1.基本定理1

$$\boxed{If\ F'(x) = f(x), then\ \int_a^b f(x)dx = F(b) - F(a)}$$

> $F = \int f(x)dx$。

我们还可以用下面的notation来表示F(b)-F(a)：

$$\boxed{F(x)\Big|_a^b = F(x)\Big|_{x=a}^{x=b}}$$

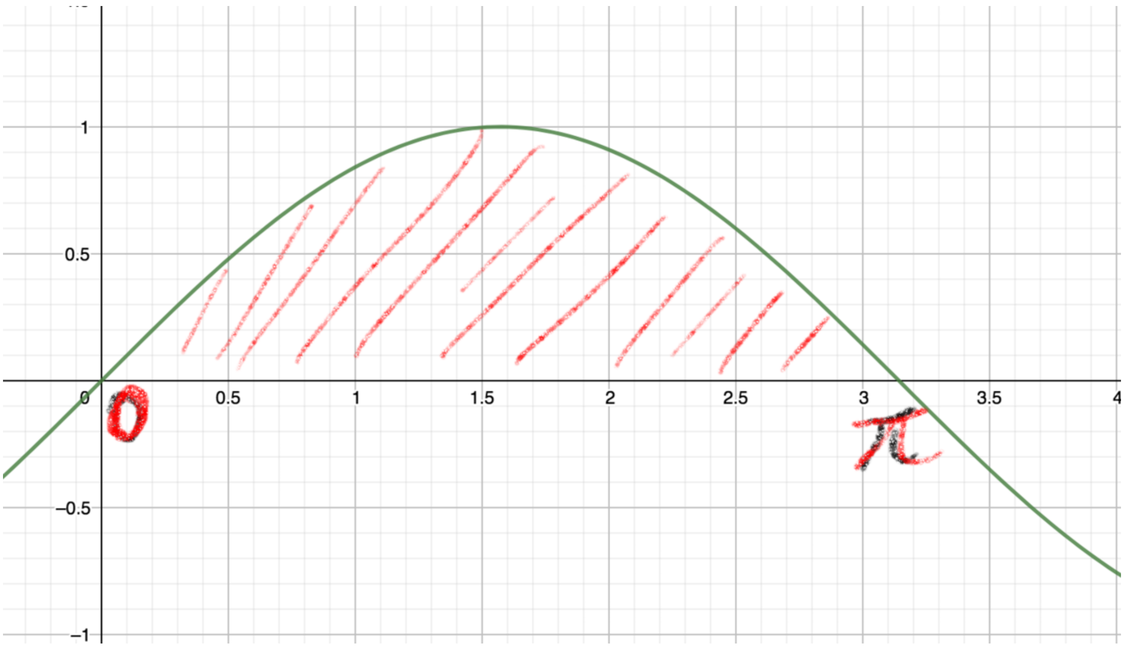
则基本定理1也可以表示为：

$$\boxed{\int_a^b f(x)dx = F(x)\Big|_a^b}$$

例1：$f(x) = x^2$

$$\boxed{\begin{aligned}&F'(x) = x^2\\&F(x) = x^3/3\\&\int_a^b x^2dx = \frac{x^3}{3}\Big|_a^b = \frac{b^3}{3} - \frac{a^3}{3}\\&if\ a = 0, \int_a^b x^2dx = \frac{b^3}{3}\end{aligned}}$$

例2：求$f(x) = \sin x$下红色阴影的面积：



$$\int_0^{\pi} \sin x \cdot dx = (-\cos x)\Big|_0^{\pi} = (-\cos\pi) - (-\cos 0) = -(-1) - (-1) = 2$$

例3：$$\int_0^1 x^{100} dx = \frac{x^{101}}{101}\Big|_0^1 = \frac{1}{101}$$

2. 基本定理1的直观解释

x(t)是t时刻的位置，$x'(t) = \frac{dx}{dt} = V(t)$是t时刻的速度。基本定理告诉我们：

$$\int_a^b \underbrace{V(t)dt}_{\text{速度}} = \underbrace{x(b) - x(a)}_{\text{运动的距离}}$$

