Paper Replication:
"Forecasting the term structure of government bond yields"
Diebold and Li (2006)*

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Contents

*Diebold, Francis X., and Canlin Li. "Forecasting the term structure of government bond yields." Journal of econometrics 130.2 (2006): 337-364.

1 Introduction

In this essay I'm replicating the paper "Forecasting the term structure of government bond yields" by Diebold and Li (2006). This essay comes along with a detailed Jupiter notebook file that allows to replicate all the results in this essay and the complete dataset.

This paper adopts an out-of-sample forecasting approach, utilizing the Nelson and Siegel (Nelson, C.R., Siegel, A.F., 1987. Parsimonious modeling of yield curve. Journal of Business 60, 473–489) exponential components framework in order to model the term structure of bond yields. This method reduces the yield curve to a three-dimensional parameter that evolves over time, which can be interpreted as factors. Unlike traditional factor analysis that estimates both factors and loadings, this framework imposes structure on the factor loadings, enhancing the precision of factor estimation. The authors interpret these factors as level, slope, and curvature, propose autoregressive models for them, and use these models to forecast the yield curve.
Their findings indicate that these models provide one-year-ahead forecasts that are significantly more accurate than traditional benchmarks.

2 Theoretical model

Before going into the estimation of the model and the forecast, we need to define some theoretical objects: the discount curve, the forward curve and the yield curve.
Let $P_t(\tau)$ be the price of a τ-period discount bond, which is the present value at time t of 1$ receivable τ periods ahead. Let $y_t(\tau)$ denote its continuously compounded zero-coupon nominal yield to maturity. We define the discount curve as:

$$P_t(\tau) = e^{-\tau y_t(\tau)}$$

and from this curve we can obtain the instantaneous nominal forward rate curve, defined as:

$$f_t(\tau) = -\frac{P_t'(\tau)}{P_t(\tau)}$$

Then the relation between the yield and the forward rate is:

$$y_t(\tau) = \frac{1}{\tau}\int_0^\tau f_t(u)du$$

therefore the zero-coupon yield is an average of forward rates.

The problem is that in practice the discount curve, the forward curve and the yield curve are observed and therefore they must be estimated from the bond prices.
In this paper the authors use the method due to Fama and Bliss to construct the yields (for further details Fama, E., Bliss, R., 1987. The information in long-maturity forward rates. American Economic Review 77, 680–692.) and obtain a set of "unsmoothed Fama-Bliss yields".

2.1 The Nelson-Siegel yield curve

The functional form proposed by Nelson and Siegel (1987) is a parsimonious three component exponential approximation. The forward rate curve is:

$$f_t(\tau) = \beta_{1t} + \beta_{2t}e^{-\lambda_t\tau} + \beta_{3t}\lambda_t e^{-\lambda_t\tau}$$

and the corresponding yield curve is:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \tag{1}$$

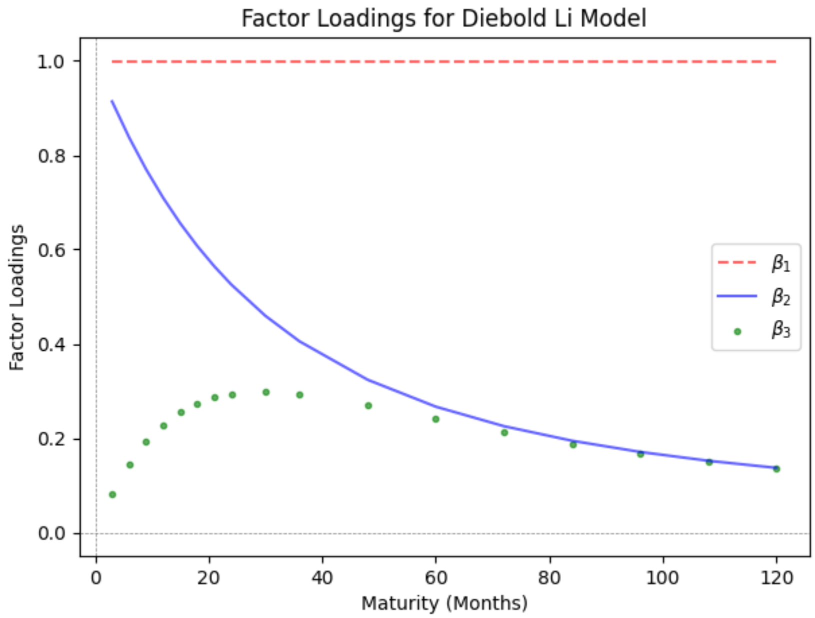

Figure 1:

