Exercise 2: Up-Conversion and Pulse Shaping

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Christoph Studer (studer@ethz.ch)
Stefan M. Moser (moser@isi.ee.ethz.ch)

- **Exercise: Thursday, 17 October 2024**
- This exercise will be assisted in CAB G11 from 8:15 h to 10:00 h
- Problem 4 requires a computer with MATLAB (or Octave) installed
- **Due date:** Monday, 28 October 2024, 08:00 h
- **Submission instructions:** Upload your solutions (answers and MATLAB code) to Moodle as a *single* zip-file with a file name `⟨last-name⟩-⟨first-name⟩-e2.zip`. Your homework should consist of a single pdf file for the written answers and one MATLAB script per problem part. The MATLAB file must be named according to the syntax `E2_P4_1.m` (for "Exercise 2 Problem 4 Part 1").
- **Rules:** You may collaborate with other students on this exercise, but each student must turn in their own assignment. You must specify the names of your collaborators! Copying answers from previous years' exercises is strictly forbidden.

Problem	Maximum Points	Points Received
1	20	
2	10	
3	30	
4	40	
Total	**100**	

Problem 1: Linearity and Time Invariance of Up-Conversion (20pts)

Let $s_{\mathrm{b}}(t) \in \mathbb{C}$, $t \in \mathbb{R}$, be a complex baseband signal. Up-conversion of $s_{\mathrm{b}}(\cdot)$ to a carrier frequency of f_{c} Hz can be written as

$$s(t) = \sqrt{2}\Re\left\{s_{\mathrm{b}}(t)\, e^{j2\pi f_{\mathrm{c}} t}\right\}, \quad t \in \mathbb{R}, \tag{1}$$

where $s(t) \in \mathbb{R}$, $t \in \mathbb{R}$, is the real-valued passband signal.

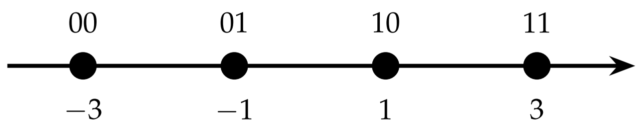
Part 1 (10pts). Is the up-conversion process in (1) linear? Explain your answer with math.

Part 2 (10pts). Is the up-conversion process in (1) time invariant? Explain your answer with math.

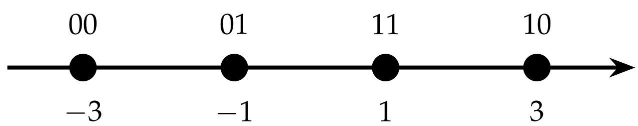
Problem 2: Robustness of Various Bit Labelings (10pts)

Order the following bit labelings (a, b, and c) of 4-PAM in terms of the associated bit error rate (BER). List the one with the lowest BER first. Assume that the dominating error event is confusing neighboring constellation points. Explain your decisions.

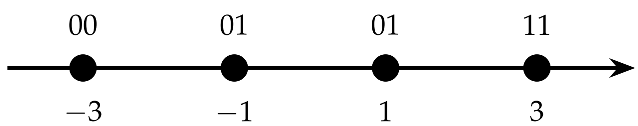
(a)



(b)



(c)



Problem 3: Properties of BPSK-Modulated Baseband Signals (30pts)

Consider a random process $\mathsf{s}_{\mathrm{b}}(t)$ resulting form a time-domain signal that uses random BPSK symbols with rectangular pulse shapes. Concretely, let the bits d_n be IID Bernoulli distributed with equal probability for 0 and 1, $\mathsf{d}_n \sim \mathcal{B}(1/2)$. Let us define the mapping from bits to symbols as $\mathsf{d}_n = 0$ to $\mathsf{x}_n = 1$ and $\mathsf{d}_n = 1$ to $\mathsf{x}_n = -1$, and use rectangular pulse shapes of duration T defined as

$$g(t) = \begin{cases} 1 & \text{for } |t| < \frac{T}{2}, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Then, the resulting time-domain baseband signal is given by

$$\mathsf{s}_\mathrm{b}(t) = \sum_{n=-\infty}^{\infty} \mathsf{x}_n \, g(t - nT). \tag{3}$$

The goal of this problem is to analyze the properties of this random process.

Part 1 (10pts). Compute the *mean* $\mu_{\mathsf{s}_\mathrm{b}}(t)$ and the *covariance function* $K_{\mathsf{s}_\mathrm{b}}(t, t')$ of the random process $\mathsf{s}_\mathrm{b}(t)$.

Part 2 (10pts). Is the random process $\mathsf{s}_\mathrm{b}(t)$ *wide-sense stationary* (WSS)?

Part 3 (10pts). Compute the spectrum of $K_{\mathsf{s}_\mathrm{b}}(t, t')$ at $t' = 0$ using the continuous-time Fourier transform.

Problem 4: Eye Diagrams in MATLAB (40pts)

Part 1 (30pts). Write a MATLAB script that generates an eye diagram for raised cosine (with $\beta = 0.5$), triangular, and rectangular pulse shapes. Create one figure that contains four subplots, showing

1. a single pulse,
2. the frequency response of a single pulse,
3. a pulse-shaped random BPSK transmit signal, and finally
4. the resulting eye diagram.

We suggest to structure the MATLAB script as follows. First, generate a random BPSK sequence $\mathsf{x}_n \in \{-1, 1\}$, $n = 1, \dots, B$, with $B = 100$ bits. Then, create a time vector $\boldsymbol{t}$ for the interval $[-T, BT]$ with sufficient time resolution, e.g., $\Delta t = 10^{-3}$. Compute the transmit signal $\boldsymbol{s}$ by superposition of all B shifted-by-T pulses, which are evaluated at the time instants as specified by the time vector $\boldsymbol{t}$ and multiplied by x_n. The eye diagram can be plotted by overlaying all T-shifted snippets of length $2T$, which are extracted from the transmit signal $\boldsymbol{s}$. Plot the single pulse in the range $[-4T, 4T]$.

Part 2 (10pts). Discuss the results of your MATLAB script. What are the pros and cons of each pulse shape? Argue based on the horizontal and vertical eye openings, and the corresponding frequency spectrum.