Quantifying the Impact of Childcare Subsidies on Social Security

Lidia Cruces
Universidad Carlos III de Madrid
lcruces@eco.uc3m.es

F. Javier Rodríguez-Román
University of Edinburgh & Universidad Carlos III de Madrid
frodrigu@ed.ac.uk

February 15, 2021 –First draft: Februrary 15 2021

Abstract

Female labour force participation and fertility levels directly impact social security, especially when it relies on a pay-as-you-go scheme. In this paper, we quantify the impact of childcare subsidisation policies on a PAYG social security system. We build an overlapping generations model in which women decide how many children to have, the allocation of childcare time among different alternatives and their labour force participation along their life cycle. We calibrate the model to Spanish data and use it to experiment with different childcare subsidisation policies. We find that childcare subsidies increase mother's labour force participation and fertility minimally. Therefore, they have a negative effect on the present value of social security budget balance.

1 Introduction

Higher female labour force participation and decreased fertility levels are two of the most significant changes in women's socio-economic behaviour in developed countries in recent decades. Social security systems are directly affected by them, especially those that rely on pay-as-you-go schemes. Increased female labour force participation brings in more contributions to Social Security in the short run. In the future, this means additional pension entitlements once these women retire. Moreover, a drop in fertility levels causes the old-age dependency ratio to increase eventually (absent migration). As the share of pensioners over contributors rise, pressure on the system's finances builds.

Naturally, changes in female labour force participation and fertility are not independent of each other. They are jointly determined. A policy that affects this joint decision has the potential affecting the finances of Social Security.

In this paper, we document the state of institutions related to childcare in Spain and conclude that the introduction of childcare subsidies for women with children aged 0-3 years old can stimulate fertility and female labour force participation. Three main reasons underlie this conclusion. First, like in most developed countries, there is a family gap in female labour force participation (mothers participate less in the labour market). Second, women in this economy consume less childcare than needed according to their labour force participation status. Women with children aged in contrast 3-6 have access to public childcare and use it extensively. Additionally, this gap does not seem to be covered by access to informal unpaid childcare (provided by other family members). Third, the positive gap between desired and realised fertility implies that women are having fewer children than they want. By stimulating fertility and female labour force participation, such a policy can improve Social Security's budget balance.

We quantify the effect of the introduction of such a policy via an overlapping generations model in which women choose fertility levels at the beginning of life. In subsequent periods they decide on labour force participation, and if they have young children, childcare allocation among maternal, informal and out-of-pocket alternatives. At the same time, the government runs a Social Security PAYG system financed with a payroll tax and pays retirement benefits.

We calibrate the model to Spanish data and Social Security regulations. We then use this model to perform a policy experiment consisting of introducing partial (50%) or full (100%) childcare subsidies for women with children aged 0 to 3.

Our main finding is that the introduction of childcare subsidies for children 0-3 years old has small effects on female labour force participation and fertility decisions, which translates into a negative impact on the present value of the budget balance of Social Security.

In particular, the subsidies induce an increase in part-time and a decrease in full-time work among women aged 33-39 with children aged 0-3, with a small net positive effect on total participation. The total fertility rate increases marginally, from 1.25 in our baseline economy to 1.26 under partial subsidies and 1.27 under full subsidies. These small changes in fertility are mainly driven by more women having one child instead of none.

The combined effects of the subsidies on female labour force participation and fertility are to worsen the Social Security budget balance in all future periods, which lowers its present value. The effect of fertility is too small to compensate for the immediate cost that the subsidies entail. Therefore, the policy fails to deliver the benefit hypothesised above.

We contribute to different strands of the literature. We are broadly related to the extensive literature on the relationship between fertility, women's labour market outcomes, and policies relevant to it. Among the papers that rely on structural, dynamic models to study the issues at hand are Erosa et al. (2002), Da Rocha and Fuster (2006) and Adda et al. (2017). Unlike these, we do not model the labour market in detail because we do not consider search frictions or occupational choices. In contrast, we introduce a detailed Social Security system. The model economy is related to the recent papers Bick (2016), Laun and Wallenius (2021) and Hannusch et al. (2019). Women's labour force and childcare utilisation decisions during their child-rearing years are heavily inspired by Bick (2016). We add to this by modelling non-child rearing years, retirement benefits, and changing the way in which fertility decisions are made, which relies on data on desired fertility. Different from Hannusch et al. (2019) we allow women to decide use less than their endowment of informal childcare. It may be that some mothers who value maternal childcare time highly do not use all the informal childcare endowment. Children arrive deterministically to the household after the fertility decision is made, as in Laun and Wallenius (2021). That is, we do not study the timing and spacing of multiple children. Even though the last two papers introduce pensions, they do not quantify the effect of family policies on the system's finances.

Our paper relates as well to the literature on Spanish child related policies. Two papers in particular are important to us. González (2013) studies the impact of a sizable universal child benefit introduced in Spain between 2007 and 2010 on fertility and early maternal labour supply. Nollenberger and Rodríguez-Planas (2015) study the effect of the introduction in Spain of universal public preschool provision for children aged three and older in the 1990s on mother's labour force participation. We implement these policies in our structural model to compare the results as an external validity check, and find that the results are qualitatively similar, although they differ in magnitude[1]. A third study by Sánchez-Mangas and Sánchez-

[1]The long-run effects of family policies obtained through structural models may be smaller than the short-run effects found by quasi-experimental models, as pointed out by Adda et al. (2017).

Marcos (2008) analyses the introduction of a monthly cash benefit to working mothers with children under three years.

Finally, this paper builds on a long tradition in macroeconomics research on sustaining social security systems. This literature uses life-cycle models, pioneered by Auerbach et al. (1987). We contribute to the papers that analyse potential policy reforms that look for a self-financed pension system. Some of the proposals are the partial privatisation (Nishiyama and Smetters (2007)), the mandatory fully funded Social Security System financed with a consumption tax (Kotlikoff et al. (2007)), the retirement age delay (Díaz-Giménez and Díaz-Saavedra (2009) in Spain, Imrohoroğlu and Kitao (2012) and Kitao (2014) in the U.S.), the increase in the income tax, the decrease in the wage replacement rate or to make the system benefits to fall one-to-one with income above a threshold level as in Kitao (2014). Cruces (2021) shows in a context of exogenous fertility that women have helped finance social security in the short run in Spain. In contrast, in her projections for the long run, this is no longer true. Our paper's novelty is that we consider childcare subsidisation as a policy to achieve sustainability, which has not been considered before.

The rest of the paper is structured as follows. In section 2, we discuss the Spanish context, introduce some facts about female labour force participation and childcare allocation, and describe Spain's family policies during the years that we are considering. Section 3 describes the model. Section 4 discusses the calibration and the model's performance. In section 5, we present our policy experiments. Section 6 concludes.

2 Family policies in Spain, female labour force participation and childcare usage.

This section briefly discusses Spanish policies related to labour gender equality and work/life balance. Then, we describe women's labour market behaviour when they become mothers and the usage of childcare in Spain. For this section and the remainder of the paper, we rely mainly on data from the *Encuesta de Condiciones de Vida* (ECV henceforth) of the Spanish National Statistics Institute, *INE*. The ECV is an annual survey. It consists of a rotating panel that interviews families for four years. The *INE* provides cross-sectional and longitudinal weights that allow computing statistics representing the whole Spanish population. We use the years 2011 to 2019 of the survey.

Family policies. The main regulatory instrument to facilitate the compatibility between family and work is the Spanish family and workers' labour life reconciliation (1999). This law includes different mechanisms that aim to balance labour and family. The current legislation entitles mothers and fathers with 16 and 4.3[2] paid leave weeks of full wage replacement rate. This parental leave carries full job protection only for the first year. For the last two years of the leave, only a return to a similar job or job of the same category is guaranteed. Besides, parents can be entitled to unpaid leave to take care of their children. The length of this leave determines whether the return to the same work is guaranteed. Short leaves[3] assure it, while for long unpaid leaves just a return to a similar job is warranted.

Nevertheless, the Spanish legislation does not consider financial transfers. The Spanish government does not provide any generalised childcare cash benefits to parents. Instead, there is a childcare provision in terms of public schooling. For children younger than three years old, regional authorities are in charge of its provision while for older children, it is the Spanish government who supplies it. The difference in who the supplier is, implies an unbalanced provision across ages and regions. On the one hand, the enrollment rate in early childhood education and care services from 0 to 2-year-old is low compared to three-years-old children. For instance, in 2017, the enrollment rates were 36.4% and more than 90%, respectively. On the other hand, there exists unequal public kindergarten provision across regions. This fact is well-documented in González (2004).

The lack of childcare benefits in Spain put significant pressure on parents, especially on women. Families are forced to use informal childcare, to enrol their kids in private kindergartens or/and to change their labour market behaviour when children are born.

Labour force participation. The *ECV* includes a labour force participation variable for each month over the last year. Taking full-time work to be 1, part-time work to be 0.5 and being out of the labour force to be zero, we compute the average labour force participation for each woman in the sample each year. If this average is above 0.75, we assign the woman to full-time work, if it is between 0.25 and 0.75 we assign her to part-time work and if it is below 0.25 we assign her to be out of the labour force for that year.

In panels (a) and (b) of Figure 1, we plot the 2011-2019 average fractions of part-time and full-time women by age. Panel (a) shows those fractions among mothers and panel (b) among childless women. Both full-time and part-time work fall with age among mothers and childless women. Women in older cohorts with lower labour force attachment partially explain this result. Panel (c) plots the difference or gap between full and part-time work between mothers and childless women, plus the participation gap, which is the sum of the two. Unsurprisingly, there is a large (-15 percentage points) participation gap around age 30,

[2]The number of weeks assigned equated across gender in 2021.

[3]An unpaid leave is considered as a short one if it is less than one year out of the labour market.

that closes steadily and becomes nearly zero after age 50. This is the well known "family gap" in female labour force participation. The part-time gap is positive, meaning that mothers are more likely to work part-time. The full-time gap is negative, which means the opposite. Both also close with time, although older mothers are still somewhat more likely to work part-time and less likely to work full time than childless women.

Childcare. In Figure 2 we plot the average hours per week spent at preschool for children aged 0 to 3 and 3 to 6 by labour force participation of the mother. There are several things to notice. First, as expected, the children of women who work full time spend more time in preschool than those who work part-time, and they spend more time than those out of the labour force. Second, children aged 3 to 6 spend more time at preschool than children aged 0 to 2 regardless of their mother's labour force participation status. Third, the difference in average preschool hours between children of working mothers and non-working mothers is much smaller among children aged 3 to 6. Children of women who work full-time aged 0 to 3 spend almost twice as many hours in preschool on average than children of women who are not in the labour force. Among children aged 3 to 6, the difference is only about 10%.

This difference is mostly a result of universal public preschool coverage for children older than three, introduced in the late 1990s. It comes as no surprise that most children between 3 and 6 spend almost 30 hours per week at preschool since public preschools are in general open for six hours per day[4].

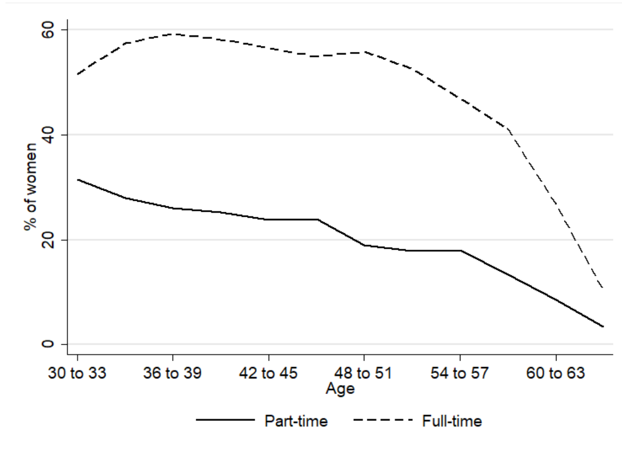
Preschool is not the only source of childcare tapped by working mothers, as it covers only about 15 of the 30-40 hours per week, and about 10 of the 15-20 hours per week required by full-time and part-time working mothers with children aged 0 to 3, respectively. The *ECV* includes information on childcare received by the child without any payment in return. Grandparents, aunts, uncles, older siblings, extended family or neighbours might provide this childcare. We call it informal childcare. We computed average hours spent under informal childcare for children aged 0 to 3 and 3 to 6 by the mother's labour force participation. The results are shown in Figure 3.

The results are surprisingly low. Adding the average preschool and informal care hours still leaves some required childcare unaccounted for. Likely the informal childcare is under-reported in the survey. Moreover, the father's childcare is not measured separately in the *ECV*, although it is unlikely that this solves the issue since most fathers work full time.