The method adopted in the original paper and in this replication is the following: for any point in time t, we have a set of yields of different maturities, to which we fit the Nelson-Siegel yield curve. Notice that this curve is a discount curve that begins at one when $\tau = 0$ and approaches zero as $\tau \to \infty$.
In Equation 1 the coefficients are interpreted as three latent dynamic factors while the regressors are the factor loadings. The parameter λ_t governs the rate of the exponential decay: the higher is the value of λ_t the faster is the decay.
The factor loading on β_{1t} is 1, on β_{2t} starts at 1 but decays quickly to 0 and the loading on β_{3t} starts at 0, increases and the decreases to 0 again. The betas can therefore be interpreted respectively as long-term, short-term and medium-term factors. This is clear by looking at Figure 1.
In particular the three factors can also be interpreted as level, slope and curvature of the yield curve respectively, by noting that:

- level: $y_t(\infty) = \beta_{1t}$
- slope : $y_t(120) - y_t(3) = -0.78\beta_{2t} + 0.06\beta_{3t}$
- curvature : $2y_t(24) - y_t(3) - y_t(120) = 0.00053\beta_{2t} + 0.37\beta_{3t}$

Note also that the functional form that is adopted is in principle consistent with some characteristics that the yield curve must respect in order to represent historical facts:

1. Average yield curve increasing and concave
2. Yield curve can assume different shapes over time.
3. Stronger persistence of yield dynamics with respect to spread dynamics
4. Short term more volatile then the long end
5. Stronger persistence of long rates than short rates

3 Estimation and forecast

In this section we'll see the estimation of the yield curve and then we will perform an out-of-sample forecast by means of different models and compare the forecast statistics of these models to the one obtained with the Nelson-Siegel exponential approximation.

3.1 The Data

The data consists in end of month unsmoothed Fama-Bliss zero yields from January 1985 to December 2000. For each date (month), there are 17 observed yield, one for each maturity: 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, 120 months. In Figure 2 there's a 3D plot of the yield curve

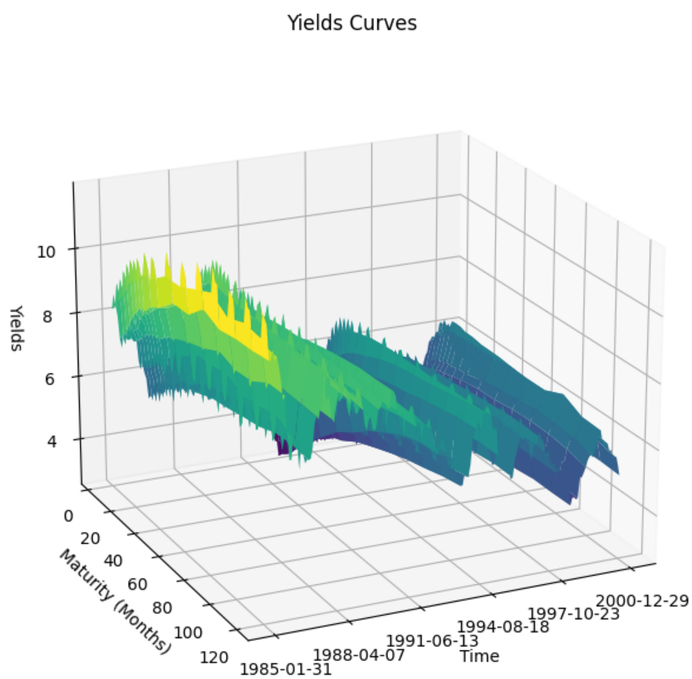

Figure 2: 3D plot of the data

data where it is possible to see the variation in terms of level slope and curvature.
In Table 1 are reported the descriptive statistics for the yields. As we can see the long rates are less volatile and more persistent, and the typical yield curve is upward sloping. The level is highly persistent but doesn't vary a lot with respect to its mean while the slope is less persistent but varies moderately. Curvature is the least persistent factor but it's the one that varies the most with respect to its mean.
In figure 3 there is the plot of the median yield curve along with pointwise interquartile ranges. It's clear that the median curve is upward sloping and is less volatile in the short-term than in the long-term.

	Mean	Std.dev	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	5.630	1.488	2.732	9.131	0.978	0.569	-0.079
6	5.785	1.482	2.891	9.324	0.976	0.555	-0.042
9	5.907	1.492	2.984	9.343	0.973	0.545	-0.005
12	6.067	1.501	3.107	9.683	0.969	0.539	0.021
15	6.225	1.504	3.288	9.988	0.968	0.527	0.060
18	6.308	1.496	3.482	10.188	0.965	0.513	0.089
21	6.375	1.484	3.638	10.274	0.963	0.502	0.115
24	6.401	1.464	3.777	10.413	0.960	0.481	0.133
30	6.550	1.462	4.043	10.748	0.957	0.479	0.190
36	6.644	1.439	4.204	10.787	0.956	0.471	0.226
48	6.838	1.439	4.308	11.269	0.951	0.457	0.294
60	6.928	1.430	4.347	11.313	0.951	0.464	0.336
72	7.082	1.457	4.384	11.653	0.953	0.454	0.372
84	7.142	1.425	4.352	11.841	0.948	0.448	0.391
96	7.228	1.413	4.433	11.512	0.953	0.467	0.416
108	7.270	1.428	4.429	11.664	0.953	0.475	0.426
120 (level)	7.254	1.432	4.443	11.663	0.953	0.467	0.428
Slope	1.624	1.213	-0.752	4.060	0.961	0.405	-0.049
Curvature	-0.081	0.648	-1.837	1.602	0.896	0.337	-0.015

Table 1: Descriptive statistics of data. The authors defined the level as the 120 months yields, slope as the difference between the 120 motnhs and the 3 months yield and the curvature as two times the 24 months yield minus the 3 months and the 120 months.

