Gottesman-Kitaev-Preskill bosonic error correcting codes: a lattice perspective

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Alexander von Humboldt
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Outline

1) Introduction and motivation

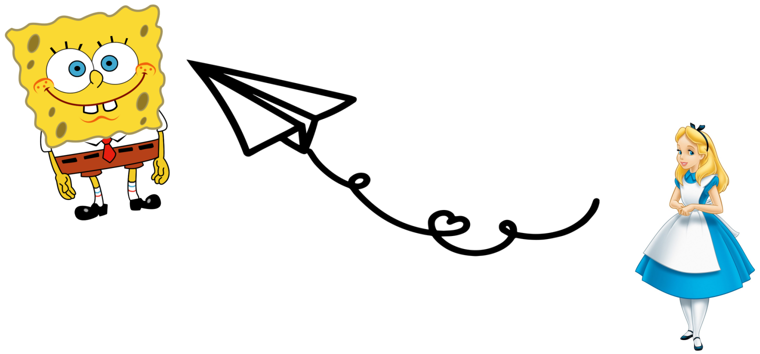
2) Some definitions : Lattices in a nutshell

3) Results : Lattice bases and symplectically equivalent codes

4) Conclusions

Part 1 : Introduction and motivation

(Quantum) Error correction and harmonic oscillators



Information always encoded in phys. syst.

⇨ Always subject to **noise**

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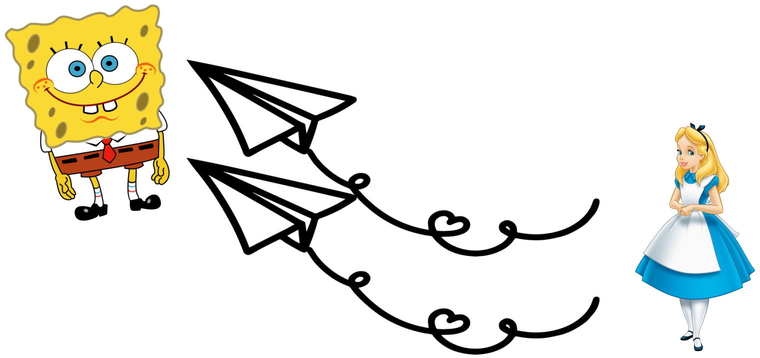

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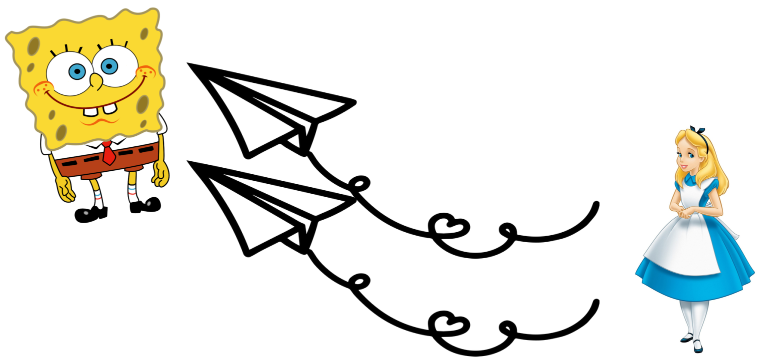

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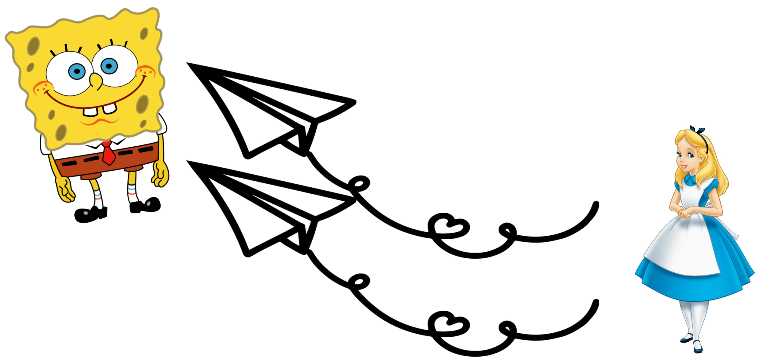

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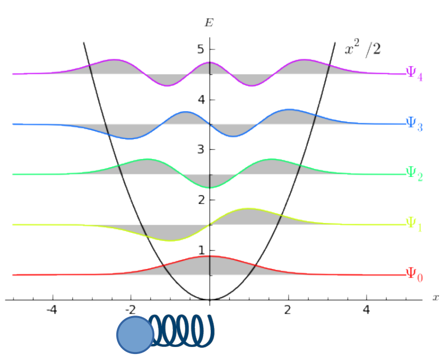

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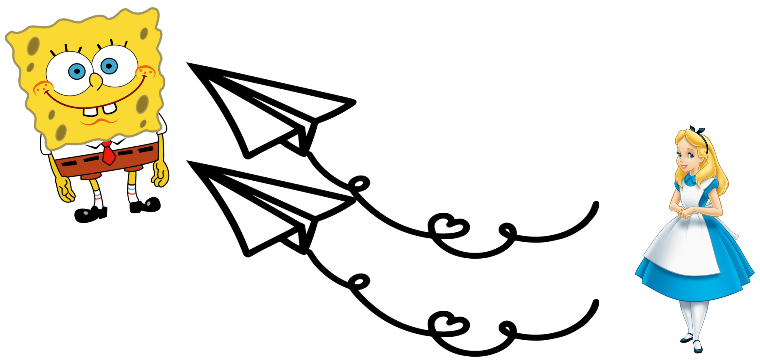

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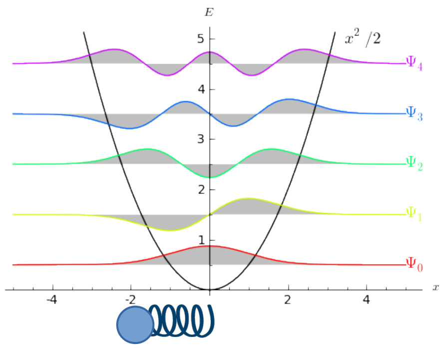

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$$\hat{\boldsymbol{x}} = (q_1, \ldots, q_n, p_1, \ldots, p_n)^T$$

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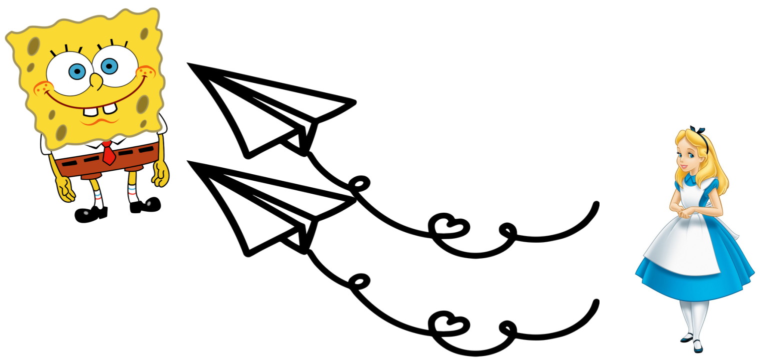

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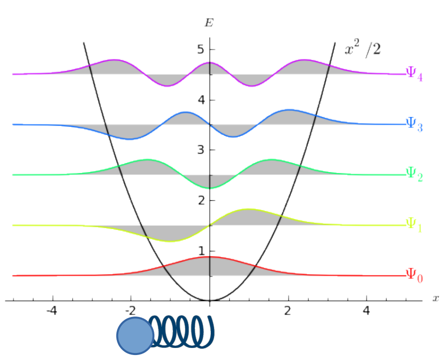

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Infinite dimensional space!

How to exploit?

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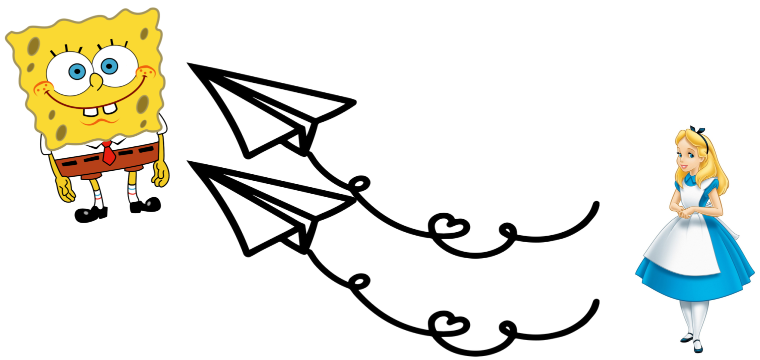

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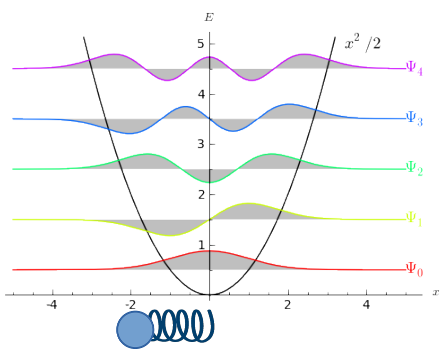

