González (2013) studies the impact of a sizeable universal child benefit introduced in Spain between 2007 and 2010 on fertility and early maternal labour supply. Following a regression discontinuity design (RDD), she found that this policy encourages fertility, and mothers to stay out of the labour force for longer after childbirth. Nollenberger and Rodríguez-Planas (2015) investigate the effect of the introduction in Spain of universal public preschool provision for children aged three and older in the 1990s on mother's labour force participation. The strategy relies on a DDD approach for this natural experiment. They find that two mothers enter the labour market for every ten additional children enrolled in public childcare.

We implement some version of the last two policies in the structural model to compare the results as an external validity test. Namely, a one-time payment of 2500 euros for mothers giving birth and a counterfactual analysis in which public preschool provision is not present. The results are qualitatively similar. However, they differ in magnitude[4].

Finally, this paper builds on a long tradition in macroeconomics research on sustaining social security systems. This literature uses life-cycle models, pioneered by Auerbach and Kotlikoff (1987). This study contributes to the body of research that proposes and quantifies potential policy reforms that seek to help finance social security systems. This proposals include partial privatisation (Nishiyama and Smetters, 2007), mandatory fully funded social security systems financed with a consumption tax (Kotlikoff et al., 2007), retirement age delay (Díaz-Giménez and Díaz-Saavedra (2009) in Spain, Imrohoroğlu and Kitao (2012) and

[2]In particular, before the policy, families could deduct 1200 euros each for the first and second child, and 1800 for each subsequent child. The tax reform increased these deductions to 1400 for the first, 1500 for the second, 2200 for the third and 2300 euros for the subsequent children.

[3]From 300 euros to 1200 euros per child.

[4]The long-run effects of family policies obtained through structural models may be smaller than the short-run effects found by quasi-experimental models, as pointed out by Adda et al. (2017).

Kitao (2014) in the US), increasing the payroll tax, decreasing the wage replacement rate or making the benefits fall one-to-one with income above a threshold level (Kitao, 2014). Cruces (2021) shows that women's increased labour force participation in Spain has helped finance social security in the short run, but will generate pressure in the long run. Her model takes fertility as exogenous, however. This paper's novelty is that it assesses childcare subsidisation as a policy to achieve sustainability, which has not been considered before.

The rest of the paper is structured as follows. Section 2 discusses some empirical facts on female labour force participation around childbearing years, childcare utilisation and fertility choices, as well as the state of child-related policies in Spain. Section 3 describes the model. Section 4 discusses the calibration and model performance. Section 5 presents the policy experiments. Section 6 concludes.

2 Data, empirical facts and policy environment

In this section we briefly discuss Spanish policies related to labour gender equality and work/life balance. Then, we proceed to discuss women's labour market behaviour when they become mothers and the usage of childcare in Spain. This section and the remainder of the paper relies mainly on data from the *Encuesta de Condiciones de Vida* (ECV henceforth) of the Spanish National Statistics Institute, *INE*. The ECV is an annual survey that consists of a rotating panel in which families are interviewed for four years. *INE* provides cross-sectional and longitudinal weights that allow for computing statistics representing the whole Spanish population. We use data from 2011 to 2019.

2.1 Family policies in Spain

The main regulatory instrument to facilitate the compatibility between family and work is a bill from 1999 called the Spanish family and workers' labour life reconciliation. This law includes different mechanisms that aim to balance work and family, mainly regarding parental leaves. The current legislation entitles mothers with 16 and fathers with 4.3[5] paid leave weeks with full wage replacement. This parental leave includes full job protection during the first year. For the next two years, only a return to a similar or to a job of the same category is guaranteed. Besides, parents are entitled to unpaid leave to take care of their children. Returning to the same position after a short leave[6] is guaranteed, while only a return to a similar position is guaranteed after a long one.

[5]The number of weeks assigned was equated across genders in 2021.

[6]An unpaid leave is considered as a short one if it is less than one year.

The Spanish legislation currently does not consider direct cash transfers to families with children. However, between 2007-2010 the government provided a cash benefit of 2500 euros for each child born or adopted. This policy was faded out as part of the austerity measures after the 2008 economic crisis.

Childcare provision depends on the age of the child. For children younger than three years old, regional authorities are in charge, while for older children, it is the Spanish government who supplies it. The difference in who the supplier is, implies an unbalanced provision across ages and regions. The enrolment rate in early childhood education and care services from 0 to 2-year-old is low compared to three-year-old children. There exists unequal public kindergarten provision across regions. This fact is well-documented in González (2004). The lack of public childcare provision for younger children in Spain puts significant pressure on parents, especially on women. Families are forced to use informal childcare, to enrol their kids in private kindergartens and/or to change their labour market behaviour when children are born.

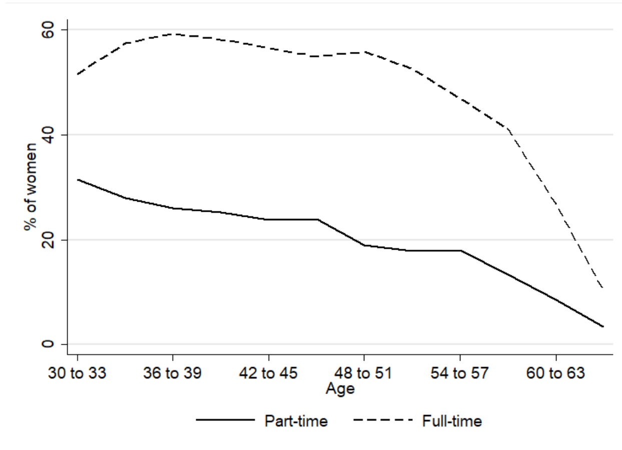
2.2 Female labour force participation

The *ECV* includes a labour force participation variable for each month over the last year. Taking full-time work to be 1, part-time work to be 0.5 and being out of the labour force to be zero, we compute average labour force participation for each woman in the sample in each year. If this average is above 0.75, we assign the woman to full-time work, if it is between 0.25 and 0.75, we assign her to part-time work, and if it is below 0.25, we assign her to be out of the labour force for that year.

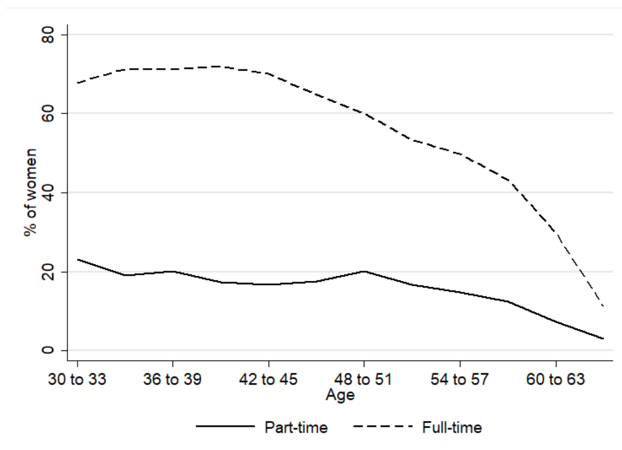
Panels (a) and (b) of Figure 1 show the 2011-2019 average fractions of part-time and full-time working women by age. Panel (a) shows those fractions among mothers and panel (b) among childless women. Both full-time and part-time work fall with age among mothers and childless women. Women in older cohorts with lower labour force attachment partially explain this result. Panel (c) plots the difference or gap between full and part-time work between mothers and childless women, plus the participation gap, which is the sum of the two. Unsurprisingly, there is a large (-15 percentage points) participation gap around age 30 that closes steadily and becomes nearly zero after age 50. This is the well known "family gap" in female labour force participation. The part-time gap is positive, meaning that mothers are more likely to work part-time. The full-time gap is negative, which means the opposite. Both close with time, although older mothers are still somewhat more likely to work part-time and less likely to work full time than childless women.

Figure 1: Labour force participation of woman by age (30-65) and motherhood status in Spain, 2011-2019

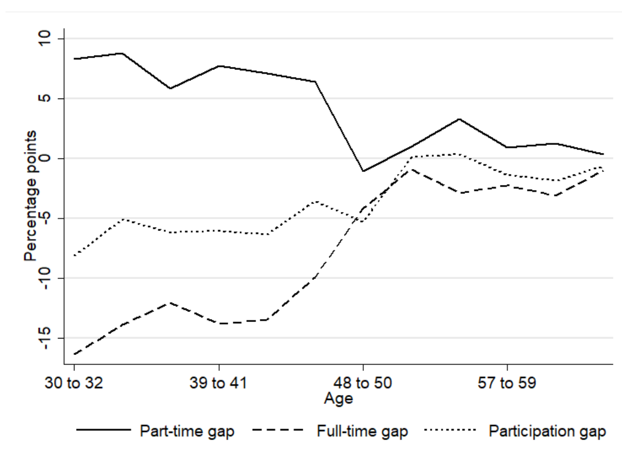
(a) Mothers



(b) Childless



(c) Gap



Source: Author's work with data from *INE's Encuesta de Condiciones de Vida*.

2.3 Childcare

We plot the average hours per week spent at preschool for children aged 0 to 3 and 3 to 6 by labour force participation of the mother in figure 2. There are several things to notice. First, as expected, the children of women who work full time spend more time in preschool than those who work part-time, and they spend more time than those out of the labour force. Second, children aged 3 to 6 spend more time at preschool than children aged 0 to 2 regardless of their mother's labour force participation status. Third, the difference in average preschool hours between children of working mothers and non-working mothers is much smaller among children aged 3 to 6. Children of women who work full-time aged 0 to 3 spend almost twice as many hours in preschool on average than children of women who are not in the labour force. Among children aged 3 to 6, the difference is only about 10%.

This difference is mostly a result of universal public preschool coverage for children older than three, introduced in the late 1990s. It comes as no surprise that most children between 3 and 6 spend almost 30 hours per week at preschool since public preschools are in general open for six hours per day[7].

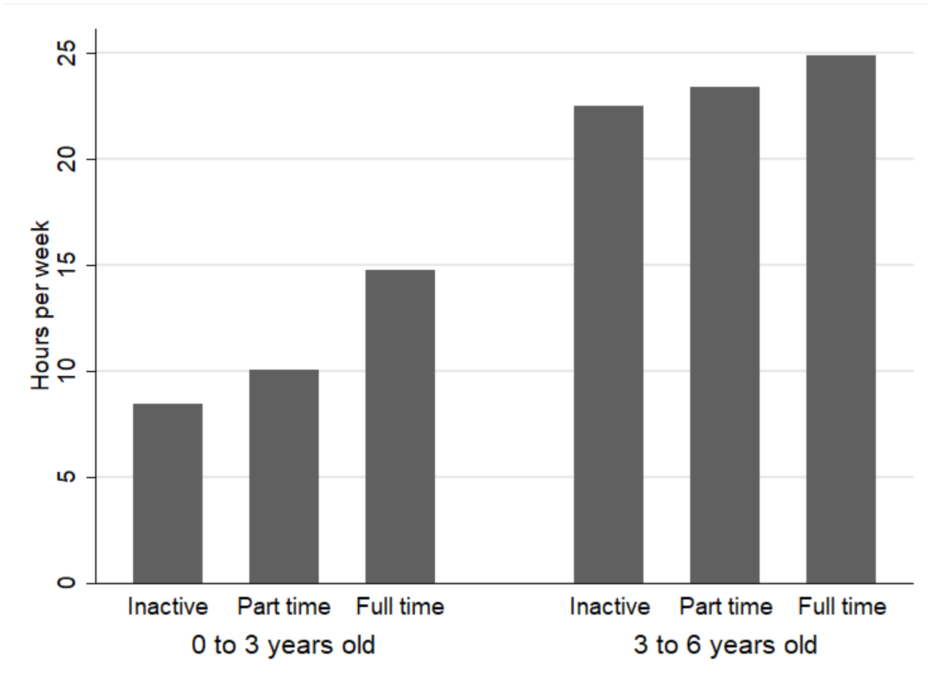
Preschool is not the only source of childcare used by working mothers, as it covers only about 15 of the 30-40 hours per week, and about 10 of the 15-20 hours per week required by full-time and part-time working mothers with children aged 0 to 3, respectively. The *ECV* includes information on childcare received without any payment in return. Grandparents, aunts, uncles, older siblings, extended family or neighbours might provide this childcare. This is called here informal childcare. Average hours spent under informal childcare for children aged 0 to 3 and 3 to 6 by the mother's labour force participation are shown in Figure 3.