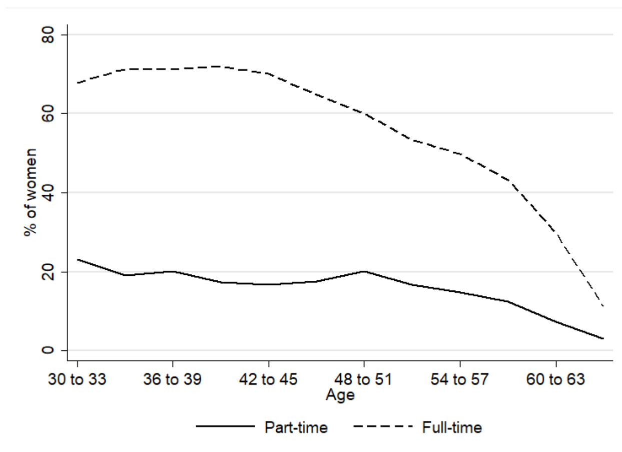
[4]Average preschool hours are below 30 because some children aged 3 to 6 were not yet eligible at the time of the survey to attend public preschools. The reason is that they were not aged three by January 1st. Others may be already attending primary schooling, which is why the average is below 30. Suppose we compute this for children aged 4 and 5 at the time of the survey. These children are eligible for public preschool and not for primary. In that case, we obtain an average of 30 hours for all levels of labour force participation of the mother.

Figure 1: Labour force participation of woman by age (30-65) and motherhood status in Spain, 2011-2019

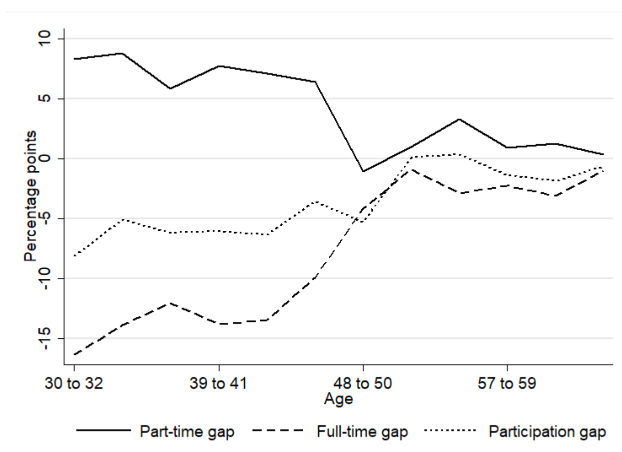
(a) Mothers



(b) Childless

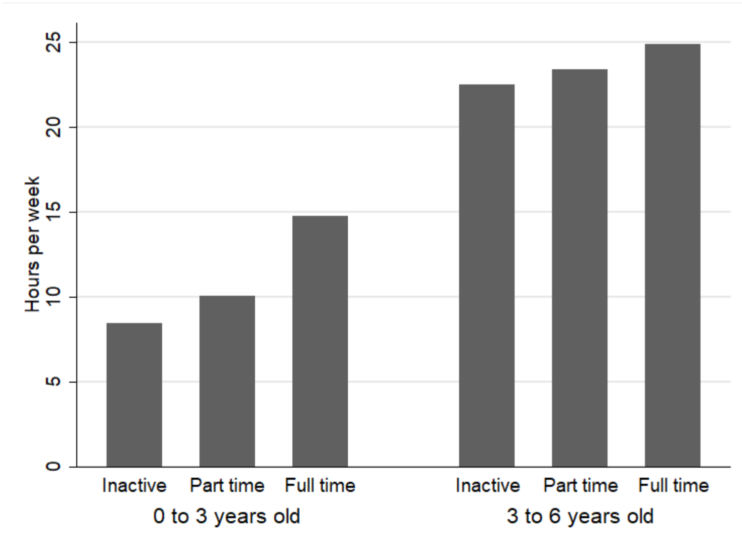


(c) Gap



Source: author's work with data from *INE*.

Figure 2: Preschool usage by age of child and labour force participation status of mother (2011-2019)



Source: author's work with data from *INE*.

There are other childcare sources measured in the *ECV*, but none of them is significant on average. To this end, we took advantage of the occasional survey *Encuesta del uso del tiempo*, provided by INE between 2009 and 2010. Even though it might be out-of-date, it is useful to get an idea regarding the childcare gender gap between parents.

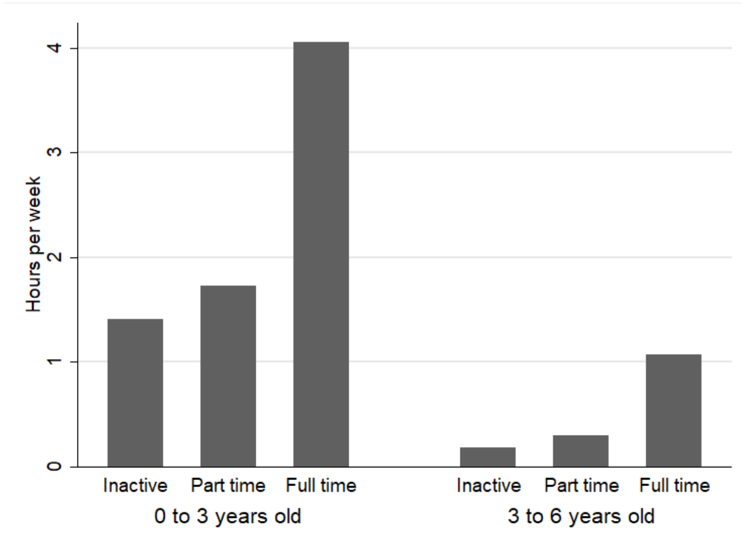
The average gender gap was 86.57%. In other words, mothers provided 86.57% more of childcare compared to fathers. Once we control by participation in the labour market, we get three remarkable results. First, in two-earner households[5], this gap slightly decreases, reaching 80.42%. These results show that the gender gap in childcare is persistent and robust once we focus on households where both spouses work. Second, this gap dramatically increases in households where mothers are out of the labour force. In particular, it is 174.09%. Third, in households where women are in the labour market, men now employ more childcare than women. Therefore the gap is equal to -14.53%.

To sum up, the evidence points that mothers of children aged 0 to 3 are in a problematic childcare position. This comes from the insufficient access to affordable or free options and the persistent gender norms that we measured with the gender gap in childcare time.

Desired fertility vs realised fertility. Like in most European countries, Spanish women would like to have more children than they do. In 2018, the Spanish Statistics Institute ran

[5]Households in which both spouses are in the labour market.

Figure 3: Informal childcare usage by age of child and labour force participation status of mother (2011-2019)



Source: author's work with data from *INE*.

a new iteration of its fertility survey *Encuesta de Fecundidad*, henceforth *EdF* (the previous one is dated from 1999). Table 2 shows the realised and desired fertility for women aged 40 to 44, and the difference between them. We choose this age group because most of these women already completed their fertility at this point, meaning that the difference with desired fertility is definitive. The most popular fertility choice is two children, both in desired and realised terms.

Nevertheless, the data shows that the fraction of childless women exceeds by more than two times the fraction of women who declare not wanting to have any children. Almost ten percentage points form the gap between women with a single child and women declaring having only one child. Less than half the fraction of women that want three children have three children. These results show that there is a considerable number of women whose completed fertility is lower than their ideal one.

An additional question of interest is whether most of the gap between desired and realised fertility is due to some women being unable to have children (the extensive margin), or whether some mothers would like to have more children (the intensive margin). In Table 2 we show realised and desired fertility for mothers and childless women aged 40 to 44, and the difference between these two. First, notice that very few mothers declare not wanting children at all. This result shows that few women go beyond their desired fertility level. We find this reasonable because contraception and abortion are legal and widely available in

Table 1: Desired and realised fertility, women aged 40 to 44 in 2018 in Spain

Number of children	Realised	Desired	Difference
0	18.99%	7.90%	11.09%
1	24.96%	15.20%	9.76%
2	43.79%	49.75%	-5.96%
3 or more	12.26%	27.15%	-14.89%

Source: *Encuesta de Fecundidad 2018, INE.*

Spain. Second, the number of desired children is much higher for mothers than for childless women. Thus, even among mothers, there is a difference between realised and the desired number of children. The fact that the number of women with two children coincides with the number of women declaring to want two children does not mean that every woman in this category is satisfied with her fertility. Likely a fraction of them would like to have three children, while a fraction of those with one would like to have two.

Table 2: Desired fertility by motherhood status, women aged 40 to 44 in 2018 in Spain

Number	Mothers			Childless		
of children	Realised	Desired	Difference	Realised	Desired	Difference
0	0.00%	0.22%	-0.22%	100.00%	40.65%	59.35%
1	30.81%	13.97%	16.84%	0.00%	20.47%	-20.47%
2	54.06%	54.14%	-0.08%	0.00%	31.00%	-31.00%
3 or more	15.13%	31.67%	-16.54%	0.00%	7.88%	-7.88%

Source: *Encuesta de Fecundidad 2018, INE.*

There are many potential reasons why women may choose not to have their desired number of children. The *EdF* collects information on the reasons that explain why women have fewer children than their desired number. The most common ones for women aged 40 to 44 are "work and work-family balance reasons" and "economic reasons".

Given the evidence laid out here, we think that there is room for a childcare subsidisation policy targeted at women with children aged 0 to 3 years old to affect mother's labour force participation and fertility levels. These changes, in turn, would affect the social security budget. To quantify the magnitude of these changes, we develop the model described in section 3.

3 The model

We study an economy populated by overlapping generations of households that derive utility from consumption, leisure, the number of children they have, and the time spent with their children (if any).

At the beginning of its life, each household decides the number of children they have. Then, each household is composed of a woman, her husband and children. Children depend on the chosen fertility level and the household's age (children eventually grow up and leave).

Working-age households make labour force participation decisions. Young children impose childcare needs upon parents that they need to cover with a combination of their own time, out-of-pocket childcare purchased in the market, and the unpaid childcare time is available to them. Therefore, households with young children need to balance labour force participation decisions and childcare needs.

Households eventually retire from the workforce. There are no savings, but the government runs a social security PAYG scheme financed with contributions from current working-age households in the form of a payroll tax. Retired households receive benefits that depend on women and husbands' income back when they were in the labour force. In the rest of the section, we describe the model in detail.

3.1 Demographics, endowments and heterogeneity

The economy is populated by overlapping generations. At each period t, a new generation of individuals is born. We assume that each household is composed of a man ($i = h$), a woman ($i = f$) and up to three children. We abstract from modelling singles [6], divorce or remarriage. For simplicity, we assume that both spouses are of the same age, and they face no uncertainty regarding life expectancy. At age $j = 1$, they begin life as potential workers, they retire after the mandatory retirement age, $j = R$, and they die with probability one at age $j = J$.

Individuals enter the economy with no assets, and they are endowed with one unit of time. This can be devoted to market work n_j^i, leisure l_j^i, and time spent with their children h_j^m. With this, households in our economy are either workers, out of the labour force or retirees. These employment status for an individual i, are denoted by $E^i = \{\omega, o, r\}$, respectively. During the working periods, $j \in [1, R]$ the employment status of all men is workers while women can decide between being workers or out of the labour force.

[6] We focus on the relationship between female labour force participation and number of children. By not modelling single households, we are not including the 10.1% of Spanish households that were mono-parental in 2019. Within mono-parental households, 82% were composed of women and her children.

At age $j = 1$, spouses receive a correlated income shock ϵ_1^i. This income shock will determine labour behaviour in the first period. After that, workers receive an endowment of efficiency labour units each period. This endowment has two components. First, a deterministic component which depends on the accumulated experience, x_j^i. Second, a stochastic component is given by an income shock of ϵ_j^i. This shock evolves stochastically as an autoregressive process of order 1. Therefore, the market income of a husband and her wife in each working-age period can be expressed as:

$$\ln y_j^i = \eta^i + \eta_1^i x_j^i + \eta_2^i (x_j^i)^2 + \epsilon_j \tag{1}$$

At each period t, the household heterogeneity comes from the age, j, employment status, E^i, pension rights p_j^i, labour market income, y_j^i and preferences regarding children. The heterogeneity in preferences is discussed in the calibration.

3.2 Timing and choice variables

Households live for J periods, where the length of each period is three years. We divide the household's lifespan into three different stages: the working-age child-rearing, the working-age non-child-rearing and the retirement. Each of them lasts for six periods. In other words, it encompasses the following group of ages: 30-47, 48-65 and 66-83. Working-age child-rearing stage refers to periods in which a household might have young children. The working-age non-child-rearing periods include those ages in which women do not have young children and are still in the labour market. Finally, the retirement stage is the one in which both spouses are retired. We denote these three stages as J_1, J_2 and J_3, respectively.

