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Optimization Methods in Business Analytics

Hazardous Waste Management

*A capacitated multi-vehicle periodic Vehicle Routing Problem to
optimize the weekly collection of Healthcare Waste*

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1 Introduction

This project concerns the optimized management of hazardous and special waste transportation, with a particular focus on the hospital setting. Such waste includes infectious, toxic, flammable, or biological substances, whose disposal must comply with strict regulations regarding environmental and health safety.

The objective is to determine the optimal route and collection frequency for a fleet of vehicles serving multiple hospital centers, minimizing the total distance traveled while also considering critical risk-related variables. In the event of an accident, the materials being transported could cause serious harm to public health and the environment. Specific constraints have also been included to ensure compliance with current regulations, particularly those outlined in the *ADR Agreement* (European Agreement concerning the International Carriage of Dangerous Goods by Road), which imposes volume limits, vehicle requirements, safety equipment standards, personnel training, and traceability.

Through this project, the aim is to propose an optimal logistics solution that combines operational efficiency with compliance with regulatory and safety requirements.

2 Problem Description and Bi-Objective Strategy

This study was conducted in the province of Brescia, where 14 hospital facilities were identified, managed by the three main Local Health Authorities (ASSTs): ASST Brescia, ASST Garda, and ASST Franciacorta.

The core problem is modeled as a variant of the Capacitated Vehicle Routing Problem (CVRP), in which a fleet of vehicles departs from a central depot to collect waste from hospitals. The VRP objective is to minimize the total travel time while respecting vehicle capacity limits. The planning horizon covers a full operational week, requiring route scheduling across multiple days.

This classical formulation is extended with specific constraints related to ADR regulations, which govern the road transport of dangerous goods. In particular, the most relevant constraints include:

- the type of vehicle required, depending on the nature of the waste being transported (e.g., standard or refrigerated vehicles);
- compatibility between waste types: some materials cannot be transported together in the same vehicle due to chemical or biological safety concerns.

In addition to regulatory compliance, our approach incorporates a second objective: alongside minimizing travel time, we also aim to minimize risk exposure associated with traveling each segment of the road network. This risk-aware adjustment will discourage the selection of risky routes in favor of safer choices and reduce both the likelihood and the potential impact of critical incidents. However, rather than integrating this dual objective directly into the VRP formulation, the risk minimization mechanism is handled externally, during the preprocessing of the input data.

Specifically, to balance transport efficiency with the need to reduce hazards related to the movement of dangerous substances, penalties were introduced for certain road categories, based on:

- type of road (e.g., limited traffic zones, presence of tunnels or underpasses);
- surface quality (e.g., asphalt, dirt, gravel);
- proximity to sensitive locations, such as schools or public parks.

Finally, a cost matrix representing a proxy for travel time was then precomputed using Dijkstra’s algorithm over the risk-adjusted network. The resulting *risk-aware* cost matrix serves as the input to the VRP model, allowing the solver to focus solely on minimizing total cost—while implicitly balancing route efficiency and environmental safety.

2.1 Waste Classification

Healthcare waste is classified based on its origin, potential hazards, and required treatment or disposal procedures. In the European Union, waste is systematically categorized using the European Waste Catalogue (EWC), also known as *Codice Europeo dei Rifiuti* (CER). This standardized system facilitates traceability, regulatory compliance, and proper handling across all waste management stages.

The main categories of healthcare waste considered in this study include:

A. Non-toxic, Non-flammable, Non-corrosive Chemical Waste EWC/CER: 18 01 07¹

This group includes chemical substances that are unreactive under normal conditions and pose minimal risks to health or the environment. While not classified as hazardous, these materials still require dedicated disposal pathways due to their medical origin.

Examples: Expired drugs, expired saline solutions, pH-neutral cleaning agents, non-reactive laboratory reagents.

B. Toxic, Flammable and/or Corrosive Chemical Waste EWC/CER: 18 01 06

This includes chemicals that are dangerous due to their toxicity, reactivity, flammability, or corrosiveness. These substances require special handling procedures and must be treated as hazardous.

Examples: Formaldehyde, xylene, strong acids or bases, flammable solvents used in diagnostic labs or research.

C. Infectious Waste EWC/CER: 18 01 03

Infectious waste comprises materials contaminated by blood, body fluids, or other potentially infectious agents. This type of waste poses biological risks and requires sterilization or incineration.

Examples: Used syringes, contaminated bandages, microbiological cultures, used personal protective equipment (PPE) from isolation wards.

D. Special Waste (Including Anatomical Parts and Ashes) EWC/CER: 18 01 02

This category includes biological or human-derived waste that requires separate treatment due to ethical, sanitary, or regulatory concerns.

Examples: Amputated body parts, pathological tissues, ashes from incinerated biomedical waste.

E. General Healthcare Waste EWC/CER: 18 01 04

General waste from healthcare facilities is non-hazardous and resembles municipal waste, yet it can't be managed as ordinary household waste. Due to its origin in clinical settings, it may still pose minimal risks of contamination and require specific traceability.

Examples: Clean disposable materials such as unused masks, packaging of sanitary materials,

uncontaminated empty containers such as saline and disinfectant bottles, cleaning materials such as disposable cloths, mops.

2.2 Compatibility of Hazardous Waste Types According to ADR Regulations

According to ADR regulations, there are strict restrictions on the coexistence of hazardous waste types within the same vehicle, especially when dealing with infectious, flammable, or biologically sensitive substances. The following table summarizes the compatibility between different waste categories and serves as a critical reference for implementing the Vehicle Routing Problem (VRP) in our approach. Although the same modeling framework will be used, it must be applied separately to waste classified in categories B and D to comply with the relevant safety and regulatory requirements.

| Waste Category | A | B | C | D | E |
|----------------|---|---|---|---|---|
| A | | ✗ | ✓ | ✗ | ✓ |
| B | ✗ | | ✗ | ✗ | ✗ |
| C | ✓ | ✗ | | ✗ | ✓ |
| D | ✗ | ✗ | ✗ | | ✗ |
| E | ✓ | ✗ | ✓ | ✗ | |

Table 1: Compatibility matrix for waste categories

2.3 Road Assessment

To ensure the safe transportation of hazardous waste, our model integrates a comprehensive risk evaluation of the road network. Our approach applies penalties to road segments with unfavorable conditions based on multiple risk factors, with the ultimate goal of identifying the safest possible routes for hazardous material transport.

