Least Squares

Dr François Pitié

Problem 1. Which of the following models with input x_1, x_2 , parameters w_1, w_2 and noise $\epsilon \sim \mathcal{N}(0, \sigma^2)$, are linear in the parameters and can be used as such for Least Squares:

- 1. $y = w_0 + w_1 x^2 + \epsilon$
- $2. \ y = w_0 x^{w_1} + w_2 + \epsilon$
- 3. $y = \exp(w_0 + w_1 x) + \epsilon$
- 4. $\log(y) = w_0 + w_1 x + \epsilon$

Problem 2. Which of the following models with input x_1, x_2 , parameters w_1, w_2 and noise $\epsilon \sim \mathcal{N}(0, \sigma^2)$, are linear in the parameters and can be used as such for Least Squares:

- 1. $y = \sin(x_1 w_1 + w_2) + \epsilon$
- 2. $y = \log(x_1)w_1 + \log(x_2)w_2 + \epsilon$
- 3. $y = w_1 x_1^2 + \epsilon$
- 4. $y = w_1^2 x_1 + \epsilon$

Problem 3. For n real numbers x_1, \dots, x_n , what is the value \hat{x} that minimises the sum of squared distances from x to each x_i :

$$\hat{x} = \arg\min_{x} \sum_{i=1}^{n} (x_i - x)^2$$

[4C16] Tutorial page 2/1

Problem 4. For a linear model $\mathbf{y} = \mathbf{X}\mathbf{w} + \boldsymbol{\epsilon}$, derive, in a matrix form, the expression of the least square error. That is, for $E(\mathbf{w}) = \boldsymbol{\epsilon}^{\top} \boldsymbol{\epsilon}$ derive the expression of $\min_{\mathbf{w}} E(\mathbf{w})$.

Problem 5. An autoregressive model is when a value from a time series is regressed on previous values from that same time series.

$$x_t = w_0 + \sum_{i=1}^p w_i x_{t-i} + \varepsilon_t$$

write the design matrix for this problem.

Problem 6. Consider the linear model $y = w_0 + w_1x$. We want to bias w_1 towards the value $\hat{w_1}$. Write a loss function that achieves this.