The results are surprisingly low. Adding the average preschool and informal care hours still leaves some required childcare unaccounted for. Likely the informal childcare is under-reported in the survey. Father's childcare time is not measured separately in the *ECV*, although it is unlikely that this solves the issue since most fathers work full time. There are other childcare sources measured in the *ECV*, but none of them is significant on average.

To get a better idea of the father's contribution to childcare, *INE's* occasional *Encuesta del uso del tiempo*, i.e. Time Use Survey (TUS), is used. The last iteration of the survey ran between 2009 and 2010. Contrary to the American Time Use Survey (ATUS), the Spanish

[7] Average preschool hours are below 30 because some children aged 3 to 6 were not yet eligible at the time of the survey to attend public preschools. The reason is that they were not aged three by January 1st. Moreover, others may be already attending primary schooling. When we compute average preschool hours for children aged 4 and 5 at the time of the survey, who are eligible for public preschool and not for primary, we obtain an average of 30 hours for all levels of labour force participation of the mother.

Figure 2: Preschool usage by age of child and labour force participation status of mother (2011-2019)



Source: Author's work with data from *INE's Encuesta de Condiciones de Vida*.

one collects data for each household member[8]. Thus, it allows to study how parents split chilcare duties.

The average gender gap was -86.57%[9]. In other words, mothers provide 86.57% more of childcare compared to fathers. On average mothers spend 183 minutes per day looking after children while fathers spend 98 minutes. After controlling by labour market participation, three results stand out. First, the childcare gender gap is very close to the average among two-earner households[10], at -80.42%. Second, this gap dramatically increases to -174.09% in households where mothers are out of the labour force. Third, in households where women are the sole income provider, men perform slightly more childcare than women, as the gap among these households is 14.53%. A great majority of the households are two-earner, meaning that on average mothers do a great deal more childcare than fathers.

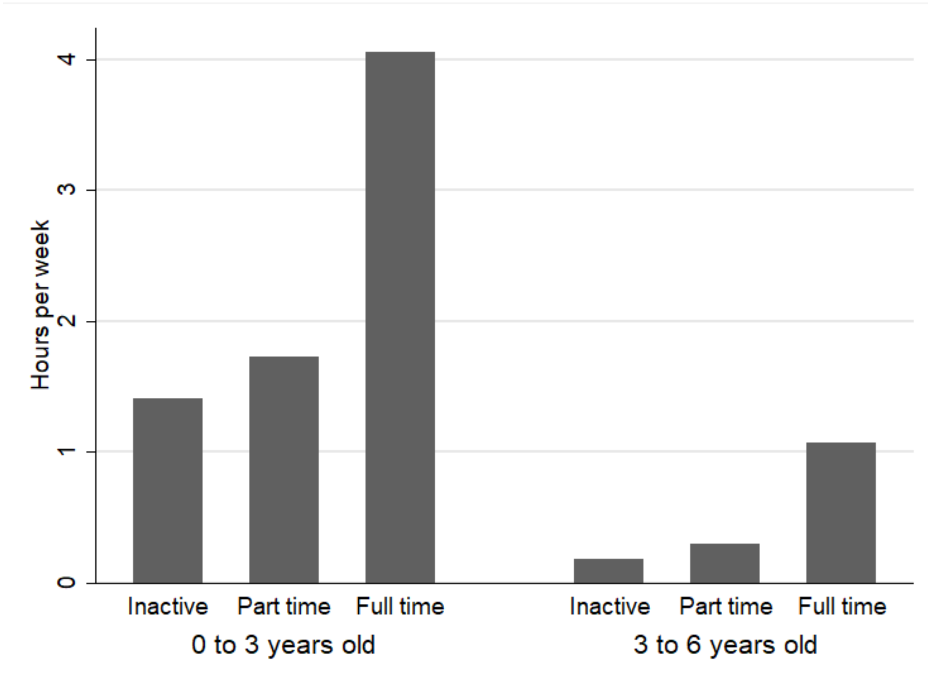
To sum up, the evidence points that mothers of children aged 0 to 3 are in a problematic childcare position. This stems from insufficient access to affordable options and persistent gender norms measured through the childcare gender gap.

[8]The ATUS collects data for one individual of each household.

[9]Defined as $(1 - \frac{\text{Mother's time}}{\text{Father's time}}) \times 100\%$.

[10]Households in which both spouses participate in the labour market.

Figure 3: Informal childcare usage by age of child and labour force participation status of mother (2011-2019)



Source: Author's work with data from *INE's Encuesta de Condiciones de Vida*.

2.4 Desired versus realised fertility

Like in most European countries, Spanish women would like to have more children than they do. In 2018, the Spanish Statistics Institute ran a new iteration of its fertility survey *Encuesta de Fecundidad*, henceforth *EdF* (the previous one is dated from 1999). Table 2 shows the realised and desired fertility for women aged 40 to 44, and the difference between them. We choose this age group because most of these women already completed their fertility at this point, meaning that the difference with desired fertility is definitive. The most popular fertility choice is two children, both in desired and realised terms.

Nevertheless, the data shows that the fraction of childless women is more than double the fraction of women who declare not wanting to have any children. There is gap of almost ten percentage points between the fraction of women with a single child and the fraction that declares wanting only one child. Less than half of the women that want three children have three children. These results show that there is a considerable number of women whose completed fertility is lower than their ideal one.

An additional question of interest is whether most of the gap between desired and realised fertility is due to some women being unable to have children (the extensive margin), or

Table 1: Desired and realised fertility, women aged 40 to 44 in 2018 in Spain

Number of children	Realised	Desired	Difference
0	18.99%	7.90%	11.09%
1	24.96%	15.20%	9.76%
2	43.79%	49.75%	-5.96%
3 or more	12.26%	27.15%	-14.89%

Source: *Encuesta de Fecundidad 2018, INE.*

Table 2: Desired fertility by motherhood status, women aged 40 to 44 in 2018 in Spain

Number	Mothers			Childless		
of children	Realised	Desired	Difference	Realised	Desired	Difference
0	0.00%	0.22%	-0.22%	100.00%	40.65%	59.35%
1	30.81%	13.97%	16.84%	0.00%	20.47%	-20.47%
2	54.06%	54.14%	-0.08%	0.00%	31.00%	-31.00%
3 or more	15.13%	31.67%	-16.54%	0.00%	7.88%	-7.88%

Source: *Encuesta de Fecundidad 2018, INE.*

whether some mothers would like to have more children (the intensive margin). Table 2 shows realised and desired fertility for mothers and childless women aged 40 to 44, and the difference between the two. First, notice that very few mothers declare not wanting children at all. This result shows that few women go beyond their desired fertility level. This seems reasonable because contraception and abortion are legal and widely available in Spain. Second, the number of desired children is much higher for mothers than for childless women. Thus, even among mothers, there is a difference between realised and the desired number of children. The fact that the number of women with two children coincides with the number of women declaring to want two children does not mean that every woman in this category is satisfied with her fertility. Likely a fraction of them would like to have three children, while a fraction of those with one would like to have two.

There are many potential reasons why women may choose not to have their desired number of children. The *EdF* asks women why they have fewer children than their desired number. The most common reasons given by women aged 40 to 44 are "work and work-family balance reasons" and "economic reasons".

Given the evidence laid out here, it seems that there is room for a childcare subsidisation policy targeted at women with children aged 0 to 3 years old to boost mother's labour force participation and fertility levels. These changes, in turn, would affect the social security budget. To quantify the magnitude of these changes, we develop the model described in the next section.

3 The model

We study an economy populated by overlapping generations of households that derive utility from consumption, leisure, the number of children they have, and the time spent with their children (if any).

Each household consists at least of a woman and her husband. At the beginning of life a fertility level is chosen. Conditional on this choice, children arrive and leave[11] the household following a fixed schedule. That is, households choose total fertility, but not the timing of births.

Working-age households make labour force participation decisions. Young children impose childcare needs upon parents that they need to cover with a combination of their own time, out-of-pocket childcare purchased in the market, and unpaid childcare time available to them. Therefore, households with young children need to balance labour force participation decisions and childcare needs.

Households eventually retire from the workforce. There are no savings, but the government runs a social security PAYG scheme financed with contributions from current working-age households in the form of a payroll tax. Retired households receive benefits that depend on women and husbands' income back when they were in the labour force. In the rest of the section, we describe the model in detail.

3.1 Demographics, endowments and heterogeneity

At each period t, a new generation of individuals is born and another one dies. Since the prices faced by each cohort are the same, the life-cycle decisions are the same for households with identical states. Therefore, the time subscript is omitted from household variables.

By the age that modelling of household decisions begins, we assume that individuals are already sorted into couples formed by a woman $i = f$ and her husband $i = m$. The model abstracts from singles[12], divorce and remarriage. Spouses are assumed to be the same age.

[11] Children eventually grow up and leave.

[12] This excludes 10.1% of Spanish households that were mono parental in 2019, out of which 82% were composed of women and her children. This households would likely respond very differently than two-parent households to child related policies, but introducing them would considerably complicate the model and it is left for future work.

At age $j = J_0$ households make a fertility decision and start working-age life. Mandatory retirement occurs at $j = J_R$, and the household dies with probability one at $j = J$, i.e. life expectancy is deterministic. The life cycle of the household is therefore divided into $W = J_R - J_0$ working-age, and $R = J - J_R + 1$ retirement periods.

Households start working-age with no assets, and they are unable to save their period's income or borrow from the future. They are endowed with one unit of time in each period, which can be devoted to work, leisure and looking after children. At age $j = J_0$, spouses receive a pair of correlated income shocks $(\epsilon^f_{J_0}, \epsilon^m_{J_0})$. This shocks then evolves following an AR(1) process:

$$\begin{aligned} \epsilon^f_j &= \phi^f \epsilon^f_{j-1} + \nu^f_j \\ \epsilon^m_j &= \phi^m \epsilon^m_{j-1} + \nu^m_j \end{aligned} \quad \text{with} \quad \begin{bmatrix} \nu^f_j \\ \nu^m_j \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma^2_{\nu^f} & \rho \\ \rho & \sigma^2_{\nu^m} \end{bmatrix}\right), \tag{1}$$

Apart from having different income shocks, households differ in their preferences over children. There are two sources of heterogeneity regarding fertility preferences: over the desired number of children and over the intensity of those preferences. The exact functional form this takes in the model is discussed in subsection 3.3.

3.2 Timing and choice variables

The length of each model period corresponds to three years. Period $j = J_0$ is set to start around age 30, while $W = 18$ and $R = 13$. That is, the working-age stage in the model spans ages 30-66, while the retirement stage goes from ages 66 to 84. For households with children, the working-age stage is further subdivided into two stages: child-rearing and non child-rearing. The former lasts while there are children younger than 12 years old present in the household.

In what follows, the choice variables in each stage are described.

Prior to $j = J_0$. Before starting the first period and after a couple is formed, the household decides on the number of children they will have. This decision is denoted by b, with $b \in \{0, 1, 2, 3\}$. That is, couples can have up to three children. If $b > 0$, the first child arrives at $j = 1$. If $b > 1$, a child is born in every subsequent period until the chosen number of children is reached. This implies a spacing between siblings of three years.

Working-age stage. In every working-age period, that is for $j < J_R$, households face a discrete labour supply choice. The alternatives available are full-time employment, part-time employment and staying out of the labour force. It is assumed that husbands are always employed full time. The labour force supply decision of the woman in period j is therefore simply denoted by $n_j \in \{0, \frac{1}{4}, \frac{1}{2}\}$. Taking non-sleeping hours to be 16 per day, this corresponds to full and part time employment taking 8 and 4 hours per day, respectively.

Child-rearing stage. If the household chooses a positive number of children, it enters working-age, child-rearing stage at $j = J_0$. The number of periods spent in this stage depends only on the chosen fertility, since the arrival of children is pre-determined. Let $W_c(b)$ be the number of child-bearing periods. Then,

$$W_c(b) = \begin{cases} 4 & \text{if } b = 1 \\ 5 & \text{if } b = 2 \\ 6 & \text{if } b = 3 \end{cases}.$$

The youngest child imposes a time constraint on parents. They must be looked after at all times. Therefore, additionally to deciding on the woman's labour supply, child-rearing households must decide how to cover their childcare needs. Various sources are considered: mother's own time, other relatives' time (including the father and grandparents), formal institutions such as kindergartens and schools. We aggregate them into three types of childcare: mother's own time (h_j^m), unpaid childcare (h_j^u) and out-of-pocket childcare purchased in the market (h_j^p). By choosing two of them, the household is implicitly choosing the third via the time constraint. Therefore, from here on h_j^m and h_j^p are the two additional choice variables in the child-rearing stage.

