Cyber Planner Code Report v0.1.0 (C++ Version)

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***Abstract*—The code report is abstract enough that no additional abstract is needed**

I. Introduction

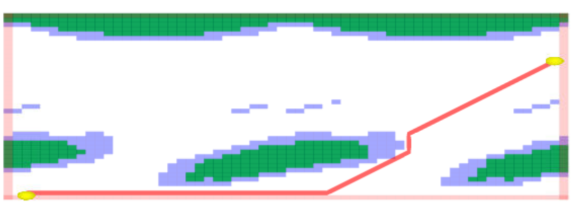
WHAT can I say? What can I say? What can I say? What can I say? What can I say?

Fujun Ruan
January 31, 2025

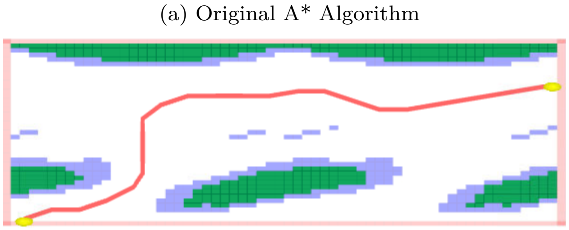
II. Path Planning

A. Improved A* Algorithm

As Image 1a shown, the original A* algorithm may always find a path that very close to the obstacles. This is often led by the heuristic function. However, if any errors occured in estimate or control, the robot may collide with the obstacles. To avoid this, we introduce artificial potential field to the cost function.



(a) Original A* Algorithm



(b) Improved A* Algorithm

Fig. 1: Difference between original and improved A* algorithm

Every grid in the map has a potential value. The potential value is calculated by the distance to the obstacles in a dynamic programming way. It can find us a heuristic path, but it is still discrete. To make the path continuous, we need to interpolate the path.

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See Z. Zhang from https://github.com/zhangzrjerry
See Y. Chen from https://github.com/mirrorcy
See R. Xu from https://github.com/rockyxrq

B. Polynomial Curve

The key idea of a polynomial curve is to make the discrete path continuous, not only in position but also in velocity and acceleration.

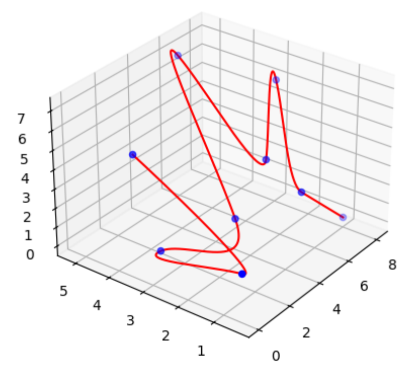


Fig. 2: 3D Cubic Spline

$$
\begin{aligned}
p_i &\in \mathcal{P} \\
p_i &= [x_i, y_i],\ \forall i \in [0, n] \\
q_{i,x}(s) &= a_{i,x}s^3 + b_{i,x}s^2 + c_{i,x}s^1 + d_{i,x} \\
&\forall i \in [0, n-1], s \in [0, 1)
\end{aligned} \tag{1}
$$

s is the normalized representation of the arc length. The minimum order of polynomial to make the path continuous in position, velocity, and acceleration is 3. And to ensure the continuous, we need

$$
\begin{cases}
q_{i-1,x}(1) = q_{i,x}(0) = x_i \\
q'_{i-1,x}(1) = q'_{i,x}(0) \\
q''_{i-1,x}(1) = q''_{i,x}(0)
\end{cases}
, \forall i \in [1, n] \tag{2}
$$

If the object is stationary in the beginning and at the end, the velocity should be $q'_0(0) = q'_n(1) = 0$. By solving these equations we can uniquely determine a polynomial to fit those points.

$$
\begin{cases}
a_i = x_{i+2} - 2x_{i+1} + x_i \\
b_i = -x_{i+2} + 2x_{i+1,k} - x_i \\
c_i = x_{i+1} - x_i \\
d_i = x_i
\end{cases}
, \forall i \in [1, n-1] \tag{3}
$$