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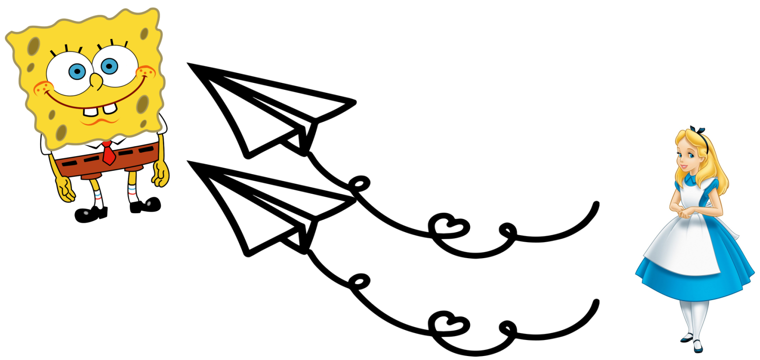

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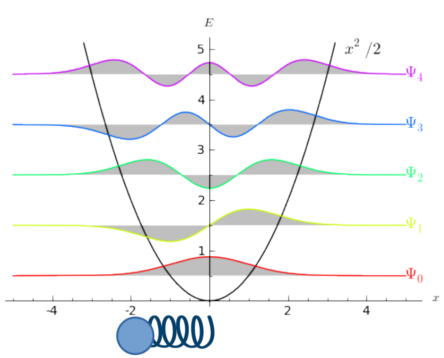

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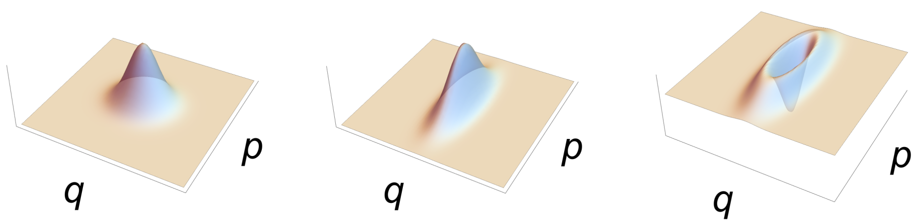
In phase space:

Wigner Function



Quasi-probability distribution

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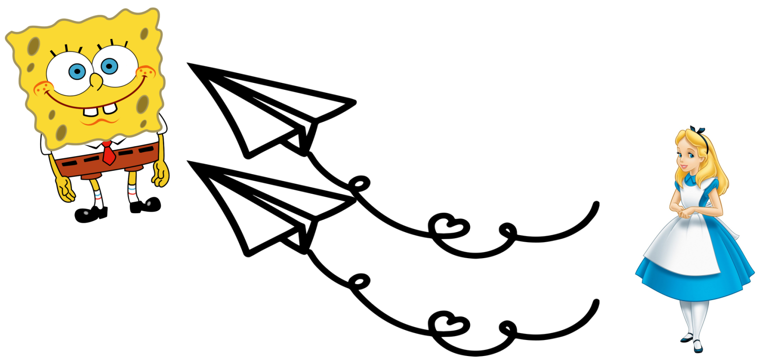

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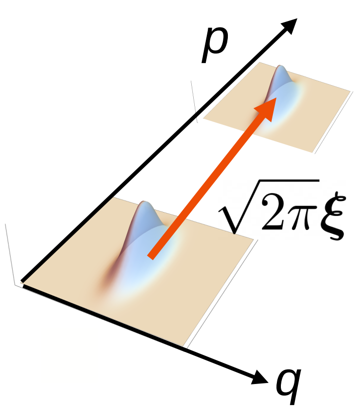

Displacements :

$$D^\dagger(\boldsymbol{\xi})\hat{\boldsymbol{x}}D(\boldsymbol{\xi}) = \hat{\boldsymbol{x}} + \sqrt{2\pi}\boldsymbol{\xi}$$

$$D(\boldsymbol{\xi})D(\boldsymbol{\eta}) = e^{-i2\pi\boldsymbol{\xi}^T J\boldsymbol{\eta}}D(\boldsymbol{\eta})D(\boldsymbol{\xi})$$

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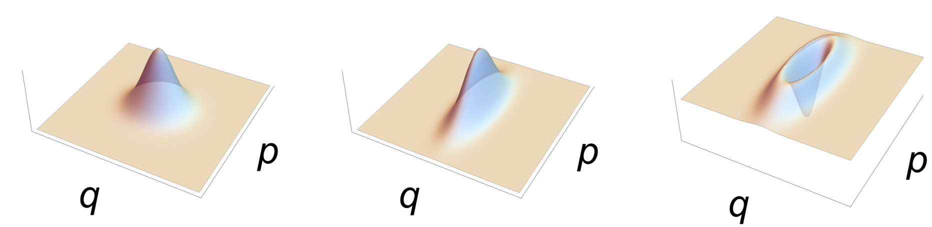

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“Logical subspace”: finite photon number, odd/even photon number, or...**translation symmetries**!

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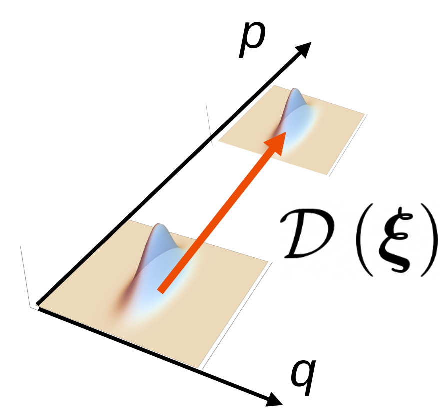

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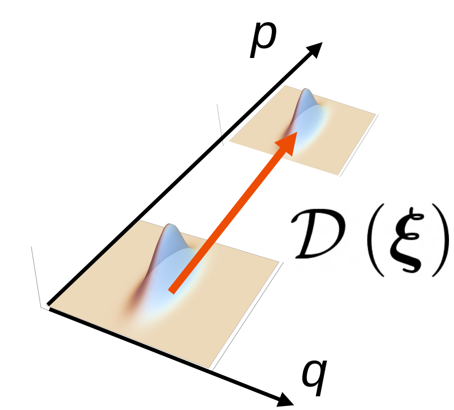

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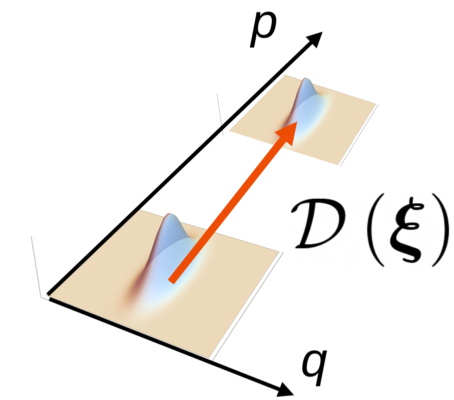

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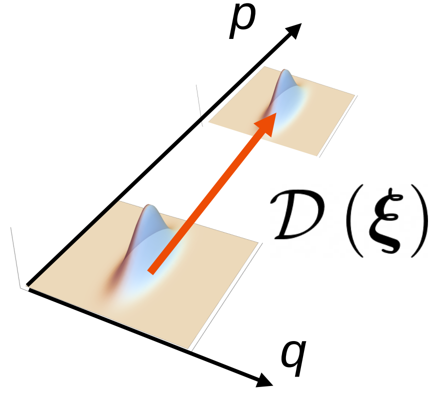

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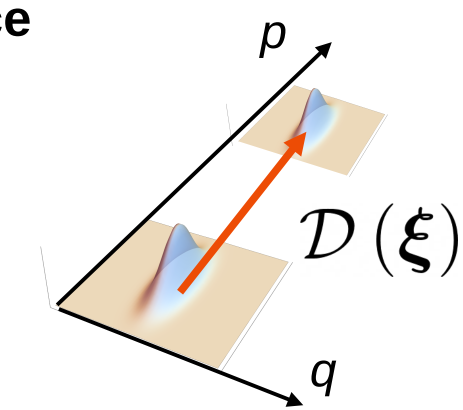

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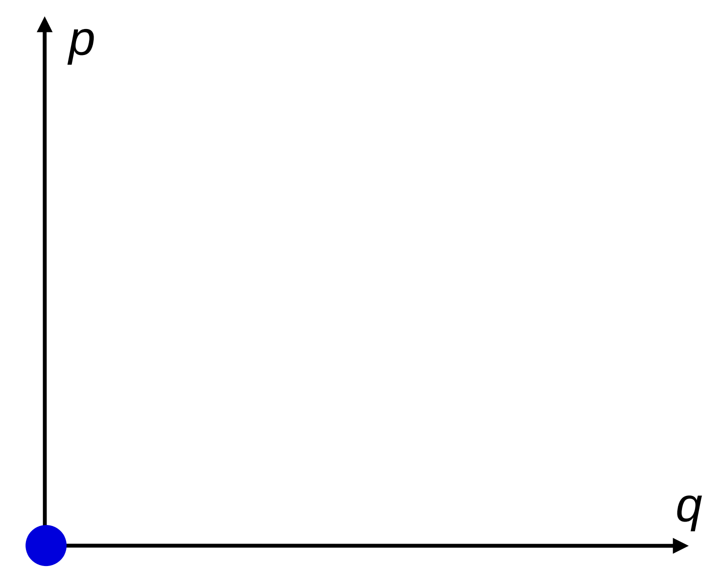