In what follows, we detail the choice variables that we model and at which stage they are made. In all periods, independent of the age, the household decides the consumption level c_j taking into consideration that children deflate consumption. See Table 3 for the timing of the women's decisions.

At the beginning of life. Before starting the first stage and after a couple is formed, the household decides the number of children they will have. Children are modelled as a discrete choice. To this end, they can choose between being a married couple without children, with one, two or three children, $b \in \{0, 1, 2, 3\}$. We assume that the first child is born in the first period in case of being a couple with children. Moreover, siblings are born in consecutive periods. This assumption implies that the spacing between siblings is three years.

Child-rearing stage. Young children impose a time constraint on mothers. We denote by young children those aged between 0 and 12. The age of the children is indicated by $a \in \{1, 2, 3, 4\}$[7]. Therefore, in the working-age child-rearing stage $j \in J_1$, mothers should decide how to take care of their young children. They have to be accompanied by an adult all time. We model three caregivers: mothers, other relatives (as fathers or grandparents) and formal institutions (kindergartens). We denote these three as mother childcare time h^m, informal childcare h^i, and out-of-pocket childcare purchased in the market h^p. This implies that $\forall j \in J_1$ the following equation must be satisfied:

$$h_j^m + h_j^i + h_j^p = 1 \tag{2}$$

Two features are relevant to note here. First, informal childcare provision increases with children's age and second, the out-of-pocket childcare cost decreases with the number of young siblings.

The endowment of unpaid childcare is bounded below and above. The lower bound is determined by the mandatory schooling m, while the upper bound in unpaid childcare available is denoted by κ. Both limits are a function of the number of young children the household has b, and the child's age a. Then, the following constraint must hold in the working-age child-rearing stages, $\forall j \in J_1$:

$$m_j(b, a) \leq h_j^i \leq \kappa_j(b, a) \tag{3}$$

The out-of-pocket childcare cost is a function of the childcare market price, q, the number and the age of young children in the household, and the childcare subsidisation, Υ. This subsidy is capped above by the schooling hours, s. The functional form of this paid childcare cost for young children is :

$$\lambda_j = q\sqrt{b}h_j^p - \Upsilon q \min[h_j^p, s] \tag{4}$$

Working stages. In the working stages, $j \in [J_1, J_2]$, individuals face a discrete choice of market hours. They can choose between being full-time workers, part-time workers or out of the labour force. We represent this by $n_j^i \in \{0, \frac{1}{4}, \frac{1}{2}\}$. We assume that men's labour supply is exogenous in both the intensive and extensive margins to ease the computation. Specifically, husbands always work full time in the first two stages of their life. Therefore, $n_j^h = \frac{1}{2}$ $\forall j \in [J_1, J_2]$. From now, we simplify the notation by avoiding the superscript f for women's choice variables.

[7] $a = 1$ includes ages 0-2 and $a = 4$ ages 10-12.

With all, the per-period time constraint for women is given by

$$1 = \begin{cases} l_j + n_j + h_j^m & \text{if } j \in J_1 \\ l_j + n_j & \text{if } j \in J_2 \\ l_j & \text{if } j \in J_3 \end{cases} \tag{5}$$

where h_j^m is the time mothers spend with their children. Therefore, for childless women this term equals to zero.

Table 3: Timing of decisions

Before period 1	Draw an initial income shock vector
	Make the fertility decision
Working-age child-rearing	labour force participation and childcare
	Enjoy utility
Working-age non -child-rearing	labour force participation decision
	Enjoy utility
Retirement	Obtain the retirement pension
	Enjoy utility

3.3 Social security

There is a pay-as-you-go social security system that taxes labour earnings to current workers and provides pension benefits to retired individuals. The social security tax rate, τ is proportional to labour earnings up to a cap, d_1. This is the maximum taxable earnings in the economy. The social security contributions of individual i are given with

$$T(y_j^i, n_j^i) = \tau \min[y_j^i n_j^i, d_1]. \tag{6}$$

The retirement pension a worker is entitled to, p, depends on previous taxable labour earnings. In particular, the formula for the retirement benefits depends on four components. First, it depends on age. To receive a retirement pension, the individual should be older than R. Second, eligibility. To be eligible to receive a retirement pension, the individual's life cycle contributions should be greater than N_c. Third, the pension will be calculated as an average of the individual's gross taxable wages over the last N_R periods of working-age life before retirement. Four, it depends on the wage replacement rate, θ. It is increasing in the number of worked years, χ, until it reaches 100%.

The retirement benefits of an individual who is retired at age $j = R$ is determined according to the following formula:

$$p = \min[\bar{p}, \max[\hat{p}, p]], \tag{7}$$

where $\bar{p}$ is the maximum and p the minimum retirement benefits. The interior solution is given as

$$\hat{p} = \theta \frac{\sum_{j=R-N_R}^{R} \min(\tau d_1, \tau y_j)}{N_R} \tag{8}$$

where d_1 is the maximum taxable earnings stabilised by law and the replacement rate θ follows:

$$\theta = \begin{cases} 0 & \text{if } \chi < a_1 \\ \theta_0 + \triangle\theta_1(\chi - a_1) & \text{if } \chi \in (a_1, a_2) \\ \theta_2 + \triangle\theta_3(\chi - a_2) & \text{if } \chi \in (a_2, a_3) \\ 1 & \text{if } \chi > a_3. \end{cases} \tag{9}$$

$$ss(y_j^i, n_j^i) = \Omega \min[y_j^i n_j^i, d_1]$$

Each period t the government or social security administration in our model collects payroll taxes on gross labour income Ω, and pays the retirement benefits.

The Social Security revenue at period t is denoted by SS_t^R. This is computed as the sum of all social security contributions $ss(y_j^i, n_j^i)$ from the working-age cohorts, $j \in [1, R]$ at period t. Then the resources of the social security is expressed as

$$SS_t^R = \sum_{j=1}^{R} N_{j,t} \sum_{i=\{f,h\}} ss(y_{j,t}^i, n_{j,t}^i), \tag{10}$$

where $N_{j,t}$ denotes the share of population of age j at period t as a fraction of the total population in the economy.

In the same line, the Social Security expenditure at period t, SS_t^X is a function of all the retirement pensions, $p_{j,t}^i$ paid to the retired population , $j \in [R+1, J]$. Then this expenditure follows:

$$SS_t^X = \sum_{j=R+1}^{J} N_{j,t} \sum_{i=\{f,h\}} pj, t^i. \tag{11}$$

The balance of social security is then:

$$SS_t = SS_t^R - SS_t^X \tag{12}$$

where SS_t can be positive, when the revenue exceeds the expenditure or negative in the opposite situation.

3.4 Preferences

From the previous section, it is clear that we are modelling just women's decisions on labour force participation, labour supply, fertility, childcare and consumption. Therefore, women are the sole decision-maker, they have all the bargaining power in the couple. The utility function of a woman varies with the stage of her life and whether she has children or not.

Mothers. Women's utility comes from different sources. At age j they derive utility from consumption c_j, leisure l_j, and a component related with children Γ, which depends on their age:

$$u_j\left(c_j, l_j, b, h_j^m\right) = \frac{\left(\frac{c_j}{\psi(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} + \Gamma. \tag{13}$$

where $\psi(b)$ is the OECD equivalence scale that depends on the number of children.

The children's utility component Γ, includes three terms. First, the disutility generated by the difference between realised and desired fertility b^*. Second, the disutility generated by having children. This component is needed to match the share of households without children. Third, the time spent with young children, h^m. Young children provide this last extra utility. In other words, it is only available if $j \in J_1$. The functional form of Γ as a function of age is:

$$\Gamma = \begin{cases} -\delta_2 |b - b^*|^{\gamma_2} + \zeta + \delta_3 \left(h_j^m\right)^{\gamma_3} & \text{if } j \in J_1 \\ -\delta_2 |b - b^*|^{\gamma_2} + \zeta & \text{if } j \in J_2 \\ 0 & \text{if } j \in J_3 \end{cases} \tag{14}$$

Childless women. A childless woman derives utility from her consumption's share c_j and leisure time, l_j. As women do not participate in the labour market in the retirement stage, leisure time is equal to one, i.e. $l_j = 1$. Therefore, the instantaneous utility $\forall j$ is:

$$u_j\left(c_j, l_j\right) = \frac{\left(c_j/\psi(0)\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1}. \tag{15}$$

where $\psi(0)$ is the OECD equivalence scale when the number of children is zero, $b = 0$.

3.5 Budget constraint

The per-period budget constraint is given by:

$$c_j = \begin{cases} y_j n_j - T(y_j, n_j) + y_j^h n_j^h - T(y_j^h) - \lambda_j & \text{if } j \in J_1 \\ y_j n_j - T(y_j, n_j) + y_j^h n_j^h - T(y_j^h) & \text{if } j \in J_2 \\ p_j + p_j^h & \text{if } j \in J_3 \end{cases} \tag{16}$$

where $T(y_j^i, n_j^i)$ represents the social security contributions of individual i at age j, n_j^i is the labour supply and p_j^i is the retirement benefit. We do not introduce the labour supply for husbands as it is constant and equal to one half.

3.6 Household problem in recursive form

We will describe the dynamic problem faced by households in each stage, starting with the last one. For simplicity, we do not include the value function for women without children. The main differences are: first, the utility function does not depend on the number of children b, second, they do not choose an allocation of childcare in the first stage, $j \in J_1$ and third, they never face the fertility decision.

Retirement stage. In this stage, women choose their consumption allocation. This choice depends only on the wife's pension p_j, and the husband's p_j^h. The taxable earnings and the experience determine these two. The value function of retired women at age $j \in J_3$ is given by:

$$
\begin{aligned}
V_j(x_j, \bar{y}_j, \bar{y}_j^h; b) = \max_{c_j} \quad & u_j\left(c_j, b\right) + \beta\, \mathbb{E}\left[V_{j+1}\left(x_{j+1} \bar{y}_{j+1}, \bar{y}_{j+1}^h; b\right)\right] \\
& \text{s.t.} \\
& c_j = p_j + p_j^h \\
& x_{j+1} = x_j \\
& c_j \geq 0
\end{aligned}
$$

Working age non-child-rearing stage. In this stage, women decide their consumption and labour supply. They can be out of the labour force, $n_j = 0$, part-time workers $n_j = \frac{1}{2}$ or full-time workers $n_j = \frac{1}{4}$. The differences between mothers and women without children are the utility component for children and the equivalence scale. The value function of a working-age non-child-rearing woman at age $j \in J_2$ is given by:

$$
\begin{aligned}
V_j(\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j, \bar{y}_j^h; b) = \max_{c_j, n_j \in \{0, \frac{1}{4}, \frac{1}{2}\}} \quad & u_j\left(c_j, n_j, b\right) + \beta\, \mathbb{E}\left[V_{j+1}\left(\epsilon_{j+1}, \epsilon_{j+1}^h, x_{j+1}, \bar{y}_{j+1}, \bar{y}_{j+1}^h; b\right)\right] \\
& \text{s.t.} \\
& c_j = y_j n_j - T(y_j, n_j) + y_j^h - T^h(y_j^h) \\
& \ln y_j = \eta + \eta_1 x_j + \eta_2 x_j^2 + \epsilon_j \\
& \ln y_j^h = \eta^h + \eta_1^h\left(j-1\right) + \eta_2^h\left(j-1\right)^2 + \epsilon_j^h \\
& \epsilon_{j+1} = \phi \epsilon_j + \nu_j \\
& \epsilon_{j+1}^h = \phi^h \epsilon_j^h + \nu_j^h \\
& l_j + n_j = 1 \\
& x_{j+1} = x_j + n_j \\
& c_j \geq 0, \quad n_j \geq 0
\end{aligned}
$$