In natural form [12].

$$\begin{bmatrix} 2 & 1 & & & \\ 1 & 4 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 4 & 1 \\ & & & 1 & 2 \end{bmatrix} \begin{bmatrix} D_0 \\ D_1 \\ \vdots \\ D_{m-1} \\ D_m \end{bmatrix} = \begin{bmatrix} 3(x_1 - x_0) \\ 3(x_2 - x_0) \\ \vdots \\ 3(x_m - x_{m-2}) \\ 3(x_m - x_{m-1} \end{bmatrix} \quad (4)$$

In case,

$$\begin{cases} a_i = 2(x_i - x_{i+1}) + D_i + D_{i+1} \\ b_i = 3(x_{i+1} - x_i) - 2D_i - D_{i+1} \\ c_i = D_i \\ d_i = x_i \end{cases} \quad (5)$$

C. Continuous Potential Field

In section II.A, we proposed a method to calculate the approximate and discrete potential field. However, the potential field often cause the path a severe shaking. Although you can set the potential field below a certain threshold to zero, this still does not make the system satisfactory.

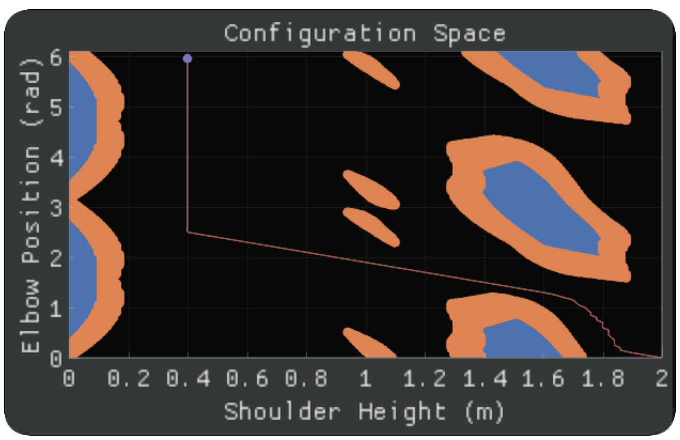


Fig. 3: The path is not smooth and has sharp jitters.

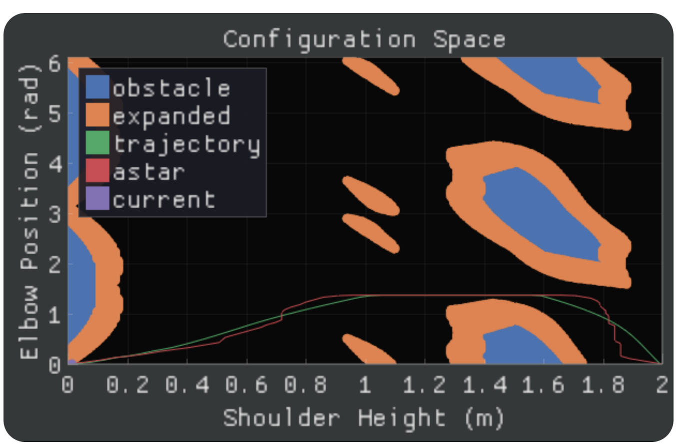


Fig. 4: The difference between the discrete and continuous potential field.

III. Trajectory Planning

Classical method to deal with the time parameterization problem in FRC is called forward and backward pass. Its essence is based on the trapezoidal curve or S-curve model, adjusting the time scale so that the motion planning meets the given constraints. However, it is only able to deal with 1 DOF planning problem. It has difficulty in multi-DOF trajectory planning, especially when it is not in Cartesian space [7], [5].