3.3 Preferences

The basic preference structure follows Bick (2016), who in turn points to Greenwood et al. (2003) and Jones et al. (2010) as previous examples. However, we introduce important differences regarding utility related to children.

As in Bick (2016), the utility that the household maximises is that of the woman, which can be interpreted either as her being the sole decision maker, or her having full bargaining power. Therefore, the gender subscript i is dropped for all choice variables. The utility function varies over the life cycle for mothers, depending on whether she is in the child-bearing stage or not. The utility function in the various stages is described below.

Child-bearing mothers. At age j, a woman that has children ($b > 0$) and is of child-bearing age ($j \leq W_c(b)$) derives utility from consumption c_j, leisure l_j, time spent caring for her children h_j^m and number of children $\Gamma(b)$:

$$u_j\left(c_j, l_j, h_j^m; b\right) = \frac{\left(\frac{c_j}{\psi_j(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} + \delta_3 \left(h_j^m\right)^{\gamma_3} + \Gamma(b), \tag{2}$$

where $\psi_j(b)$ is an age-dependent consumption equivalence scale, and:

$$\Gamma(b) = -\delta_2 |b - b^*|^{\gamma_2} + \zeta. \tag{3}$$

That is, utility is the sum of two constant relative risk aversion terms for consumption and leisure, a term related to childcare provided by the mother and a term related to the number of children. The first two are standard, but the last two deserves some discussion.

Child bearing women derive utility from providing childcare (spending time with their children). That means they face a three-way trade off regarding their time: each unit of time can be used to work, which increases consumption, enjoy leisure or care after young children. The parameters δ_3 and γ_3 determine how much women value time with children, and how fast the marginal utility from it falls (if $\gamma_3 < 1$) or rises (if $\gamma_3 > 1$).

Utility from the number of children has two components. The first component depends negatively on the absolute value of the difference between desired and realised fertility. That is, there is a utility penalty from having a number of children different to the ideal one. The parameter $\delta_2 > 0$ determines how large this penalty is, while γ_2 determines the curvature of the penalty with respect to the difference between desired and realised fertility. The second term is a parameter ζ, that could be positive or negative, which is a fixed utility cost or gain from motherhood.

Non child-bearing mothers. At age j, a woman that has children ($b > 0$) and is not of child-bearing age ($j > W_c(b)$) derives utility only from consumption c_j, leisure l_j and number of children $\Gamma(b)$:

$$u_j\left(c_j, l_j; b\right) = \frac{\left(\frac{c_j}{\psi_j(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} + \Gamma(b). \tag{4}$$

That is, women past child bearing stage still enjoy utility from the number of children (and in the same amount as child bearing women), but since they don't need to be looked after any more they do not derive utility from childcare they provide themselves.

Childless women. At age j, a childless woman's utility is:

$$u_j\left(c_j, l_j; 0\right) = \frac{\left(\frac{c_j}{\psi(0)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} - \delta_2 b^{*\gamma_2}. \tag{5}$$

The first two terms are identical to those for mothers, with the equivalence scale evaluated at $b = 0$. The third term is equivalent to the first component of the term related to the number of children for mothers, as $-\delta_2|0 - b^*|^{\gamma_2} = -\delta_2 b^{*\gamma_2}$. That is, childless women do experience disutility from the children that they did not have but want to have. Notice that this term is 0 if $b^* = 0$. That is, women that do not want to have children and do not have them do not experience any utility penalty. The term is negative if $b > 0$. This can be interpreted as fertility regret or dissatisfaction.

Preference heterogeneity. Parameters γ_0, δ_1, γ_1, δ_2, δ_3 and γ_3 are the same for all women. However, δ_2, ζ, and desired fertility b^* are allowed to vary across women. Notice that if all women had the same preferences for children, either with this or other utility additively separable specification, all variation in fertility would have to come from differences the initial income shock vector, as is the case in Bick (2016). This is a bit too restrictive, as variation in fertility within households with similar incomes and/or education levels is indeed observed.

3.4 Income and constraints

During working-age stage, full-time gross income depends on gender i, accumulated experience x_j^i and the current shock ϵ_j^i. Since we assume husbands work full time in every period, $x_j^m = j - J_0$, $\forall j < J_R$, and the gender subscript for women's experience can be dropped, $x_j^f = x_j$, $\forall j < J_R$. Full-time gross income for a woman and her husband at age j, with experience x_j and income shocks $\left(\epsilon_j^f, \epsilon_j^m\right)$ is:

$$\begin{aligned} \ln y_j^f &= \eta^f + \eta_1^f x_j + \eta_2^f x_j^2 + \epsilon_j^f \\ \ln y_j^m &= \eta^m + \eta_1^m \left(j - J_0\right) + \eta_2^m \left(j - J_0\right)^2 + \epsilon_j^m. \end{aligned} \tag{6}$$

As there are no savings, consumption will be equal to net income minus out-of-pocket childcare costs, if any. Net income is equal to actual gross earnings minus taxes. For women, actual gross earnings are full-time gross earnings multiplied by the labour supply choice times two as $n_j = 1/2$ denotes full-time work. That is, a woman working part time earns half as much as she would if she worked full time. If retired, the household consumes its pension benefits, which are defined at time J_R and depend on prior average earnings at the time of retirement, $\bar{y}^f_{J_R}$ and $\bar{y}^m_{J_R}$. Moreover, in the case of the woman, the pension depends on how many years she worked, i.e. her experience at age J_R, x_{J_R}. Pension benefits are denoted by $p_f\left(\bar{y}^f_{J_R}, x_{J_R}\right)$ and $p_m\left(\bar{y}^m_{J_R}\right)$. The budget constraint in each stage of life is therefore:

$$c_j = \begin{cases} 2n_j y^f_j + y^m_j - T\left(y^f_j, y^m_j, n_j\right) - \lambda_j\left(h^p_j, b\right) & \text{if } b > 0 \text{ and } j < J_0 + W_c(b) \\ 2n_j y^f_j + y^m_j - T\left(y^f_j, y^m_j, n_j\right) & \text{if } b = 0 \text{ or } b > 0 \text{ and } J_0 + W_c(b) \leq j < J_R \\ p_f\left(\bar{y}^f_{J_R}, x_{J_R}\right) + p_m\left(\bar{y}^m_{J_R}\right) & \text{if } j \geq J_R \end{cases},$$

where $T\left(y^f_j, y^m_j, n_j\right)$ is the tax payable by the household, and $\lambda_j\left(h^p_j, b\right)$ is the cost of out-of-pocket childcare, which takes the form:

$$\lambda_j\left(h^p_j, b\right) = q\left(1 - \Upsilon\right)\sqrt{b}h^p_j.$$

The cost of out-of-pocket childcare is therefore a concave function of the number of children, where the price per unit of time is denoted by q, and $\Upsilon \in [0, 1]$ denotes the fraction of the cost that is subsidised.

The time constraints faced by a working-age woman depends on whether she has children, and if she does on whether she is in child-bearing stage. Retired women don't work and can use all of the time for leisure. In general, the endowment of time is normalised to 1 in every period. The sum of hours spent working, enjoying leisure and taking care of children (if any) must be 1:

$$\begin{cases} l_j + n_j + h^m_j = 1 & \text{if } \text{ and } j < J_0 + W_c(b) \\ l_j + n_j = 1 & \text{if } b = 0 \text{ or } b > 0 \text{ and } J_0 + W_c(b) \leq j < J_R \\ l_j = 1 & \text{if } j \geq J_R \end{cases}$$

Moreover, there is an additional constraint imposed by young children on child-bearing mothers. So, if $b > 0$:

$$h_j^m + h_j^u + h_j^p = 1 \quad \forall j < J_0 + W_c(b).$$

There is an endowment of unpaid childcare available to the household that sets an upper bound for h_j^u. Likewise, mandatory schooling sets a lower bound for unpaid childcare use. Both the endowment and the amount of mandatory schooling time depend on the age of the children, which is a function of age of the household and number of children. Formally:

$$m_j(b) \leq h_j^u \leq \kappa_j(b),$$

where $m_j(b)$ is the amount of mandatory schooling and $\kappa_j(b)$ the endowment of unpaid childcare.

3.5 Social security and population dynamics

There is a pay-as-you-go social security system that taxes current working-age population's earnings and provides pension benefits to retired individuals. The social security tax rate, Ω is proportional to gross earnings up to a cap, d_1. The tax rate that is levied on the worker is denoted by τ. Total social security contributions attributable to a household with full-time earnings y_j^f and y_j^m, and choice of labour supply for the woman n_j are:

$$\mathcal{C}\left(y_j^f, y_j^m, n_j\right) = \Omega\left[\min\left(2n_j y_j^f, d_1\right) + \min\left(y_j^m, d_1\right)\right],$$

while the contributions payable by the household, which should be deducted from gross income in the household's budget constraint, are:

$$T\left(y_j^f, y_j^m, n_j\right) = \tau\left[\min\left(2n_j y_j^f, d_1\right) + \min\left(y_j^m, d_1\right)\right].$$

The retirement income for a household is the sum of the woman's and her husband's pensions, which depends on $\bar{y}_{J_R}^f$, x_{J_R} and $\bar{y}_{J_R}^m$.

The formula to compute the pensions reflects Spanish Social Security rules. In general, the pension is equal to a fraction (called the wage replacement rate) of the average taxable earnings over the last N_R periods of working-age life. The wage replacement rate depends on the number of years worked over the life cycle. Since it is assumed that all men work full time in every period, the wage replacement rate is the same for all men, and it is denoted θ_m. For women, it depends on the experience accumulated upon retirement, and it is denoted $\theta\left(x_{J_R}\right)$. Moreover, there are maximum and minimum pensions, denoted by $\bar{p}$ and $\underline{p}$, respectively.

The first step to compute the pensions is to establish the average taxable earnings:

$$\begin{aligned}
\bar{y}_{J_R}^f &= \frac{\sum_{j=R-N_R-1}^{R-1} \min\left\{d_1, 2n_j y_j^f\right\}}{N_R} \\
\bar{y}_{J_R}^m &= \frac{\sum_{j=R-N_R-1}^{R-1} \min\left\{d_1, y_j^m\right\}}{N_R}.
\end{aligned}$$

Then, the actual pensions are computed as:

$$\begin{aligned}
p\left(\bar{y}_{J_R}^f, x_{J_R}\right) &= \min\left\{\bar{p}, \max\left\{\theta_f\left(x_{J_R}\right)\bar{y}_{J_R}^f, \underline{p}\right\}\right\} \\
p\left(\bar{y}_{J_R}^m, x_{J_R}\right) &= \min\left\{\bar{p}, \max\left\{\theta_m \bar{y}_{J_R}^m, \underline{p}\right\}\right\}
\end{aligned}$$

To describe aggregate social security revenue and expenditure per time period, it is necessary to describe first how population evolves in time. We assume that there is an initial population, the fraction of which is aged j at time $t = 0$ is given by $\mathcal{P}_0^j$. Let $\mathcal{F}(b)$ denote the fraction of women that have $b \in \{1, 2, 3\}$ children. Notice that there is no time subscript, since the same fraction of women are going to have each number of children in every cohort. Starting from $\left\{\mathcal{P}_0^j\right\}_{j=1}^J$, population evolves according to:

$$\mathcal{P}_{t+1}^j = \begin{cases} \sum_{k=0}^{2}\left[\sum_{b=k+1}^{3}\mathcal{F}(b)\,\mathcal{P}_t^{J_0+k}\right] & \text{if } j = 1 \\ \mathcal{P}_t^{j-1} & \text{if } 1 < j \leq J \end{cases}.$$

That is, as a result of the sequential arrival of children to the household, the size of the new cohort $j = 1$ at time $t+1$ is equal to the fraction of women in cohort J_0 at time t having children, $\sum_{b=1}^{3}\mathcal{F}(b)\,\mathcal{P}_t^{J_0}$ plus the fraction of women from cohort $J_0 + 1$ at time t that chose to have two and three children $\sum_{b=2}^{3}\mathcal{F}(b)\,\mathcal{P}_t^{J_0+1}$ plus the fraction of women from cohort $J_0 + 2$ at time t that chose to have three children $\mathcal{F}(3)\,\mathcal{P}_t^{J_0+2}$. The size of cohort aged j at $t+1$ is just the size of cohort aged $j-1$ at t, as there is no inter-year mortality. Notice that $\sum_{j=1}^{J}\mathcal{P}_t^j \neq 1$ in general. That is, total population can grow or fall depending on fertility, but it is normalised to 1 at $t = 0$.

