EE4C08 - Tutorial 1
Image and Video Processing
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1. (a) Derive the difference equations for the following linear filters with transfer functions as shown. You can assume that z_1 and z_2 correspond to the horizontal and vertical 2D z-transform variables respectively.

$$
\begin{aligned}
H_1(z_1, z_2) &= \frac{1}{1 - 0.99z_1^{-1} - 0.9z_2^{-1} + 0.891z_1^{-1}z_2^{-1}} \\
H_2(z_1, z_2) &= 0.64z_1^{-1}z_2^{-1} + 0.8z_1^{-1} + 0.64z_1^{-1}z_2 + 0.8z_2^{-1} + 1.0+ \\
&\quad 0.8z_2 + 0.64z_1z_2^{-1} + 0.8z_1^{1} + 0.64z_1z_2 \\
H_3(z_1, z_2) &= 1 - z_1^{-1} \\
H_4(z_1, z_2) &= -z_2^{-1}z_1^{-1} + z_1^{-1} + -z_2^{-1} + 1 \\
H_5(z_1, z_2) &= \frac{1}{1 - 0.99z_1^{-1} - 1.0z_2^{-1} + 1.98z_1^{-1}z_2^{-1}}
\end{aligned}
$$

   (b) For each of the filters above, classify them as IIR or FIR. State in addition their causality i.e. whether they are causal, non-causal or semi-causal.

   (c) Classify the filters as separable and non-separable. In the case of separable filters, find the row and column filters.

   (d) A video processing system requires a vertical edge strength measurement of an input image. Which of the above filters would you use for this?

2. A signal $f_l(h,k)$ is processed with a variety of filters with convolution masks as shown below. The results of processing are shown in figure 1, *but not in the right order.* The images have also been modified so that the mid-gray value represents the 0 value, darker pixels indicatwe negative values and brighter values indicate positive values. By observing the filter masks, and using your knowledge of what filters do to images, identify the corresponding filter and output image. Explain your selection.

$$p_1(h,k) \equiv [1 - 1]$$

$$p_2(h,k) \equiv \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$p_3(h,k) \equiv \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$p_4(h,k) \equiv \begin{bmatrix} 0.0013 & 0.0086 & 0.0159 & 0.0086 & 0.0013 \\ 0.0086 & 0.0581 & 0.1076 & 0.0581 & 0.0086 \\ 0.0159 & 0.1076 & 0.1993 & 0.1076 & 0.0159 \\ 0.0086 & 0.0581 & 0.1076 & 0.0581 & 0.0086 \\ 0.0013 & 0.0086 & 0.0159 & 0.0086 & 0.0013 \end{bmatrix}$$