Working age child-rearing stage. In this stage, women decide consumption and labour supply. Additionally, mothers choose childcare allocation. They decide the time spent with their children h_j^m, the time use of informal childcare h_j^i and the out-of-pocket childcare time h_j^p. Then, the value function of a working-age child-rearing woman at age $j \in J_1$, is given by:

$$
\begin{aligned}
V_j(\epsilon_j, \epsilon_j^h, x_j, \bar{y}_j, \bar{y}_j^h; b) = & \max_{c_j, n_j \in \{0, \frac{1}{4}, \frac{1}{2}\}, h_j^m, h_j^p, h_j^i} \quad u_j\left(c_j, n_j, b, h_j^m\right) + \beta\, \mathbb{E}\left[V_{j+1}\left(\epsilon_{j+1}, \epsilon_{j+1}^h, x_{j+1}, \bar{y}_{j+1}, \bar{y}_{j+1}^h; b\right)\right] \\
& \text{s.t.} \\
& c_j = y_j n_j - T(y_j, n_j) + y_j^h - T^h(y_j^h) - \lambda_j \\
& \lambda_j = q\sqrt{b} h_j^p - \Upsilon q \min[h_j^p, s] \\
& \ln y_j = \eta + \eta_1 x_j + \eta_2 x_j^2 + \epsilon_j \\
& \ln y_j^h = \eta^h + \eta_1^h (j-1) + \eta_2^h (j-1)^2 + \epsilon_j^h \\
& \epsilon_{j+1} = \phi \epsilon_j + \nu_j \\
& \epsilon_{j+1}^h = \phi^h \epsilon_j^h + \nu_j^h \\
& l_j = 1 - n_j - h_j^m \\
& h_j^m + h_j^p + h_j^i = 1 \\
& x_{j+1} = x_j + n_j \\
& c_j \geq 0, n_j \geq 0, \quad 0 \leq h_j^m, \quad 0 \leq h_j^p \\
& m_j(b, a) \leq h_j^i \leq \kappa_j(b, a)
\end{aligned}
$$

Fertility decision. At age $j = 1$ and before any other choices, women decide the number of children they will have. This decision is a function of income shocks. Then, the problem face by a potential mother is:

$$
\underset{b \in \{0,1,2,3\}}{\text{Max}} V_0(\epsilon_0, \epsilon_0^h, 0, 0, 0; b)
$$

4 Calibration and model performance

In this section, we explain the calibration strategy we follow. First, we specify how each period works in the model. Second, we set some parameters exogenously. This includes the parameters governing Social Security. Third, another set of parameters is estimated without solving the model, notably the income process and childcare price. Finally, a third set of parameters is chosen by solving the economy in equilibrium. Since there is no closed-form solution for the model, we rely on the method of simulated moments.

4.1 Model's periods and population dynamics

Households are assumed to enter the economy at the age of 30 and live up to age 83 with no uncertainty regarding this. For simplicity, all households are retired at the age of 65, the legal retirement age in Spain until 2013.

For computational reasons, one model period is three years. Hence, the first age (j=1) corresponds to ages 30-32. In stage-terms imply: the working-age child-rearing periods J_1, includes ages 30-47, the non-child-rearing periods J_2, ages 48-65 and the retirement J_3, the last 18 years. The total number of periods that a household lives J, is 18 while 12 are working-age periods.

The initial population is taken from *INE* for the year 2020. We aggregate the cohorts into brackets of 3 years, to match the model period's length. The size of a newborn cohort in period $t+1$ is computed as the total fertility rate resulting from the model times the size of the cohort aged 30 to 33 in period t, divided by two as we only model women.

4.2 Parameters chosen before solving the model

Earning ability process. This section intends to discuss the methodology and the goals of the labour income process estimation.

The goal of our paper is to introduce critical determinants for fertility choices. The paper's introduction of income risk is motivated by the findings in Sommer (2016). Her paper shows that increasing labour market risk and more persistent adverse spells cause parents to postpone childbirth. Then, they work and save more. However, this is not the only determinant. Returns to experience are introduced to measure the effect of an individual's labour market decisions on their next earnings. As in the model, women's labour supply is endogenous, this term will play a central role in the trade-off between working more or having and raising children.

Regarding the methodology, earnings risk aims to capture the risk faced by individuals in the labour market and their risk when they decide to be out. This is justified by the fact that we do not model unemployment explicitly. To this end, we measure the gross yearly income. It is computed as the income per worked day times 365. This avoids the over-representation of women in part-time and temporary jobs. This income measure includes all income received by an individual but for pensions, prizes from games and non-levied income. See the appendix for a more in-depth explanation of the methodology.

The labour income process is estimated to capture the impact of work experience on female-male gender wage gaps. In this paper, we take the functional form assumption in Bick (2016) for the labour income process of men's, and we apply it to both men and women.

Therefore, the main difference concerning Bick (2016) is that we can estimate women's labour income from the data without making further assumptions.

The individual earning ability process of men and women, y_j^h and y_j, for ages $j \in \{J_1, J_2\}$ is a concave function that depends on the worker's experience. As husbands always work full-time, it is equivalent to say that it depends on age. However, this is not the case for women. If they choose to work part-time, the accumulated experience is half of the full-time women. Therefore, it is calculated as

$$\begin{aligned}\ln y_j &= \eta + \eta_1 x_j + \eta_2 x_j^2 + \epsilon_j \\ \ln y_j^h &= \eta^h + \eta_1^h (j-1) + \eta_2^h (j-1)^2 + \epsilon_j^h\end{aligned}$$

where the persistent shocks ϵ_j^h and ϵ_j follows the following AR(1) process:

$$\begin{aligned}\epsilon_j &= \phi \epsilon_{j-1} + \nu_j \\ \epsilon_j^h &= \phi^h \epsilon_{j-1}^h + \nu_j^h\end{aligned}, \quad \begin{bmatrix} \nu_j \\ \nu_j^h \end{bmatrix} \sim N\left(\begin{matrix} \mu \\ \mu^h \end{matrix}, \begin{bmatrix} \sigma_\nu^2 & \rho \\ \rho & \sigma_{\nu^h}^2 \end{bmatrix} \right).$$

In order to estimate the previous equations we use the Spanish Continuous Working Life Sample (MCVL). See Table 4 for a summary of the parameters. The methodology is discussed in the Appendix.

Table 4: Income process parameters

Parameter	**Value**	**Parameter**	**Value**
η_0	9.286	η_0^h	9.533
η_1	0.0326	η_1^h	0.0318
η_2	-0.00028	η_0^h	-0.00045
ϕ	0.655	ϕ^h	0.654
ρ	0.0342	σ, σ^h	03701

Household preferences. Given the instantaneous utility function discussed previously, we calibrate some parameters in this function to match different labour supply and fertility moments. Table 8 shows the results obtained for the labour and fertility calibration.

The subjective discount factor is taken as $\beta = (1/1.04)^3 = 0.89$, which is the time discount factor in Kydland and Prescott (1982). At the beginning of life, households have a probability of desiring $b^* \in \{0, 1, 2, 3\}$ children. These probabilities are directly extracted from the ECV. See section 2 for the methodology. Table 5 summarizes the parametrization of these.

Table 5: Preference parameters

	Parameter	Value
Three-year discount factor	β	0.89
OECD equivalence scale	ψ	$1.7 + \sqrt(b)$
	$b^* = 0$	7.9
Desired fertility	$b^* = 1$	15.2
	$b^* = 2$	49.75
	$b^* = 3$	27.15

Social Security and taxation. The Social Security parameters' baseline calibration for the contributory pensions is based on the General Regimen of the Social Security System. The legislation that we introduce in the model is the one previous to the pension reform in 2013. We are not modelling the latest pension reform introduced in Spain. This assumption was taken due to the computational complexity of changing legislation over time as this reform has been progressively introduced since then. Nevertheless, we justify this assumption with the following two arguments. First, in 2019 the majority of pensioners retired before 2013, under the previous-law conditions. Second, the new pension reform has been progressively incorporated and it will be totally implemented by 2027. Therefore, the changes in the law are minor for the pensioners that retired between 2013 and 2019. For instance, we are incorporating the legal retirement age as 65 years old, while the one for the workers retired in 2019 is 65 years and eight months.

The social security parameters are displayed in Table 6. The retirement age, R is set to 65 years old, the legal retirement age in Spain. The number of contribution periods that count towards eligibility and the retirement benefit computation, N_c and N_R are set to 15 years to reflect current social security law. The replacement rate parameters show the increasing pattern with the number of contributed years. When the individual's worked years are lower than a_1 the Spanish legislation sets the replacement rate to 0. Finally, retirement pensions are introduced with boundaries, following the law.

The Spanish Social Security collects a proportional tax on the covered earnings in order to finance pensions. Covered earnings are computed as the worker's gross labour income with a cap above and a tax-exempt minimum. As in Díaz-Giménez and Díaz-Saavedra (2009), we do not include the tax-exempt minimum. In 2019, the payroll tax was 28.3%, 23.6% was attributed to firms while the remaining 4.7% was levied on workers. We denote the share attributed to firms as τ^c, to workers as τ and the sum of both as Ω. The following function determines social security contributions of individual i at age j:

$$ss(y_j^i, n_j^i) = \Omega \min[y_j^i n_j^i, d_1]$$

where d_1 is the maximum contribution base. In 2019, d_1 was 56.981,4 per year. As in our model one period represents three years, $d_1 = 170.944, 2$. Moreover, the worker's tax is τ, their social security contributions are:

$$T(y_j^i, n_j^i) = \tau \min[y_j^i n_j^i, d_1].$$

The parameters of the contributions to the Social Security System are taken from the Social Security database and are summarized in Table 6 .

Table 6: Social Security parameters

	Parameter	**Value**
Regular retirement age	R	13
Minimum contributed periods (in years)	N_c	5
Regulatory base (in years)	N_R	5
Total payroll tax	Ω	28.3%
Payroll tax levied on workers	τ	16.6%
Maximum taxable yearly earnings	d_1	56981.4
Penalties (in years)		
	a_1	15
	a_2	25
	a_3	35
	θ_1	50
	$\triangle\theta_1$	3
	θ_2	80
	$\triangle\theta_2$	2
Pension bounds (yearly euros)		
Minimum retirement pension	p_t	9081.8
Maximum retirement pension	$\hat{p}_t$	37566.76

Childbearing. In the model, we assume that young children should be looked after by an adult all day. Therefore in the household has a child aged $a \in [1, 4]$, they choose who will take care of him/her. This decision depends on four factors: first, the price of the out-of-pocket childcare in the market, second, the endowment of informal childcare, third, the childcare subsidy provided by the government and four, the household's preferences regarding children. This last factor was previously explained[8].

Following Sánchez Martín and Sánchez Marcos (2010), the price of out-of-pocket childcare q, is computed as the 33% of average earnings in the economy. Regarding the endowment bounds of informal childcare, we make two assumptions. On the one hand, universal schooling is provided for children older than three years old. This implies that the lower bound in the mandatory schooling is 0 if $a = 1$ and positive otherwise. In addition, we assume there is not a one to one substitution between schooling and full-time jobs. We assume that the schooling time provision is 30 hours per week while a full-time job is 40. On the other hand, informal childcare provision is equal to 0.25 independent of the child's age. In this baseline economy, we aim to replicate as closely as possible the Spanish economy. To this end, childcare subsidies Υ, are equal to zero. Table 7 summarizes the parameter values for childbearing.

Table 7: Childbearing parameters

	Parameter	**Value**
Price of out-of-pocket childcare	q	7031.4 euros
Lower bound in informal childcare provision	$m(a = 1)$	0
	$m(a > 1)$	0.375
Upper bound in informal childcare provision	$\kappa(a = 1)$	0.25
	$\kappa(a > 1)$	0.625
Childcare subsidy	Υ	0

4.3 Parameters chosen by solving the model

Since the utility function is additively separable, the utility term associated with children's mere presence does not affect labour supply decisions conditional on children's number. We take advantage of this to calibrate separately the rest of the parameters, namely γ_0, γ_1, γ_3, δ_1 and δ_3. We take the observed distribution of children across women and use the method of simulated moments to choose these parameters to replicate several labour supply moments along the life cycle. Even though all parameters affect all moments, it is worth to discuss the chosen targeted moments and how they relate to specific parameters.

The curvature of consumption γ_0 is informative about differences in labour force participation between women with different number of children through the potential monetary costs that childcare entails and through consumption adjustment via the equivalence scale.

[8] See the section for the functional form for preferences.

Therefore we include the fraction of women with children aged 6 to 12 who work full time and the fraction of childless women the same age working full-time.

The curvature of leisure γ_1 is informative about women's labour supply decisions with different household income levels at the intensive margin (part-time versus full time). As a target for this parameter, we choose the fraction of women with children aged 6 to 12 working part-time, and the fraction of childless women the same age working part-time.

The utility weight of leisure δ_1 is informative for labour force participation decisions, i.e. labour supply at the extensive margin. Thus we include as a target the total labour force participation for childless women (lifetime average).

The parameters associated with childcare δ_3 and γ_3 govern the utility mothers derive from spending time with children, and are therefore informative about labour force supply for women with children and their childcare utilization. So we include the labour force participation of women with children aged 0-2 and the paid childcare utilization of women that do not work.