Let $\bar{\mathcal{C}}_j$ be the average contribution to social security attributable to households aged $j \in [J_0, J_R - 1]$. Aggregate social security revenue in period t is then:

$$SS_t^R = \sum_{j=J_0}^{J_R-1} \mathcal{P}_t^j \bar{\mathcal{C}}_j.$$

Likewise, let $\bar{\mathcal{B}}_j$ be the average pension received by households aged $j \in [J_R, J]$. Aggregate social security expenditure in period t is:

$$SS_t^X = \sum_{j=J_R}^{J} \mathcal{P}_t^j \bar{\mathcal{B}}_j.$$

Finally. the balance of the social security in period t is:

$$SS_t = SS_t^R - SS_t^X.$$

3.6 Household problem in recursive form

In this subsection we formally write and discuss the household problem in recursive form, starting from the last stage of life and going backwards.

Retirement stage. Households need not to make any decisions in the retirement stage. In each period, the household consumes its pension income, which depends on x_{J_R}, $\bar{y}^f_{J_R}$ and $\bar{y}^m_{J_R}$. Women do not work or take care of any children, so all time is devoted to leisure, i.e. $l_j = 1$ for $j \in [J_R, J]$. Therefore, the value of retirement at time J_R is:

$$\begin{aligned} V_{J_R}\left(x_{J_R}, \bar{y}^f_{J_R}, \bar{y}^m_{J_R}; b\right) &= \sum_{j=J_R}^{J} \beta^{j-J_R} u_j \left[p_f\left(\bar{y}^f_{J_R}, x_{J_R}\right) + p_m\left(\bar{y}^m_{J_R}\right), 1; b\right] \\ &= \frac{1-\beta^R}{1-\beta} u_j \left[p_f\left(\bar{y}^f_{J_R}, x_{J_R}\right) + p_m\left(\bar{y}^m_{J_R}\right), 1; b\right] \end{aligned}$$

Working-age non child-rearing/working-age childless stage. In each period, women observe theirs and their husband's income shocks and decide on their labour supply, by optimally choosing to be either out of labour force, work part time or work full time. By working more, they are able to consume more in the current period, accumulate more labour market experience that potentially increases income in future periods and increase their average earnings for a higher pension (if the current period counts towards its calculation, i.e. if $j \geq J_R - N_R$). However, the leisure component of utility goes down. The problem solved by households in this stage is:

$$
\begin{aligned}
V_j\left(\epsilon_j^f, \epsilon_j^m, x_j, \bar{y}_j^f, \bar{y}_j^m; b\right) = \max_{\substack{c_j \geq 0 \\ n_j \in \{0, \frac{1}{4}, \frac{1}{2}\}}} \quad & u_j\left(c_j, l_j; b\right) + \beta\,\mathbb{E}\left[V_{j+1}\left(\epsilon_{j+1}^f, \epsilon_{j+1}^m, x_{j+1}, \bar{y}_{j+1}^f, \bar{y}_{j+1}^m; b\right)\right] \\
\text{s.t.} \quad & \\
& c_j = 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) \\
& l_j + n_j = 1 \\
& \ln y_j^f = \eta^f + \eta_1^f x_j + \eta_2^f x_j^2 + \epsilon_j^f \\
& \ln y_j^m = \eta^m + \eta_1^m \left(j - J_0\right) + \eta_2^m \left(j - J_0\right)^2 + \epsilon_j^m \\
& \epsilon_j^f = \phi^f \epsilon_{j-1}^f + \nu_j^f \\
& \epsilon_j^m = \phi^m \epsilon_{j-1}^m + \nu_j^m \\
& x_{j+1} = x_j + 2n_j \\
& \bar{y}_{j+1}^f = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}_j^f N_R + \min\left\{d_1, 2n_j y_j^f\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases} \\
& \bar{y}_{j+1}^m = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}_j^m N_R + \min\left\{d_1, y_j^m\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases}
\end{aligned}
$$

Working-age child-rearing. The problem faced by child-rearing women is similar to the one solved by women past this stage, but it is more complicated. Additionally to choosing labour supply, they face an additional constraint regarding childcare. Providing it themselves provides utility, but reduces leisure. Purchasing it out-of-pocket reduces consumption. Notice that, conditional on a labour supply decision, childcare allocation is a static problem. Appendix A.1 shows that it has a unique solution under reasonable parameter values, and characterises this solution. The problem solved by households in this stage is:

$$
\begin{aligned}
V_j(\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j^f, \bar{y}_j^m; b) = & \max_{\substack{c_j \geq 0 \\ n_j \in \{0, \frac{1}{4}, \frac{1}{2}\} \\ h_j^m, h_j^p, h_j^u \geq 0}} u_j\left(c_j, l_j, h_j^m; b\right) + \beta\, \mathbb{E}\left[V_{j+1}\left(\epsilon_{j+1}, \epsilon_{j+1}^h, x_{j+1}, \bar{y}_{j+1}^f, \bar{y}_{j+1}^m; b\right)\right] \\
& \text{s.t.} \\
& c_j = 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j\left(h_j^p, b\right) \\
& \lambda_j\left(h_j^p, b\right) = q\left(1 - \Upsilon\right)\sqrt{b} h_j^p \\
& l_j + n_j + h_j^m = 1 \\
& h_j^m + h_j^p + h_j^u = 1 \\
& m_j(b) \leq h_j^u \leq \kappa_j(b) \\
& \ln y_j^f = \eta^f + \eta_1^f x_j + \eta_2^f x_j^2 + \epsilon_j^f \\
& \ln y_j^m = \eta^m + \eta_1^m\left(j - J_0\right) + \eta_2^m\left(j - J_0\right)^2 + \epsilon_j^m \\
& \epsilon_j^f = \phi^f \epsilon_{j-1}^f + \nu_j^f \\
& \epsilon_j^m = \phi^m \epsilon_{j-1}^m + \nu_j^m \\
& x_{j+1} = x_j + 2n_j \\
& \bar{y}_{j+1}^f = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}_j^f N_R + \min\left\{d_1, 2n_j y_j^f\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases} \\
& \bar{y}_{j+1}^m = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}_j^m N_R + \min\left\{d_1, y_j^m\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases}
\end{aligned}
$$

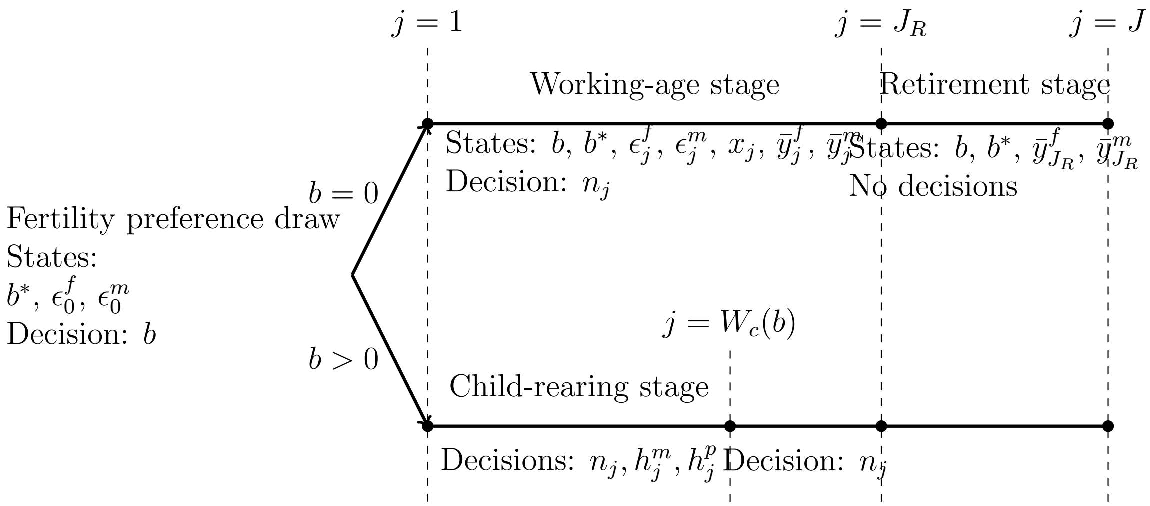
Fertility decision. Prior to age $j = J_0$, the woman draws her fertility preferences, observes hers and her husband's initial income shocks, and decides the total number of children she will bear. From then on, b becomes a state for the household. The problem at this stage is:

$$
\max_{b \in \{0,1,2,3\}} V_{J_0}\left(\epsilon_0^f, \epsilon_0^n, x_0, \bar{y}_0^f, \bar{y}_0^m; b\right)
$$

where $x_0 = 0$, $\bar{y}_0 = 0$ and $\bar{y}_0^h = 0$.

Figure 4 shows states and decisions through the life cycle.

Figure 4: States and decisions through the life cycle



4 Calibration

The calibration strategy we follow can be divided in steps. First, we choose a set of parameters exogenously, either by taking them from previous literature, directly from the data, or from Spanish law, as is the case for the parameters governing social security. We estimate a second set of parameters without solving the model, notably the income process and the price of childcare. Finally, we choose the remaining parameters by solving the model in equilibrium. Since there is no closed-form solution, we rely on the method of simulated moments.

4.1 Parameters chosen exogenously

The set of parameters chosen exogenously includes the subjective discount factor β, the functional form of the equivalence scale $\psi(\cdot)$, the initial population $\left\{\mathcal{P}_0^j\right\}_{j=1}^J$, the probability distribution for the draw of desired number of children b^* and the parameters for social security contributions and benefit calculations.

The subjective discount factor is set to be $\beta = (1/1.04)^3 = 0.89$, which is the standard time discount factor used in macro since Kydland and Prescott (1982). The equivalence scale for household consumption we use is the OECD scale, i.e. $\psi(b) = 1.7 + \sqrt{b}$. The initial population is taken from *INE* for the year 2020. We aggregate cohorts into brackets of 3 years, to match the model period's length. The probability distribution for the desired number of children draw we use is the fraction of women aged 40-44 that declares wanting 0, 1, 2 and 3 or more children in *INE*'s 2018 *Encuesta de Fecundidad* presented in Table 1.

Social security. The parameters governing social security contribution and retirement pension calculation are set to match as closely as possible the Spanish *Régimen General de la Seguridad Social* (RGSS), which is the regime that covers most of the workers in the country. The rules we use for the baseline calibration are the ones in place in 2013, which excludes the latest pension reforms. This is done to avoid changing legislation over time, as the reforms have been introduced progressively. Moreover, since some of the dispositions will only be implemented by 2027, the reforms have little impact for a large share of women making labour force participation decisions during the period covered by the data.

The retirement age set by the RGSS is 65 years. As J_0 is set to start at age 30 and there are 18 three-year working-age periods, retirement in the model J_R happens when the person turns 66 years old, which does not perfectly match the law, but is close. The number of contribution periods that count towards retirement pension computation in the law is 15 years, which fits perfectly with the last 5 working-age periods in the model. That is, $N_R = 5$.

The replacement rate defined in the RGSS $\theta_{RGSS}(\chi)$ depends on the number of years that the worker has contributed χ, where part-time work for a year counts as half of full-time work over the same period, according to the schedule:

$$\theta_{RGSS}(\chi) = \begin{cases} 0 & \text{if } \chi < 15 \\ 0.5 + 0.03\,(\chi - 15) & \text{if } 15 \leq \chi < 25 \\ 0.8 + 0.02\,(\chi - 25) & \text{if } 25 \leq \chi < 35 \\ 1 & \text{if } \chi \geq 35 \end{cases}.$$

Again, since there are 12 three-year working-age periods and it is assumed that husbands work full time always, all men have the same replacement rate $\theta_m = 1$. For women, it is a function of accumulated labour market experience x_j and follows the following schedule:

$$\theta_f(x_j) = \begin{cases} 0 & \text{if } x_j < 5 \\ 0.5 + 0.03\,(x_j - 5) & \text{if } 5 \leq x_j \leq 8 \\ 0.8 + 0.02\,(x_j - 8) & \text{if } 8 < x_j < 12 \\ 1 & \text{if } x_j = 12, \end{cases}$$

where again, the fit between model periods and the kinks in the schedule that determines the replacement rate is not perfect, but is the closest possible given the three year model periods.