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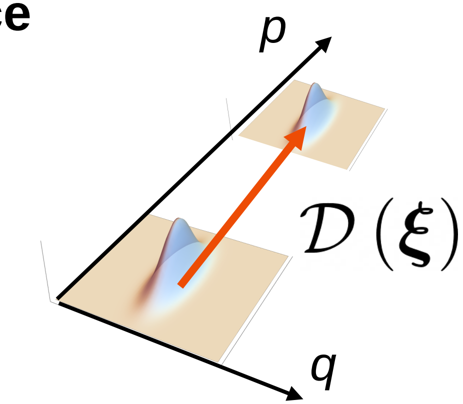

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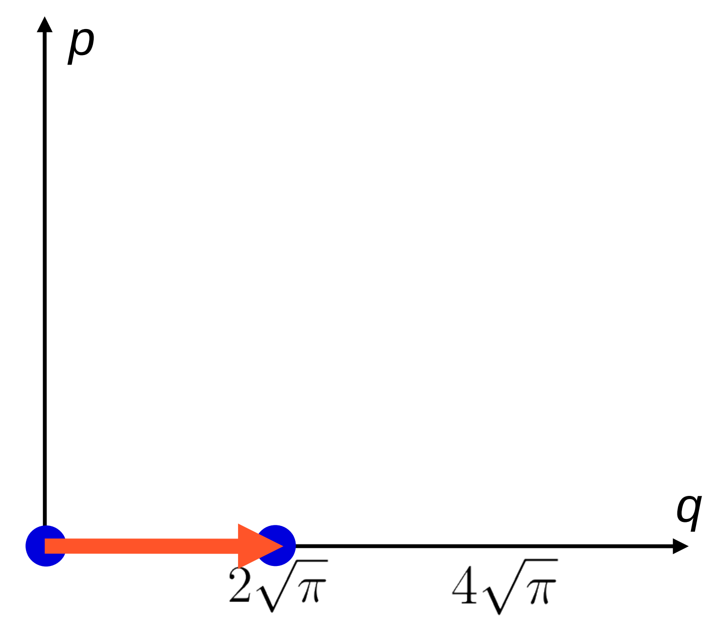

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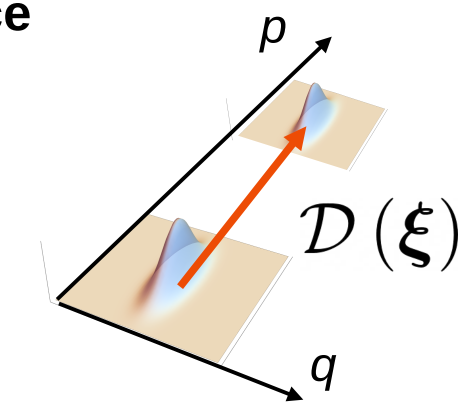

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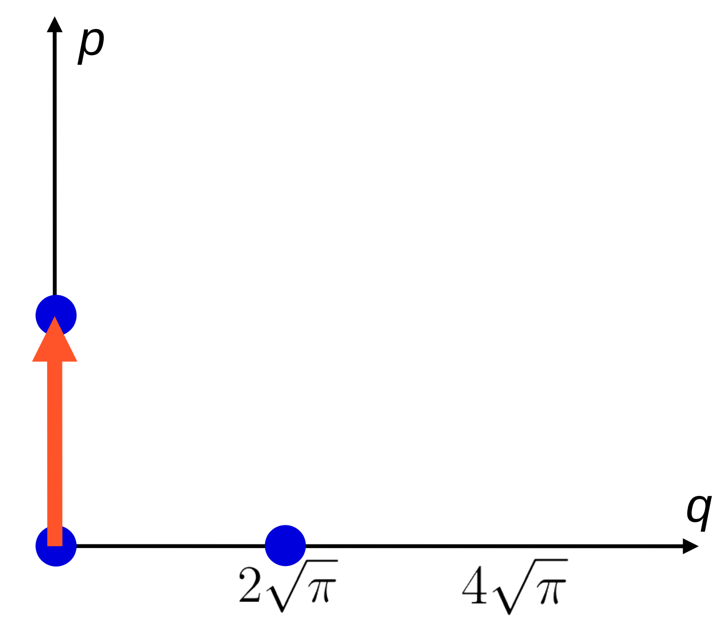

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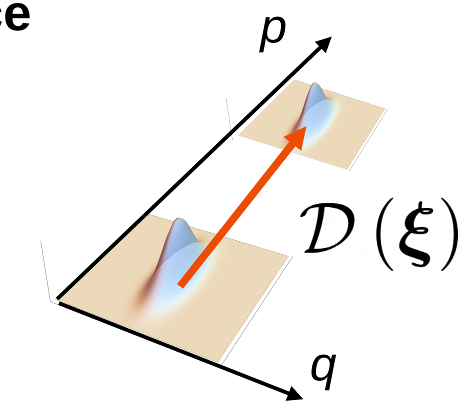

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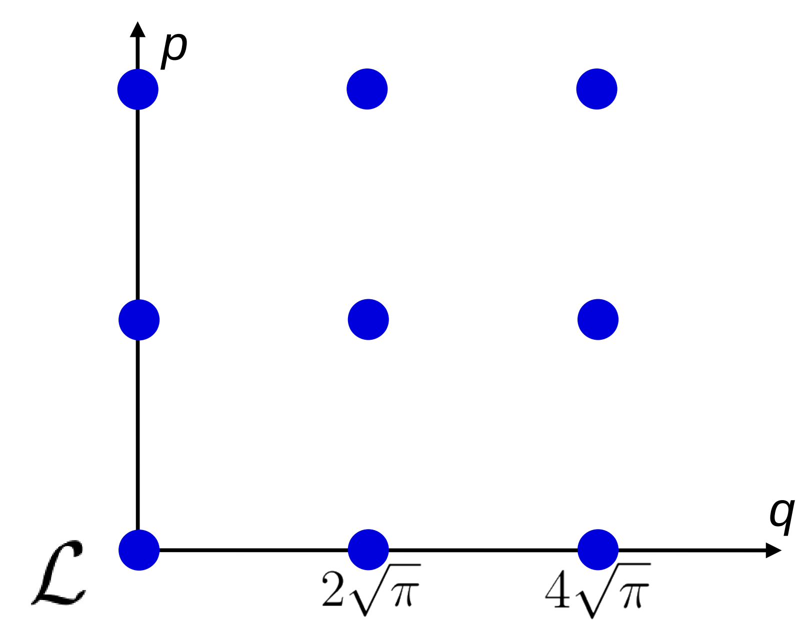

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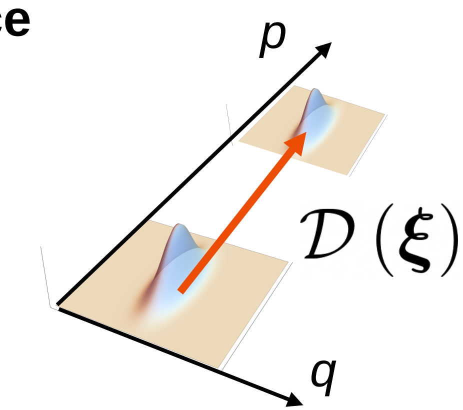

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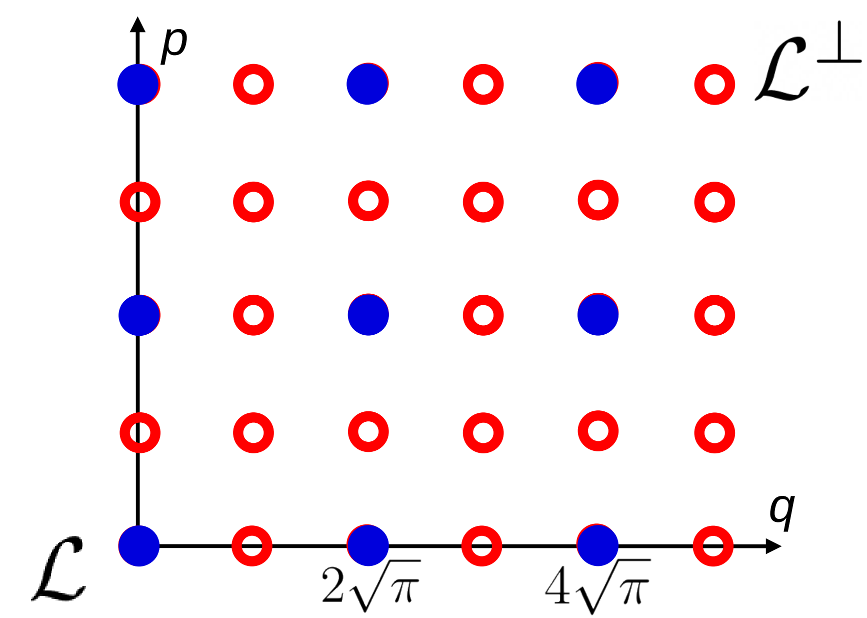