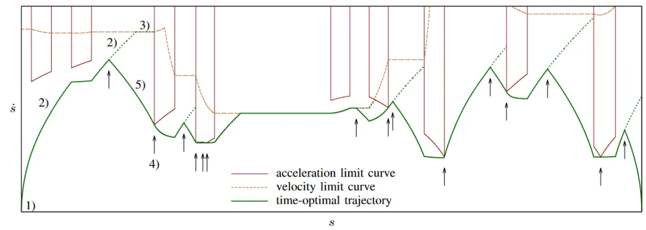


Fig. 5: Phase-plane trajectory[15]

Team 6328 proposed a Kairos solver in 2023. In their 23 season machine and code, they used an intake active avoidance strategy to avoid anti-collision. This allowed their solver to complete planning in a simpler way. They use linear interpolation as the initial position of waypoints, and then adjusts the position of waypoints to make the speed, acceleration, and torque meet the constraints based on the assumption that the time interval between each waypoint remains unchanged[10].

Our approach is very different from that of Team 6328. Instead of globally adjusting the time scale by assuming that the time intervals between points are consistent, we express the function of time with respect to arc length in a discretized form and parameter.

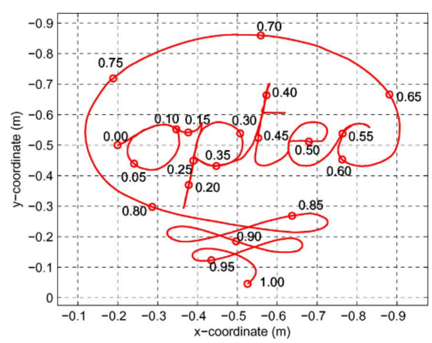


Fig. 6: The curve is uniquely determined by the arc length[2].

A. Time Optimal Path Parameterization

The total time of the trajectory is given by the integration of the whole path.

$$T = \int_0^T 1\mathrm{d}t = \int_0^L \frac{1}{\frac{\mathrm{d}s}{\mathrm{d}t}} \mathrm{d}s \quad (6)$$

Velocity and acceleration constraints at elevator position x and arm radian θ are

$$
\begin{aligned}
-v_{x,\max} \leq v_x \leq v_{x,\max} \\
-a_{x,\max} \leq a_x \leq a_{x,\max} \\
-v_{\theta,\max} \leq v_\theta \leq v_{\theta,\max} \\
-a_{\theta,\max} \leq a_\theta \leq a_{\theta,\max}
\end{aligned}
\tag{7}
$$

Given by chain rules, we have

$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{\mathrm{d}x}{\mathrm{d}s} \cdot \frac{\mathrm{d}s}{\mathrm{d}t} \\
\frac{\mathrm{d}^2x}{\mathrm{d}t^2} &= \frac{\mathrm{d}^2x}{\mathrm{d}s^2} \cdot \frac{\mathrm{d}s}{\mathrm{d}t} + \frac{\mathrm{d}x}{\mathrm{d}s} \cdot \frac{\mathrm{d}^2s}{\mathrm{d}t^2} \\
\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{\mathrm{d}\theta}{\mathrm{d}s} \cdot \frac{\mathrm{d}s}{\mathrm{d}t} \\
\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} &= \frac{\mathrm{d}^2\theta}{\mathrm{d}s^2} \cdot \frac{\mathrm{d}s}{\mathrm{d}t} + \frac{\mathrm{d}\theta}{\mathrm{d}s} \cdot \frac{\mathrm{d}^2s}{\mathrm{d}t^2}
\end{aligned}
\tag{8}
$$

Denote

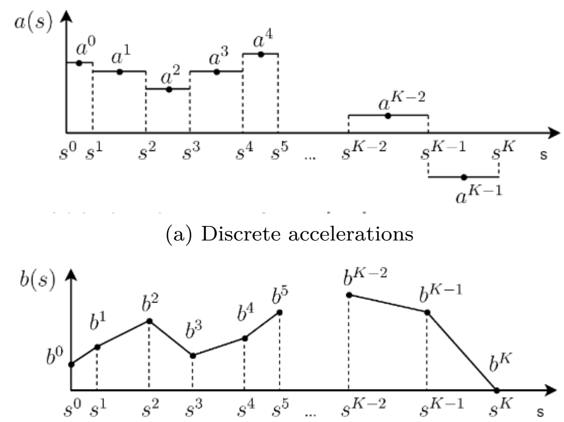
$$
a(s) = \frac{\mathrm{d}^2s}{\mathrm{d}t^2}, b(s) = \left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 \tag{9}
$$