The RGSS collects a proportional tax on covered earnings in order to finance pensions. Covered earnings are defined as the worker's gross labour income with a cap above and a tax-exempt minimum. Following Díaz-Giménez and Díaz-Saavedra (2009), the tax-exempt minimum is not included. In 2019, the total payroll tax was 28.3%, where 11.7% was levied on the employer and the remaining 16.6% was levied on workers. Therefore, Ω is set to 28.3% and τ to 16.6%. The cap on earnings for contribution calculation was 56 981 euros per year. As in the model one period represents three years, d_1 is set to 170 944. Finally, the minimum pension set out in the law is 9 081 euros per year, which comes to 27 243 euros in three years, while the maximum one is 37 566 euros per year, or 112 698 per model period. Table 3 summarises the social security parameters used.

Table 3: Social security parameters

Parameter	**Description**	**Value**
J_R	Retirement age (model periods)	22
N_R	Regulatory base (model periods)	5
Ω	Total payroll tax (%)	28.3
τ	Payroll tax levied on workers (%)	16.6
d_1	Maximum taxable period earnings (euros/model period)	170 944
$\underline{p}$	Minimum retirement pension (euros/model period)	27 243
$\bar{p}$	Maximum retirement pension (euros/model period)	112 698

4.2 Parameters estimated without solving the model

We estimate the next set of parameters using data from various sources, without having to solve the model. This includes the income process and the price of childcare. The estimation is discussed in that order.

The income process. The income process described by equations 1 and 6 is estimated using the *Muestra Continua de Vidas Laborales* (MCVL), the Spanish Social Security database on working life histories.

The methodology followed is based on Bick (2016). Mincer (1974) equations are estimated separately for men and women to obtain the parameters of the income process, and then the residuals of the Mincer regressions are regressed on the first lag to estimate the persistence of the income shocks. Since the data is at the individual level, it is not possible to directly estimate the correlation between wife and husband's income shocks, ρ. As in Bick (2016) and Attanasio et al. (2008), the value of 0.25 that Hyslop (2001) estimated for the United States is used. For the numerical solution of the model, the income shock process is discretised using the methodology proposed by Tauchen and Hussey (1991). Table 4 presents the results of the estimation.

Table 4: Income process parameters

Parameter	**Value**	**Parameter**	**Value**
η_0^f	9.2860	η_0^m	9.5330
η_1^f	0.0326	η_1^m	0.0318
η_2^f	-0.0003	η_2^m	-0.0004
ϕ^f	0.6550	ϕ^m	0.6540
σ^f	0.0342	σ^m	0.0370

The price of childcare. Following Sánchez-Mangas and Sánchez-Marcos (2008), the price of out-of-pocket childcare q, is computed so that a couple aged J_0 with average income shocks in which both spouses work full time and that has one child attending full time spends 33% of their earnings on childcare. That is, q solves:

$$q = 2 \times 0.33 \left[3 \left(e^{\eta_0^f} + e^{\eta_0^m} - T \left(e^{\eta_0^f}, e^{\eta_0^m}, \frac{1}{2} \right) \right) \right].$$

The result of this calculation comes down to 37 053 euros per model period, or 12 351 euros per year.

4.3 Parameters estimated by solving the model

Since the utility function is additively separable, the utility term associated with children's mere presence does not affect labour supply decisions conditional on the number of children chosen. To take advantage of this, the rest of the parameters, namely γ_0, γ_1, γ_2, γ_3, δ_1 and δ_3, are calibrated separately. Taking the observed distribution of children across women, the method of simulated moments is used to choose these parameters to replicate several labour supply moments along the life cycle. Even though all parameters affect all moments, it is worth to discuss the chosen targeted moments and how they relate to specific parameters.

The curvature of consumption γ_0 is informative about differences in labour force participation between women with different number of children through the potential monetary costs that childcare entails and through consumption adjustment via the equivalence scale.

Therefore we include the fraction of women with children aged 6 to 12 who work full time and the fraction of childless women the same age working full-time as targets.

The curvature of leisure γ_1 is informative about women's labour supply decisions with different household income levels at the intensive margin (part-time versus full time). As a target for this parameter, we include the fraction of women with children aged 6 to 12 working part-time, and the fraction of childless women the same age working part-time.

The utility weight of leisure δ_1 is informative for labour force participation decisions, i.e. labour supply at the extensive margin. Thus we include the total labour force participation for childless women (lifetime average) as a target.

The parameters associated with childcare δ_3 and γ_3 govern the utility mothers derive from spending time with children, and are therefore informative about labour force supply for women with children and their childcare utilization. So, we include labour force participation of women with children aged 0-2 and the paid childcare utilization of women that do not work as targets.

Once we choose the labour supply parameters, it is possible to compute lifetime utility, excluding the term related to children. This term will be different for each fertility level because of the equivalence scale and the time constraints imposed by children.

To finish the calibration, a set of parameters needs to be set such that the fraction of households that choose to have 0, 1, 2 and 3 children coincides with the number of households that have them in the data.

Intuitively, having more children lowers lifetime utility because they lower equivalent consumption and leisure. Fertility parameters need to be such that households are compensated for this difference, just in the right amounts on average so that their fertility decisions are close to the observed ones.

If all households have the same fertility parameters, variation in fertility decisions is based only on the initial income shocks and differences in the desired number of children. This is not enough to generate the observed distribution of children. Therefore, each household's parameters δ_2 and ζ are drawn from an exponential distribution. That is, households are heterogeneous in how much they care for departures from their ideal level of fertility b^* and how much they care about having children at all. The parameters that we estimated are the underlying means of those draws.

4.4 Calibration results

The calibration results for the baseline economy are shown in Table 5. Parameters and moments related to labour force participation and childcare, and those related to fertility decisions are shown separately.

The estimated curvature parameters for consumption and leisure γ_0 and γ_1 are relatively close to 1. This implies that the consumption and leisure utility terms are close to being logarithmic. Therefore, changes in the earnings potential have a muted effect on labour force participation, as income and substitution effects would roughly cancel each other.

The model is able to reproduce quite closely the total labour force participation of women whose youngest child is 0 to 3 years old, and the childcare usage of women that do not work and that have a child aged 0 to 3 years old. In the baseline economy, the share of women working full time is lower than in the data, while the share of women working part-time is higher for both women with the youngest child aged 6-12 and women aged 48-59. However, the total labour force participation is close for women with children aged 6-12. Childless women work too much with respect to the data. These results seem to suggest that γ_1 should be set at a higher value and δ_1 at a lower one, according to the previous discussion as to how each parameter affects each of the moments. However, in reality, all parameters affect all moments. The model is over identified as there are more moments than parameters. Changing γ_1 and δ_1 affects its ability to reproduce the labour force participation and childcare usage of women with children 0-2, which are very important given the nature of the question of this paper.

It is also likely that a few things are missing in the current setting. The effect of experience on young women is probably underestimated. Currently, the cost of non-participation is just missed experience. However, early-career interruptions can be particularly costly. Moreover, the high prevalence of zero-hour contracts in Spain could increase the value of labour force participation early on. Working full time may increase the likelihood of converting one's contract into an indefinite contract in the future, which would factor in the labour force participation decision. Allowing for these kinds of effects within the model is left for future work. Reflecting this in a tractable manner within the framework laid out here has proven to be challenging.

In terms of fertility, the model matches the share of childless women closely, but overestimates the fraction of women with one child and underestimates the fraction of women with two with respect to the data. This is partly compensated by a slightly higher fraction of women with three children so that the total fertility rate is only slightly below the one in the data.

Table 5: Baseline calibration results

Parameter	Value	Target(s)	Model	Data
Labour force participation and childcare				
γ_0	0.968	Share full-time work	32.72%	47.03%
		(women with youngest child 6-12)		
		Share full-time work	30.39%	41.68%
		(women aged 48-59)		
γ_1	1.011	Share part-time work	44.04%	20.32%
		(women with youngest child 6-12)		
		Share part-time work	44.77%	13.41%
		(women aged 48-59)		
γ_3	0.730	Labour force participation	69.81%	71.28%
		(women with youngest child 0-3)		
δ_1	1.062	Average lifetime LFP	83.03%	66.49%
		(childless women)		
δ_3	2.330	Weekly paid childcare hours	9.74	8.80
		(non-working women with child 0-2)		
Fertility				
		Share of women with:		
ζ	0.10	0 children	24.50%	25.50%
		1 child	35.40%	27.40%
δ_2	1.60	2 children	30.50%	39.80%
γ_2	3.00	3 children	9.60%	7.30%
		Total fertility rate	1.25	1.29

5 Policy experiments

In this section, the model is used to test three different policies. The first two have been implemented in the past by the Spanish government. The third one has not. The first to be discussed are the one implemented in the past.

5.1 Universal childcare coverage for children aged 3-6 and baby checks

In an attempt to compare the quantitative model to empirical evidence of child-related policies' effectiveness, we test in the model two policies that have been assessed in the empirical literature.

The first one is the universal offering of public childcare for 3-6-year-old children in Spain's late 1990s. This policy's impact was studied by Nollenberger and Rodríguez-Planas (2015) using its staggered implementation across Autonomous Communities as a natural experiment. They find that the policy raised maternal employment by 9.6% for women with children in this age group, with the results for women older than 30 and with two or more children being larger, 15.3% and 14.9% respectively, and no effect on fertility.

Since in the baseline calibration there is a provision of 30 hours per week of schooling for children aged 3-6, such provision is removed and the model is solved with the same parameters used in the baseline. The differences in labour force participation for women with children aged 3-6 and in fertility between the economy with no provision of schooling and the baseline economy are then computed. This can be compared to the results in Nollenberger and Rodríguez-Planas (2015). There are some caveats, though. In the model, women with children aged 3-6 are either 33 to 36, 36 to 39 or 39 to 42 years old by definition. Moreover, those aged 33 to 36 have one or two children. Those aged 36 to 39 have two or three, and those aged 39 to 42 have three children.

Table 6 presents the results of the exercise. Labour force participation of older women (which are also the ones that have more children) reacts strongly to the removal of the schooling provision. However, the magnitude of the effect is much larger than in Nollenberger and Rodríguez-Planas (2015) though. The effect on fertility in the model is very small, while they don't find any.

The second policy we test in the model is the "cheque bebé" (baby check). This was a one-time cash transfer upon the birth of 2500 euros offered to parents by the Spanish government between 2007 and 2010. González (2013) evaluates this policy empirically, and finds negatives effect on labour force participation after childbirth and a significant increase in fertility.

Table 6: The effect of removing free provision of childcare for children aged 3 to 6.

Outcome	**Baseline**	**No free scholing 3-6**	**Difference**
Labour force participation			
(women with child 3 to 6):			
Aged 33-36	69.67%	71.99%	2.33%
Aged 36-39	65.59%	48.51%	-17.08%
Aged 39-42	50.00%	35.00%	-15.00%
Fertility			
Share of women with:			
0 children	24.50%	24.30%	-0.20%
1 child	35.40%	35.50%	0.10%
2 children	30.50%	30.20%	-0.30%
3 children	9.60%	10.00%	0.40%
Total fertility rate	1.25	1.26	0.01

The policy is tested in the model by introducing the cash benefit mentioned above in the period in which a child is born. Similarly, we solve the model with the same parameters used in the baseline. We then compute the difference in labour force participation and fertility outcomes with respect to the baseline.

Again, there are some caveats to this. The most important is that there are no savings in the model, and therefore the cash benefit has to be consumed within the period in which it is received. This may not a very serious caveat for two reasons. First, if the families are credit constraint and expect their earnings to increase in the future, they would consume the cash benefit immediately. Second, the model period is three years, so the cash benefit is spread out at least over that period.

The results of the exercise are reported in Table 7. The baby check has a negligible effect on the youngest mothers' labour force participation, but increases part-time relative to full-time employment for mothers aged 33 to 36 and then lowers labour force participation for mothers aged 36-39 and 39 42. In a sense, this is similar to the previous exercise: older mothers are more affected than young ones. Moreover, the results are qualitatively similar to González (2013), at least for older mothers. The baby check leads to a tiny increase in total fertility, driven by more women having three children. In her study, González finds that the baby check increased births by 6 per cent. In the model, this is much lower. One reason may be that part of the effect captured by her is anticipated births. This effect would be more substantial if people expected the policy to end (which it did, although they did not have the certainty it would at the time). In the model, households cannot anticipate births, which would lead to a difference in results even if they expected the policy to end.