$$p_5(h,k) \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

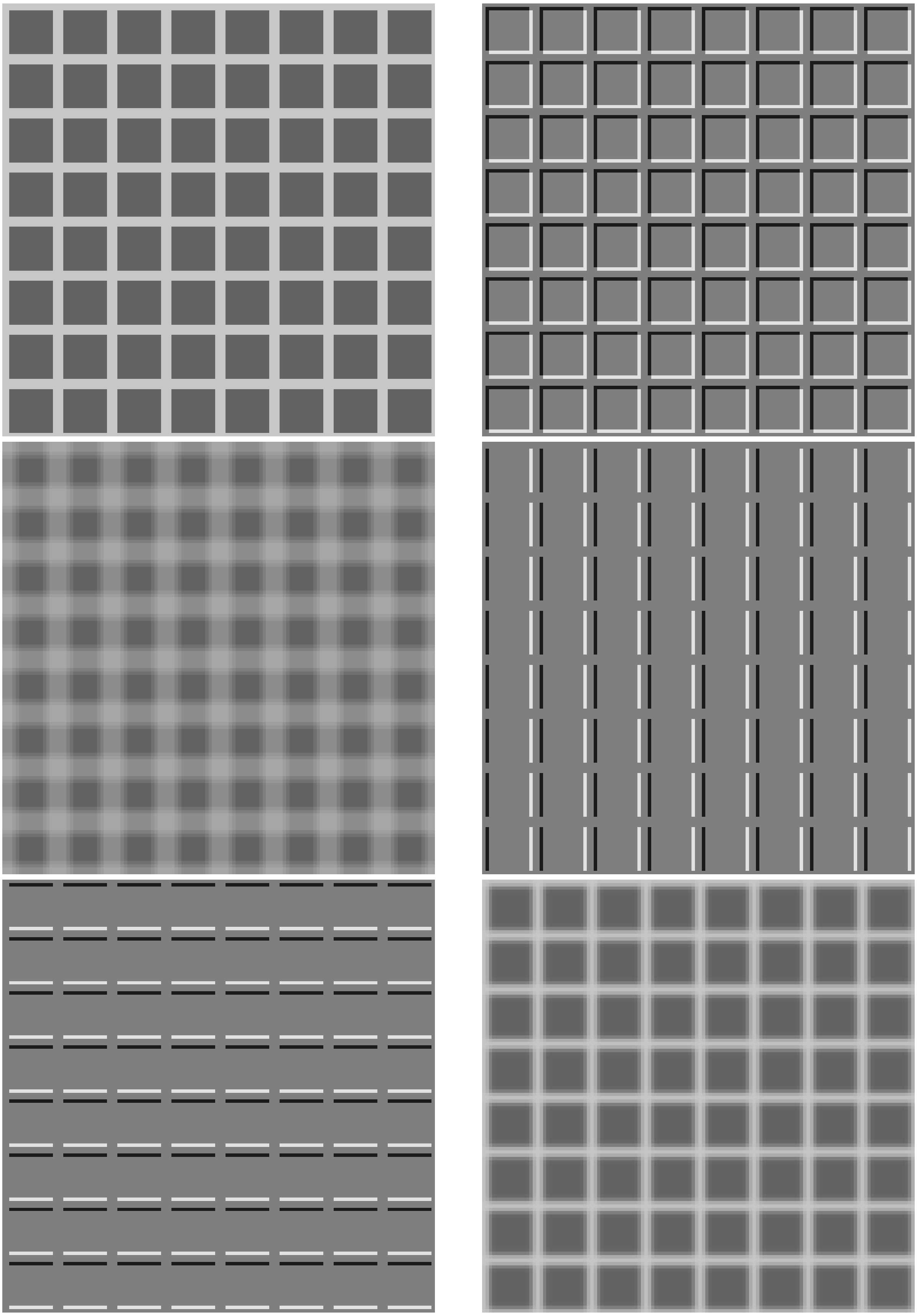

Figure 1: Top Left: $f(h, k)$, Clockwise from top right: Output A, B, C, D, E

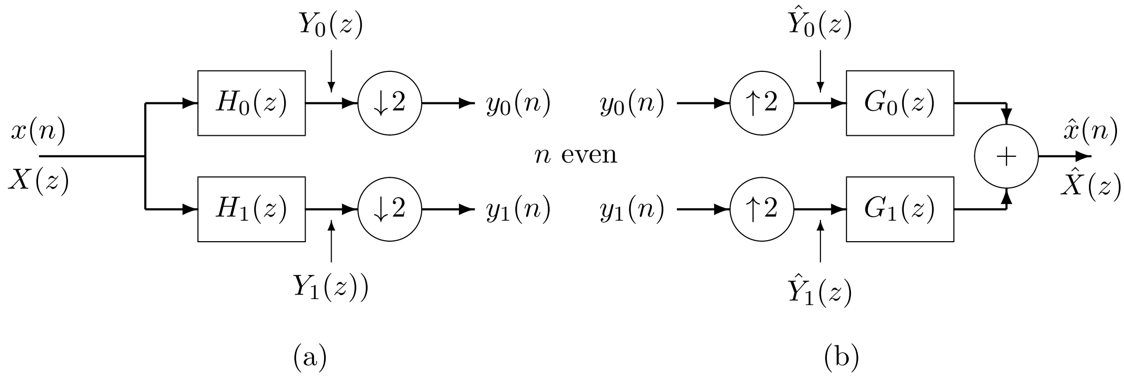

Figure 2: Two-band filter banks for analysis (a) and reconstruction (b).

3. The system shown in figure 2 shows the filters used in the basic unit of a Perfect Reconstruction filterbank. In this course you know that the following expressions must hold for Perfect Reconstruction.

$$G_0(z)H_0(z) + G_1(z)H_1(z) \equiv 2 \tag{1}$$

and

$$G_0(z)H_0(-z) + G_1(z)H_1(-z) \equiv 0 \tag{2}$$

Show that the LeGall 3-5 tap filters satisfy the Perfect Reconstruction requirement. The LeGall filters are as below.

$$\begin{aligned} H_0(z) &= \frac{1}{2}(z + 2 + z^{-1}) \\ G_0(z) &= \frac{1}{8}(-z^2 + 2z + 6 + 2z^{-1} - z^{-2}) \\ G_1(z) &= \frac{1}{2}z(-z + 2 - z^{-1}) \\ H_1(z) &= \frac{1}{8}z^{-1}(-z^2 - 2z + 6 - 2z^{-1} - z^{-2}) \end{aligned} \tag{3}$$

4. The images on the left of figure 3 show a series of images of differing content and on the right (but not in order) are the 20× Log magnitudes of their 2D FT's. Find the correct signal and transform picture pairs, explaining your choices.

