1. 导数的法则

特定公式（specific）： $f'(x) \qquad (f(x)=x^n, \frac{1}{x})$

通用法则（general）：

- 加法法则： (u+v)' = u'+v'
- 数乘法则： (Cu)' = Cu', 其中C为常数
- 乘积法则：(uv)' = u'v + uv'

$$\boxed{\begin{aligned}(uv)' &= \frac{\Delta(uv)}{\Delta x}\\ &= \frac{u(x+\Delta x)v(x+\Delta x)-u(x)v(x)}{\Delta x}\\ &= \frac{(u(x+\Delta x)-u(x))v(x+\Delta x)+u(x)(v(x+\Delta x)-v(x))}{\Delta x}\\ &= \frac{(\Delta u)v(x+\Delta x)+u(x)(\Delta v)}{\Delta x}\\ &= \frac{\Delta u}{\Delta x}v(x+\Delta x)+u(x)\frac{\Delta v}{\Delta x}\\ \Delta x\to 0 \Rightarrow \frac{du}{dx}v+u\frac{dv}{dx}\\ &= u'v+uv'\end{aligned}}$$

- 除法法则：$(\mathrm{u/v})' = \frac{\mathrm{u'v-uv'}}{\mathrm{v^2}}$

$$\boxed{\begin{aligned}\Delta(\frac{u}{v}) &= \frac{u+\Delta u}{v+\Delta v}-\frac{u}{v}\\ &= \frac{uv+(\Delta u)v-uv+u\Delta u}{(v+\Delta v)v}\\ &= \frac{(\Delta u)v+u\Delta u}{(v+\Delta v)v}\\ \therefore \frac{\Delta(\frac{u}{v})}{\Delta x} &= \frac{\frac{\Delta u}{\Delta x}v-u\frac{\Delta v}{\Delta x}}{(v+\Delta v)v}\\ \Delta x\to 0 \Rightarrow \frac{\frac{du}{dx}v-v\frac{dv}{dx}}{v^2} && \#\text{因为连续性，}\Delta x\text{趋向0时，}\Delta v\text{也趋向于0}\\ &= \frac{u'v-uv'}{v^2}\end{aligned}}$$

2. 求三角函数

正弦函数的导数：

$$
\begin{aligned}
\frac{d}{dx}\sin x &= \frac{\sin(x+\Delta x)-\sin x}{\Delta x} \\
&= \frac{\sin x\cdot\cos\Delta x+\cos x\cdot\sin\Delta x-\sin x}{\Delta x} && \#\text{根据两角和公式将sin展开} \\
&= \sin x(\frac{\cos\Delta x-1}{\Delta x})+\cos x(\frac{\sin\Delta x}{\Delta x}) && \#\text{用夹逼定理或洛必达公式求极限} \\
\Delta x\to 0\text{ 时} &= \sin x\cdot 0+\cos x\cdot 1 \\
&= \cos x
\end{aligned}
$$

> 正弦两角和公式:
> $\sin(a+b)=\sin a\cdot\cos b+\cos a\cdot\sin b$

余弦函数的导数:

$$
\begin{aligned}
\frac{d}{dx}\cos x &= \frac{\cos(x+\Delta x)-\cos x}{\Delta x} \\
&= \frac{\cos x\cdot\cos\Delta x-\sin x\cdot\sin\Delta x-\cos x}{\Delta x} \\
&= \cos x(\frac{\cos\Delta x-1}{\Delta x})-\sin x(\frac{\sin\Delta x}{\Delta x}) && \#\text{重组} \\
\\
\Delta x\to 0\text{ 时} &= \cos x\cdot 0-\sin x\cdot 1 \\
&= -\sin x
\end{aligned}
$$

> 余弦两角和公式:
> $\cos(a+b)=\cos a\cdot\cos b-\sin a\cdot\sin b$

3. 用导数法则来解题

$$
\begin{aligned}
&\frac{d}{dx}(x^n\sin(x)) \\
&=(x^n)'\sin(x)+x^n(\sin(x))' \\
&=nx^{n-1}\sin(x)+x^n\cos(x)
\end{aligned}
$$

$$
\frac{d}{dx}(\frac{1}{x^n})=\frac{0-1v'}{x^{2n}}=-x^{-2n}v'=-x^{-2n}\cdot nx^{n-1}=-nx^{-n-1}
$$