The problem is described by

$$
\begin{aligned}
\min_{a(s),b(s)} \quad & \int_0^L \frac{1}{\sqrt{b(s)}}\mathrm{d}s \\
\text{s.t.} \quad & b'(s) = 2a(s) \\
& x'(0)\sqrt{b(0)} = 0 \\
& x'(L)\sqrt{b(L)} = 0 \\
& \theta'(0)\sqrt{b(0)} = 0 \\
& \theta'(L)\sqrt{b(L)} = 0 \\
& -b(s) \leq 0 \\
& [x'(s)]^2 b(s) \leq v_{x,\max}^2 \\
& [\theta'(s)]^2 b(s) \leq v_{\theta,\max}^2 \\
& x''(s)b(s) + x'(s)a(s) \leq a_{x,\max} \\
& -x''(s)b(s) + x'(s)a(s) \leq a_{x,\max} \\
& \theta''(s)b(s) + \theta'(s)a(s) \leq a_{\theta,\max} \\
& -\theta''(s)b(s) + \theta'(s)a(s) \leq a_{\theta,\max}
\end{aligned}
\tag{10}
$$

1) Discrete Case: Considering the problem in discrete case, we have

We obtain the discretized object function



(a) Discrete accelerations

(b) Discrete velocities

Fig. 7: Discrete function of time with respect to arc length[2].

$$
\begin{aligned}
\min_{a,b} \quad & \sum_{k=1}^{K} \frac{2(s^{k+1} - s^k)}{\sqrt{b^{k+1}} + \sqrt{b^k}} \\
\text{s.t.} \quad & \frac{b^{k+1} - b^k}{s^{k+1} - s^k} = 2a^k && \forall k \in [0, K-1] \\
& x'(s^0)\sqrt{b^0} = 0 \\
& x'(s^K)\sqrt{b^K} = 0 \\
& \theta'(s^0)\sqrt{b^0} = 0 \\
& \theta'(s^K)\sqrt{b^L} = 0 \\
& -b^k \leq 0 && \forall k \in [0, K] \\
& [x'(s^k)]^2 b^k \leq v_{x,\max}^2 && \forall k \in [0, K] \\
& [\theta'(s^k)]^2 b^k \leq v_{\theta,\max}^2 && \forall k \in [0, K] \\
& x''(s^k)b^k + x'(s^k)a^k \leq a_{x,\max} && \forall k \in [0, K-1] \\
& -x''(s^k)b^k + x'(s^k)a^k \leq a_{x,\max} && \forall k \in [0, K-1] \\
& \theta''(s^k)b^k + \theta'(s^k)a^k \leq a_{\theta,\max} && \forall k \in [0, K-1] \\
& -\theta''(s^k)b^k + \theta'(s^k)a^k \leq a_{\theta,\max} && \forall k \in [0, K-1]
\end{aligned}
\tag{11}
$$

2) Second-Order Cone: To rewrite the problem to a second-order conic programming form, we first try to bound nonlinear term $\sqrt{b^k}$ with c^k, such that $\sqrt{b^k} \geq c^k, k \in [0, K]$. Going a step further, we introduce one more slack variable d^k, which satisfies $\frac{1}{c^{k+1}+c^k} \leq d^k, k \in [0, K-1]$.