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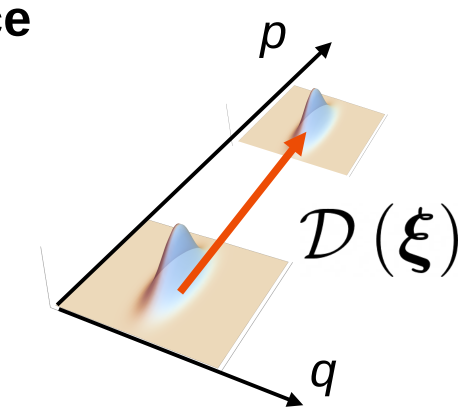

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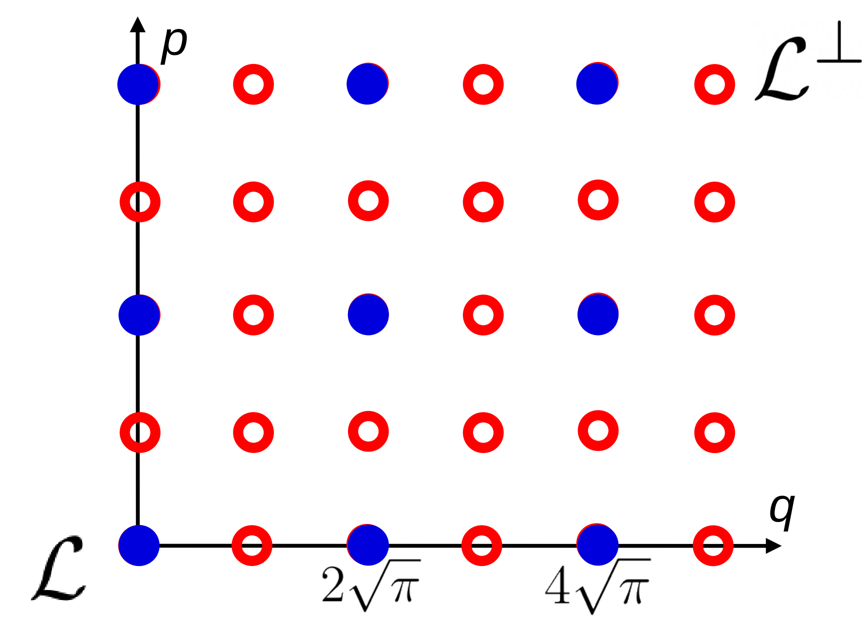

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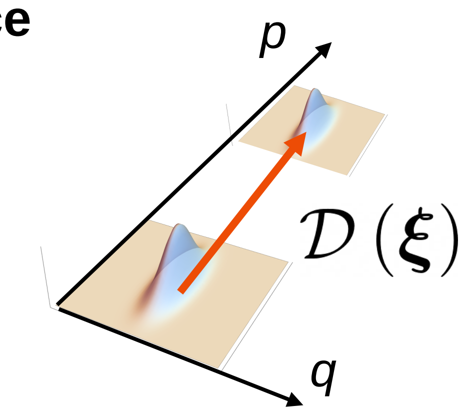

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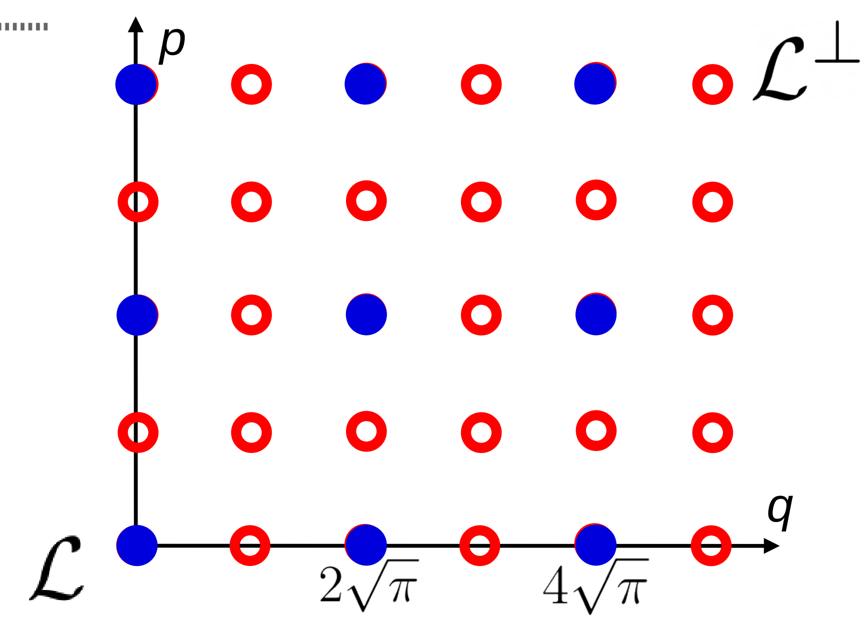
- Good protection against common noise processes
  Albert et al, PRA 97 (2018)



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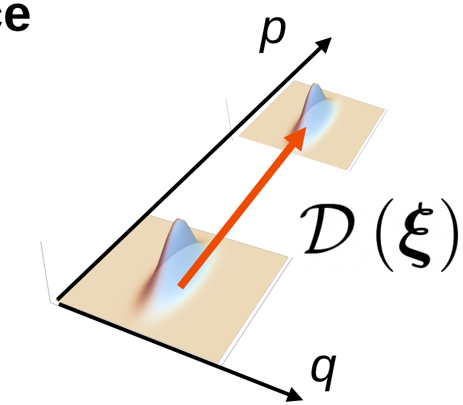

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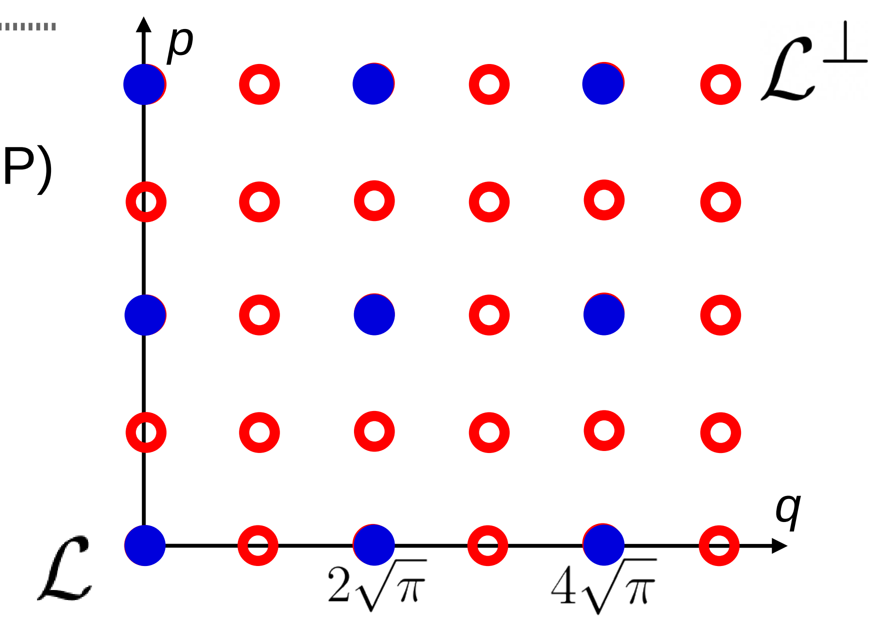

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Encoding qubits on a lattice

“Logical subspace”: finite photon number, odd/even photon number, or...**translation symmetries**!

Grid codes: **stabilized** by (commuting) **displacement** operators → underlying **lattice**

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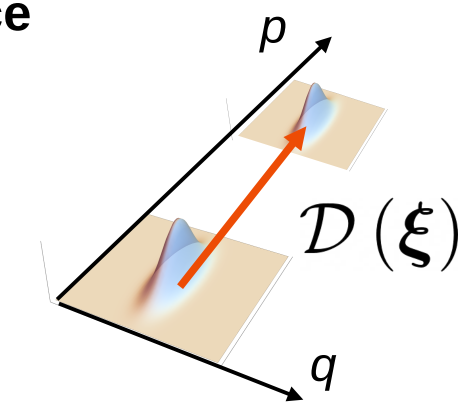

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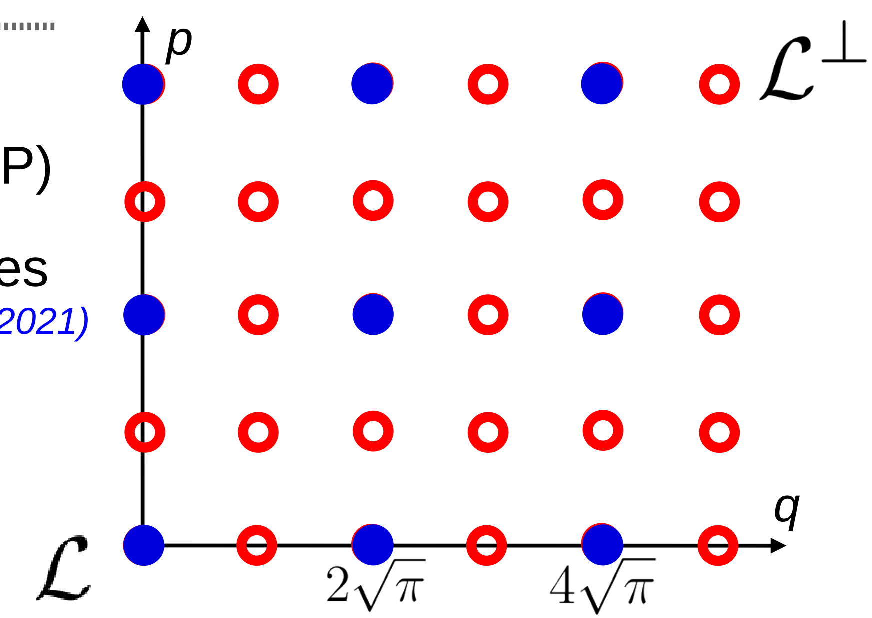