Once we have the labour supply parameters, we can compute lifetime utility, excluding the term related to children. This term will be different for each fertility level because of the equivalence scale and the time constraints imposed by children.

To finish the calibration, we need to compute a set of parameters such that the fraction of households that choose to have 0, 1, 2 and 3 children coincides with the number of households that have them in the data.

Intuitively, having more children lowers lifetime utility because they lower equivalent consumption and leisure. We need the fertility parameters to be so that they compensate households for this difference, just in the right amounts on average so that their fertility decisions are close to the observed ones.

If all households have the same fertility parameters, variation in fertility decisions is based only on the initial income shocks and differences in the desired number of children. This is not enough to generate the observed distribution of children. Therefore, each household's parameters δ_2 and ζ are drawn from an exponential distribution. That is, households are heterogeneous in how much they care for departures from their ideal level of fertility b^* and how much they care about having children at all. The parameters that we estimate are the underlying means of those draws.

4.4 Model performance.

Matched moments. The calibration results for the baseline economy are shown in Table 8. We separate those parameters and moments related to labour force participation and childcare, and those related to fertility decisions.

The estimated curvature parameters for consumption and leisure γ_0 and γ_1 are relatively close to 1. This implies that the consumption and leisure utility terms are close to being logarithmic. Therefore, changes in the earnings potential have a muted effect on labour force participation, as income and substitution effects would roughly cancel each other.

We can reproduce quite closely the total labour force participation of women whose youngest child is 0 to 3 years old, and the childcare usage of women that do not work and that have a child aged 0 to 3 years old. In our baseline economy, the share of women working full time is lower than in the data, while the share of women working part-time is higher for both women with the youngest child aged 6-12 and women aged 48-59. However, the total labour force participation is close for women with children aged 6-12. Childless women work too much with respect to the data. These results seem to suggest that γ_1 should be set at a higher value and δ_1 at a lower one, according to our previous discussion as to how each parameter affects each of the moments. However, in reality, all parameters affect all moments. The model is over identified as we have more moments than parameters. Changing γ_1 and δ_1 affects its ability to reproduce the labour force participation and childcare usage of women with children 0-2, which are very important given the nature of our question.

It is also likely that we are missing a few things in our current setting. We are probably underestimating the effect of experience on young women. Currently, the cost of non-participation is just the missed experience. However, early-career interruptions can be particularly costly. Moreover, the high prevalence of zero-hour contracts in Spain could increase the value of labour force participation early on. Working full time may increase the likelihood of converting one's contract into an indefinite contract in the future, which would factor in the labour force participation decision. We are working into allowing for these kinds of effects within the model. Reflecting this in a tractable manner within our framework has proven to be challenging.

In terms of fertility, the model matches the share of childless women closely, but overestimates the fraction of women with one child and underestimates the fraction of women with two with respect to the data. This is partly compensated by a slightly higher fraction of women with three children so that the total fertility rate is only slightly below the one in the data.

Comparison to previous literature. In an attempt to compare our quantitative model to empirical evidence of child-related policies' effectiveness, we tested in it two policies that have been assessed in the empirical literature.

The first one is the universal offering of public childcare for 3-6-year-old children in

Table 8: Baseline calibration results

Parameter	**Value**	**Target(s)**	**Model**	**Data**
Labour force participation and childcare				
γ_0	0.968	Share full-time work (women with youngest child 6-12)	32.72%	47.03%
		Share full-time work (women aged 48-59)	30.39%	41.68%
γ_1	1.011	Share part-time work (women with youngest child 6-12)	44.04%	20.32%
		Share part-time work (women aged 48-59)	44.77%	13.41%
γ_3	0.730	Labour force participation (women with youngest child 0-3)	69.81%	71.28%
δ_1	1.062	Average lifetime LFP (childless women)	83.03%	66.49%
δ_3	2.330	Weekly paid childcare hours (non-working women with child 0-2)	9.74	8.80
Fertility				
		Share of women with:		
ζ	0.10	0 children	24.50%	25.50%
		1 child	35.40%	27.40%
δ_2	1.60	2 children	30.50%	39.80%
γ_2	3.00	3 children	9.60%	7.30%
		Total fertility rate	1.25	1.29

Spain's late 1990s. This policy's impact was studied by Nollenberger and Rodríguez-Planas (2015) using its staggered implementation across Autonomous Communities as a natural experiment. They find that the policy raised maternal employment by 9.6% for women with children in this age group, with the results for women older than 30 and with two or more children being larger, 15.3% and 14.9% respectively. They find no effect on fertility.

Since in our baseline calibration there is a provision of 30 hours per week of schooling for children aged 3-6, we remove such provision and solve the model with the same parameters used in the baseline. We then compute the difference in labour force participation for women with children aged 3-6 and fertility, between the economy with no provision of schooling and the baseline economy. This we can compare to the results in Nollenberger and Rodríguez-Planas (2015). There are some caveats, in any case. In our model, women with children aged 3-6 are either 33 to 36, 36 to 39 or 39 to 42 years old by definition. Moreover, those aged 33 to 36 have one or two children. Those aged 36 to 39 have two or three, and those aged 39 to 42 have three children.

Table 9 presents the results of the exercise. Labour force participation of older women (which are also the ones that have more children) reacts strongly to the removal of the schooling provision. The magnitude of the effect is much larger than in Nollenberger and Rodríguez-Planas (2015) though. Similarly to them, we find no effects on fertility.

Table 9: The effect of removing free provision of pre-schooling for children aged 0 to 3.

Outcome	**Baseline**	**No free scholing 3-6**	**Difference**
Labour force participation			
(women with child 3 to 6):			
Aged 33-36	69.67%	71.99%	2.33%
Aged 36-39	65.59%	48.51%	-17.08%
Aged 39-42	50.00%	35.00%	-15.00%
Fertility			
Share of women with:			
0 children	24.50%	24.30%	-0.20%
1 child	35.40%	35.50%	0.10%
2 children	30.50%	30.20%	-0.30%
3 children	9.60%	10.00%	0.40%
Total fertility rate	1.25	1.26	0.01

The second policy that we test in our model is the "cheque bebé" (baby check). This was a one-time cash transfer upon the birth of 2500 euros offered to parents by the Spanish government between 2007 and 2010. González (2013) evaluates this policy empirically, and finds negatives effect on labour force participation after childbirth and a significant increase in fertility.

We test this policy in our model by introducing the cash benefit mentioned above in the period in which a child is born. Similarly, we solve the model with the same parameters used in the baseline. We then compute the difference in labour force participation and fertility outcomes with respect to the baseline.

Again, there are some caveats to this. The most important is that there are no savings in our model, and therefore the cash benefit has to be consumed within the period in which it is received. We believe this is not a very serious caveat for two reasons. First, if the families are credit constraint and expect their earnings to increase in the future, they would consume the cash benefit immediately. Second, our model period is three years, so the cash benefit is spread out at least over that period.

We report the results of the exercise in Table 10. The baby check has a negligible effect on the youngest mothers' labour force participation, but increases part-time relative to full-time employment for mothers aged 33 to 36 and then lowers labour force participation for mothers aged 36-39 and 39 42. In a sense, this is similar to our previous exercise: older mothers are more affected than young ones. Moreover, the results are qualitatively similar to González (2013), at least for older mothers. The baby check leads to a tiny increase in total fertility, driven by more women having three children. In her study, González finds that the baby check increased births by 6 per cent. In our model, this is much lower. One reason may be that part of the effect captured by her is anticipated births. This effect would be more substantial if people expected the policy to end (which it did, although they did not have the certainty it would at the time). In our model, households cannot anticipate births, which would lead to a difference in results even if they expected the policy to end.

The main take from these two exercises is that female labour force participation and fertility responses to the two policies tested in the model are qualitatively similar to the empirical literature's effects. Although there are quantitatively important differences. We get stronger labour force participation effects than Nollenberger and Rodríguez-Planas (2015), but more muted fertility effects than González (2013).

5 Policy experiments

The main policy experiments that we consider in this paper consist of subsidising childcare for mothers with children aged 0-3 years old. We implement partial and full subsidisation, i.e. the government covering 50% and 100% of childcare costs for up to 30 hours per week.

Table 10: The effect of a cash benefit per child born.

Outcome	Baseline	Baby check	Difference
Labour force participation			
Part time share			
(all mothers):			
Aged 30-33	53.77%	54.19%	0.41%
Aged 33-36	43.97%	49.08%	5.11%
Aged 36-39	51.13%	39.27%	-11.86%
Aged 39-42	37.48%	30.50%	-6.99%
Aged 42-45	47.81%	46.86%	-0.96%
Aged 45-48	49.14%	49.48%	0.34%
Full time share			
(all mothers):			
Aged 30-33	19.60%	19.50%	-0.10%
Aged 33-36	25.70%	21.60%	-4.10%
Aged 36-39	16.16%	15.71%	-0.45%
Aged 39-42	36.03%	37.57%	1.54%
Aged 42-45	38.54%	38.61%	0.07%
Aged 45-48	33.91%	34.03%	0.12%
Fertility			
Share of women with:			
0 children	24.50%	24.30%	-0.20%
1 child	35.40%	35.50%	0.10%
2 children	30.50%	30.20%	-0.30%
3 children	9.60%	10.00%	0.40%
Total fertility rate	1.25	1.26	0.01

Table 11 presents the results of our policy experiments.

Table 11: Policy experiment outcomes

Outcome	**Baseline**	**Partial**	**Full**
Female labour force participation			
Part time share			
(women with child 0 to 2):			
Aged 30-33	53.77%	53.89%	54.19%
Aged 33-36	42.14%	45.77%	57.07%
Aged 36-39	15.63%	34.38%	59.18%
Full time share			
(women with child 0 to 2):			
Aged 30-33	19.60%	19.63%	19.50%
Aged 33-36	27.93%	27.61%	17.62%
Aged 36-39	25.00%	10.42%	7.14%
Childcare use			
Hours per week (women with child 0 to 2):			
Aged 30-33	25.88	26.52	27.14
Aged 33-36	26.18	27.47	27.05
Aged 36-39	20.14	19.71	24.12
Fertility			
Share of women with:			
0 children	24.50%	24.10%	23.60%
1 child	35.40%	35.70%	36.10%
2 children	30.50%	30.60%	30.50%
3 children	9.60%	9.60%	9.80%
Total fertility rate	1.25	1.26	1.27
Present value of SS budget			
% of average working-age family's income	40.10%	39.53%	38.88%

In terms of female labour force participation, the subsidies lead to a shift from full-time to part-time work among older mothers with younger children. This leads to a minimal increase in total labour force participation for mothers aged 33 to 39, as shown in Figure 4.

Unsurprisingly, the childcare subsidies lead to an increase in childcare usage among women with young children for all age brackets. The increase is not very large, being less than 10% in all cases except for women aged 36-39 when moving from no subsidies to full subsidies.

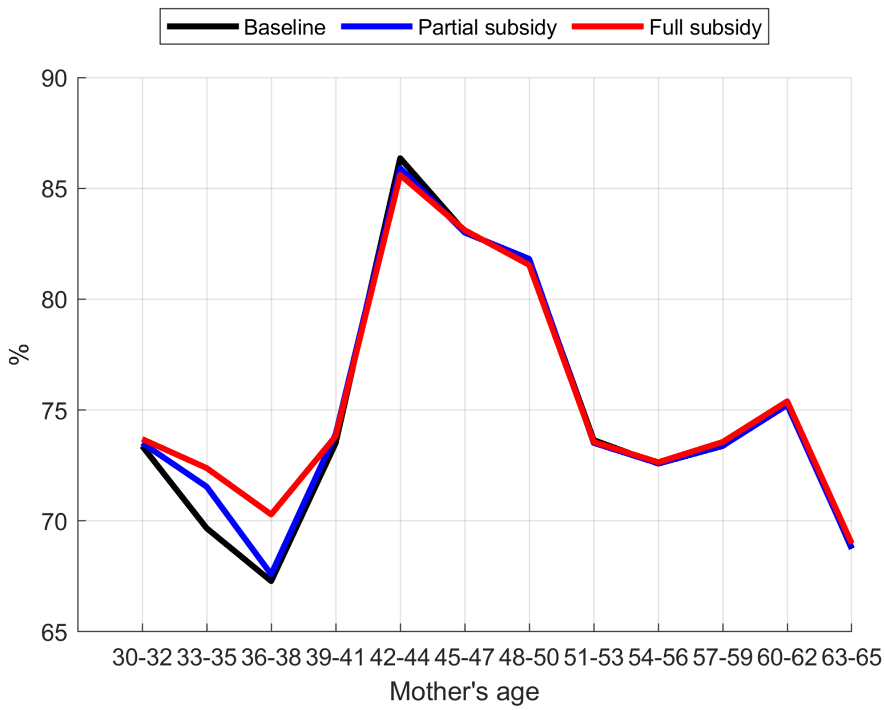


Figure 4: Labour force participation of mothers by age.