Table 7: The effect of a cash benefit per child born.

Outcome	**Baseline**	**Baby check**	**Difference**
Labour force participation			
Part time share (all mothers):			
Aged 30-33	53.77%	54.19%	0.41%
Aged 33-36	43.97%	49.08%	5.11%
Aged 36-39	51.13%	39.27%	-11.86%
Aged 39-42	37.48%	30.50%	-6.99%
Aged 42-45	47.81%	46.86%	-0.96%
Aged 45-48	49.14%	49.48%	0.34%
Full time share (all mothers):			
Aged 30-33	19.60%	19.50%	-0.10%
Aged 33-36	25.70%	21.60%	-4.10%
Aged 36-39	16.16%	15.71%	-0.45%
Aged 39-42	36.03%	37.57%	1.54%
Aged 42-45	38.54%	38.61%	0.07%
Aged 45-48	33.91%	34.03%	0.12%
Fertility			
Share of women with:			
0 children	24.50%	24.30%	-0.20%
1 child	35.40%	35.50%	0.10%
2 children	30.50%	30.20%	-0.30%
3 children	9.60%	10.00%	0.40%
Total fertility rate	1.25	1.26	0.01

The main take from these two exercises is that female labour force participation and fertility responses to the two policies tested in the model are qualitatively similar to the empirical literature's effects. However, there are quantitative differences. The model features stronger labour force participation effects than Nollenberger and Rodríguez-Planas (2015), but more muted fertility effects than González (2013).

5.2 Childcare subsidies for children aged 0-3

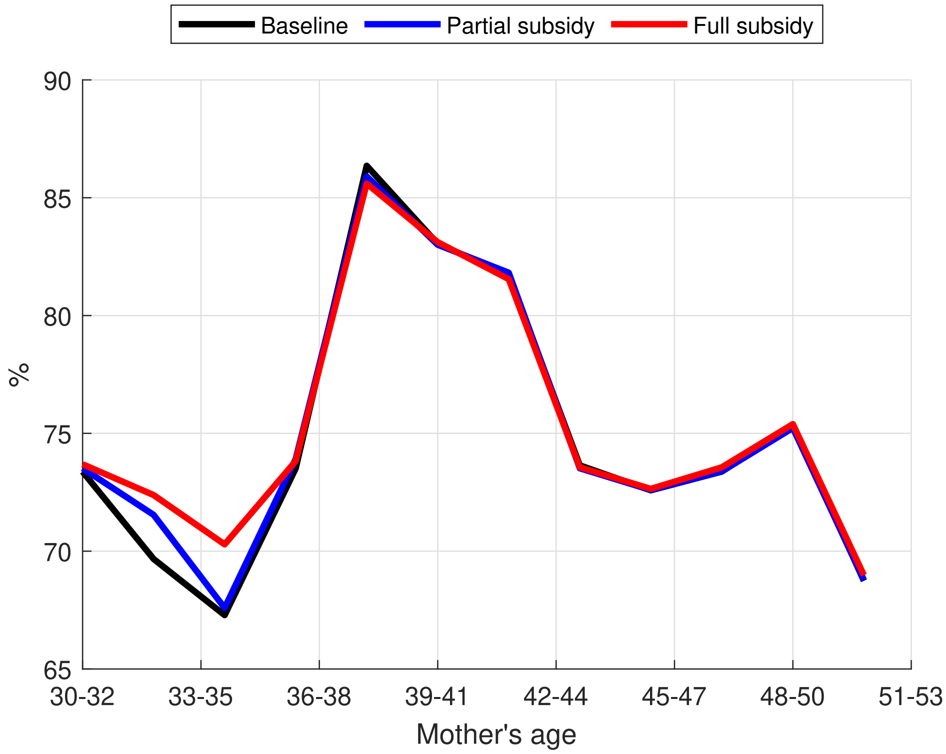
The main policy experiments considered in this paper consist of subsidising childcare for mothers with children aged 0-3 years old. We implement partial and full subsidisation, i.e. the government covering 50% and 100% of childcare costs for up to 30 hours per week. Table 8 presents the results of this policy experiment.

Table 8: The effect os subsidising childcare for children aged 0 to 3

Outcome	Baseline	Partial	Full
Female labour force participation			
Part time share			
(women with child 0 to 2):			
Aged 30-33	53.77%	53.89%	54.19%
Aged 33-36	42.14%	45.77%	57.07%
Aged 36-39	15.63%	34.38%	59.18%
Full time share			
(women with child 0 to 2):			
Aged 30-33	19.60%	19.63%	19.50%
Aged 33-36	27.93%	27.61%	17.62%
Aged 36-39	25.00%	10.42%	7.14%
Childcare use			
Hours per week (women with child 0 to 2):			
Aged 30-33	25.88	26.52	27.14
Aged 33-36	26.18	27.47	27.05
Aged 36-39	20.14	19.71	24.12
Fertility			
Share of women with:			
0 children	24.50%	24.10%	23.60%
1 child	35.40%	35.70%	36.10%
2 children	30.50%	30.60%	30.50%
3 children	9.60%	9.60%	9.80%
Total fertility rate	1.25	1.26	1.27
Present value of SS budget			
% of average working-age family's income	40.10%	39.53%	38.88%

In terms of female labour force participation, the subsidies lead to a shift from full-time to part-time work among older mothers with younger children. This leads to a minimal increase in total labour force participation for mothers aged 33 to 39, as shown in Figure 5.

Figure 5: Labour force participation of mothers by age under different childcare subsidisation policies for children aged 0-3.



Unsurprisingly, the childcare subsidies lead to an increase in childcare usage among women with young children for all age brackets. The increase is not very large, being less than 10% in all cases except for women aged 36-39 when moving from no subsidies to full subsidies.

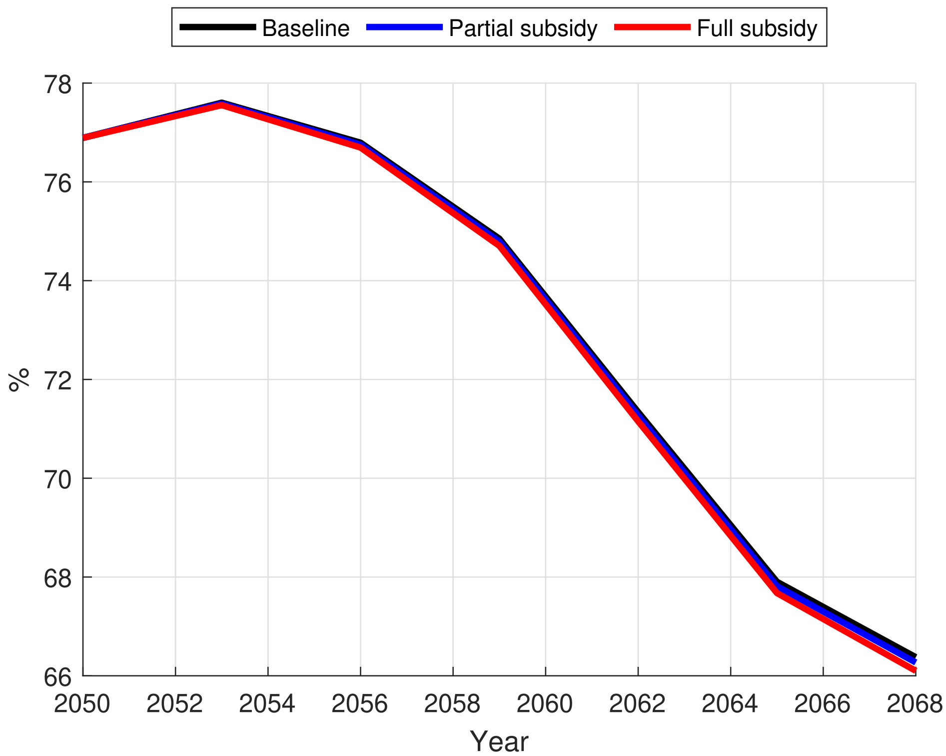
The effect of childcare subsidies on fertility is small. The fraction of childless women falls with respect to the baseline under both partial and full subsidies. The fraction of women with three children increases very slightly with full subsidies. Total fertility rate increases only marginally, from 1.25 children per woman in the baseline to 1.27 in the full subsidies case. The subsidies may lead to welfare benefits by allowing for a lower mismatch between desired and realised fertility. Moreover, if the subsidies were budget-neutral or budget-positive for the social security, it would be possible to claim unequivocally that they raise welfare.

However, the policy is not budget neutral. The present value of the social security budget balance falls as a percentage of the average working-age family's income[13]. Expenditures for the social security administration increase immediately, albeit not by a lot since the subsidies'

[13]The present value of the Social Security balance is positive in the baseline economy. In reality, this value is negative. Payroll taxes laid out in the law (28.3%) are used to calculate social security revenues. This is likely too high. However, estimating the effective contribution to Social Security as a function of gross income using the administrative in the *MCVL* data set was beyond the scope of the paper.

cost is small compared to the regular outlays of social security (the retirement benefits). This lowers the net budget in the present and near-present periods. On the other hand, the increase in the total fertility rate induced by the subsidies barely affects the future's old-age dependency ratio. This can be seen in Figures 6 and 7.

Figure 6: Projected old dependency ratio under different childcare subsidisation policies (2050-2068)



Unfortunately, the experiments' main takeaway is that childcare subsidies do not achieve any of the goals hypothesised before. The response of the mother's labour force participation is not the expected one, and the subsidies fail to generate a large enough fertility response to be budget-neutral for the social security administration.

5.3 Discussion

What drives the findings of the policy experiments? The childcare subsidies induce an income effect and a substitution effect. On the one hand, they increase households' real income with children aged 0-3, which should lead to higher consumption, leisure, and time spent with children. On the other hand, they make childcare less expensive. This increases the price of leisure relative to consumption, inducing the household to decrease leisure, increasing consumption, and increasing the price of time spent with children relative to leisure, which

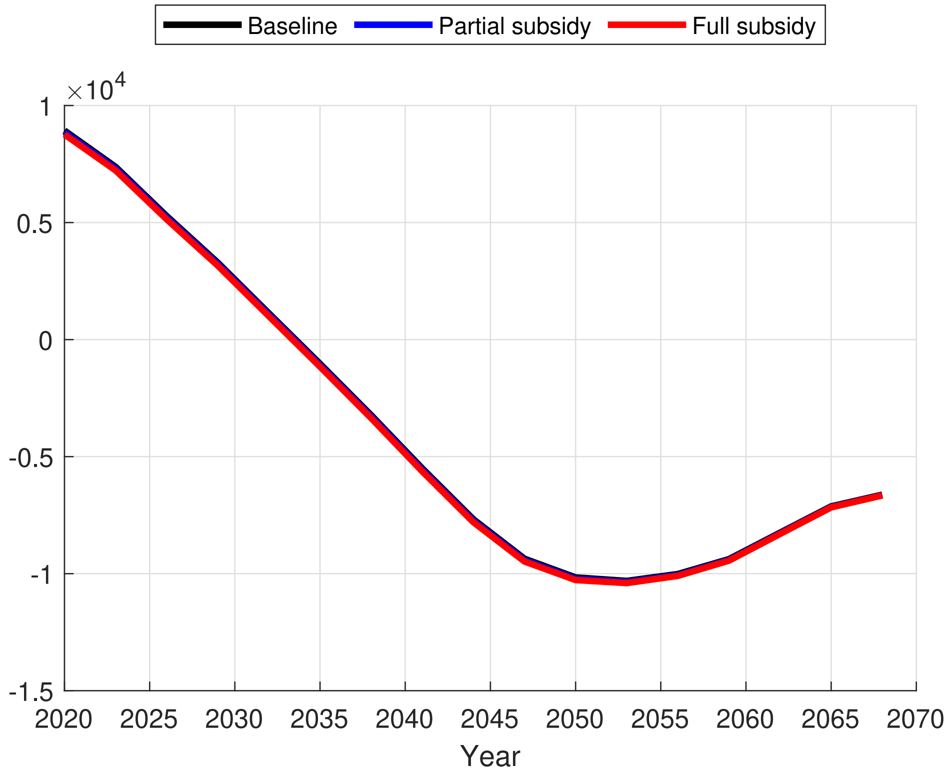


Figure 7: Social Security budget balance

should decrease time spent with children and increase leisure. Notice that, at the margin, childcare is still as expensive as before for women that want to consume many hours of it since the subsidy is only 30 hours per week. Therefore, it is costly for women working full time. Since γ_0 and γ_1 are both close to one, the income and substitution effects should roughly cancel each other out for leisure.