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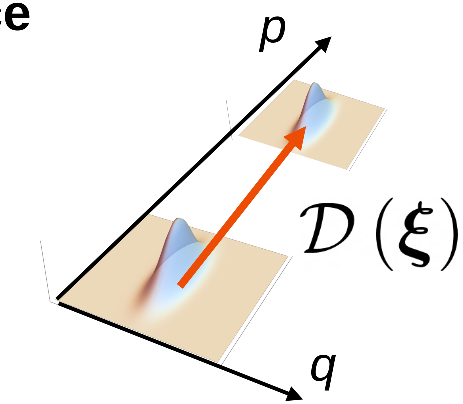

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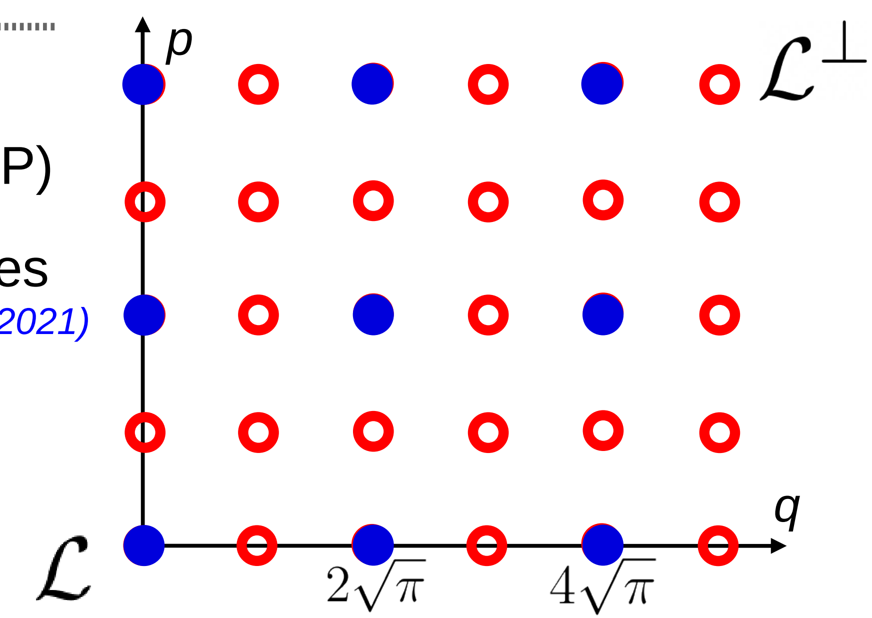

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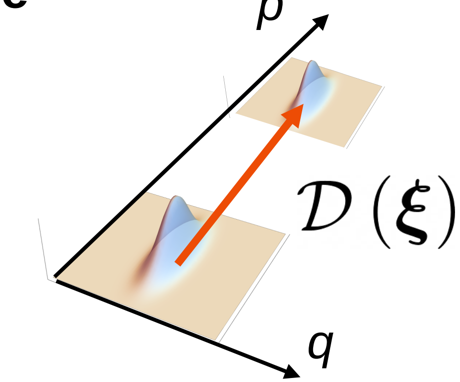

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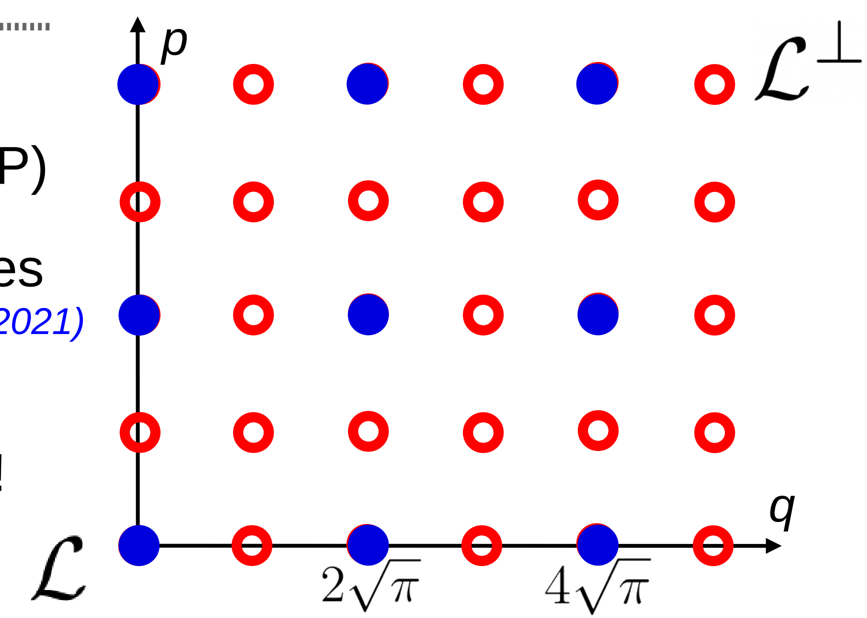
  Flühmann et al, Nature 566 (2019) *Campagne-Ibarcq et al, Nature 584 (2020)*



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For exponential noise suppression: more oscillators

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1. Code properties from lattice bases
2. Symplectic operations
3. Distance bounds for GKP codes
4. Decoding problem and Θ functions
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Part 2 : Definitions

Lattices and stabilizer group

Start from

$$\mathcal{S} = \langle D(\boldsymbol{\xi}_1), \ldots, D(\boldsymbol{\xi}_{2n}) \rangle \quad \leftrightarrow \quad M = (\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{2n})^T$$

$$\Rightarrow \boxed{\mathcal{S} \simeq \mathcal{L} = \left\{ \boldsymbol{\xi} \in \mathbb{R}^{2n} \middle| \boldsymbol{\xi}^T = \boldsymbol{a}^T M,\ \boldsymbol{a} \in \mathbb{Z}^{2n} \right\}}$$

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$$\boxed{A_{jk}} = \left(M J M^T \right)_{jk} \in \mathbb{Z} \quad \Rightarrow \quad \text{Symplectically integral lattices}$$

Symplectic Gram matrix

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Symplectic Gram matrix

Each linearly independent generator “discretizes” 1 direction in phase space

$\Rightarrow$ Focus on full rank lattices: $\det M \neq 0 \Rightarrow d < \infty$ Logical dimension

Gottesman, Kitaev, Preskill PRA 64 (2001)

Lattices and logical operations

Logical Pauli operators :

all displacements that commute with stabilizers

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all vectors that have integer symplectic product with lattice vectors

$$\left\{ \boldsymbol{\xi}^{\perp} \in \mathbb{R}^{2n} \mid \left(\boldsymbol{\xi}^{\perp}\right)^{T} J \boldsymbol{\xi} \in \mathbb{Z} \, \forall \boldsymbol{\xi} \in \mathcal{L} \right\}$$

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Dual lattice generator (*finite logical dimension*) : $M^\perp = (JM^T)^{-1}$

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One can show that the logical dimension *d* is computed from:

$$d^2 = |\mathcal{L}^{\perp}/\mathcal{L}| = |\det M|/|\det M^{\perp}| = \det A = (\det M)^2$$

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Part 3 : Results

Lattice bases

Basis (stabilizer generators) not unique given lattice (code)

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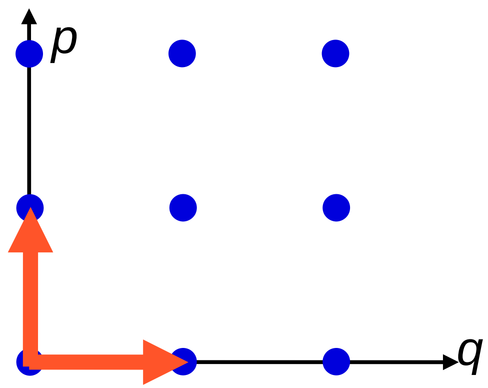