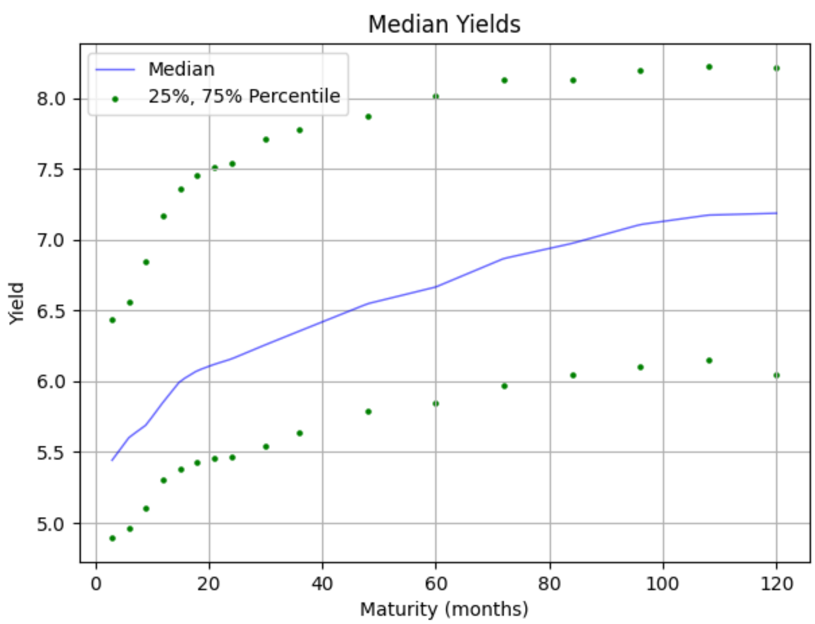

Figure 3: Median yield curve with interquartile ranges (25% and 75% percentile)

3.2 Estimation

The three factors curve to fit is Equation 1. Instead of estimating all the parameters $\theta = \{\beta_{1t}, \beta_{2t}, \beta_{3t}, \lambda_t\}$, the authors set $\lambda_t = 0.0609$, which is the value that maximizes the loading on the medium-term factor at a maturity of exactly 30 months.
This assumption on the level of λ_t allows us to compute the values of the two regressors and estimate the betas by means of a simple ordinary least squares regression for each month t. This let us to obtain a time series of estimates of $\{\beta_{1t}, \beta_{2t}, \beta_{3t}\}$ and the corresponding residuals.
In Figure 4 we can see that the fitted curve for the mean yields of each maturity agree quite well with the true means.

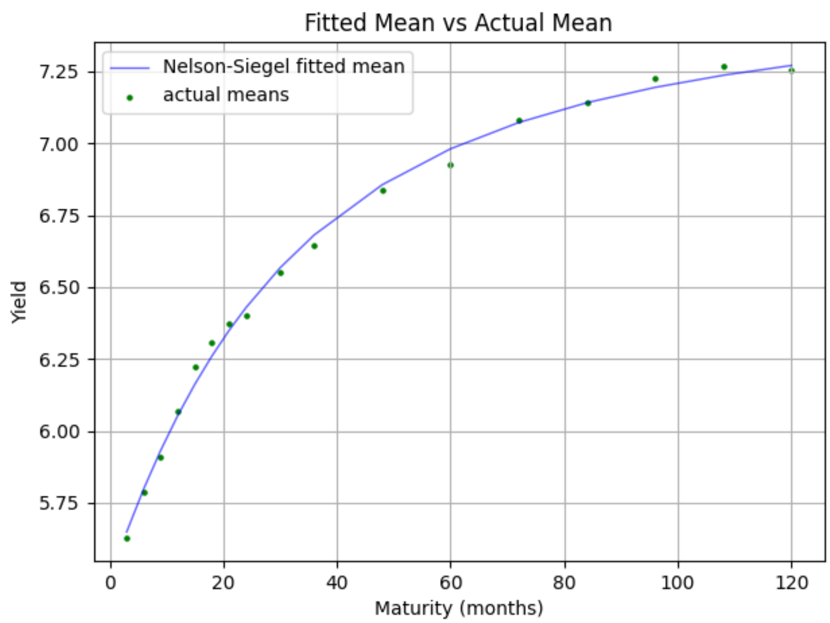

Figure 4: Actual and fitted average yields curve

In figure Figure 5 there 4 plots of the Nelson-Siegel fitted curve against the true data for some selected dates. We can see that this model allows for yield curves of any shape: upward sloping, downward sloping, humped and inverted humped. The model can have some problems when there are multiple interior minima and maxima (bottom-right plot of Figure 5).
In Table 2 there are the descriptive statistics of residuals from the estimation o the betas (factors). Notice that the residual autocorrelations inidctaes that pricing errors are persistent. Regardless of the method used for the estimation of the term structure, some discrepancy between the actual and the estimated bond prices will arise. According to the authors this discrepancy are due to persistent tax and/or liquidity effects. The good news is that since they persist, they should be absent in fitted yield changes.

