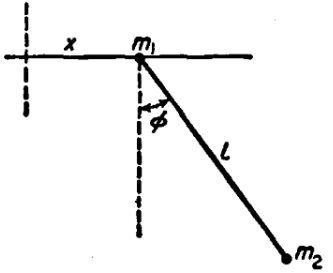


Exercises marked with (*) have extra difficulty, don't hesitate to ask for help.

1. **Pendulum with free point of support** [Landau §5 ex. 2]

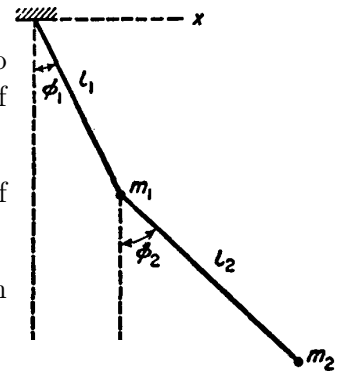
Particle of mass m_2 is hanging from a rigid bar of length ℓ and negligible mass. On the other end there is a device of mass m_1 linked to a horizontal bar, and it's free to move horizontally along the x axis. The device allows the hanging bar to span any angle φ respect to the vertical axis.



- Write expressions for kinetic energy, T and potential, V , as functions of the generalized coordinates suggested by the figure.
- Verify that if you fix the position of mass m_1 you recover the expressions of T and V of an ideal pendulum.

2. **Coplanar double pendulum** [Landau §5 ex. 1]

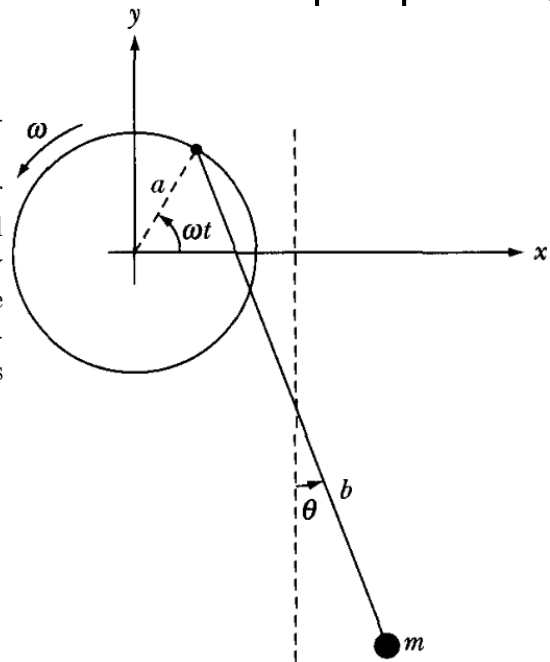
A rigid bar of length ℓ_1 of negligible mass has a particle of mass m_1 attached to one end. There is a second bar of negligible mass hanging from the first one, of length ℓ_2 , with a particle of mass m_2 attached to the other end too.



- Write expressions for kinetic energy, T and potential, V , as functions of the generalized coordinates suggested by the figure.
- Verify that you recover the expressions of T and V of an ideal pendulum if you set $m_1 = 0$, $\varphi_1 = \varphi_2 = \varphi$ and $\ell_1 = \ell_2 = \frac{\ell}{2}$.

3. (*) **Pendulum with rotating point of support** [Marion (e) ex. 7.5] [Landau §5 ex. 3]

A particle of mass m is attached to the end of a rigid bar of length b . The point of support is linked to a vertical circle of radius a and it rotates with constant frequency ω . It is assumed that all positions lie in the same plane and the mass of the bar is negligible. Calculate the kinetic energy, T , and potential V , of the particle of mass m .



4. (*) **Coupled weights rotating about a vertical axis** [Landau §5 ex. 4]

Particle with mass m_2 moves on a vertical axis and the whole system rotates about this axis with a constant angular velocity Ω . This particle is linked to two particles of mass m_1 through bars of length a and negligible mass, and at the same time these particles are linked to the fixed point A through identical bars, forming the variable angle θ respect to the vertical axis. Calculate the kinetic energy of each of the three particles and find a compact expression of the kinetic energy of the whole system.