$$
\begin{aligned}
\left\| \begin{matrix} b^k - 1 \\ 2c^k \end{matrix} \right\|_2 \leq b^k + 1 \Leftrightarrow \begin{bmatrix} b^k + 1 \\ b^k - 1 \\ 2c^k \end{bmatrix} \in \mathcal{Q}^3 \\
\left\| \begin{matrix} c^k + c^{k+1} - d^k \\ 2 \end{matrix} \right\|_2 \leq c^k + c^{k+1} + d^k \\
\Leftrightarrow \begin{bmatrix} c^k + c^{k+1} + d^k \\ c^k + c^{k+1} - d^k \\ 2 \end{bmatrix} \in \mathcal{Q}^3
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&\min_{a,b} \sum_{k=1}^{K} \frac{2(s^{k+1}-s^k)}{\sqrt{b^{k+1}}+\sqrt{b^k}} \Leftrightarrow \\
&\min_{a,b,c,d} \sum_{k=1}^{K} 2d^k(s^{k+1}-s^k) \\
&\text{s.t. } \left\| \begin{matrix} c^k + c^{k+1} - d^k \\ 2 \end{matrix} \right\|_2 \leq c^k + c^{k+1} + d^k \\
&\quad\;\; \left\| \begin{matrix} b^k - 1 \\ 2c^k \end{matrix} \right\|_2 \leq b^k + 1
\end{aligned} \tag{13}
$$

Rewriting the constraints of the optimization problem, especially the nonlinear terms $\sqrt{b_k}$, we get the final objective function [2].

$$
\min_{a,b,c,d} \sum_{k=1}^{K} 2(s^{k+1}-s^k)d^k
$$

Subject to

$$
\begin{aligned}
\begin{bmatrix} c^k + c^{k+1} + d^k \\ c^k + c^{k+1} - d^k \\ 2 \end{bmatrix} &\in \mathcal{Q}^3 & \forall k \in [0, K-1] \\
\begin{bmatrix} b^k + 1 \\ b^k - 1 \\ 2c^k \end{bmatrix} &\in \mathcal{Q}^3 & \forall k \in [0, K] \\
2(s^{k+1}-s^k)a^k + b^k - b^{k+1} &= 0 & \forall k \in [0, K-1] \\
[x'(s^0)]^2 b^0 &= 0 \\
[x'(s^K)]^2 b^K &= 0 \\
[\theta'(s^0)]^2 b^0 &= 0 \\
[\theta'(s^K)]^2 b^L &= 0 \\
-b^k &\leq 0, & \forall k \in [0, K] \\
[x'(s^k)]^2 b^k &\leq v_{x,\max}^2 & \forall k \in [0, K] \\
[\theta'(s^k)]^2 b^k &\leq v_{\theta,\max}^2 & \forall k \in [0, K] \\
x''(s^k)b^k + x'(s^k)a^k &\leq a_{x,\max} & \forall k \in [0, K-1] \\
-x''(s^k)b^k + x'(s^k)a^k &\leq a_{x,\max} & \forall k \in [0, K-1] \\
\theta''(s^k)b^k + \theta'(s^k)a^k &\leq a_{\theta,\max} & \forall k \in [0, K-1] \\
-\theta''(s^k)b^k + \theta'(s^k)a^k &\leq a_{\theta,\max} & \forall k \in [0, K-1]
\end{aligned} \tag{14}
$$

All constraints are divided into second-order cone constraints, equality constraints and inequality constraints [4].

$$
\begin{aligned}
\min_x \;& c^T x \\
\text{s.t. } & A_i x + b_i \in \mathcal{K}_i \\
& Gx = h \\
& Px \leq q
\end{aligned} \tag{15}
$$

3) Augmented Lagrangian Method: Therefore, we can reformulate the problem using the Augmented Lagrangian method for Symmetric Cones.

$$
\begin{aligned}
\mathcal{L}_\rho(x,\mu,\lambda,\eta) =& c^T x + \\
& \frac{\rho}{2} \sum_{i=1}^{m} ||P_{\mathcal{K}_i}(\frac{\mu_i}{\rho} - A_i x - b_i)||^2 + \\
& \frac{\rho}{2} ||Gx - h + \frac{\lambda}{\rho}||^2 + \\
& \frac{\rho}{2} ||\max[Px - q + \frac{\eta}{\rho}, 0]||^2
\end{aligned} \tag{16}
$$