我们知道：

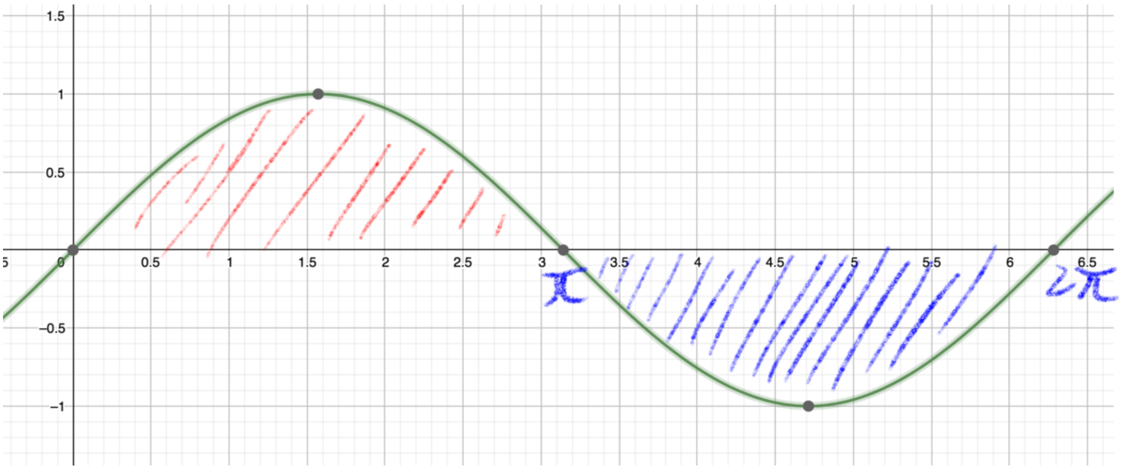
$$\int_a^b V(t)dt \approx \sum_{i=a}^{b} V(t_i)\Delta t$$

在时刻a到b之间的无限个Δt的时刻内的速度和即移动的距离。

3. 定积分的真正解释

例3：$\int_0^{2\pi} \sin x$

$$\boxed{\int_0^{2\pi} \sin x = (-\cos x)\Big|_0^{2\pi} = -\cos 2\pi - (-\cos 0) = -1 - (-1) = 0}$$



定积分的真正解释：它是x轴上方的面积，减去x轴下方的面积。就像上图展示的那样，

当曲线在x轴上方时，它的面积是正的；当曲线在x轴下方时，它的面积是负的。

4. 积分的性质

1. $$\boxed{\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx}$$
2. $$\boxed{\int_a^b cf(x)dx = c\int_a^b f(x)dx, (c\, is\, constant)}$$
3. $$\boxed{\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx}$$

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4. $\boxed{\int_a^a f(x)dx = 0}$

5. $\boxed{\int_a^b f(x)dx = -\int_b^a f(x)dx}$

6. 积分估计：$\boxed{If \;\; f(x) \le g(x), \;\; then \; \int_a^b f(x)dx \le \int_a^b g(x)dx}$

例子：$e^x \ge 1, x \ge 0$

$$\boxed{\begin{array}{l} \int_0^b e^x dx \ge \int_0^b 1dx, (b \ge 0) \\ \\ \int_0^b e^x dx = e^x \Big|_0^b = e^b - e^0 = e^b - 1 \\ \int_0^b 1dx = b \\ \\ e^b - 1 \ge b \Leftrightarrow e^b \ge 1 + b \end{array}}$$

7. 变量替换。$\boxed{\begin{array}{l} u = u(x), du = u'(x)dx \\ \int_{u1}^{u2} g(u)du = \int_{x1}^{x2} g(u(x))u'(x)dx \end{array}}$

例：

$$
\int_1^2 (x^3+2)^5 x^2 dx
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$$
u = x^3 + 2, du = 3x^2 dx
$$
$$
x^2 dx = \frac{du}{3}
$$
$$
u_1 = 1^3 + 2 = 3, u^2 = 2^3 + 2 = 10
$$

$$
\int_1^2 (x^3+2)^5 x^2 dx = \int_3^{10} u^5 \frac{1}{3} du = \frac{1}{18} u^6 \Big|_3^{10} = \frac{1}{18}(10^6 - 3^6)
$$

2. 基本定理2

第18课