Proximity to Sensitive Areas: A critical aspect of risk assessment involves the proximity to sensitive locations. Roads within a 50-meter radius of parks and schools are subject to specific penalties due to higher population density, particularly of vulnerable groups, higher pedestrian activity and greater potential consequences in case of accidents or material release.

To incorporate the risk factor in the form of a penalty, the travel time for each road segment was calculated by dividing the road length by its maximum allowed speed. The base travel time was then multiplied by 1.5 if the road was classified as being near a park or a school:

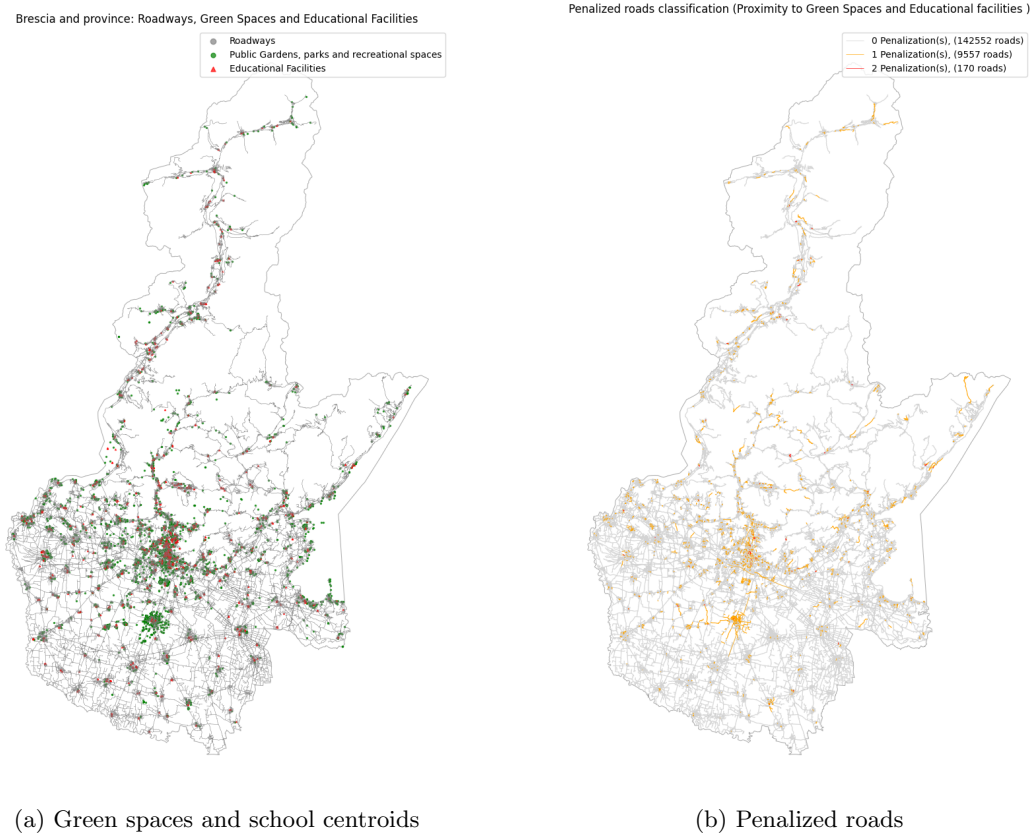
$$penalization = travel\ time \times 0.5 \quad (1)$$

¹List of European Waste Codes available on: <https://www.ecochim.it/elenco-codici-cer/>

$$penalizedtraveltime = travel\ time + penalization \quad (2)$$

This represents a 50% increase in the effective travel time for routing purposes, effectively discouraging the algorithm from selecting these higher-risk segments. When a road segment is located near both a school and a park simultaneously, the penalty is applied twice, resulting in a substantial disincentive for route selection.

Figure 1: Proximity penalization



Road Network Attributes: our multi-criteria risk evaluation framework takes into account the various road types and network attributes available in the dataset. Each attribute enabled a detailed assessment of each road segment:

- **surface:** describes the surface of the road. Certain surface types imply higher risks for hazardous materials transportation due to their instability and increased accident probability. (e.g. "gravel", "dirt", "sand", "ground", "cobblestone", "grass");
- **smoothness:** measures the regularity of the road surface, directly affecting vehicle stability and control during transportation;
- **tracktype:** provides supplementary information about road conditions (e.g. "grade1" for a high-grade paved road, "grade5" for a low-quality trail);

