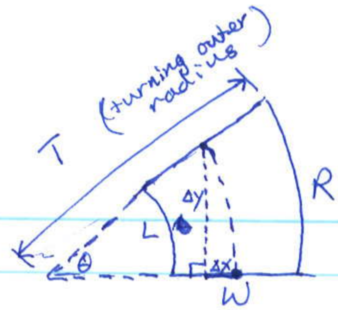


By similar figures,

$$\frac{T - W}{T} = \frac{L}{R}$$

$$RT - RW = LT$$

$$(R-L)T = RW$$

$$T = \frac{RW}{R-L}$$

$$\boxed{\theta = \frac{R}{T} = \frac{R-L}{W}}$$

Turning radius at the center is $T - \frac{W}{2} = \frac{2RW - W(R-L)}{2(R-L)} = \frac{W(R+L)}{2(R-L)}$

$$= \frac{R+L}{2\theta}$$

$$\boxed{\begin{aligned} \Delta y &= \left(\frac{R+L}{2\theta}\right)\sin\theta \\ \Delta x &= -\left(\frac{R+L}{2\theta}\right)(1-\cos\theta) \end{aligned}}$$

However, one must be careful about numerical stability as $\theta \to 0$.
Luckily, calculus helps us with taylor expansions.

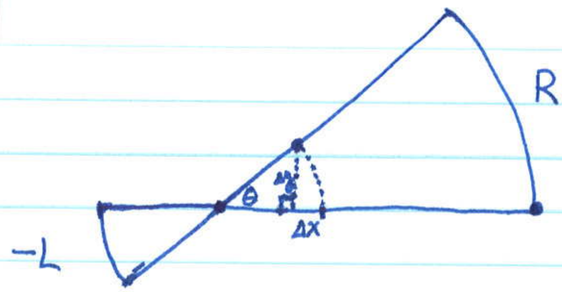
$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$\Delta y \approx \left(\frac{R+L}{2\theta}\right)\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right) = \left(\frac{R+L}{2}\right)\left(1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \cdots\right)$$

$$\Delta x = -\left(\frac{R+L}{2\theta}\right)\left(1 - 1 + \frac{\theta^2}{2!} - \frac{\theta^4}{4!} + \cdots\right) = -\left(\frac{R+L}{2}\right)\left(\frac{\theta}{2!} - \frac{\theta^3}{4!} + \cdots\right)$$

What if $R > 0 > L$? Let W = length of axle (same as before)



Then the center of rotation is $\frac{RW}{R+(-L)}$ from the right wheel.

So, $$\boxed{\theta = \frac{R}{\frac{RW}{R+(-L)}} = \frac{R-L}{W}}$$ (still works!)

T_c, turning radius of center of axle is:

$$\frac{RW}{R+(-L)} - \frac{W}{2} = \frac{2RW - W(R-L)}{2(R-L)} = \frac{W(R+L)}{2(R-L)} = \frac{R+L}{2\theta}.$$

$$\boxed{\begin{aligned} \Delta y &= \left(\frac{R+L}{2\theta}\right)\sin\theta = \ldots \quad \text{taylor expansion} \\ \Delta x &= \left(\frac{R+L}{2\theta}\right)(\cos\theta - 1) = \ldots \quad \text{''} \end{aligned}}$$

Same formula as before!

If $|L| > |R|$, then flip the sign on Δx? Apparently not!