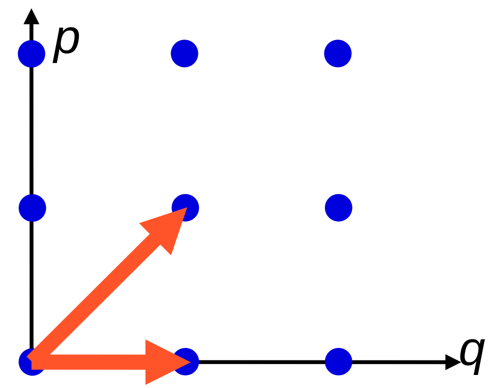

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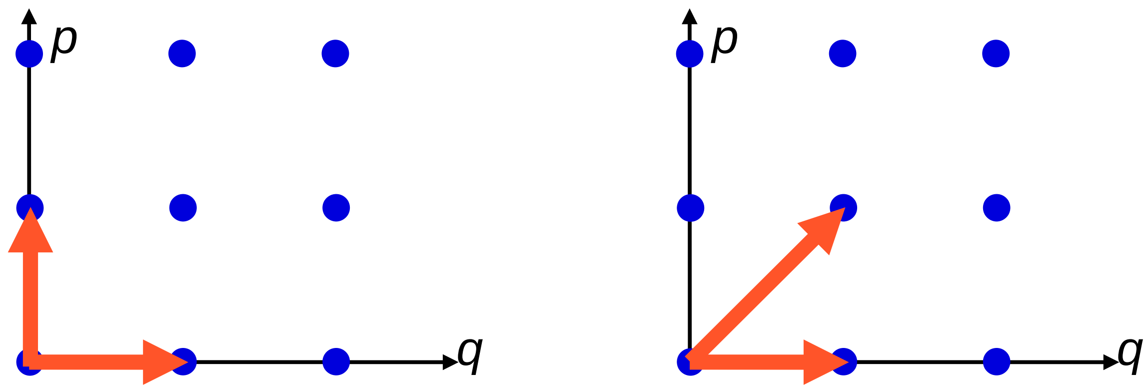

⇒ Exploit basis choice and properties to study codes

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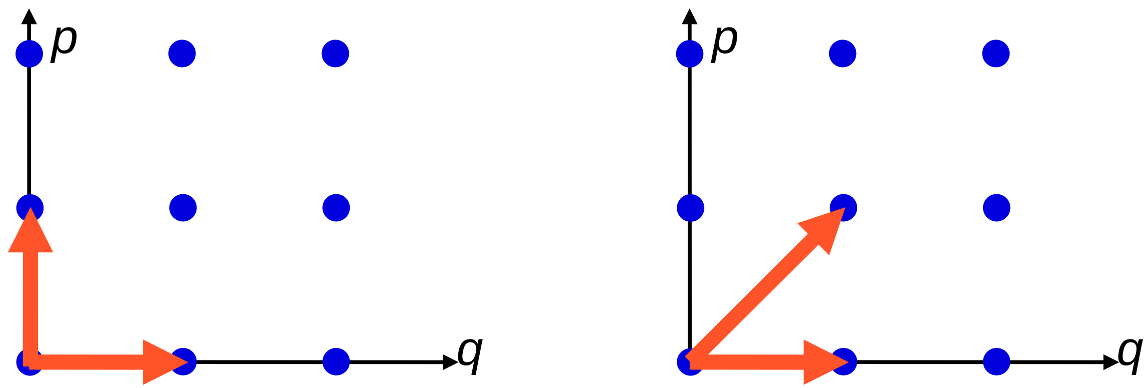

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Theorem (Hadamard's bound):

$$\boxed{\text{Let } C = \max_j ||\boldsymbol{\xi}_j||,\ d = 2^k = \det M. \text{ Then } k \leq 2n \log_2 C}$$

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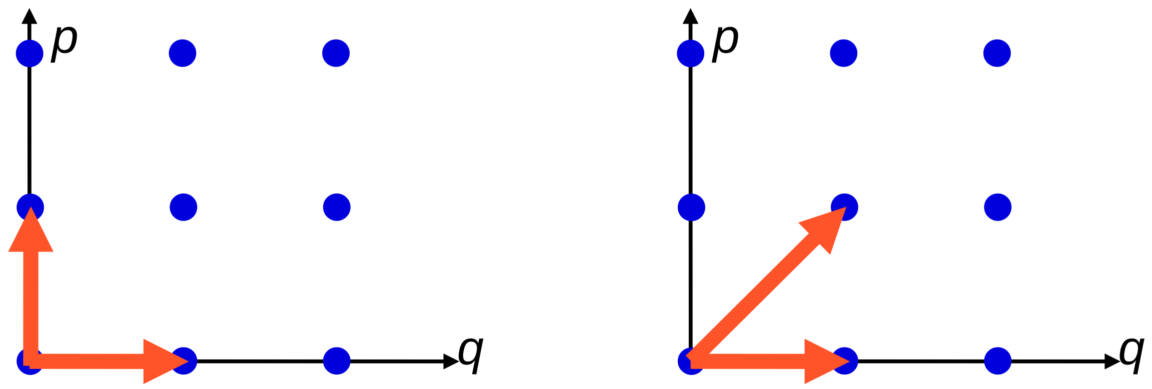

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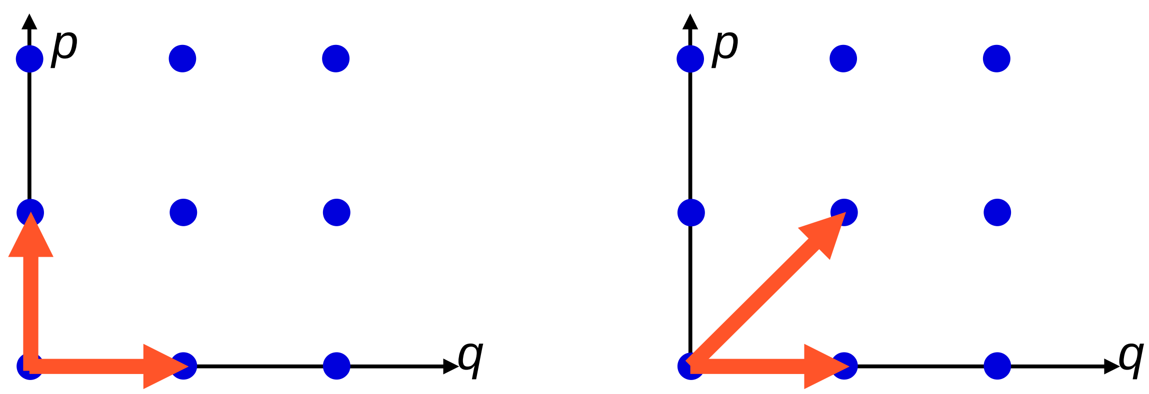

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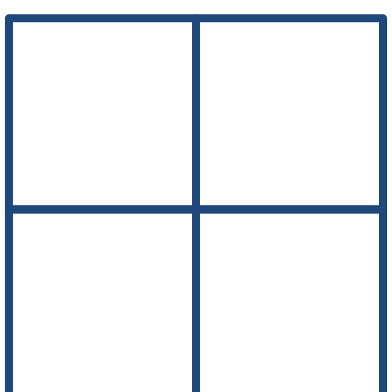
⇨ Encoding ratio related to "hardness" of measuring generators *in the chosen basis*

Minimal stabilizer sets for concatenated codes

Concatenation :

Example : L=3 GKP-surface code (*n* = *9* modes, logical dimension *d* = *2*)



qubits/modes on vertices

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Concatenation : 1) fix a qubit in each of n modes (logical dimension d = 2^n)
(think 1 square lattice for each mode) → $2n$ generators

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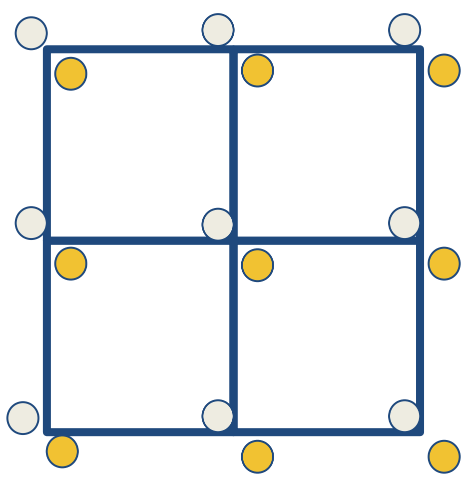

qubits/modes on vertices

○ $\exp\left(i2\sqrt{\pi}\hat{q}_j\right)$
● $\exp\left(-i2\sqrt{\pi}\hat{p}_j\right)$
GKP qubits

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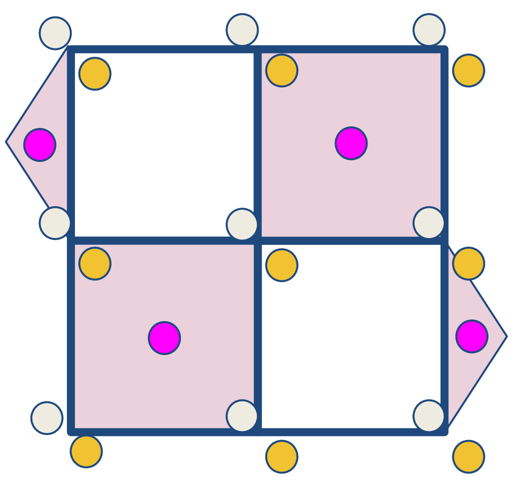

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GKP qubits

$$\prod_{l(j)} X^{(L)}_{l(j)} = \prod_{l(j)} D\left(\boldsymbol{\xi}_{l(j)}\right)$$