The effect of childcare subsidies on fertility is small. The fraction of childless women falls with respect to the baseline under both partial and full subsidies. The fraction of women with three children increases very slightly with full subsidies. Total fertility rate increases only marginally, from 1.25 children per woman in the baseline to 1.27 in the full subsidies case. We are working on providing numbers for the effect of the subsidies on the mismatch between desired and realised fertility, which could drive some welfare benefits. This would be especially important if the subsidies were budget-neutral or even budget-positive for Social Security. It would allow us to make a strong statement about the convenience of childcare subsidies.

However, they are not. The present value of the Social Security budget balance falls as a percentage of the average working-age family's income[9]. Expenditures for the Social Security administration increase immediately, albeit not by a lot since the subsidies' cost is small compared to the regular outlays of Social Security (the retirement benefits). This lowers the net budget in the present and near-present periods. On the other hand, the increase in the total fertility rate induced by the subsidies barely affects the future's old-age

[9]The present value of the Social Security balance is positive in our baseline economy. In reality, this value is negative. We use the payroll tax laid out in the law (28.3%) to calculate Social Security revenues. This is likely too high. We are working on estimating the effective contribution to Social Security as a function of gross income using our administrative *MCVL* data set.

dependency ratio. This can be seen in Figures 5 and 6.

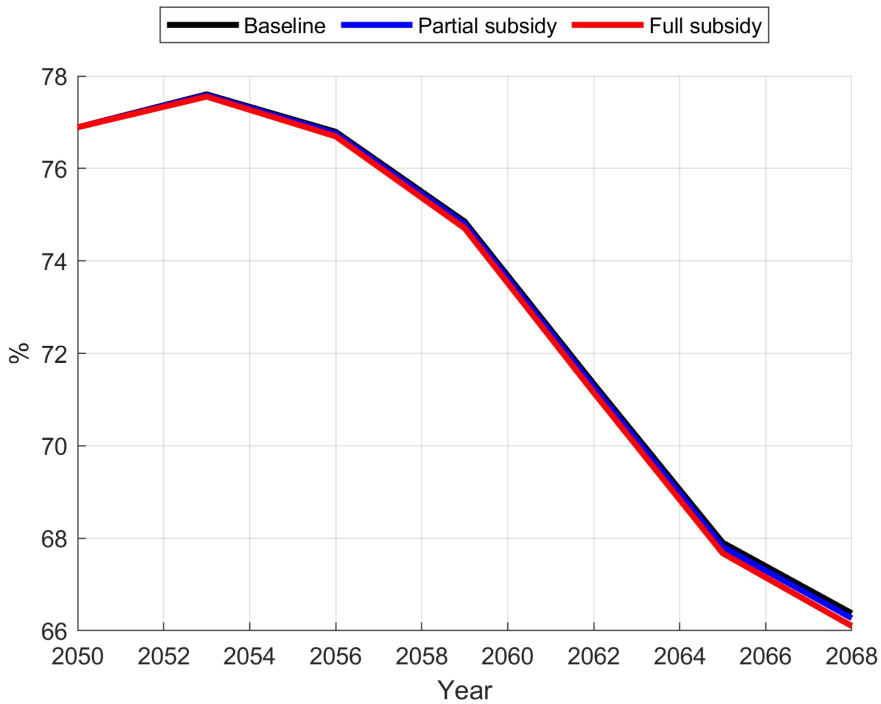


Figure 5: Old dependency ratio (2050-2068)

Unfortunately, our experiments' main takeaway is that childcare subsidies do not achieve any of the goals we have hoped for. The response of the mother's labour force participation is not what we expected. The subsidies fail to generate a large enough fertility response to be budget-neutral for the Social Security Administration.

5.1 Discussion

What drives the findings of our policy experiments? The childcare subsidies induce an income effect and a substitution effect. On the one hand, they increase households' real income with children aged 0-3, which should lead to higher consumption, leisure, and time spent with children. On the other hand, they make childcare less expensive. This increases the price of leisure relative to consumption, inducing the household to decrease leisure, increasing consumption, and increasing the price of time spent with children relative to leisure, which should decrease time spent with children and increase leisure. Notice that, at the margin, childcare is still as expensive as before for women that want to consume many hours of it since the subsidy is only 30 hours per week. Therefore, it is costly for women working full time. Since γ_0 and γ_1 are both close to one, the income and substitution effects should

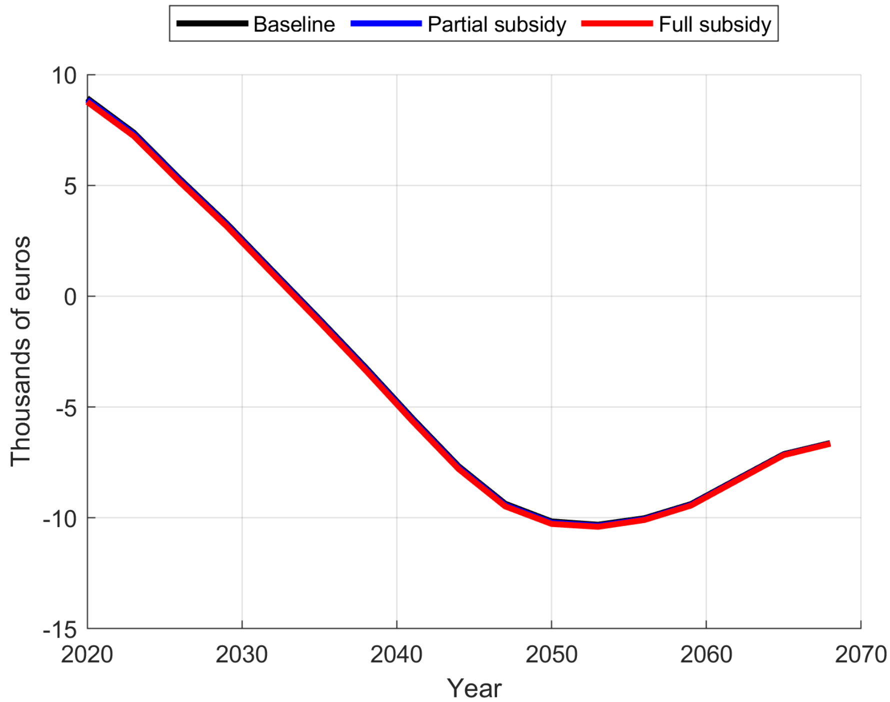


Figure 6: Social Security budget balance

roughly cancel each other out for leisure.

The childcare subsidies have a negligible effect on the labour force participation of women aged 30-33 with children aged 0-2. Since childcare also increases, this means that time spent with children decreases. Therefore, for these women, the substitution effect dominates the income effect on their children's time.

For older women, the subsidies induce women to increase time spent with children. They would tend to have higher consumption since they are older and therefore, husbands' incomes and their own earnings potential are higher. Therefore, they care more about the time spent with children.

The effect of the subsidies on fertility depends heavily on the fertility parameters. These are chosen to match the distribution of children across households. Intuitively, δ_2 controls how much households care more about departures from their desired fertility, while ζ matters for the decision between having children or not. The latter seems to be playing a more important role here, that is, the subsidies can convince some (small) fraction of women to have children, but not many to have more children.

6 Conclusions

This paper quantifies the effect of childcare subsidisation policies on fertility, female labour force participation, and social security budgets. To this end, we study an overlapping generations economy, in which women initially choose fertility and then the labour force participation and childcare along their life-cycle. We calibrate the model to Spanish data and experiment by introducing partial (50%) and full (100%) childcare subsidies for women with children aged 0-3, since childcare for children 3-6 is already free and universal in Spain.

The paper's main takeaway is that the childcare subsidies do not have a strong positive effect on the mother's labour force participation and fertility. This somewhat consistent with previous literature. Moreover, the subsidies affect the budget balance of Social Security negatively: more is spent on them than is recovered through future lower old-age dependency ratios. Mainly, the positive impact of the subsidies on fertility is too small to offset their immediate cost.

The analysis presented here has two major caveats. First, women choose the number of children at age 30, and they are matched with men in a deterministic way. Therefore, this framework is not suitable for analysing how childcare subsidies affect the timing of childbirth. Second, we abstract from modelling important Spanish labour market features, like high unemployment and fixed-term contracts' pervasiveness.

These two features, potentially interact in a non-trivial way to determine women's response to childcare subsidies. Since the effects of shifts in childbirth timing on social security contributions over the life cycle are probably small compared to changes in total fertility, we think the first caveat may be minor. Besides, to the extent that the income process that we impose accounts for the effects of fixed-term contracts and high unemployment, we can also claim we are missing only second-order effects through the second.

Therefore, we believe that these mechanisms are outside the scope of the questions that we ask in this paper. Nevertheless, further research is necessary to understand them in their own right.

Appendix

The childcare allocation problem

Given a choice of labour force participation n_j, and state variables ϵ_j, ϵ_j^h, x_j and b the childcare allocation problem faced by a working-age child rearing mother at age j is:

$$
\begin{aligned}
&\max_{h_j^m, h_j^p, h_j^i} \frac{\left(\frac{c_j}{\psi(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} - \delta_2 |b - b^*|^{\gamma_2} - \zeta + \delta_3 \left(h_j^m\right)^{\gamma_3} \\
&\text{subject to} \\
&c_j = Y_j\left(\epsilon_j, \epsilon_j^h, x_j, n_j\right) - \lambda_j \\
&\lambda_j = q\sqrt{b} h_j^p - \Upsilon q \min[h_j^p, s] \\
&\ln y_j = \eta + \eta_1 x_j + \eta_2 x_j^2 + \epsilon_j \\
&\ln y_j^h = \eta^h + \eta_1^h (j-1) + \eta_2^h (j-1)^2 + \epsilon_j^h \\
&\epsilon_{j+1} = \phi \epsilon_j + \nu_j \\
&\epsilon_{j+1}^h = \phi^h \epsilon_j^h + \nu_j^h \\
&l_j = 1 - n_j - h_j^m \\
&h_j^m + h_j^p + h_j^i = 1 \\
&x_{j+1} = x_j + n_j \\
&c_j \geq 0, n_j \geq 0, \quad 0 \leq h_j^m, \quad 0 \leq h_j^p \\
&m_j(b,a) \leq h_j^i \leq \kappa_j(b,a)
\end{aligned}
$$

where h_j^m is the childcare provided by the mother, h_j^p is paid childcare purchased in the market and h_j^i in unpaid childcare (childcare provided by the father or other family members, time spent at school, etc.).

Consumption c_j is equal to the household's net income $Y_j\left(\epsilon_j, \epsilon_j^h, x_j, n_j\right)$ minus childcare costs $\lambda_j(h_j^p, n)$, where the childcare cost function is:

$$\lambda_j = q\sqrt{b} h_j^p - \Upsilon q \min[h_j^p, s]$$

Leisure l_j is equal to one minus paid labour time n_j and mother provided childcare time. The amount of unpaid childcare h_j^i is bounded below by the amount of mandatory schooling $m_j(b,a)$. This is a function of the age (and hence the age of the children) and the number of children. It is bounded above by the total unpaid childcare available to the mother, $\kappa_j(b, a)$. Since mandatory schooling is included in total available unpaid childcare, $m_j(b, a) \leq \kappa_j(b, a)$. Mother childcare provision and paid childcare utilization must be non negative. Consumption and leisure must be strictly positive. Finally, total childcare time must be equal to 1, that is, children must be looked after every moment of the day.

Not well-defined problem if children are alone. It is possible that for a given labour force participation choice n_j the mother is unable to satisfy the children's time constraint. Define $\bar{h}_j^p$ as:

$$\bar{h}_j^p = \max\{0, \sup\{h_j^p : Y_j\left(\epsilon_j, \epsilon_j^h, x_j, n_j\right) - \lambda_j(h_j^p, n) > 0\}\},$$

that is, $\bar{h}_j^p$ is the maximum amount of paid childcare that the household can afford. Likewise, $\bar{h}_j^m = 1 - n_j$ is the maximum childcare time that the mother can provide given that her labour force participation is n_j. If

$$\bar{h}_j^m + \bar{h}_j^p + \kappa_j(b, a) \leq 1,$$

then the choice of labour force participation n_j is unfeasible for the mother, and the childcare problem is not well defined.