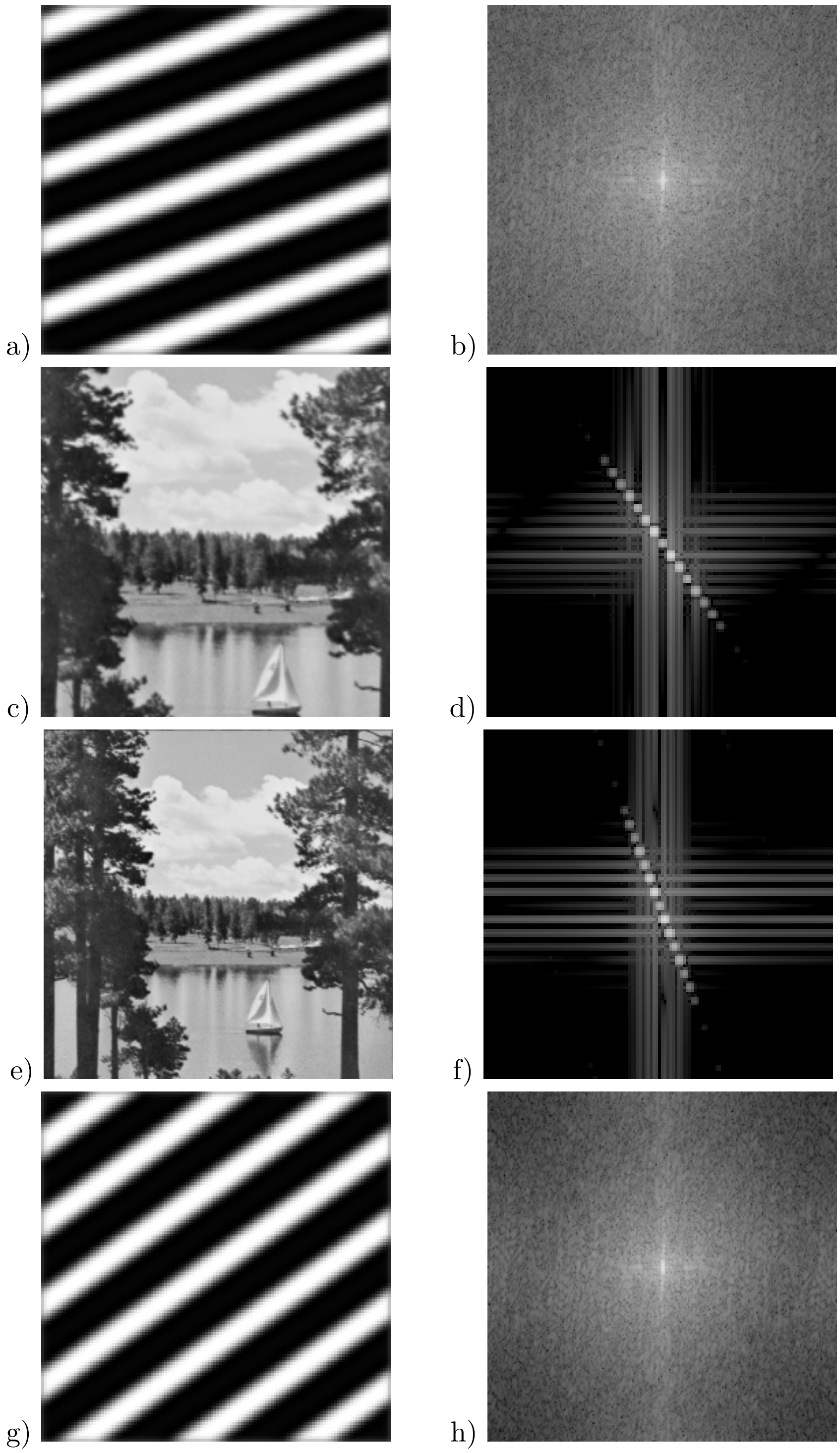

Figure 3: *A mixture of pictures and their 2D Log Power Spectra estimated using the 2D DFT*