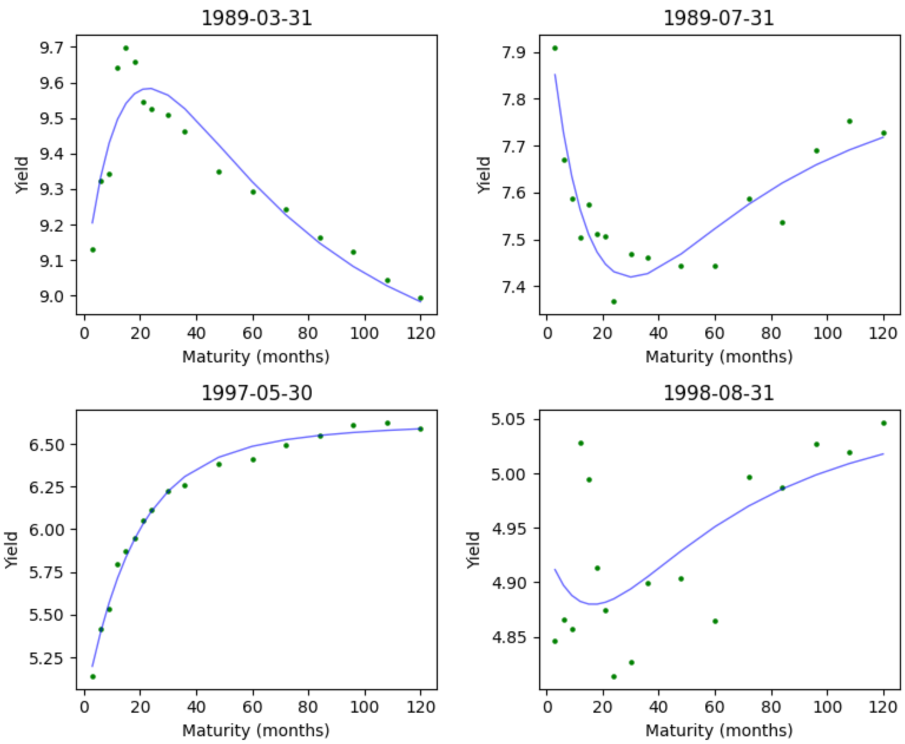

Figure 5: Fitted yield curves for selected dates (blue line) together with the actual yields (green dots)

	Mean	Std.Dev	Min	Max	MAE	RMSE	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	-0.018	0.080	-0.332	0.156	0.061	0.082	0.778	0.157	-0.360
6	-0.013	0.042	-0.141	0.218	0.032	0.044	0.290	0.257	-0.046
9	-0.026	0.062	-0.201	0.218	0.052	0.067	0.704	0.216	-0.247
12	0.013	0.080	-0.160	0.267	0.064	0.081	0.562	0.321	-0.265
15	0.063	0.050	-0.063	0.242	0.067	0.080	0.650	0.139	-0.069
18	0.048	0.035	-0.048	0.165	0.052	0.059	0.494	0.183	-0.139
21	0.026	0.030	-0.091	0.101	0.033	0.039	0.369	-0.044	-0.011
24	-0.027	0.045	-0.190	0.082	0.037	0.053	0.667	0.211	0.056
30	-0.017	0.036	-0.200	0.098	0.029	0.039	0.398	0.072	-0.058
36	-0.037	0.046	-0.203	0.128	0.047	0.059	0.598	0.052	-0.017
48	-0.018	0.065	-0.204	0.230	0.053	0.067	0.753	0.238	-0.319
60	-0.053	0.058	-0.199	0.186	0.066	0.078	0.755	-0.019	-0.177
72	0.010	0.080	-0.134	0.399	0.056	0.081	0.900	0.277	-0.162
84	0.001	0.062	-0.259	0.263	0.044	0.061	0.581	0.016	-0.000
96	0.033	0.048	-0.202	0.251	0.047	0.058	0.635	0.131	-0.120
108	0.033	0.046	-0.161	0.132	0.047	0.057	0.664	0.087	-0.173
120	-0.017	0.071	-0.256	0.164	0.057	0.073	0.633	0.254	-0.068

Table 2: Residuals descriptive statistics

In Figure 6 I plotted the time series for $\{\beta_{1t}, \beta_{2t}, \beta_{3t}\}$ together with the level, slope and curvature retrieved from the data as defined before (level defined as $y_t(120)$).
It's clear that the three factors in this model have the interpretation of level slope and curvature.