$P_{\mathcal{K}}$ is the projection of a vector on a symmetric cone

$$
P_{\mathcal{K}}(v) = \arg\min_{x \in \mathcal{K}} ||v - x||^2 \tag{17}
$$

In second order cone case,

$$
P_{\mathcal{K}=\mathcal{Q}^n}(v) = \begin{cases} 0, & v_0 \leq -||v_1||_2 \\ \frac{v_0 + ||v_1||_2}{2||v_1||_2}(||v_1||_2, v_1)^T, & |v_0| < ||v_1||_2 \\ v, & v_0 \geq ||v_1||_2 \end{cases} \tag{18}
$$

And the gradient of the augmented Lagrangian function is given by

$$
\begin{aligned}
\nabla\mathcal{L}_\rho(x,\mu,\lambda,\eta) =& c - \rho \sum_{i=1}^{m} A_i^T P_{\mathcal{K}_i}(\frac{\mu_i}{\rho} - A_i x - b_i) + \\
& \rho G^T(Gx - h + \frac{\lambda}{\rho}) + \\
& \rho P^T \{\max[Px - q + \frac{\eta}{\rho}, 0]\}
\end{aligned} \tag{19}
$$

So we can use a L-BFGS method to solve this convex and unconstrained function [3], [1], [16].

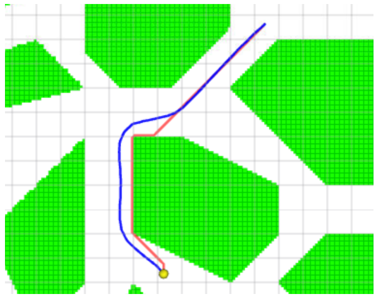
$$
\begin{aligned}
x &\leftarrow \arg\min_x \mathcal{L}_\rho(x,\mu,\lambda,\eta) \\
\mu_i &\leftarrow P_{\mathcal{K}_i}(\mu_i - \rho(A_i x + b_i)) \\
\lambda &\leftarrow \lambda + \rho(Gx - h) \\
\eta &\leftarrow \max[\eta + \rho(Px - q), 0] \\
\rho &\leftarrow \min[(1+\gamma)\rho, \beta]
\end{aligned} \tag{20}
$$

γ is the growth rate of ρ and β is the upper bound of ρ, which is typically 10^3.

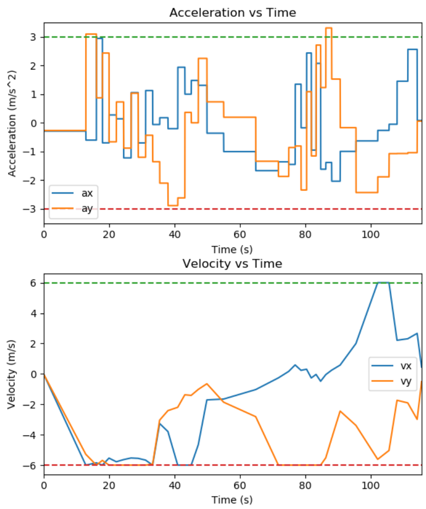
B. Constraints on voltage and current

Can we go one step further? Although we can set constraints for speed and acceleration in segments instead of setting a global maximum value, it is difficult to exhaust all possibilities and enumerate all the required constraints. A conservative constraint will waste control resources.

On the contrary, under the guidance of the feedforward model, speed and acceleration together constitute a feasible control domain. Under the guidance of the theory of control allocation, we redesign the speed and acceleration constraints into voltage and current constraints. Doing so enables the model to dynamically adjust the energy distribution on speed and acceleration, maximize the use of control resources on the field.



(a) The corresponding path



(b) Velocity and acceleration under constraints

Fig. 8: Our previous work showing constraints on velocity and acceleration[19].