From here on we assume:

$$\bar{h}_j^m + \max\{0, \bar{h}_j^p\} + \kappa_j(b, a) > 1.$$

Lemmas

Lemma 6.1 *If $\{h_j^m, h_j^p, h_j^i\}$ is a solution to the childcare allocation problem, then either:*

1. $h_j^p = 0$ *and* $h_j^i < \kappa_j(b, a)$*, or*
2. $h_j^p = 0$ *and* $h_j^i = \kappa_j(b, a)$*, or*
3. $h_j^p > 0$ *and* $h_j^i = \kappa_j(b, a)$*.*

Proof. Assume that $\{h_j^m, h_j^p, h_j^i\}$ solves the childcare allocation problem, with $h_j^p > 0$ and $h_j^i < \kappa_j(b, a)$. There exists ϵ such that $\epsilon > 0$ and $\epsilon < \min\{h_j^p, \kappa_j(b, a) - h_j^i\}$. Allocation $\{h_j^m, \epsilon, h_j^i + \epsilon\}$ provides a higher utility to the household and satisfies all constraints. The result follows by contradiction. ■

Lemma 6.1 just states that the mother would first exhaust all available unpaid childcare before drawing on paid childcare, as the former does not affect utility in any way, while the latter decreases it by reducing consumption.

An implication of lemma 6.1 is that we can rewrite the childcare allocation problem in terms of mother provided childcare only:

$$\begin{aligned} &\max_{h_j^m} \quad u_{n_j}\left(h_j^m\right) \\ &\text{subject to} \\ &0 \leq h_j^m \leq 1 - m_j(b,a) \\ &1 - \bar{h}_j^p - \kappa_j(b,a) < h_j^m < 1 - n_j \end{aligned} \tag{17}$$

where:

$$u_{n_j}\left(h_j^m\right) = \frac{\left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j\left(h_j^m\right)^{1-\gamma_1} - 1}{1-\gamma_1} \\ - \delta_2|b-b^*|^{\gamma_2} - \zeta + \delta_3\left(h_j^m\right)^{\gamma_3}$$

and

$$c_j(h_j^m) = \begin{cases} Y_j(\epsilon_j, \epsilon_j^h, x_j, n_j) - \lambda_j(1 - h_j^m - \kappa_j(b,a), n) & \text{if } h_j^m \leq 1 - \kappa_j(b,a) \\ Y_j(\epsilon_j, \epsilon_j^h, x_j, n_j) & \text{if } h_j^m > 1 - \kappa_j(b,a) \end{cases}$$
$$l_j = 1 - n_j - h_j^m.$$

Define the following interval from the constraints of the problem:

$$I_{h_j^m} = \left[\max\{0, 1 - \bar{h}_j^p - \kappa_j(b,a)\}, \min\{1 - m_j(b,a), 1 - n_j\right].$$

Lemma 6.2 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the derivative of $u_{n_j}\left(h_j^m\right)$ exists almost everywhere in the interval $I_{h_j^m}$.*

Proof. If $h_j^m > 1 - \kappa_j(b,a)$, then:

$$u'_{n_j} = -\delta_1 l_j (h_j^m)^{-\gamma_1} + \delta_3 \gamma_3 (h_j^m)^{\gamma_3 - 1}.$$

For $\gamma_1 > 0$, the first term on the right hand side exists as long as

$$l_j = 1 - n_j - h_j^m > 0 \iff h_j^m < 1 - n_j,$$

and for $0 < \gamma_3 < 1$, the second term on the right hand side exists as long as $h_j^m > 0$.

If $h_j^m < 1 - \kappa_j(b,a)$, then

$$u'_{n_j} = \left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0} \frac{1}{\psi(b)} \frac{\partial \lambda_j \left(h_j^p, b\right)}{\partial h_j^p} - \delta_1 l_j (h_j^m)^{-\gamma_1} + \delta_3 \gamma_3 (h_j^m)^{\gamma_3 - 1}.$$

We already know that the second and third terms exist almost everywhere in $I_{h_j^m}$. For $\gamma_0 > 0$, the first term on the right hand side exists as long as:

$$c_j(h_j^m) > 0 \iff h_j^m > 1 - \bar{h}_j^p - \kappa_j(b,a).$$

Finally, u'_{n_j} does not exist at $h_j^m = 1 - \kappa_j(b,a)$.

Therefore, u'_{n_j} exists everywhere in $I_{h_j^m}$ except at $h_j^m = 1 - n_j$, $h_j^m = 0$, $h_j^m = 1 - \bar{h}_j^p - \kappa_j(b,a)$ and $h_j^m = 1 - \kappa_j(b,a)$ whenever they belong to $I_{h_j^m}$. ■

Lemma 6.3 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the second derivative of $u_{n_j}\left(h_j^m\right)$ exists and is negative almost everywhere in $I_{h_j^m}$.*

Proof. If $h_j^m > 1 - \kappa_j(b,a)$, then:

$$u''_{n_j} = -\gamma_1 \delta_1 (1 - n_j - h_j^m)^{-\gamma_1 - 1} + \delta_3 \left(\gamma_3 - 1\right) \gamma_3 (h_j^m)^{\gamma_3 - 2}.$$

For $\gamma_1 > 0$, the first term on the right hand side exists as long as

$$1 - n_j - h_j^m > 0 \iff h_j^m < 1 - n_j.$$

For $0 < \gamma_3 < 1$, the second term on the right hand side exists as long as $h_j^m > 0$. Both elements are negative for $\gamma_1 > 0$ and $0 < \gamma_3 < 1$ as long as the expression is valid, therefore the whole expression exists and is negative everywhere except at $h_j^m = 1 - n_j$ and $h_j^m = 0$ in $I_{h_j^m}$.

If $h_j^m < 1 - \kappa_j(b,a)$, then:

$$u''_{n_j} = -\gamma_0 \left(\frac{c_j(h_j^m)}{\psi(b)} \right)^{-\gamma_0 - 1} \frac{1}{\psi(b)} \frac{\partial \lambda_j \left(h_j^p, n \right)}{\partial h_j^p} \\ - \gamma_1 \delta_1 (1 - n_j - h_j^m)^{-\gamma_1 - 1} + \delta_3 \left(\gamma_3 - 1 \right) \gamma_3 (h_j^m)^{\gamma_3 - 2}.$$

We already know the last two terms on the left hand side exist and are negative almost everywhere in $I_{h_j^m}$, so we need to concern ourselves only with the first term. For $\gamma_0 > 0$ the expression exists as long as

$$c_j(h_j^m) > 0 \iff h_j^m > 1 - \bar{h}_j^p - \kappa_j(b,a),$$

and is negative as long as it exists since $\left(\frac{c_j(h_j^m)}{\psi(b)} \right)^{-\gamma_0 - 1} > 0$, $\frac{1}{\psi(b)} > 0$ and $\frac{\partial \lambda_j \left(h_j^p, n \right)}{\partial h_j^p} > 0$. Therefore, the whole expression exists and is negative everywhere except at $h_j^m = 1 - n_j$, $h_j^m = 0$ and $h_j^m = 1 - \bar{h}_j^p - \kappa_j(b,a)$ in $I_{h_j^m}$.

Finally, the second derivative does not exist at $h_j^m = 1 - \kappa_j(b,a)$. ■

Lemma 6.4 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the first derivative of $u_{n_j}\left(h_j^m\right)$ is strictly decreasing in $I_{h_j^m}$.*

Proof. We know from lemma 6.3 that u''_{n_j} is negative almost everywhere in $I_{h_j^m}$, with the notable exception of $h_j^m = 1 - \kappa_j(b,a)$. If $1 - \kappa_j(b,a)$ is not an interior point of $I_{h_j^m}$, the result follows automatically. Suppose that $1 - \kappa_j(b,a)$ is an interior point of $I_{h_j^m}$. Since

$$\left(\frac{c_j(h_j^m)}{\psi(b)} \right)^{-\gamma_0} \frac{1}{\psi(b)} \frac{\partial \lambda_j \left(h_j^p, n \right)}{\partial h_j^p} > 0,$$

then,

$$\lim_{h_j^m \to 1 - \kappa_j(b,a)^-} u'_{n_j} > \lim_{h_j^m \to 1 - \kappa_j(b,a)^+} u'_{n_j}.$$

Thus, u'_{n_j} is strictly decreasing in $I_{h_j^m}$. ■

Theorem unique solution of the childcare problem.

Theorem 6.5 *If $\gamma_0 > 1$, $\gamma_1 > 1$ and $0 < \gamma_3 < 1$, the childcare allocation problem has a unique solution in $I_{h_j^m}$.*

Proof.

Suppose that $\exists\ h_j^m \in I_{h_j^m} : u'_{n_j}(h_j^p) = 0$. From lemma 6.4 we know that this point is unique, and by standard marginal arguments we know that it necessarily is a maximum of u_{n_j} in $I_{h_j^m}$.

Now suppose there is no such point. Notice that:

$$\lim_{h_j^m \to \inf I_{h_j^m}+} u'_{n_j} = \begin{cases} \lim_{h_j^m \to 0^+} u'_{n_j} = \infty & \text{if} \quad 0 \geq 1 - \bar{h}_j^p - \kappa_j(b,a) \\ \lim_{h_j^m \to 1-\bar{h}_j^p-\kappa_j(b,a)^+} u'_{n_j} = \infty & \text{if} \quad 0 < 1 - \bar{h}_j^p - \kappa_j(b,a). \end{cases}$$

Suppose further that $1 - n_j < 1 - m_j(b,a)$. Since

$$\lim_{h_j^m \to 1-n_j+} u'_{n_j} = -\infty,$$

then $1 - \kappa_j(b,a) < 1 - n_j$, for otherwise $u'_{n_j}(h_j^p) = 0$ for some $h_j^p \in I_{h_j^m}$ as u'_{n_j} would be a strictly decreasing continuous function that takes both positive and negative values in $I_{h_j^m}$. Moreover, using the same argument it follows that:

$$\begin{aligned} u'_{n_j}(h_j^m) > 0 \quad & \forall h_j^m \in \left(\inf I_{h_j^m}, 1 - \kappa_j(b,a)\right) \\ u'_{n_j}(h_j^m) < 0 \quad & \forall h_j^m \in \left(1 - \kappa_j(b,a), 1 - n_j\right), \end{aligned}$$

which implies that $h_j^m = 1 - \kappa_j(b,a)$ is the unique solution to the childcare allocation problem.

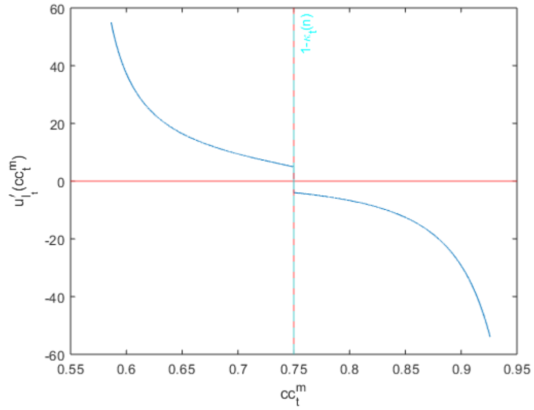
Assume now that $1 - m_j(b,a) < 1 - n_j$. There are two possibilities: $u'_{n_j}(1 - m_j(b,a)) < 0$, or $u'_{n_j}(1 - m_j(b,a)) \geq 0$. Suppose first that the former is true. Then, by the same argument used twice before, it follows that $1 - \kappa_j(b,a) < 1 - m_j(b,a)$, that:

$$\begin{aligned} u'_{n_j}(h_j^m) > 0 \quad & \forall h_j^m \in \left(\inf I_{h_j^m}, 1 - \kappa_j(b,a)\right) \\ u'_{n_j}(h_j^m) < 0 \quad & \forall h_j^m \in \left(1 - \kappa_j(b,a), 1 - m_j(b,a)\right), \end{aligned}$$

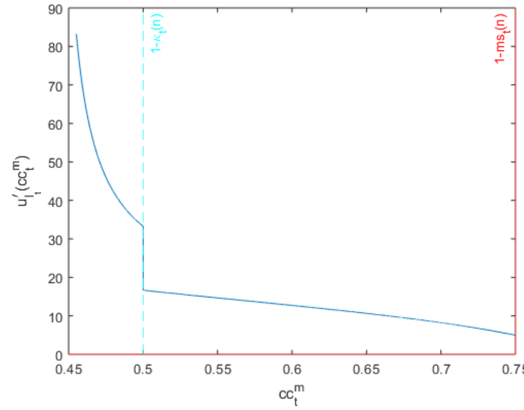
and hence that $h_j^m = 1 - \kappa_j(b,a)$ is the unique solution to the childcare allocation problem.