Higher level code

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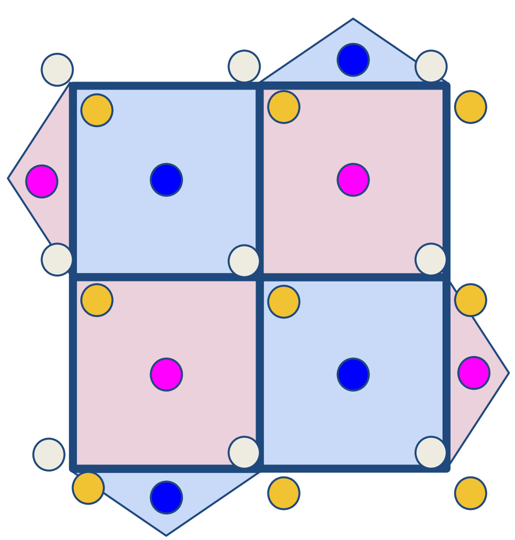

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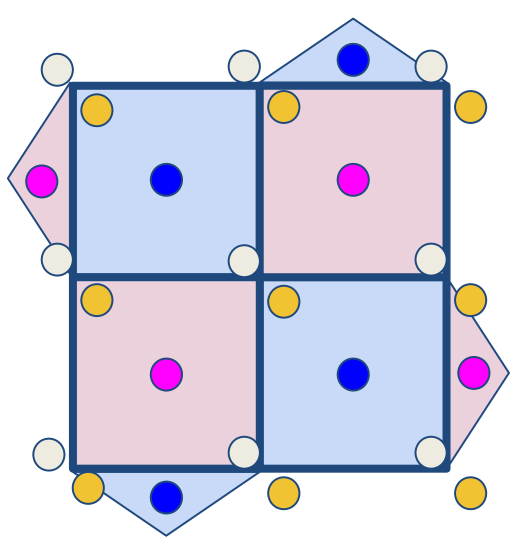
Example : L=3 GKP-surface code (n = 9 modes, logical dimension d = 2)



qubits/modes on vertices

$2n + n - 1 = \mathbf{26}$ stab.

$\exp\left(i2\sqrt{\pi}\hat{q}_j\right)$

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GKP qubits

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Higher level code

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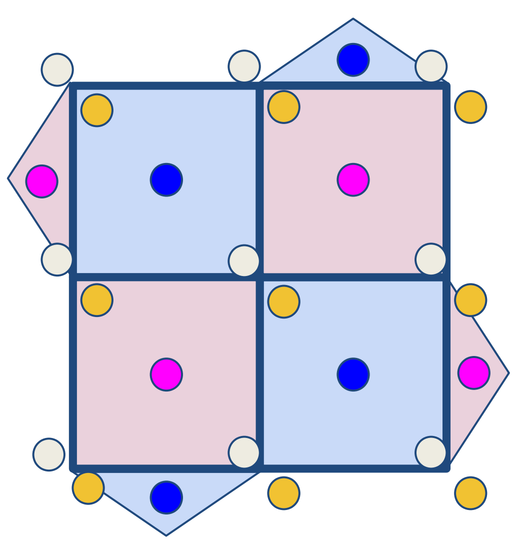

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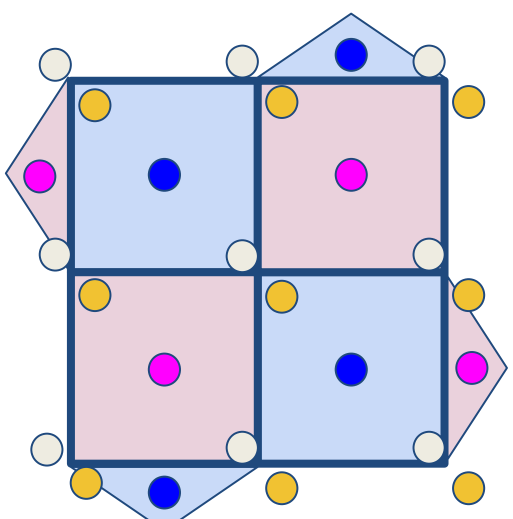

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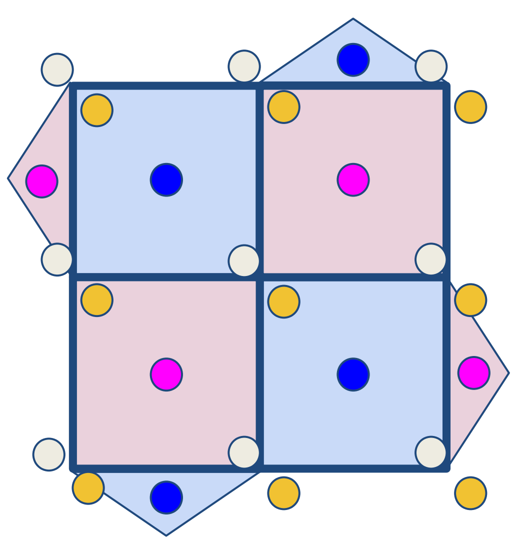

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$$M_{\mathrm{conc}} = \begin{pmatrix} M_{\mathrm{GKP}} \\ M_{\mathrm{Q}} \end{pmatrix}$$

⇩ lattice basis reduction

M_{min}

$2n =$ ***18*** stab. gen.

can do respecting weights
and geometric locality!

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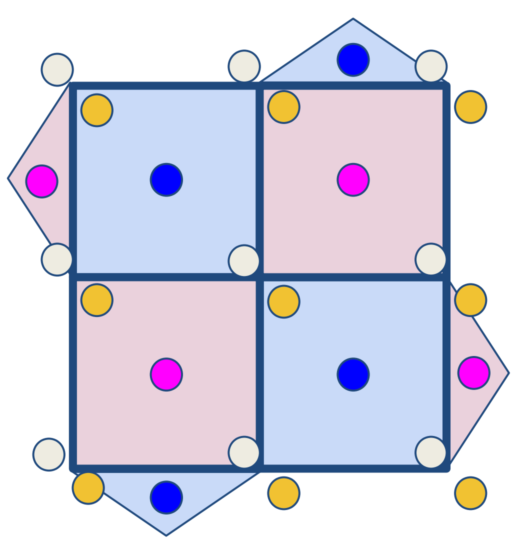

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Symplectic operations

$$U_S = \exp\left(-i\hat{\boldsymbol{x}}^T H \hat{\boldsymbol{x}}\right)$$

$$U_S \hat{\boldsymbol{x}} U_S^\dagger = S\hat{\boldsymbol{x}} \qquad S \in \mathrm{Sp}(2n), \quad SJS^T = J$$

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$$\boxed{\text{Given } \mathcal{L}(M), \mathcal{L}(N),\ \exists S \mid M = NS^T \text{ iff } MJM^T = NJN^T \text{ (in canonical form)}}$$

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$$A_{M,N} = \begin{pmatrix} 0 & D \\ -D & 0 \end{pmatrix}$$

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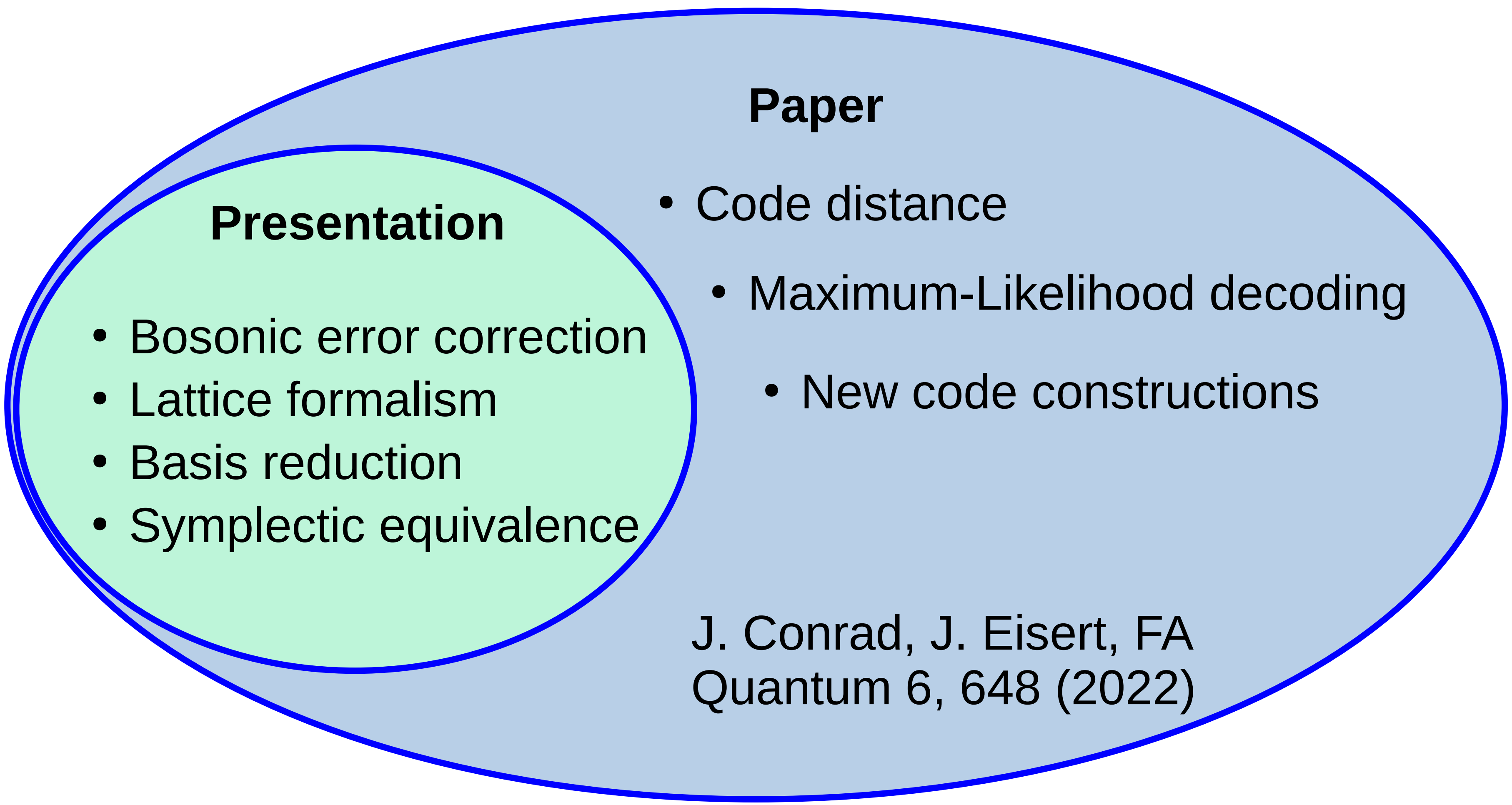
one qubit in mode 1, a fixed state on others