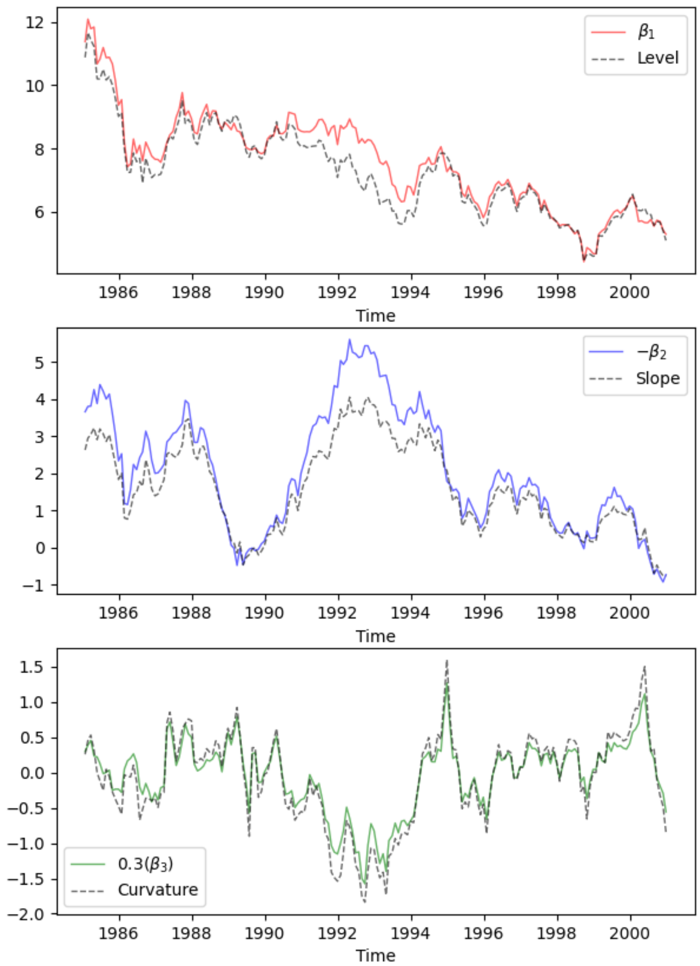

Figure 6: Estimated level, slope and curvature against empirical level slope and curvature.

Furthermore the correlation between the factors and the empirical counterparts (l_t, s_t, c_t) are:

$\rho(\hat{\beta}_{1t}, l_t)$	0.970
$\rho(\hat{\beta}_{2t}, s_t)$	-0.990
$\rho(\hat{\beta}_{3t}, c_t)$	0.990

In Table 3 and in the left part of Figure 7 (left column) there are the descriptive statistics of the estimated betas (factors). The firts factor is the more persistent and the second factor is more persistent than the third: this is perfectly in line with the iterpretation of the factors. Moreover The augmented Dick-Fuller test (ADF) suggests that the first two facotrs have unit roots while the third doesn't.

	Mean	Std.Dev	Min	Max	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{\beta}_{1t}$	7.580	1.520	4.427	12.089	0.957	0.511	0.454	-2.133
$\hat{\beta}_{2t}$	-2.099	1.604	-5.616	0.919	0.969	0.452	-0.082	-1.338
$\hat{\beta}_{3t}$	-0.164	1.681	-5.251	4.233	0.901	0.354	-0.007	-3.542

Table 3: Descriptive statistics of estimated factors along with autocorrelation at displacemt of 1, 12 and 30 months and ADF test.