Elevator feedforward model, i_e means the ratio from motor radian to elevator position meter:

$$V_E = K_g + K_S \cdot \text{sign}(\dot{d}) + \frac{K_v}{i_e} \cdot \dot{d} + \frac{K_a}{i_e} \cdot \ddot{d} \tag{21}$$

Arm feedforward model, i_a means the ratio from motor radian to arm radian:

$$V_A = K_g \cdot \cos(\theta) + K_S \cdot \text{sign}(\dot{\theta}) + \frac{K_v}{i_a} \cdot \dot{\theta} + \frac{K_a}{i_a} \cdot \ddot{\theta} \tag{22}$$

The motor current at a given voltage and speed is:

$$I = -\frac{K_v}{R} \cdot \omega + \frac{1}{R} \cdot V \tag{23}$$

Notice that we used c^k to bound the lower side of $\sqrt{b^k}$. This is because

$$\frac{2(s^{k+1} - s^k)}{\sqrt{b^{k+1}} + \sqrt{b^k}} \le \frac{2(s^{k+1} - s^k)}{c^{k+1} + c^k} \le 2(s^{k+1} - s^k)d^k \tag{24}$$

In case, we introduce a second order cone to handle the chain constraints. The equality stands only when $\sqrt{b^k} = c^k$. How about directly let $\sqrt{b^k} = c^k$ instead of $\sqrt{b^k} \ge c^k$? So that,

$$\frac{2(s^{k+1} - s^k)}{\sqrt{b^{k+1}} + \sqrt{b^k}} = \frac{2(s^{k+1} - s^k)}{c^{k+1} + c^k} \le 2(s^{k+1} - s^k)d^k \tag{25}$$

We introduce a dual variable ν, to bound the quadratic term. The problem is modeled by the following.

$$\begin{aligned} \mathcal{L}_\rho(x, \mu, \nu, \lambda, \eta) = & c^T x + \\ & \frac{\rho}{2} \sum_{i=1}^{m} ||P_{\mathcal{K}_i}(\frac{\mu_i}{\rho} - A_i x - b_i)||^2 + \\ & \frac{\rho}{2} \sum_{j=1}^{q} ||x^T J_j x - r_j^T x + \frac{\nu_j}{\rho}||^2 + \\ & \frac{\rho}{2} ||Gx - h + \frac{\lambda}{\rho}||^2 + \\ & \frac{\rho}{2} ||\max[Px - q + \frac{\eta}{\rho}, 0]||^2 \end{aligned} \tag{26}$$

$$\begin{aligned} \nabla \mathcal{L}_\rho(x, \mu, \nu, \lambda, \eta) = & c - \rho \sum_{i=1}^{m} A_i^T P_{\mathcal{K}_i}(\frac{\mu_i}{\rho} - A_i x - b_i) + \\ & \rho \sum_{j=1}^{q} (x^T J_j - r_j^T + \frac{\nu_j}{\rho})[(J_j^T + J_j)x - r_j] + \\ & \rho G^T (Gx - h + \frac{\lambda}{\rho}) + \\ & \rho P^T \{\max[Px - q + \frac{\eta}{\rho}, 0]\} \end{aligned} \tag{27}$$

In case J is symmetric, $J^T + J = 2J$.

$$\begin{aligned} & x \leftarrow \arg\min_x \mathcal{L}_\rho(x, \mu, \nu, \lambda, \eta) \\ & \mu_i \leftarrow P_{\mathcal{K}_i}(\mu_i - \rho(A_i x + b_i)) \\ & \nu_j \leftarrow \nu_j + \rho(x^T J_j x - r_j^T x) \\ & \lambda \leftarrow \lambda + \rho(Gx - h) \\ & \eta \leftarrow \max[\eta + \rho(Px - q), 0] \\ & \rho \leftarrow \min[(1 + \gamma)\rho, \beta] \end{aligned} \tag{28}$$

Till now, we obtain all the math tools to solve our problem.

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