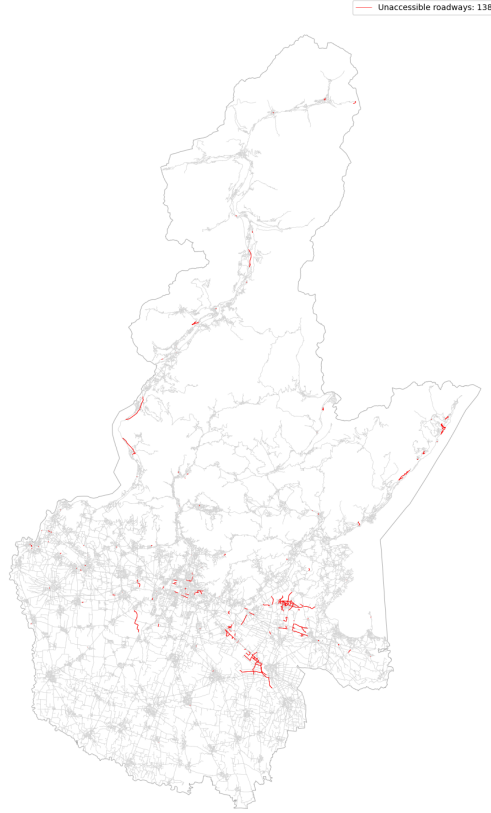
- **access:** indicates who can access the road. For example, it can specify whether access is restricted to certain scope (e.g., “yes,” “no,” “residents”, “delivery”, “customers”);
- **highway:** functional road classification (e.g., primary, secondary, residential, busway);
- **lanes:** indicates the number of lanes of the road. Roads with multiple lanes generally provide safer transportation options through better traffic flow and emergency maneuvering options;
- **width:** carriageway width (roads <3.5 m are classified as higher risk due to limited space for evasive actions);
- **bridge:** indicates whether the road crosses a bridges, which increase severity in case of accidents;
- **tunnel:** indicates the presence of tunnels or underpasses which imply limited ventilation and evacuation options, presenting heightened risks for hazardous material incidents;
- **lit:** indicates whether there is or not a lighting system;
- **hazmat:** roads where the transport of hazardous materials is prohibited by law;
- **service:** indicates the purpose of the road (e.g., “driveway,” “parking,” “fuel”). Certain service types present higher risks due to space constraints, increased traffic density, or proximity to sensitive infrastructure;
- **pvs:** indicates whether the road is reserved for public vehicles such as buses or cabs;
- **bicycle_road, footway:** specifies whether the road is designated for bicycles or pedestrians.

Exclusion Criteria: Several roads were automatically excluded from the model, regardless of other attributes. For example, roads legally prohibited for hazardous materials transportation (**hazmat=no**), Bicycle-only roads and pedestrian pathways (**bicycle road=yes, footway=yes**) and Limited traffic zones (**access = no**)

Each attribute was weighted according to its impact on transportation safety, with the sum of all weights normalized to 1.0. This normalization approach was chosen to:

- Facilitate a standardized risk scale (0-1);
- Enable straightforward comparison between different road segments;
- Allow for transparent prioritization of risk factors;
- Support mathematical consistency in the model.

Figure 2: Restricted roadways
Traffic-limited zones, pedestrian areas, bike lanes, or public transport-only roads



The weights used in the model are as follows:

$$\text{risk_weights} = \left\{ \begin{array}{ll} \text{'risk_surface'} & 0.15, \\ \text{'risk_lit'} & 0.05, \\ \text{'risk_service'} & 0.10, \\ \text{'risk_smoothness'} & 0.10, \\ \text{'risk_tracktype'} & 0.05, \\ \text{'risk_highway'} & 0.15, \\ \text{'risk_lanes'} & 0.10, \\ \text{'risk_width'} & 0.10, \\ \text{'risk_bridge'} & 0.05, \\ \text{'risk_access'} & 0.05, \\ \text{'risk_tunnel'} & 0.10. \end{array} \right.$$

The total penalization for each road segment was calculated using the following formula:

$$penalization = travel\ time \times \sum risk\ weight.$$

This approach enables the quantification of risk associated with each road segment and guides the model towards routes that minimize exposure to potentially hazardous conditions.

2.4 Vehicle Types

The proposed optimization model includes two main categories of vehicles, selected based on the type of waste being transported and the specific requirements set by ADR regulations²:

- **Standard Vehicles:** Used for transporting waste from categories (A, B, C, E).
 - **Volume capacity:** 16 or 32 m³ (16,000 or 32,000 L);
 - **Maximum load capacity:** 1,600 or 3,200 kg;
- **Refrigerated Vehicles:** Used exclusively for transporting Category D waste (sensitive special waste, such as anatomical remains). It requires cold chain maintenance for sanitary and regulatory compliance.
 - **Volume capacity:** 39 m³ (39,000 L)
 - **Features:** refrigerated chamber, thermal insulation, full ADR compliance

²UNECE, Accordo europeo relativo al trasporto internazionale di merci pericolose su strada (ADR). Edizione 2023

3 Estimation of Healthcare Waste Generation

To solve the proposed optimization problem, it was necessary to realistically estimate the daily production of healthcare waste. This estimate serves as a crucial input for modeling a periodic vehicle routing problem (PVRP) with a 7-day collection window and realistic daily variability.

3.1 Data Sources

The estimation of healthcare waste quantities was based on a combination of international benchmarks and region-specific studies. According to the World Health Organization (WHO), the average production of healthcare waste in Europe is approximately 0.8 kg per occupied bed per day. However, reports and assessments from national and regional sources—particularly ISPRA (Istituto Superiore per la Protezione e la Ricerca Ambientale) and ARPA (Agenzia Regionale per la Protezione Ambientale)—indicate that daily waste generation rates in the Lombardy region tend to exceed the European average. This is primarily due to the greater structural complexity and higher service intensity of healthcare facilities in the area. Furthermore, when surgical procedures are considered, the volume of healthcare waste increases significantly, emphasizing the need to account for both inpatient occupancy and procedural activity in waste estimation models. Table 2. shows an estimation of healthcare waste produced on a daily basis, highlighting the combined contribution from both occupied beds and surgical procedures.

| Healthcare waste produced daily | | | |
|---|----------------------------------|--|---|
| Waste Category | daily total kg (unit average) | of which produced during surgery | of which produced by occupied bed |
| A. Non-toxic, Non-flammable, Non-corrosive Chemical Waste | 0.52 | 0.5 | 0.02 |
| B. Toxic, Flammable and/or Corrosive Chemical Waste | 0.26 | 0.25 | 0.01 |
| C. Infectious Waste | 4 | 3 | 1 |
| D. Special Waste | 0.285 | 0.25 | 0.035 |
| E. General Healthcare Waste | 1 | 0.6 | 0.4 |