Finally, if $u'_{n_j}(1 - m_j(b,a)) \geq 0$, then it follows directly that the unique solution to the childcare allocation problem is $h_j^m = 1 - m_j(b,a)$. ■

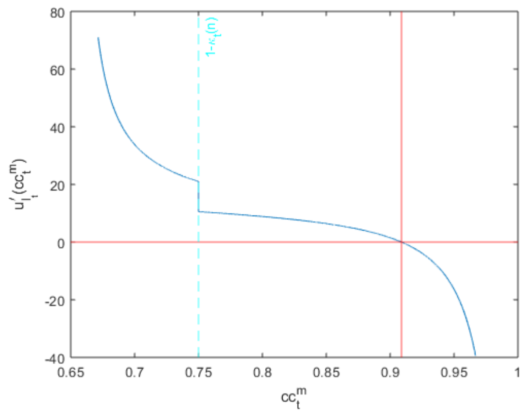
The proof of theorem 6.5 characterizes all the forms the unique solution to the childcare allocation problem can take. Figure 7 illustrates this. Panel (a) shows a situation in



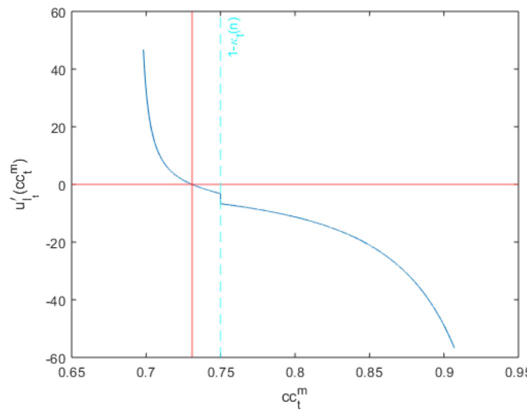
(a) Kink solution



(b) Corner solution



(c) Interior solution without paid childcare



(d) Interior solution with paid childcare

Figure 7: Solutions to the childcare allocation problem

which the solution is at the kink of the utility function, where marginal utility experiences a discrete jump. In this situation, the marginal utility of mother's childcare is positive right before reaching the unpaid childcare endowment, as it includes the term related to marginal consumption, but is negative right after. Panel (b) shows the strange but theoretically possible situation in which the mother would like to spend more time with her children but she cannot because of the limit imposed by mandatory schooling. Panels (c) and (d) show situations in which the solution is interior. In panel (c), marginal utility crosses zero after the point in which the household does not need to use paid childcare, while in panel (d) it does so before. Therefore, the household does not need to use paid childcare in the former but does in the latter.

The labour income process estimation.

The parameters of the labour income process together with the persistence of the income shock and the variance of the residual of it were estimated from the Spanish Continuous Working Life Sample (MCVL). In particular, we regress gross log yearly income by sex on experience and experience square. We also control by education and two dummies, one for part-time jobs and the other for the 2008 economic crisis. After this, we estimate the residuals of both regressions assuming that they follow an AR(1) process as it is described in the model.

In this appendix we summarize the main variables and sample restrictions that we made for the income process estimation. We use STATA codes from Roca and Puga (2017)[10]

1. **Main variables:**

   - **Gross yearly income:** refers to a very approximate measure of all income received by a person except pensions, prizes from games like the National Lottery and non-levied income. Therefore it includes labour income, unemployment benefits, extra paid hours, among others. It is extracted from the tax codes because they are uncensored. Earnings are expressed in real terms using the consumer price index of 2009.
   - **Education.** It is divided into three educational levels: less than secondary education, secondary education and university education.
   - **Part-time and fixed-term indicator.**

2. **Sample restrictions:**

   - We restricted the sample for the years 2000-2017.
   - We dropped:

     (a) Individuals younger than 25 and older than 55 years old. This eliminates individuals who haven't finished their studies and those who receive an early retirement pension.

[10] We are responsible for the possible computational mistakes.

Table 12: labour income process by sex

	Male	Female
Experience	0.0318***	0.0326***
	(319.98)	(263.54)
Experience^2	-0.000455***	-0.000280***
	(-160.10)	(-72.44)
Secondary education	0.232***	0.268***
	(437.56)	(406.81)
Tertiary education	0.477***	0.583***
	(722.69)	(831.99)
Crisis dummy	-0.00510***	0.00613***
	(-11.27)	(10.94)
Part time contract	-0.489***	-0.536***
	(-507.25)	(-805.30)
Fixed-term contract	-0.257***	-0.189***
	(-462.14)	(-285.71)
Constant	9.533***	9.286***
	(10857.95)	(9385.70)
Observations	3966582	3365913

t statistics in parentheses
Source: MCVL 2000-2017
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 13: Persistance AR(1) process by sex

	Male	Female
resid_male_lag	0.655***	
	(1037.79)	
resid_female_lag		0.654***
		(1081.15)
Observations	7332494	7332494

t statistics in parentheses
Source: MCVL 2000-2017
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 14: Residual income shock

	mean	variance
Male regression	-.0250729	.1369882
Female regression	.0272791	.1372587

Residuals of the AR(1) process
Source: MCVL 2000-2017

Algorithms

Algorithm to solve the life cycle model

Result: Policy functions
begin
  Discretize the AR(1) process for the income shocks $(\epsilon_j, \epsilon_j^h)$, Tauchen method;
  Construct a grid for the state space $\{\epsilon_j, \epsilon_j^h, x_j, b_j, Y_j\}$, using the grid values for $(\epsilon_j, \epsilon_j^h)$;
  Inputs : $\{\epsilon_j, \epsilon_j^h, x_j, b_j, Y_j\}$
  Outputs: state variables $\{x_j, \epsilon_j, \epsilon_j^h, b\}$, labour force participation choice n_j
  for Each point in the grid space $\{\epsilon_j, \epsilon_j^h, x_j, b_j, Y_j\}$ **do**
    for $j = J_1 + J_2 + 1$ **do**
      if $j = J_1 + J_2 + 1$ **then**
        Compute the value of retirement;
      end
      else if $j \in [1, J_2]$ **then**
        Solve the static childcare problem for each of the potential values of n_j of the choice set;
        Compute the flow utility for the period and retrieve the continuation value according to the rules of motion for the state variables;
        Select the value of n_j that returns the largest value;
        Record both the value and the policy;
      end
      else if $t \leq J_1$ **then**
        Compare the corresponding value with the value of the same grid point but with one child less. Select the one that returns the largest value, and record both the value and the policy.
      end
    end
  end
end

Algorithm 1: Algorithm to solve the life cycle model

Algorithm to solve the childcare allocation problem

Inputs : state variables $\{x_j, \epsilon_j, \epsilon_j^h, b\}$, labour force participation choice n_j
Outputs: value of utility v_{n_j}, optimal choice of childcare $\{h_j^m, h_j^p, h_j^i\}$

begin
Calculate the maximum mother provided childcare time and paid childcare;
$\bar{h}_j^m = 1 - n_j$;
$\bar{h}_j^p = \max\{0, \sup\{h_j^p : Y_j\left(\epsilon_j, \epsilon_j^h, x_j, n_j\right) - \lambda_j(h_j^p, b) > 0\}\}$;
if $\underline{\bar{h}_j^m + \bar{h}_j^p + \kappa_j(b) \leq 1}$ **then** choice of labour force participation n_j is unfeasible
Set $h_j^m = NaN$, $h_j^p = NaN$, $h_j^i = NaN$ and $v_{n_j} = -\infty$;
else
Attempt to find a root for u'_{n_j} in $I_{h_j^m}$;
if $\underline{\exists x \in I_{h_j^m} : u'_{n_j}(x) = 0}$ **then** x is the unique solution for the childcare allocation problem, which is interior with or without paid childcare
Set $h_j^m = x$, $h_j^i = \min\{1 - x, \kappa_j(b)\}$, $h_j^p = 1 - h_j^m - h_j^i$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^i\right)$;
else
if $\underline{1 - n_j \leq 1 - ms_j(b) \text{ or } 1 - ms_j(b) < 1 - n_j \text{ and } u'_{n_j}(1 - ms_j(b)) < 0}$ **then** there is a unique kink solution to the childcare allocation problem
Set $h_j^m = 1 - \kappa_j(b)$, $h_j^p = 0$, $h_j^i = \kappa_j(b)$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^i\right)$;
Verify that $\lim_{h_j^m \to 1-\kappa_j(b)^-} u'_{n_j} > 0$ and $\lim_{h_j^m \to 1-\kappa_j(b)^+} u'_{n_j} < 0$;
else
There is a unique corner solution to the childcare allocation problem;
Set $h_j^m = 1 - ms_j(b)$, $h_j^p = 0$, $h_j^i = ms_j(b)$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^i\right)$;
Verify that $u'_{n_j}\left(1 - ms_j(b)\right) > 0$;
end
end
end
end

Algorithm 2: Solving the childcare allocation problem

References

Adda, Jérôme, Christian Dustmann, and Katrien Stevens, "The Career Costs of Children," *Journal of Political Economy*, 2017, *125* (2), 293–337.

Auerbach, Alan J, Laurence J Kotlikoff et al., *Dynamic fiscal policy*, Cambridge University Press, 1987.

Bick, Alexander, "The quantitative role of child care for female labor force participation and fertility," *Journal of the European Economic Association*, 2016, *14* (3), 639–668.

Cruces, Lidia, "Gender gaps in the labor market and pension sustainability.," *Working Paper*, 2021.

Da Rocha, José María and Luisa Fuster, "Why Are Fertility Rates and Female Employment Ratios Positively Correlated across O.E.C.D. Countries?," *International Economic Review*, 2006, *47* (4), 1187–1222.

Díaz-Giménez, Javier and Julián Díaz-Saavedra, "Delaying retirement in Spain," *Review of Economic dynamics*, 2009, *12* (1), 147–167.

Erosa, Andrés, Luisa Fuster, and Diego Restuccia, "Fertility Decisions and Gender Differences in Labor Turnover, Employment, and Wages," *Review of Economic Dynamics*, 2002, *5* (4), 856 – 891.

González, MARÍA JOSÉ, "La escolarización de la primera infancia en España: desequilibrios territoriales y socioeconómicos en el acceso a los servicios," *El Estado del Bienestar en España, Madrid, Tecnos*, 2004, pp. 291–312.

González, Libertad, "The Effect of a Universal Child Benefit on Conceptions, Abortions, and Early Maternal Labor Supply," *American Economic Journal: Economic Policy*, 2013, *5* (3), 160–188.

Hannusch, Anne et al., "Taxing families: The impact of child-related transfers on maternal labor supply," *Unpublished Manuscript, Mannheim University*, 2019.

Imrohoroğlu, Selahattin and Sagiri Kitao, "Social security reforms: Benefit claiming, labor force participation, and long-run sustainability," *American Economic Journal: Macroeconomics*, 2012, *4* (3), 96–127.

Kitao, Sagiri, "Sustainable social security: Four options," *Review of Economic Dynamics*, 2014, *17* (4), 756–779.

Kotlikoff, Laurence J, Kent Smetters, and Jan Walliser, "Mitigating America's demographic dilemma by pre-funding social security," *Journal of monetary Economics*, 2007, *54* (2), 247–266.

Kydland, Finn E and Edward C Prescott, "Time to build and aggregate fluctuations," *Econometrica: Journal of the Econometric Society*, 1982, pp. 1345–1370.

Laun, Tobias and Johanna Wallenius, "Having it all? employment, earnings, and children," *The Scandinavian Journal of Economics*, 2021.

Martín, Alfonso R Sánchez and Virginia Sánchez Marcos, "Demographic Change and Pension Reform in Spain: An Assessment in a Two-Earner, OLG Model," *Fiscal Studies*, 2010, *31* (3), 405–452.

Nishiyama, Shinichi and Kent Smetters, "Does social security privatization produce efficiency gains?," *The Quarterly Journal of Economics*, 2007, *122* (4), 1677–1719.

Nollenberger, Natalia and Núria Rodríguez-Planas, "Full-time universal childcare in a context of low maternal employment: Quasi-experimental evidence from Spain," *Labour Economics*, 2015, *36*, 124–136.

Roca, Jorge De La and Diego Puga, "Learning by working in big cities," *The Review of Economic Studies*, 2017, *84* (1), 106–142.

Sánchez-Mangas, Rocio and Virginia Sánchez-Marcos, "Balancing family and work: The effect of cash benefits for working mothers," *Labour Economics*, 2008, *15* (6), 1127–1142.

Sommer, Kamila, "Fertility choice in a life cycle model with idiosyncratic uninsurable earnings risk," *Journal of Monetary Economics*, 2016, *83*, 27–38.