The childcare subsidies have a negligible effect on the labour force participation of women aged 30-33 with children aged 0-3. Since childcare also increases, this means that time spent with children decreases. Therefore, for these women, the substitution effect dominates the income effect on their children's time.

For older women, the subsidies induce women to increase time spent with children. They would tend to have higher consumption since they are older and therefore, husbands' incomes and their own earnings potential are higher. Therefore, they care more about the time spent with children.

The effect of the subsidies on fertility depends heavily on the fertility parameters. These are chosen to match the distribution of children across households. Intuitively, δ_2 controls how much households care more about departures from their desired fertility, while ζ matters for the decision between having children or not. The latter seems to be playing a more important role here, that is, the subsidies can convince some (small) fraction of women to have children, but not many to have more children.

6 Conclusions

This paper quantifies the effect of childcare subsidisation policies on fertility, female labour force participation, and social security budgets. To this end, an overlapping generations economy is studied, in which women initially choose fertility and then the labour force participation and childcare along their life-cycle. The model is calibrated to Spanish data and used to experiment by introducing partial (50%) and full (100%) childcare subsidies for women with children aged 0-3, since childcare for children 3-6 is already free and universal in Spain.

The paper's main takeaway is that the childcare subsidies do not have a strong positive effect on the mother's labour force participation and fertility. This is somewhat consistent with previous literature. Moreover, the subsidies affect the budget balance of Social Security negatively: more is spent on them than is recovered through future lower old-age dependency ratios. Mainly, the positive impact of the subsidies on fertility is too small to offset their immediate cost.

The analysis presented here has two major caveats. First, women choose the number of children at age 30, and they are matched with men in a deterministic way. Therefore, this framework is not suitable for analysing how childcare subsidies affect the timing of childbirth. Second, the mode abstracts from important features of the Spanish labour market, like high unemployment and the pervasiveness of fixed-term contracts.

These two features, potentially interact in non-trivial ways to determine women's response to childcare subsidies. Since the effects of shifts in childbirth timing on social security contributions over the life cycle are probably small compared to changes in total fertility, the first caveat may be minor. Besides, to the extent that the income process imposed here accounts for the effects of fixed-term contracts and high unemployment, it can also be claimed that the model is missing only second-order effects through the second.

Therefore, these mechanisms are outside the scope of the questions that are asked in this paper. Nevertheless, further research is necessary to understand them in their own right.

A Appendices

A.1 The childcare allocation problem

Given a choice of labour force participation n_j, and state variables ϵ_j^f, ϵ_j^m, x_j and b the childcare allocation problem faced by a working-age child rearing mother at age j is:

$$
\begin{aligned}
&\max_{h_j^m, h_j^p, h_j^u} \frac{\left(\frac{c_j}{\psi(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} - \delta_2 |b - b^*|^{\gamma_2} - \zeta + \delta_3 \left(h_j^m\right)^{\gamma_3} \\
&\text{subject to} \\
&c_j = 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j\left(h_j^p, b\right) \\
&\lambda_j\left(h_j^p, b\right) = q\left(1 - \Upsilon\right)\sqrt{b} h_j^p \\
&l_j + n_j + h_j^m = 1 \\
&h_j^m + h_j^p + h_j^u = 1 \\
&m_j(b) \leq h_j^u \leq \kappa_j(b) \\
&\ln y_j^f = \eta^f + \eta_1^f x_j + \eta_2^f x_j^2 + \epsilon_j^f \\
&\ln y_j^m = \eta^m + \eta_1^m (j - J_0) + \eta_2^m (j - J_0)^2 + \epsilon_j^m,
\end{aligned}
$$

where h_j^m is the childcare provided by the mother, h_j^p is paid childcare purchased in the market and h_j^u in unpaid childcare (childcare provided by the father or other family members, time spent at school, etc.).

Consumption c_j is equal to the household's net income $2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right)$ minus childcare costs $\lambda_j(h_j^p, b)$.

Leisure l_j is equal to one minus paid labour time n_j and mother provided childcare time. The amount of unpaid childcare h_j^u is bounded below by the amount of mandatory schooling $m_j(b)$. This is a function of the age (and hence the age of the children) and the number of children. It is bounded above by the total unpaid childcare available to the mother, $\kappa_j(b)$. Since mandatory schooling is included in total available unpaid childcare, $m_j(b) \leq \kappa_j(b)$. Mother childcare provision and paid childcare utilization must be non negative. Consumption and leisure must be strictly positive. Finally, total childcare time must be equal to 1, that is, children must be looked after every moment of the day.

It is possible that for a given labour force participation choice n_j the mother is unable to satisfy the children's time constraint. Define $\bar{h}_j^p$ as:

$$\bar{h}^p_j = \max\{0, \sup\{h^p_j : 2n_j y^f_j + y^m_j - T\left(y^f_j, y^m_j, n_j\right) - \lambda_j(h^p_j, b) > 0\}\},$$

that is, $\bar{h}^p_j$ is the maximum amount of paid childcare that the household can afford. Likewise, $\bar{h}^m_j = 1 - n_j$ is the maximum childcare time that the mother can provide given that her labour force participation is n_j. If:

$$\bar{h}^m_j + \bar{h}^p_j + \kappa_j(b, a) \leq 1,$$

then the choice of labour force participation n_j is unfeasible for the mother, and the childcare problem is not well defined.

From here on, it is assumed that:

$$\bar{h}^m_j + \max\{0, \bar{h}^p_j\} + \kappa_j(b, a) > 1.$$

Lemma A.1 *If $\{h^m_j, h^p_j, h^u_j\}$ is a solution to the childcare allocation problem, then either:*

1. $h^p_j = 0$ *and* $h^u_j < \kappa_j(b, a)$, *or*

2. $h^p_j = 0$ *and* $h^u_j = \kappa_j(b, a)$, *or*

3. $h^p_j > 0$ *and* $h^u_j = \kappa_j(b, a)$.

Proof. Assume that $\{h^m_j, h^p_j, h^u_j\}$ solves the childcare allocation problem, with $h^p_j > 0$ and $h^u_j < \kappa_j(b)$. There exists ϵ such that $\epsilon > 0$ and $\epsilon < \min\{h^p_j, \kappa_j(b, a) - h^u_j\}$. Allocation $\{h^m_j, \epsilon, h^u_j + \epsilon\}$ provides a higher utility to the household and satisfies all constraints. The result follows by contradiction. ■

Lemma **??** just states that the mother would first exhaust all available unpaid childcare before drawing on paid childcare, as the former does not affect utility in any way, while the latter decreases it by reducing consumption.

An implication of lemma A.1 is that the childcare allocation problem can be written in terms of mother provided childcare only:

$$\begin{aligned}
&\max_{h_j^m} \quad u_{n_j}\left(h_j^m\right) \\
&\text{subject to} \\
&0 \leq h_j^m \leq 1 - m_j(b) \\
&1 - \bar{h}_j^p - \kappa_j(b) < h_j^m < 1 - n_j
\end{aligned} \tag{7}$$

where:

$$u_{n_j}\left(h_j^m\right) = \frac{\left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j\left(h_j^m\right)^{1-\gamma_1} - 1}{1-\gamma_1} - \delta_2 |b - b^*|^{\gamma_2} - \zeta + \delta_3\left(h_j^m\right)^{\gamma_3}$$

and

$$c_j(h_j^m) = \begin{cases} 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j(1 - h_j^m - \kappa_j(b), n) & \text{if } h_j^m \leq 1 - \kappa_j(b) \\ 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) & \text{if } h_j^m > 1 - \kappa_j(b) \end{cases}$$
$$l_j\left(h_j^m\right) = 1 - n_j - h_j^m.$$

Define the following interval from the constraints of the problem:

$$I_{h_j^m} = \left[\max\{0, 1 - \bar{h}_j^p - \kappa_j(b)\}, \min\{1 - m_j(b), 1 - n_j\right].$$

Lemma A.2 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the derivative of $u_{n_j}\left(h_j^m\right)$ exists almost everywhere in the interval $I_{h_j^m}$.*

Proof. If $h_j^m > 1 - \kappa_j(b)$, then:

$$u'_{n_j} = -\delta_1 l_j(h_j^m)^{-\gamma_1} + \delta_3\gamma_3(h_j^m)^{\gamma_3 - 1}.$$

For $\gamma_1 > 0$, the first term on the right hand side exists as long as:

$$l_j = 1 - n_j - h_j^m > 0 \iff h_j^m < 1 - n_j,$$

and for $0 < \gamma_3 < 1$, the second term on the right hand side exists as long as $h_j^m > 0$.

If $h_j^m < 1 - \kappa_j(b)$, then

$$u'_{n_j} = \left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0} \frac{1}{\psi(b)} \frac{\partial \lambda_j\left(h_j^p, b\right)}{\partial h_j^p} - \delta_1 l_j (h_j^m)^{-\gamma_1} + \delta_3 \gamma_3 (h_j^m)^{\gamma_3 - 1}.$$

It is already known that the second and third terms exist almost everywhere in $I_{h_j^m}$. For $\gamma_0 > 0$, the first term on the right hand side exists as long as:

$$c_j(h_j^m) > 0 \iff h_j^m > 1 - \bar{h}_j^p - \kappa_j(b, a).$$

Finally, u'_{n_j} does not exist at $h_j^m = 1 - \kappa_j(b, a)$.

Therefore, u'_{n_j} exists everywhere in $I_{h_j^m}$ except at $h_j^m = 1 - n_j$, $h_j^m = 0$, $h_j^m = 1 - \bar{h}_j^p - \kappa_j(b, a)$ and $h_j^m = 1 - \kappa_j(b)$ whenever they belong to $I_{h_j^m}$. ■

Lemma A.3 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the second derivative of $u_{n_j}\left(h_j^m\right)$ exists and is negative almost everywhere in $I_{h_j^m}$.*

Proof. If $h_j^m > 1 - \kappa_j(b)$, then:

$$u''_{n_j} = -\gamma_1 \delta_1 (1 - n_j - h_j^m)^{-\gamma_1 - 1} + \delta_3 \left(\gamma_3 - 1\right) \gamma_3 (h_j^m)^{\gamma_3 - 2}.$$

For $\gamma_1 > 0$, the first term on the right hand side exists as long as

$$1 - n_j - h_j^m > 0 \iff h_j^m < 1 - n_j.$$

For $0 < \gamma_3 < 1$, the second term on the right hand side exists as long as $h_j^m > 0$. Both elements are negative for $\gamma_1 > 0$ and $0 < \gamma_3 < 1$ as long as the expression is valid, therefore the whole expression exists and is negative everywhere except at $h_j^m = 1 - n_j$ and $h_j^m = 0$ in $I_{h_j^m}$.

If $h_j^m < 1 - \kappa_j(b)$, then:

$$\begin{aligned} u''_{n_j} = &- \gamma_0 \left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0 - 1} \frac{1}{\psi(b)} \frac{\partial \lambda_j\left(h_j^p, b\right)}{\partial h_j^p} \\ &- \gamma_1 \delta_1 (1 - n_j - h_j^m)^{-\gamma_1 - 1} + \delta_3 \left(\gamma_3 - 1\right) \gamma_3 (h_j^m)^{\gamma_3 - 2}. \end{aligned}$$

It is already known that the last two terms on the left hand side exist and are negative almost everywhere in $I_{h_j^m}$, so only with the first term is of interest. For $\gamma_0 > 0$ the expression exists as long as

$$c_j(h_j^m) > 0 \iff h_j^m > 1 - \bar{h}_j^p - \kappa_j(b),$$

and is negative as long as it exists since $\left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0 - 1} > 0$, $\frac{1}{\psi(b)} > 0$ and $\frac{\partial \lambda_j\left(h_j^p, n\right)}{\partial h_j^p} > 0$. Therefore, the whole expression exists and is negative everywhere except at $h_j^m = 1 - n_j$, $h_j^m = 0$ and $h_j^m = 1 - \bar{h}_j^p - \kappa_j(b)$ in $I_{h_j^m}$.