- **Generalizes** to higher logical dimensions
- Can be used to **bound distance** of (any) given grid code (see paper)

Part 4 : Conclusions

Presentation

- Bosonic error correction
- Lattice formalism
- Basis reduction
- Symplectic equivalence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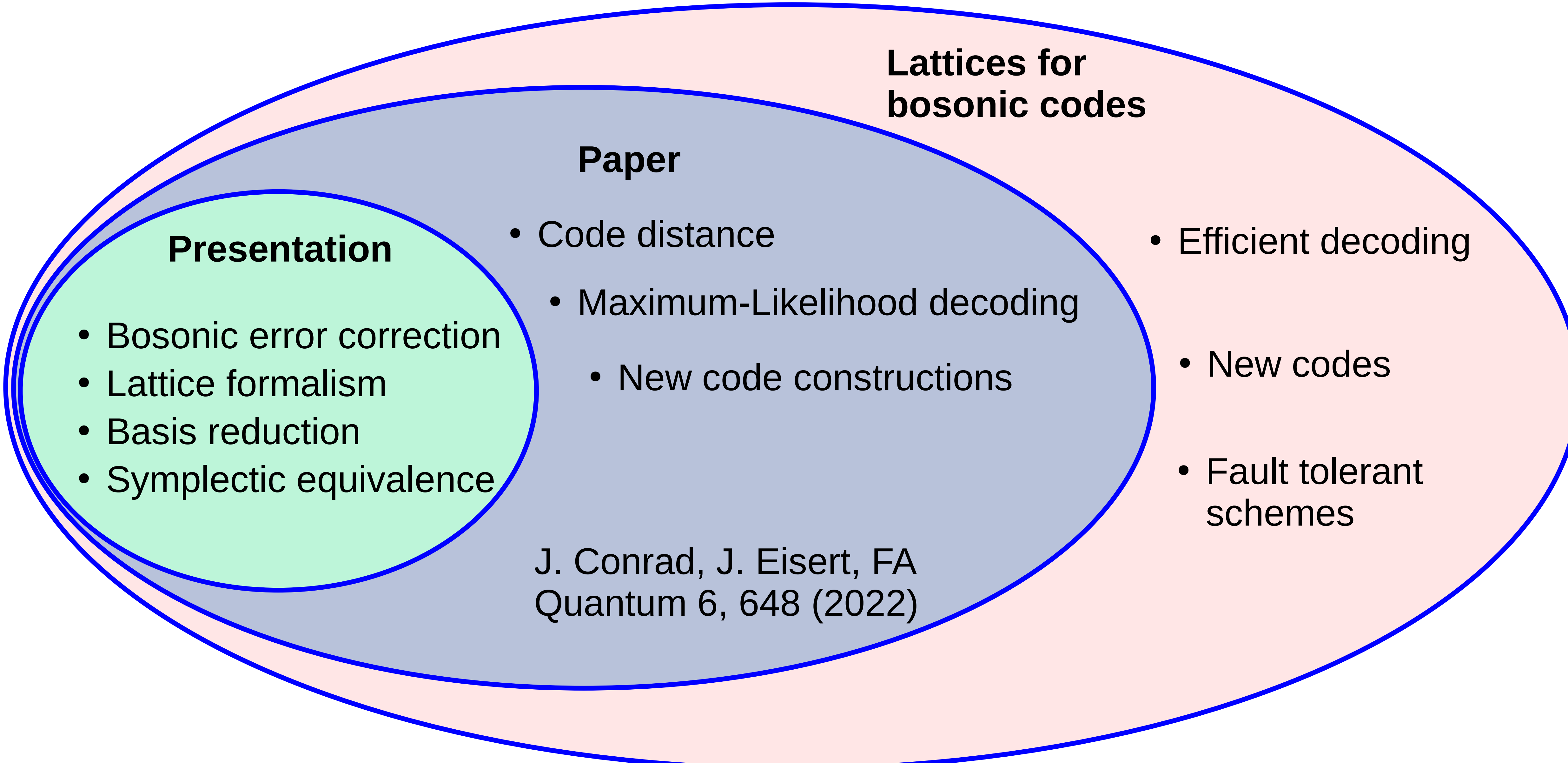
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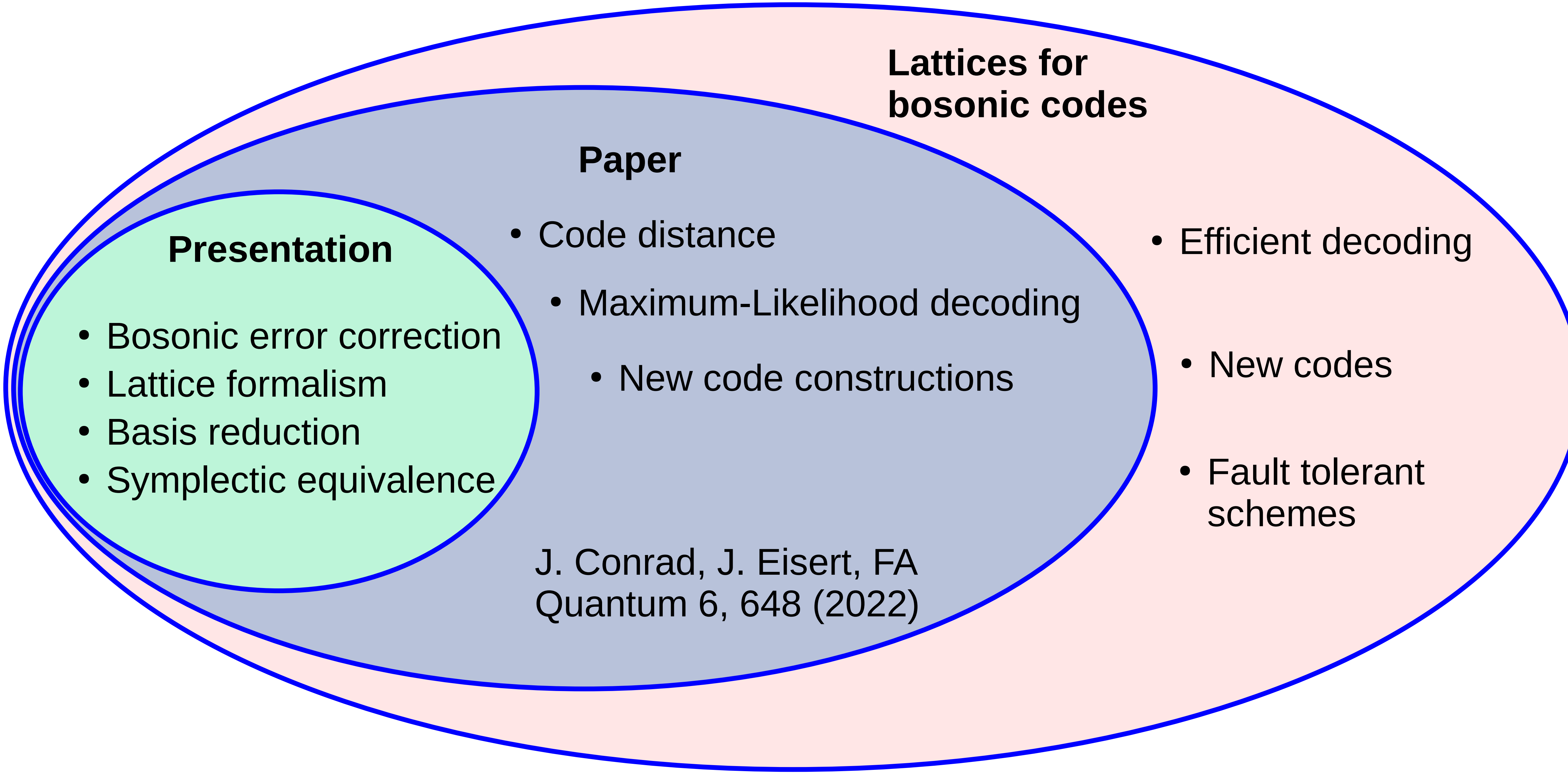
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Thank you!