Exercise 3: Discrete-Time Signals

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- **Exercise: Thursday, 24 October 2024**
- This exercise will be assisted in CAB G11 from 8:15 h to 10:00 h
- Problem 4 requires a computer with MATLAB (or Octave) installed
- **Due date:** Monday, 4 November 2024, 08:00 h
- **Submission instructions:** Upload your solutions (answers and MATLAB code) to Moodle as a *single* zip-file with a file name `⟨last-name⟩-⟨first-name⟩-e3.zip`. Your homework should consist of a single pdf file for the written answers and one MATLAB script per problem part. The MATLAB file must be named according to the syntax `E3_P4_1.m` (for "Exercise 3 Problem 4 Part 1").
- **Rules:** You may collaborate with other students on this exercise, but each student must turn in their own assignment. You must specify the names of your collaborators! Copying answers from previous years' exercises is strictly forbidden.

Problem	Maximum Points	Points Received
1	20	
2	20	
3	20	
4	40	
Total	**100**	

Problem 1: Parseval's Theorem for Bandlimited Signals (20pts)

Assume that the continuous-time signal $s(t)$, $t \in \mathbb{R}$, is bandlimited to W Hz. Show that for such signals we have the following relation

$$\int_{-\infty}^{+\infty} |s(t)|^2 \, \mathrm{d}t = T_\mathrm{s} \sum_{n=-\infty}^{+\infty} |s(nT_\mathrm{s})|^2 \tag{1}$$

with $T_\mathrm{s} = 1/(2W)$. ***Important: This result implies that the power of a transmitted continuous-time signal can be related to the scaled power of its sampled discrete-time version.***

Problem 2: Discrete Fourier Transform (20pts)

Assume that you have the following periodic discrete-time signal

$$s[n] = \frac{1}{2} \sum_{m=-\infty}^{\infty} \Big(\delta[n+1+mN] + \delta[n-1+mN] \Big), \tag{2}$$

where $N > 3$ is the period, $\delta[\cdot]$ is the Kronecker delta function, and $n \in \mathbb{Z}$.

Part 1. (10pts). Compute the length-N discrete Fourier transform (DFT) of the signal $s[\cdot]$ in (2). Try to simplify the DFT expression as much as you can.

Hint: It may help to sketch the signal from $n = 0, 1, \ldots, N-1$.

Part 2 (10pts). What is the value at $k = 0$ of the DFT of $s[\cdot]$ in (2), i.e., what is $\hat{s}[0]$?

Hint: You can either directly use the result from Part 1 or, in case you are not sure whether your result is correct or not, you have to first remember what it means to compute the DFT at $k = 0$ and then get the answer directly from (2). The latter approach also allows you to check whether your answer from Part 1 makes sense or not.

Problem 3: High-Pass Infinite-Impulse Response Filter (20pts)

Assume you want to design an infinite-impulse response (IIR) high-pass filter with the following discrete-time Fourier transform (DTFT):

$$\hat{h}\big(e^{j\omega}\big) = \begin{cases} 0 & \text{for } -\frac{\pi}{2} < \omega < \frac{\pi}{2}, \\ 1 & \text{otherwise.} \end{cases} \tag{3}$$

In words, this filter passes frequencies above $\pi/2$, but removes all frequencies below that. Compute the sample-domain coefficients $h[m]$, $m \in \mathbb{Z}$, associated with the DTFT $\hat{h}(e^{j\omega})$.

Problem 4: DFT and DTFT Analysis with MATLAB (40pts)

Let us consider the following discrete-time signal:

$$s[n] = \begin{cases} 1 & n \in \{0,1,2,3,4,5,6,7\}, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Let us also define an N-sample periodic version of this signal $s_{\mathrm{p}}[n] = s[n \bmod N]$, which repeats every N samples. In the following, we will analyze the signal $s[\cdot]$ by means of the DTFT and the DFT in order to highlight the relation between the two transforms.

Part 1 (10pts). Compute the DTFT of (4) analytically.

Part 2 (10pts). Implement a MATLAB script that plots the first 64 samples of the sample domain sequence of (4) and the magnitude of its corresponding DTFT spectrum from Part 1. Therefore, we suggest to densely sample the DTFT spectrum such that your plot appears to be continuous.

Part 3 (10pts). Plot the sample-domain signal generated from the analytical DTFT, which is sampled at $2\pi k/N$ for $k = 0, \ldots, N-1$. To visualize the periodicity of the DFT-synthesis sample-domain signal, sample the DTFT with $N = 32$, but plot the sample-domain signal for 64 samples. Remember that a sampled DTFT spectrum corresponds to the DFT of the sample-domain signal taken from one period. Thus, you can also transform the sampled DTFT spectrum to the sample-domain by an inverse DFT. Does your result correspond to $s_{\mathrm{p}}[n]$?

Part 4 (10pts). Take the sample-domain signals from Part 3 for $n = 0, \ldots, N-1$, and perform a DFT. Overlay the DFT spectrum with the plot from Part 2. Does the (discrete) DFT spectrum match the (continuous) DTFT spectrum at the sample instants $2\pi k/N$, $k = 0, \ldots, N-1$?