Finally, the second derivative does not exist at $h_j^m = 1 - \kappa_j(b)$. ■

Lemma A.4 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the first derivative of $u_{n_j}\left(h_j^m\right)$ is strictly decreasing in $I_{h_j^m}$.*

Proof. It is known from lemma A.3 that u''_{n_j} is negative almost everywhere in $I_{h_j^m}$, with the notable exception of $h_j^m = 1 - \kappa_j(b)$. If $1 - \kappa_j(b)$ is not an interior point of $I_{h_j^m}$, the result follows automatically. Suppose that $1 - \kappa_j(b)$ is an interior point of $I_{h_j^m}$. Since

$$\left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0} \frac{1}{\psi(b)} \frac{\partial \lambda_j\left(h_j^p, n\right)}{\partial h_j^p} > 0,$$

then,

$$\lim_{h_j^m \to 1 - \kappa_j(b)^-} u'_{n_j} > \lim_{h_j^m \to 1 - \kappa_j(b)^+} u'_{n_j}.$$

Thus, u'_{n_j} is strictly decreasing in $I_{h_j^m}$. ■

The preceding lemmas are now used to show that the solution to the childcare allocation problem is unique and to characterise that solution.

Theorem A.5 *If $\gamma_0 > 1$, $\gamma_1 > 1$ and $0 < \gamma_3 < 1$, the childcare allocation problem has a unique solution in $I_{h_j^m}$.*

Proof.

Suppose that $\exists\ h_j^m \in I_{h_j^m} : u'_{n_j}(h_j^p) = 0$. From lemma A.4 it is known that this point is unique, and by standard marginal arguments it follows that it necessarily is a maximum of u_{n_j} in $I_{h_j^m}$.

Now suppose there is no such point. Notice that:

$$\lim_{h_j^m \to \inf I_{h_j^m}{}^+} u'_{n_j} = \begin{cases} \lim_{h_j^m \to 0^+} u'_{n_j} = \infty & \text{if} \quad 0 \geq 1 - \bar{h}_j^p - \kappa_j(b, a) \\ \lim_{h_j^m \to 1 - \bar{h}_j^p - \kappa_j(b)^+} u'_{n_j} = \infty & \text{if} \quad 0 < 1 - \bar{h}_j^p - \kappa_j(b). \end{cases}$$

Suppose further that $1 - n_j < 1 - m_j(b, a)$. Since

$$\lim_{h_j^m \to 1 - n_j +} u'_{n_j} = -\infty,$$

then $1 - \kappa_j(b) < 1 - n_j$, for otherwise $u'_{n_j}(h_j^p) = 0$ for some $h_j^p \in I_{h_j^m}$ as u'_{n_j} would be a strictly decreasing continuous function that takes both positive and negative values in $I_{h_j^m}$. Moreover, using the same argument it follows that:

$$\begin{aligned} u'_{n_j}(h_j^m) > 0 \quad & \forall h_j^m \in \left(\inf I_{h_j^m}, 1 - \kappa_j(b)\right) \\ u'_{n_j}(h_j^m) < 0 \quad & \forall h_j^m \in \left(1 - \kappa_j(b, a), 1 - n_j\right), \end{aligned}$$

which implies that $h_j^m = 1 - \kappa_j(b)$ is the unique solution to the childcare allocation problem.

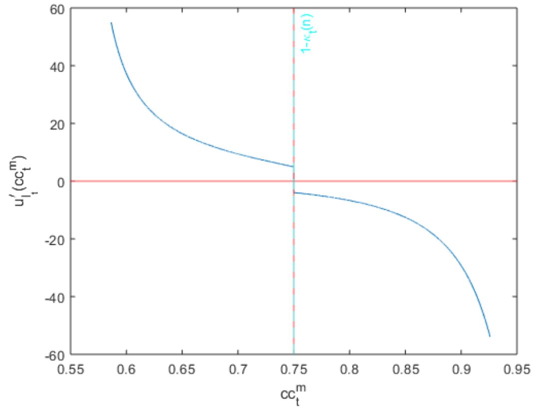
Assume now that $1 - m_j(b, a) < 1 - n_j$. There are two possibilities: $u'_{n_j}(1 - m_j(b)) < 0$, or $u'_{n_j}(1 - m_j(b)) \geq 0$. Suppose first that the former is true. Then, by the same argument used twice before, it follows that $1 - \kappa_j(b) < 1 - m_j(b)$, that:

$$\begin{aligned} u'_{n_j}(h_j^m) > 0 \quad & \forall h_j^m \in \left(\inf I_{h_j^m}, 1 - \kappa_j(b)\right) \\ u'_{n_j}(h_j^m) < 0 \quad & \forall h_j^m \in \left(1 - \kappa_j(b), 1 - m_j(b)\right), \end{aligned}$$

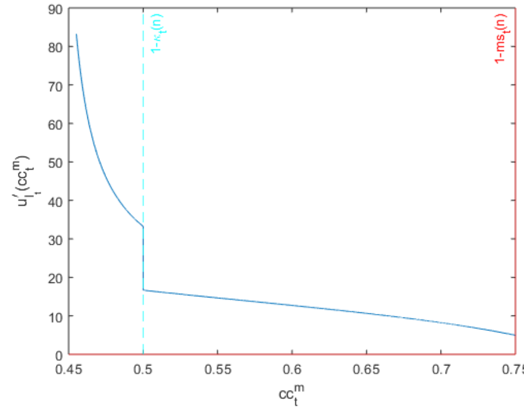
and hence that $h_j^m = 1 - \kappa_j(b)$ is the unique solution to the childcare allocation problem.

Finally, if $u'_{n_j}(1 - m_j(b)) \geq 0$, then it follows directly that the unique solution to the childcare allocation problem is $h_j^m = 1 - m_j(b)$. ■

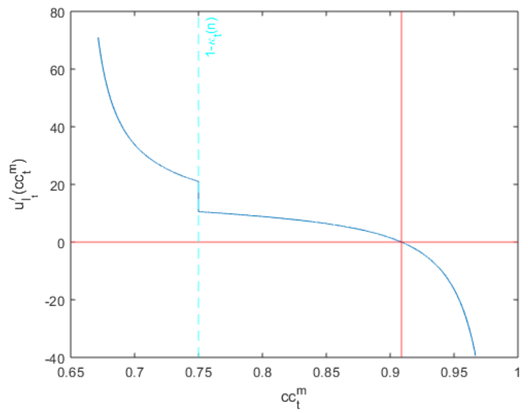
The proof of theorem A.5 characterises all the forms the unique solution to the childcare allocation problem can take. Figure 8 illustrates this. Panel (a) shows a situation in which the solution is at the kink of the utility function, where marginal utility experiences a discrete jump. In this situation, the marginal utility of mother's childcare is positive right before reaching the unpaid childcare endowment, as it includes the term related to marginal consumption, but is negative right after. Panel (b) shows the strange but theoretically possible situation in which the mother would like to spend more time with her children but she cannot because of the limit imposed by mandatory schooling. Panels (c) and (d) show situations in which the solution is interior. In panel (c), marginal utility crosses zero after the point in which the household does not need to use paid childcare, while in panel (d) it



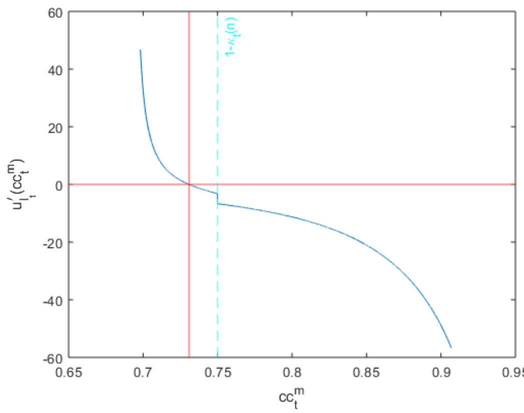
(a) Kink solution



(b) Corner solution



(c) Interior solution without paid childcare



(d) Interior solution with paid childcare

Figure 8: Solutions to the childcare allocation problem

does so before. Therefore, the household does not need to use paid childcare in the former but does in the latter.

A.2 Algorithms

Inputs : Model parameters
Outputs: Value and policy functions
begin

- Discretise the AR(1) process for the income shocks $(\epsilon_j^f, \epsilon_j^m)$ using the Tauchen method;
- Construct a grid for the state space $\{\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j^f, \bar{y}_j^m, b\}$, using the grid values for $(\epsilon_j, \epsilon_j^m)$;
- **for** $\underline{j = J_R : -1 : J_0}$ **do**
  - **if** $\underline{j = J_R}$ **then**
    - **for** Each point of the state space grid for $\{x_{J_R}, \bar{y}_{J_R}^f, \bar{y}_{J_R}^m, b\}$ **do**
      - Compute the value of retirement $V_{J_R}\left(x_{J_R}, \bar{y}_{J_R}^f, \bar{y}_{J_R}^m; b\right)$;
    - **end**
  - **end**
  - **else if** $\underline{j \in [J_0, J_R - 1]}$ **then**
    - **for** $\underline{b = 0 : 3}$ **do**
      - **if** $\underline{b > 0 \text{ and } j \leq W_c(b)}$ **then**
        - **for** Each point of the state space grid for $\{\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j^f, \bar{y}_j^m\}$ **do**
          - Solve the static childcare problem for each possible value of n_j ;
          - Compute the flow utility for the period and retrieve the continuation value according to the rules of motion for the state variables;
          - Select the choice of n_j that returns the largest value;
          - Record both the value $V_j(\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j^f, \bar{y}_j^m; b)$ and the policy ;
        - **end**
      - **end**
      - **else**
        - **for** Each point of the state space grid for $\{\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j^f, \bar{y}_j^m\}$ **do**
          - Compute the flow utility for the period and retrieve the continuation value according to the rules of motion for the state variables;
          - Select the choice of n_j that returns the largest value;
          - Record both the value $V_j(\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j^f, \bar{y}_j^m; b)$ and the policy ;
        - **end**
      - **end**
    - **end**
  - **end**
- **end**

end

Algorithm 1: Solving the working-age stages of the life cycle model

Inputs : state variables $\{x_j, \epsilon_j, \epsilon_j^m, b\}$, labour force participation choice n_j
Outputs: value of utility v_{n_j}, optimal choice of childcare $\{h_j^m, h_j^p, h_j^u\}$

begin
 Calculate the maximum mother provided childcare time and paid childcare;
 $\bar{h}_j^m = 1 - n_j$;
 $\bar{h}_j^p = \max\{0, \sup\{h_j^p : 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j(h_j^p, b) > 0\}\}$;
 if $\underline{\bar{h}_j^m + \bar{h}_j^p + \kappa_j(b) \leq 1}$ **then** choice of labour force participation n_j is unfeasible
  Set $h_j^m = NaN$, $h_j^p = NaN$, $h_j^u = NaN$ and $v_{n_j} = -\infty$;
 else
  Attempt to find a root for u'_{n_j} in $I_{h_j^m}$;
  if $\underline{\exists x \in I_{h_j^m} : u'_{n_j}(x) = 0}$ **then** x is the unique solution for the childcare allocation problem, which is interior with or without paid childcare
   Set $h_j^m = x$, $h_j^u = \min\{1 - x, \kappa_j(b)\}$, $h_j^p = 1 - h_j^m - h_j^u$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^u\right)$;
  else
   if $\underline{1 - n_j \leq 1 - m_j(b) \text{ or } 1 - m_j(b) < 1 - n_j \text{ and } u'_{n_j}(1 - ms_j(b)) < 0}$ **then** there is a unique kink solution to the childcare allocation problem
    Set $h_j^m = 1 - \kappa_j(b)$, $h_j^p = 0$, $h_j^u = \kappa_j(b)$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^i\right)$;
    Verify that $\lim_{h_j^m \to 1-\kappa_j(b)^-} u'_{n_j} > 0$ and $\lim_{h_j^m \to 1-\kappa_j(b)^+} u'_{n_j} < 0$;
   else
    There is a unique corner solution to the childcare allocation problem;
    Set $h_j^m = 1 - m_j(b)$, $h_j^p = 0$, $h_j^u = m_j(b)$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^i\right)$;
    Verify that $u'_{n_j}\left(1 - m_j(b)\right) > 0$;
   end
  end
 end
end

Algorithm 2: Solving the childcare allocation problem

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