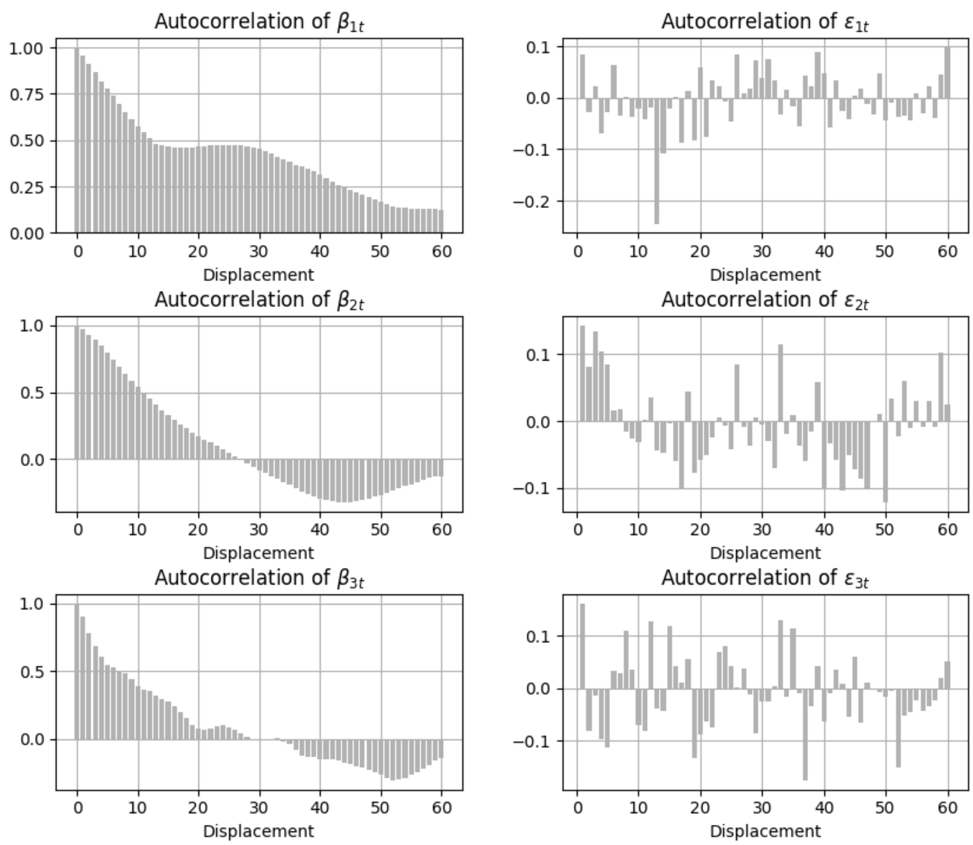

Figure 7: Left: autocorrelations of estimated factors, Right: residual autocorrelations of AR(1) model fitted to the estimated factors ($\hat{\rho}(0)$ omitted).

3.3 Forecast

In order to perfomr the forecast, the Nelson-Siegel factors are modeled as AR(1) processes. The forecasts based on the AR(1) model are:

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \hat{\beta}_{3,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

with:

$$\hat{\beta}_{i,t+h/t} = \hat{c}_i + \hat{\gamma}_i\hat{\beta}_{it} \qquad i = 1, 2, 3$$

where $\hat{c}_i$ and $\hat{\gamma}_i$ are obatined by regressing $\hat{\beta}_{it}$ on an intercept and on $\hat{\beta}_{i,t-h}$.

The authors also specify a forecast based on VAR(1) model for completeness, even though, since the variables are not highly correlated and show little cross factor interaction, we expect a multivariate model to perform more poorly the a stacked set of univariate models. The forecast based on VAR(1) model is:

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \hat{\beta}_{3,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

with:

$$\hat{\beta}_{t+h/t} = \hat{c} + \hat{\Gamma}\hat{\beta}_t$$

In particular, we can see in Figure 7 (right column) that the residuals autocorrelations are small enough to provide some evidence on the goodness of fit of the AR(1) models for level slope and curvature factors.

3.3.1 Out-of-sample forecast

In this subsection I perform a h-period-ahead out-of-sample forecast for the Nelson-Siegel three factor model. In particular, since the yield of the fitted curve depends only on the three factors, the forecast is performed on $\left\{\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\beta}_{3t}\right\}$.
The method of forecast is the following: starting form 1985:1 to 1993:12, I estimate the factors and perform an h-period-ahead forecast, then include one more observation in the sample, compute the forecast and repeat. In other words the forecast is performed recursively. The forecast horizons are $h = 1, 6, 12$.
The forecasts are computed with different models in order to be able to compare the Nelson-Siegel three factors model with other (simple) competitors[1]. The maturities for which the forecast is performed are 3, 12, 36, 60 and 120 months.
The models that I used to perform the recursive forecast are:

1. AR(1) on Nelson-Siegel factors (specified above)
2. VAR(1) on Nelson-SIegel factors (specified above)
3. AR(1) directly on bond yields:
$$\hat{y}_{t+h/t}(\tau) = \hat{c}(\tau) + \hat{\gamma}y_t(\tau)$$
4. VAR(1) directly on bond yields:
$$\hat{y}_{t+h/t}(\tau) = \hat{c} + \hat{\Gamma}y_t$$
where: $y_t = [y_t(3), y_t(12), y_t(36), y_t(60), y_t(120)]'$

[1] In the original paper the authors present 10 competitors, in this essay I will present only 3 of them

5. random walk:

$$\hat{y}_{t+h/t}(\tau) = y_t(\tau)$$

The forecast is always "no change".