Table 2: Daily average production of healthcare waste per category and per unit

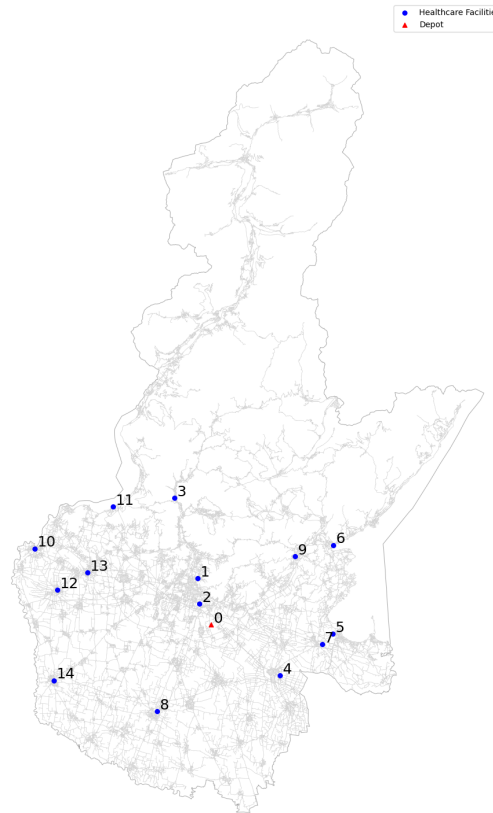
3.2 Geographical Scope

As previously mentioend, this study focuses on the province of Brescia, where we identified 14 hospital facilities managed by the three main local healthcare authorities (ASSTs): ASST

Brescia, ASST Garda, and ASST Franciacorta.

For each facility, we collected detailed data on the number of available hospital beds, the average annual number of surgical procedures, and the average annual number of hospital admissions. In cases where data on surgical procedures or hospital admissions were unavailable, we applied an approximation by scaling data from the Ospedale Civile di Brescia, the largest and most data-rich hospital in the province. Specifically, we assumed uniform characteristics across all facilities in terms of annual surgical procedures per bed and annual hospital admissions per bed, allowing us to estimate missing values through proportional ratios.

Figure 3: Target Healthcare Facilities and Depot Location



$$\text{Set of facilities} = \left\{ \begin{array}{l} \text{'1' Ospedale Civile di Brescia (ASST Brescia),} \\ \text{'2' Poliambulanza,} \\ \text{'3' Ospedale di Gardone Val Trompia(ASST Brescia,} \\ \text{'4' Ospedale di Montichiari (ASST Brescia),} \\ \text{'5' Ospedale di Desenzano del Garda(ASST del Garda),} \\ \text{'6' Ospedale di Salò (ASST del Garda),} \\ \text{'7' Ospedale di Lonato del Garda (ASST del Garda),} \\ \text{'8' Ospedale di Manerbio (ASST Garda),} \\ \text{'9' Ospedale di Gavardo (ASST del Garda),} \\ \text{'10' Ospedale di Palazzolo sull'Oglio (ASST Franciacorta),} \\ \text{'11' Ospedale di Iseo (ASST Franciacorta),} \\ \text{'12' Ospedale di Chiari (ASST Franciacorta),} \\ \text{'13' Ospedale di Rovato (ASST Franciacorta),} \\ \text{'14' Ospedale di Orzinuovi (ASST Franciacorta).} \end{array} \right.$$

3.3 Daily and Weekly Waste Production Estimation

Using the collected hospital data, we estimated the average weekly production of each type of healthcare waste per facility. The estimation is based on the combined contributions from both surgical procedures performed daily and hospital beds occupied daily.

The number of daily and weekly surgical procedures was calculated as follows:

$$Dailysurgeries = (Annualsurgicalprocedures)/365 \quad (3)$$

$$Weeklysurgeries = Dailysurgeries \times 7 \quad (4)$$

Estimating waste generated from hospital beds required additional steps. Since only occupied beds contribute to waste generation, we estimated the average number of beds occupied per day for each facility. This was achieved by applying an average bed occupancy duration of 7.4 days per admission³, which was derived from national and regional healthcare statistics and following different steps. Total annual bed-days was computed as:

$$\text{Total annual bed-days} = \text{Annual hospital admissions} \times \text{Average length of stay} \quad (5)$$

³This data refers to acute cases (e.g., myocardial infarction, pneumonia, fractures, appendicitis, post-operative complications, trauma). Long-term stays and day hospital admissions were excluded from this analysis.

Then, the daily bed occupancy rate was calculated as:

$$\text{Occupancy rate (\%)} = \left(\frac{\text{Total bed-days}}{\text{Available beds} \times 365} \right) \times 100 \quad (6)$$

From this, we derived the average number of occupied beds per day as:

$$\text{Occupied beds} = \text{Available beds} \times \text{Occupancy rate} \quad (7)$$

| Total Beds | Annual admissions | Annual bed-days (5) | Occupancy rate (%) (6) | Occupied beds daily (7) |
|------------|-------------------|---------------------|------------------------|-------------------------|
| 1546 | 71,500 | 529,100 | 93.76% | 1450 |

Table 3: Example for the facility “Ospedale Civile di Brescia (ASST Brescia)”

3.4 Simulation of Daily Waste Generation

After collecting and estimating the required input data, we simulated daily healthcare waste generation across a 7-day planning horizon. The simulation needed to reflect the pre-calculated weekly average while incorporating realistic daily fluctuations in hospital activity. Specifically, findings from Italian National Institute of Health (ISS) indicate that hospitals typically perform fewer surgeries, operate at reduced staff capacity and keep laboratories at minimal capacity on Saturdays and Sunday. For this reason, we realistically assume a 20–40% drop in waste volumes during those days.

To simulate this variability - while keeping the weekly total fixed - we developed a Python-script designed to:

- Ensure the total across 7 days of each waste type per hospital matches the estimated weekly total;
- Introduce controlled day-to-day variability to mimic real world hospital behaviour;
- Prevent unrealistic peaks or outliers by enforcing sensible lower and upper bounds on daily waste volumes.