In Table 4-5-6 there are the descriptive statistics of the forecast errors of 1-,6- and 12-period-ahead forecast respectively [2].
For each forecast horizon $h = 1, 6, 12$ the forecast error at $t + h$ is defined as:

$$y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$$

The tables report the mean and the standard deviation of the forecast error for each maturity $\tau = 3, 12, 36, 60, 120$ months, along with the root mean square error (RMSE) and autocorrelations $\hat{\rho}(h)$ and $\hat{\rho}(h + 12)$.
As we can see in Table 4, the model proposed by the authors (i.e. the Nelson-Siegel three factors model with AR(1) factor dynamics) performs only slightly better than other competitors in the 1-period-ahead forecast in terms of RMSE. Moreover the forecast errors appear to be serially correlated. In the opinion of Diebold and Li, the weak performance of the 1-period-ahead forecast comes from a variety of sources like, for example, the high persistency of the pricing errors due to illiquidity.
Looking at Table 5 we notice that the performance of the model proposed by the authors improves for the 6-periods-ahead forecast.
Moreover in the forecast performed 12-periods-ahead reported in Table 6, we notice a strong superiority of the Nelson-Siegel model in terms of RMSE.

In Table 7 I reported the Diebold-Mariano forecast accuracy comparison test statistics for the Nelson-Siegel model vs the Random walk and the direct AR(1) on yield levels along with p-values[3]. This test compares the RMSE. The null hypothesis is that of equal mean square error between two forecast models. Negative values of the statistics indicate superiority of the Nelson-Siegel three factors model.
As we can see in Table 7 the Diebold-Mariano test report universal insignificance of the RMSE differences between the model proposed by the authors and the random walk and the direct AR(1) on yield levels. On the other hand, for the 12-period-ahead forecast, we can see that the test suggests superiority of the forecast performed with the three factors model with respect to random walk and direct AR(1). In particular, 6 out of 10 of the test statistics indicate significant superiority of forecast accuracy of the three factors model in the 12-periods ahead forecast.

[2] Note that the numbers reported in Table 4-5-6 in this essay are slightly different from the numbers reported in the original paper even if the dataset is exactly the same. These differences can be due to the use of different software or some little discrepancy in the implementation of the code that performs the forecast. However both the signs and the magnitude are consistent.

[3] In the original paper, the Diebold-Marino test is computed for the three factors model vs Random walk and another model that I don't present in this essay. For this reason I computed the test vs the Random walk and the direct AR(1) on yield levels.

1 period ahead forecast

Maturity (τ)	Mean	Std	RMSE	$\hat{\rho}(1)$	$\hat{\rho}(12)$
Nelson-Siegel with AR(1) factor dynamics					
3 months	-0.009	0.162	0.162	0.189	0.042
1 year	0.054	0.226	0.233	0.387	-0.209
3 years	-0.032	0.268	0.270	0.311	-0.112
5 years	-0.069	0.273	0.282	0.308	-0.113
10 years	-0.041	0.250	0.253	0.235	-0.115
Nelson-Siegel with VAR(1) factor dynamics					
3 months	-0.068	0.165	0.178	0.200	0.092
1 year	-0.005	0.229	0.229	0.348	-0.171
3 years	-0.086	0.280	0.292	0.329	-0.111
5 years	-0.118	0.280	0.304	0.325	-0.103
10 years	-0.086	0.249	0.263	0.229	-0.109
Univariate AR(1)s on yields levels					
3 months	0.019	0.174	0.175	0.214	0.058
1 year	0.001	0.236	0.236	0.334	-0.147
3 years	-0.021	0.275	0.276	0.340	-0.128
5 years	-0.035	0.273	0.276	0.274	-0.129
10 years	-0.047	0.252	0.256	0.214	-0.143
VAR(1) on yield levels					
3 months	-0.019	0.177	0.178	0.249	0.122
1 year	-0.026	0.260	0.262	0.448	-0.162
3 years	-0.042	0.300	0.303	0.438	-0.154
5 years	-0.063	0.302	0.308	0.428	-0.132
10 years	-0.088	0.272	0.286	0.307	-0.123
Random walk					
3 months	0.033	0.176	0.179	0.220	0.053
1 year	0.021	0.239	0.240	0.340	-0.153
3 years	0.007	0.277	0.277	0.341	-0.133
5 years	-0.003	0.275	0.275	0.275	-0.131
10 years	-0.011	0.253	0.253	0.215	-0.145