3.5 Limitations and Assumptions

Despite the structured methodology, the estimation process inevitably involves several simplifications and limitations:

- Complete data was not available for every facility; missing values were estimated proportionally using a benchmark hospital;

- It was assumed that all facilities operate similarly in terms of hospital activity and department availability; however, in reality, not all hospitals have the same range of departments (e.g., some may lack surgical units or diagnostic labs);
- Daily variability was introduced while maintaining a fixed weekly total, which simplifies operational irregularities.

This simulation approach allowed us to obtain a synthetic yet evidence-informed dataset, which provides the quantitative foundation for our project.

4 Mathematical Model

4.1 Model Formulation

Problem inputs:

- $V = \{0, 1, \dots, n\}$: set of vertices, where vertex 0 represents the depot and $\{1, \dots, n\}$ represents the customers;
- c_{ij} : cost (penalized travel time) from vertex i to vertex j , for all $i, j \in V$;
- Q : vehicle capacity;
- q_i : demand of customer i , for all $i \in \{1, \dots, n\}$;
- K : set of vehicles.

Decision Variables:

- $x_{ij}^k \in \{0, 1\}$: binary variable that equals 1 if vehicle k travels directly from vertex i to vertex j , and 0 otherwise, for all $i, j \in V$ and $k \in K$;
- u_i^k : auxiliary variable representing the load of the vehicle k after serving customer i , for all $i \in \{1, \dots, n\}$.

Objective function:

$$\text{Minimize the total cost: } \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}^k$$

Subject to:

1. Each customer is visited exactly once:

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1, \quad \forall i \in \{1, \dots, n\}$$

2. Each vehicle leaves the depot:

$$\sum_{j \in V} x_{\text{DepotEntry}, j}^k = 1, \quad \forall k \in K$$

3. Each vehicle returns to the depot:

$$\sum_{i \in V} x_{i, \text{DepotExit}}^k = 1, \quad \forall k \in K$$

4. Flow conservation:

$$\sum_{i \in V} x_{ij}^k = \sum_{i \in V} x_{ji}^k, \quad \forall j \in \{1, \dots, n\}, \forall k \in K$$

5. Subtour elimination and capacity constraints:

$$u_i - u_j + Qx_{ij}^k \leq Q - q_j, \quad \forall i, j \in \{1, \dots, n\}, \forall k \in K, i \neq j$$

6. Load limits:

$$q_i \leq u_i \leq Q, \quad \forall i \in \{1, \dots, n\}$$

7. Binary decision variables:

$$x_{ij}^k \in \{0, 1\}, \quad \forall i, j \in V, \forall k \in K$$

4.2 Route Selection Using Dijkstra's Algorithm

To determine the optimal travel paths between node pairs and construct the cost matrix for the Vehicle Routing Problem (VRP) model, we employed Dijkstra's algorithm. This algorithm is particularly well-suited for road network modeling due to its simplicity and computational efficiency in finding the shortest paths from a starting node to all other nodes in graphs with non-negative edge weights. Each edge in the road network represents a road segment, with a *weight* (cost) defined as the penalized travel time, which combines actual travel duration with additional penalties related to various risk factors (e.g., proximity to schools, surface type, tunnels). This composite cost reflects both logistical and safety considerations.

The algorithm was run iteratively for each origin node (e.g., depots or hospitals) to compute the shortest path to all other destinations in the network. By doing so, we generated a complete cost matrix that captures the minimum-risk travel times between all relevant nodes. This cost matrix serves as a critical input to the VRP optimization process, guiding vehicle routing decisions by prioritizing routes with lower cumulative risk exposure and avoiding segments with higher penalties due to proximity to sensitive areas or hazardous conditions. This approach ensures that the final routing solution not only minimizes total travel time but also adheres to safety constraints, providing a risk-aware transport plan for hazardous waste collection and delivery.

An important consideration is that, in our model, the cost matrix is *asymmetric*, that is, the cost of traveling from node i to node j may differ from the cost of traveling from j to i . This asymmetry is not an error, but rather a desired and realistic characteristic of the paths found with the Dijkstra's algorithm in urban settings.

Each arc represents a road section characterized by a specific direction and weights (calculated through penalized travel time based on the factors mentioned above). In many urban and suburban context, roads are one-way or with preferential flows, making it impossible (or more expensive) to travel the same road in the opposite direction. Therefore, the asymmetry in the cost matrix reflects a more accurate modeling of real world mobility and allows our routing algorithm to generate logistically more feasible solutions, taking into account real travel conditions.

4.3 Our Approach with special Handling of Hazardous Waste of Types "B" and "D"

As previously mentioned, waste category "B" refers to chemical waste that is incompatible with other categories due to its toxicity, flammability, and corrosiveness. Similarly, type "D" waste is also incompatible with other categories, as it requires a different mode of transportation.

To reduce the computational complexity of the optimization problem, the model has been structured to operate on a day-by-day basis. Our approach consist of modeling and solving a separate VRP model for each day of the week and for each waste type group (ACE, B, and D).

Specifically, there will be:

- a dedicated VRP for "A", "C", and "E" waste (which can be grouped together) served by a fleet of suitable vehicles (3 vehicles, each with a capacity of 3200 kg);
- a dedicated VRP for type "D" waste, served by a refrigerated vehicle designed to transport this category (capacity 3900 kg);
- a dedicated VRP for type "B" waste, served by an appropriate vehicle (capacity 1600 kg).

To introduce variability and ensure the model's periodicity — that is, the linkage between different days of the week — an external operational logic is implemented for type B waste.

In particular, due to the relatively small quantity typically generated, for type **B** waste, collection does not need to occur daily, but only if the total demand exceeds a predetermined threshold. Therefore, an external trigger is essential to determine whether the VRP for type B is activated on any given day, based on the accumulated waste quantity.

Specifically, the routing problem for type B waste is activated only on days when the following condition is met:

$$\sum_{i \in V} q_i^{(B)}(t) + r_i^{(B)}(t) > \theta_B = 200 \text{ kg} \quad (8)$$

where:

- $q_i^{(B)}(t)$ is the quantity of type B waste produced at node i on day t ;
- $r_i^{(B)}(t)$ is the quantity of type B waste not collected in previous days and accumulated at node i ; This accumulated quantity is computed recursively based on the collection history. Specifically, if no collection occurs on day $t - 1$, then:

$$r_i^{(B)}(t) = r_i^{(B)}(t - 1) + q_i^{(B)}(t - 1)$$

Otherwise, if a collection is performed:

$$r_i^{(B)}(t) = 0$$

This recursive update ensures that the model accurately tracks the buildup of uncollected waste at each node, allowing the external rule to trigger collection only when necessary.

- θ_B is the minimum threshold (200 kg) required to trigger the collection.

If the condition is not met, the pickup is postponed and the entire quantity $q_i^{(B)}(t) + r_i^{(B)}(t)$ is accumulated for the following days. Conversely, if the total quantity of type "B" waste produced (including any accumulation from previous days) exceeds 200 kg, a collection vehicle is dispatched. This constraint is implemented as an *external rule* to the optimization model, acting at the decision-making level in the generation of the VRP problem for type B waste.

| nodes | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0 | 0 | 740.35 | 481.26 | 1875.63 | 1216.26 | 1371.77 | 1565.09 | 1386.27 | 1310.36 | 1199.73 | 1745.62 | 1620.66 | 1540.19 | 1321.12 | 2006.65 |
| 1 | 750.45 | 0 | 423.34 | 1154.02 | 1765.55 | 1666.02 | 2033.12 | 1680.51 | 1565.42 | 1667.76 | 1600.16 | 1522.33 | 1502.99 | 1175.65 | 2316.35 |
| 2 | 486.51 | 442.32 | 0 | 1584.52 | 1419.13 | 1319.60 | 1730.24 | 1334.09 | 1219.00 | 1364.88 | 1395.83 | 1343.84 | 1298.66 | 971.32 | 2155.63 |
| 3 | 1876.02 | 1125.57 | 1548.91 | 0 | 2771.09 | 2695.37 | 2935.47 | 2709.87 | 2657.06 | 2523.42 | 2003.45 | 1108.19 | 2010.43 | 1578.95 | 2889.70 |
| 4 | 1286.32 | 1722.75 | 1463.66 | 2787.65 | 0 | 1028.04 | 1932.90 | 678.49 | 1736.99 | 1746.65 | 2467.97 | 2514.26 | 2199.39 | 2043.46 | 2665.85 |
| 5 | 1310.64 | 1673.13 | 1305.84 | 2624.68 | 1022.02 | 0 | 1253.56 | 496.50 | 2168.08 | 1592.95 | 2282.71 | 2329.00 | 2185.54 | 1858.20 | 2962.54 |
| 6 | 1522.47 | 1919.97 | 1660.87 | 2904.10 | 2019.10 | 1338.31 | 0 | 1493.58 | 2580.18 | 638.96 | 2804.97 | 2729.90 | 2707.81 | 2380.47 | 3276.47 |
| 7 | 1325.35 | 1687.84 | 1320.55 | 2639.38 | 678.36 | 499.92 | 1404.77 | 0 | 2182.79 | 1473.63 | 2297.42 | 2343.71 | 2200.25 | 1872.91 | 2977.25 |
| 8 | 1237.49 | 1406.96 | 1039.67 | 2460.43 | 1710.84 | 1847.05 | 2414.05 | 1861.54 | 0 | 2048.69 | 2123.98 | 2170.27 | 1991.73 | 1699.48 | 1384.83 |
| 9 | 1165.81 | 1563.30 | 1304.21 | 2530.77 | 1674.86 | 1558.29 | 629.41 | 1501.01 | 2223.51 | 0 | 2448.30 | 2373.23 | 2351.14 | 2023.80 | 2919.80 |
| 10 | 1775.00 | 1701.05 | 1567.06 | 2070.48 | 2543.03 | 2355.82 | 2922.82 | 2370.31 | 2317.50 | 2557.46 | 0 | 1535.85 | 758.82 | 846.64 | 1815.61 |
| 11 | 1467.10 | 1453.52 | 1259.15 | 1066.19 | 2381.41 | 2280.04 | 2692.52 | 2294.53 | 2241.73 | 2327.16 | 1555.52 | 0 | 1562.50 | 1131.02 | 2441.77 |
| 12 | 1505.15 | 1582.42 | 1388.05 | 2050.94 | 2062.63 | 2246.16 | 2783.55 | 2260.65 | 2050.37 | 2418.19 | 784.18 | 1588.44 | 0 | 451.79 | 1313.42 |
| 13 | 1335.50 | 1261.55 | 1127.56 | 1630.99 | 2030.03 | 1916.32 | 2483.32 | 1930.81 | 1878.01 | 2117.96 | 827.03 | 1155.25 | 439.21 | 0 | 1731.05 |
| 14 | 2066.20 | 2313.04 | 2191.28 | 2909.72 | 2623.68 | 3009.92 | 3344.59 | 3024.41 | 1367.56 | 2979.24 | 1832.86 | 2518.91 | 1303.99 | 1735.06 | 0 |

Table 4: Cost-matrix between any pair of nodes i, j expressed in travel time (seconds).⁴

The final cost is obtained after applying all penalizations that apply to the road:

$$\text{cost} = \begin{cases} \text{travel time} + & \text{base travel time,} \\ \text{travel time} \times \sum \text{risk weight} + & \text{risk score based on road attributes} \\ \text{travel time} \times 0.5 + & \text{penalty if proximity to educational facilities} \\ \text{travel time} \times 0.5 & \text{penalty if proximity to green areas} \end{cases}$$

5 Results

The results gotten through this optimization problem offer a lot of insights about the strategic implications of hazardous healthcare waste collection in the province of Brescia. Through the aggregation of waste type into different categories and simplification for each day of the week, we can identify clear patterns that offer valuable insights into the behavior of the healthcare facility. These patterns can help in addressing future challenges related to the collection of special waste.