Table 4: 1 period ahead forecast errors descriptive statistics

6 periods ahead forecast

Maturity (τ)	Mean	Std	RMSE	$\hat{\rho}(6)$	$\hat{\rho}(18)$
Nelson-Siegel with AR(1) factor dynamics					
3 months	0.119	0.499	0.513	0.330	-0.205
1 year	0.142	0.656	0.671	0.117	-0.158
3 years	-0.047	0.745	0.747	-0.038	-0.202
5 years	-0.157	0.757	0.773	-0.018	-0.231
10 years	-0.221	0.674	0.709	-0.060	-0.239
Nelson-Siegel with VAR(1) factor dynamics					
3 months	-0.167	0.565	0.589	0.149	-0.088
1 year	-0.132	0.783	0.794	0.018	-0.116
3 years	-0.289	0.863	0.910	-0.049	-0.183
5 years	-0.377	0.846	0.926	-0.018	-0.218
10 years	-0.414	0.729	0.839	-0.036	-0.235
Univariate AR(1)s on yields levels					
3 months	-0.167	0.565	0.589	0.149	-0.088
1 year	-0.132	0.783	0.794	0.018	-0.116
3 years	-0.289	0.863	0.910	-0.049	-0.183
5 years	-0.377	0.846	0.926	-0.018	-0.218
10 years	-0.414	0.729	0.839	-0.036	-0.235
VAR(1) on yield levels					
3 months	-0.054	0.619	0.622	0.243	-0.129
1 year	-0.092	0.855	0.860	0.102	-0.133
3 years	-0.206	0.914	0.937	0.045	-0.186
5 years	-0.295	0.911	0.958	0.071	-0.209
10 years	-0.426	0.788	0.896	0.052	-0.219
Random walk					
3 months	0.198	0.563	0.597	0.322	-0.198
1 year	0.129	0.732	0.743	0.034	-0.113
3 years	0.032	0.833	0.833	-0.085	-0.166
5 years	-0.018	0.821	0.821	-0.086	-0.206
10 years	-0.076	0.726	0.730	-0.069	-0.226

Table 5: 6 periods ahead forecast errors descriptive statistics

12 periods ahead forecast

Maturity (τ)	Mean	Std	RMSE	$\hat{\rho}(12)$	$\hat{\rho}(24)$
Nelson-Siegel with AR(1) factor dynamics					
3 months	0.139	0.789	0.801	-0.201	-0.076
1 year	0.105	0.884	0.891	-0.274	-0.036
3 years	-0.180	0.949	0.966	-0.365	-0.021
5 years	-0.358	0.969	1.033	-0.370	-0.039
10 years	-0.505	0.881	1.016	-0.421	-0.060
Nelson-Siegel with VAR(1) factor dynamics					
3 months	-0.231	0.976	1.003	-0.298	-0.036
1 year	-0.249	1.098	1.126	-0.383	0.001
3 years	-0.491	1.105	1.209	-0.437	0.012
5 years	-0.640	1.072	1.248	-0.426	-0.011
10 years	-0.754	0.947	1.210	-0.441	-0.041
Univariate AR(1)s on yields levels					
3 months	0.120	0.858	0.867	-0.230	-0.063
1 year	-0.069	0.943	0.945	-0.354	-0.008
3 years	-0.308	1.015	1.061	-0.444	0.013
5 years	-0.441	1.011	1.103	-0.461	-0.007
10 years	-0.594	0.922	1.097	-0.443	-0.050
VAR(1) on yield levels					
3 months	-0.106	1.022	1.028	-0.263	-0.046
1 year	-0.202	1.160	1.177	-0.332	-0.014
3 years	-0.423	1.147	1.223	-0.369	-0.005
5 years	-0.576	1.121	1.260	-0.355	-0.020
10 years	-0.789	0.979	1.257	-0.373	-0.050
Random walk					
3 months	0.292	0.892	0.938	-0.240	-0.055
1 year	0.177	1.004	1.020	-0.372	0.004
3 years	0.012	1.078	1.078	-0.460	0.024
5 years	-0.075	1.070	1.072	-0.477	0.006
10 years	-0.198	0.965	0.985	-0.467	-0.029

Table 6: 12 periods ahead forecast errors descriptive statistics

Diebold-Mariano test statistics

Maturity (τ)	1-period-ahead N-S AR(1) vs RW	p-value	12-periods-ahead N-S AR(1) vs RW	p-value
3 months	-1.611	0.111	-2.289*	0.025
1 year	-0.812	0.419	-2.196*	0.031
3 years	-1.078	0.284	-2.093*	0.040
5 years	0.671	0.504	-0.940	0.351
10 years	0.034	0.973	0.954	0.343
	1-period-ahead N-S AR(1) vs AR(1)	p-value	12-periods-ahead N-S AR(1) vs AR(1)	p-value
3 months	-1.437	0.154	-1.541	0.128
1 year	-0.388	0.699	-1.334	0.186
3 years	-1.394	0.167	-2.710*	0.008
5 years	0.892	0.375	-3.385*	0.001
10 years	-0.432	0.667	-5.605*	0.000

Table 7: Diebold-Mariano test on forecast accuracy for Nelson-Siegel with AR(1) factor dynamics vs Random walk and direct AR(1) on yield levels.

4 Conclusion

In this essay, which replicates (although not completely) the procedure and the result of Diebold and Li (2005), we have reinterpreted the Nelson-Siegel yield curve as a dynamic three factor model. The factors that we have identified have the solid interpretation of level slope and curvature of the yield curve. The main advantage of this model is that it is very parsimonious.
Then we have tested the model accuracy of forecast by performing a h-step-ahead out-of-sample forecast and comparing the result with those of other competitors. Even if for what concerns 1-month-ahead forecast the model is neither better nor worse than competitors, the 1-year-ahead forecast seems promising.