As shown from the table 5 the reported travel time for each waste type and day refers to the **total penalized time** traveled by all active vehicles, as calculated from the penalized cost matrix.

The model consistently allocates all the three vehicles available for waste types A, C, and E across all seven days, which reflects the high and steady demand across the 14 healthcare facilities. The highest total penalized travel time by all three vehicles is 4 hours and 39 minutes per day, representing also the highest traveling time with respect to the other types of waste. Shortest travel time for A, C and E is shown to be on weekends (4h 27m) which might reflect the decrease in hospital activity during weekends. The key difference between weekdays and weekends lies in the **routing for node 1** ('Ospedale Civile di Brescia'): during weekdays, it is served by a unique vehicle due to the high amount of waste produced, whereas on weekends, the amount of clinical waste generated by node 1 is lower, allowing the same vehicle to serve additional hospitals within its capacity.

Type D waste, which requires refrigerated vehicles due to sanitary and ethical handling requirements, also exhibits a uniform daily collection pattern. One vehicle is dispatched each day, and the total travel time consistently hovers around 4 hours and 2 minutes. This regularity suggests a well-distributed and predictable demand that is efficiently covered by a single vehicle.

In contrast, type B waste demonstrates a sporadic collection pattern, with vehicles dispatched only on Tuesday and Thursday. This outcome results from the model's accumulation rule, which postpones collection until the total volume exceeds 200 kg. The successful implementation of this conditional logic illustrates the model's ability to adapt to variable demand while maintaining regulatory compliance and operational efficiency.

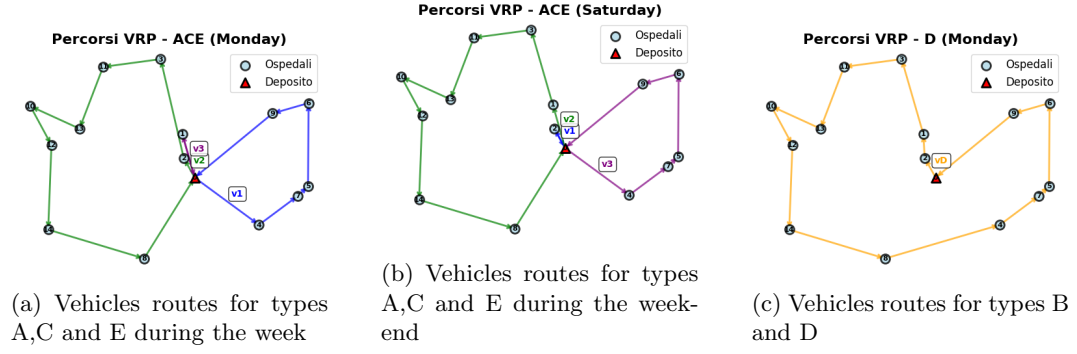
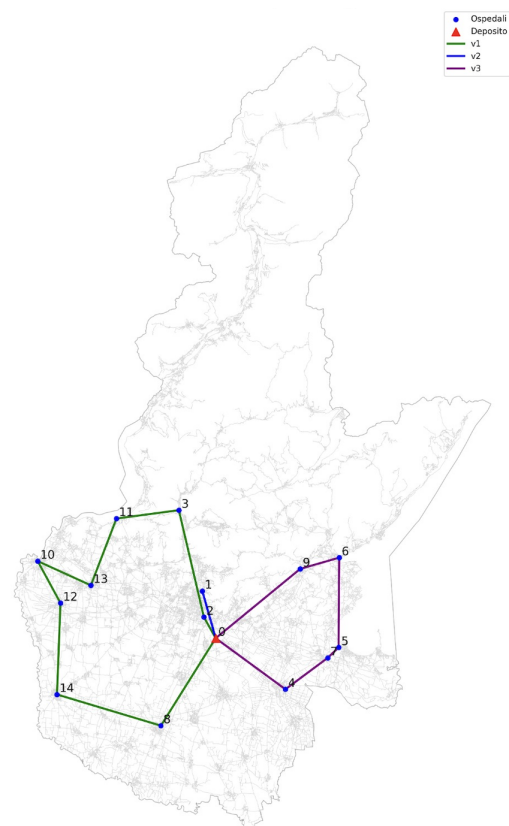


Figure 4: Road routes obtained from the results

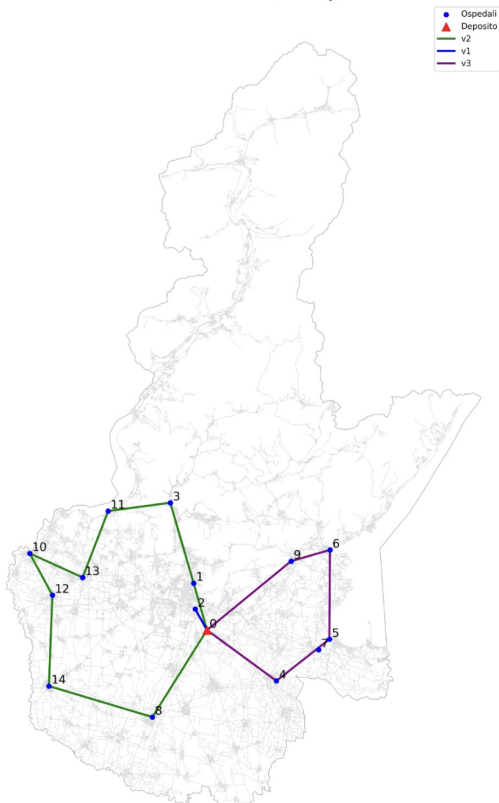
| Day | Waste Type | Vehicles Used | Travel Time |
|-----------|------------|---------------|-------------|
| Monday | ACE | 3 | 04h 39m |
| Monday | B | 0 | — |
| Monday | D | 1 | 04h 02m |
| Tuesday | ACE | 3 | 04h 39m |
| Tuesday | B | 1 | 04h 02m |
| Tuesday | D | 1 | 04h 02m |
| Wednesday | ACE | 3 | 04h 39m |
| Wednesday | B | 0 | — |
| Wednesday | D | 1 | 04h 02m |
| Thursday | ACE | 3 | 04h 39m |
| Thursday | B | 1 | 04h 02m |
| Thursday | D | 1 | 04h 02m |
| Friday | ACE | 3 | 04h 39m |
| Friday | B | 0 | — |
| Friday | D | 1 | 04h 02m |
| Saturday | ACE | 3 | 04h 27m |
| Saturday | B | 0 | — |
| Saturday | D | 1 | 04h 02m |
| Sunday | ACE | 3 | 04h 27m |
| Sunday | B | 0 | — |
| Sunday | D | 1 | 04h 02m |

Table 5: Detailed Summary of Waste Collection Solutions

Figure 5: Road routes with map background

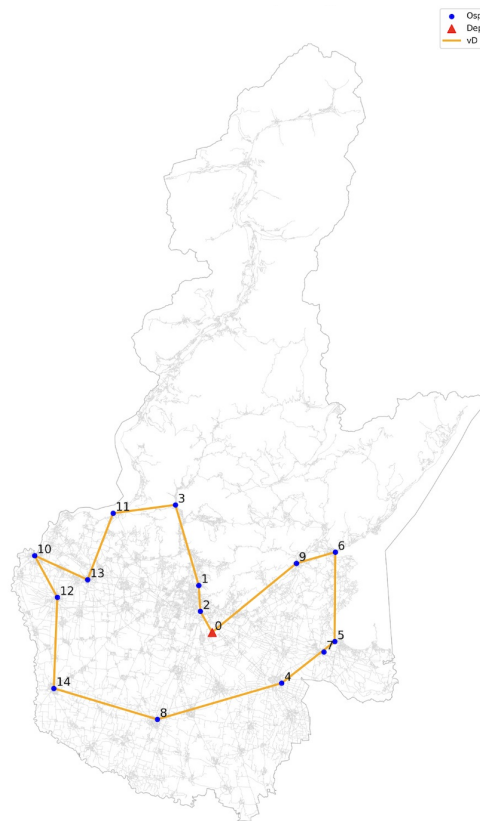


(a) Vehicles routes for types A,C and E during weekdays



(b) Vehicles routes for types A,C and E on weekends

Figure 6: Road routes for types B and D with map background



(a) Vehicles routes for types B and D

6 Limitations and Future Research

Despite the comprehensive nature of the proposed model and its operational feasibility, several limitations were identified that may affect both the accuracy and robustness of the results. These aspects offer valuable directions for future research and development.

Use of Simulated Data for Waste Estimation

One of the main limitations lies in the estimation of healthcare waste generation, which relied heavily on simulated data derived from regional averages and assumptions about hospital activity. While this approach allowed for a structured modeling process, it does not fully capture the real variability in waste production among different healthcare facilities. Future research should aim to integrate real-time or historical waste production data, ideally collected in partnership with healthcare institutions. This would allow for the validation and calibration of the simulation models, enhancing both precision and applicability.

Proximity Penalties Based on Simplified Geographical Metrics

The current methodology applies risk penalties to road segments within a 50-meter radius of schools and parks. However, these distances were calculated “as the crow flies”, ignoring the actual road network and accessibility. Furthermore, the penalization was applied uniformly across the entire road segment, regardless of the specific portion falling within the sensitive radius. This may lead to an overestimation of risk in some cases, reducing the model’s granularity and spatial accuracy. Future improvements should adopt network-based proximity calculations and restrict penalties only to the actual affected portions of the route.

Lack of Integration with Historical Accident and Traffic Data

While the model incorporates road attributes and static risk factors, it does not currently account for historical traffic incidents or real-time traffic conditions, which could significantly influence route risk and travel time. Future studies should explore the integration of traffic analytics and accident databases to create a dynamic and safety-aware cost matrix. This would allow for adaptive routing that responds to changing urban conditions.

Dependence on OpenStreetMap Data

The road network and related attributes used in this project were derived from OpenStreetMap. While this platform is widely accessible and rich in features, its crowd-sourced nature may lead to inconsistencies or missing data, particularly regarding road restrictions, surface quality, or tunnel details. Future research should consider leveraging more reliable and professionally curated datasets, possibly provided by local authorities or commercial providers, to improve the accuracy of the routing infrastructure.

Weekly Optimization Model and Dynamic Fleet Management

Another limitation of the current model is that it solves a separate routing problem for each day of the week independently. While this approach simplifies the computation and accounts for daily variability in waste production, it does not allow for cross-day optimization or the dynamic allocation of resources over the week. A promising direction for future research is the development of an integrated weekly model that considers the entire 7-day horizon in a unified framework. This would enable the optimization of vehicle dispatch schedules across the week, potentially reducing the total number of vehicles required or shifting vehicle availability in response to demand fluctuations. Such a model could better align collection frequency with actual production rates and minimize idle time or redundant trips, further improving the system's operational efficiency.

Potential for Real-Time Monitoring via IoT Technologies

To overcome the reliance on static or estimated input parameters, future implementations should consider deploying IoT sensors at healthcare facilities. These devices could enable real-time monitoring of waste production volumes and storage conditions, allowing for dynamic route optimization. In addition, the use of smart bins or containers with weight sensors would facilitate automated data collection, contributing to a more responsive and data-driven waste management system.

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