Pixel Operations / Colour Operations

4c8 Media Signal Processing

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Colour Grading

This is an example of Colour Grading. Colour manipulation can have a dramatic effect on the perception. This is a key step in visual post-production and is done by specialised artists at the very end of production.

Brightness and Contrast

The simplest operation is to apply an affine transform to the colours:

$$G = aI + b$$ here a is the gain and b the offset

This has the effect of altering the brightness and contrast of the picture.
Note there is no such a thing as an absolute brightness or contrast value.
We only talk about

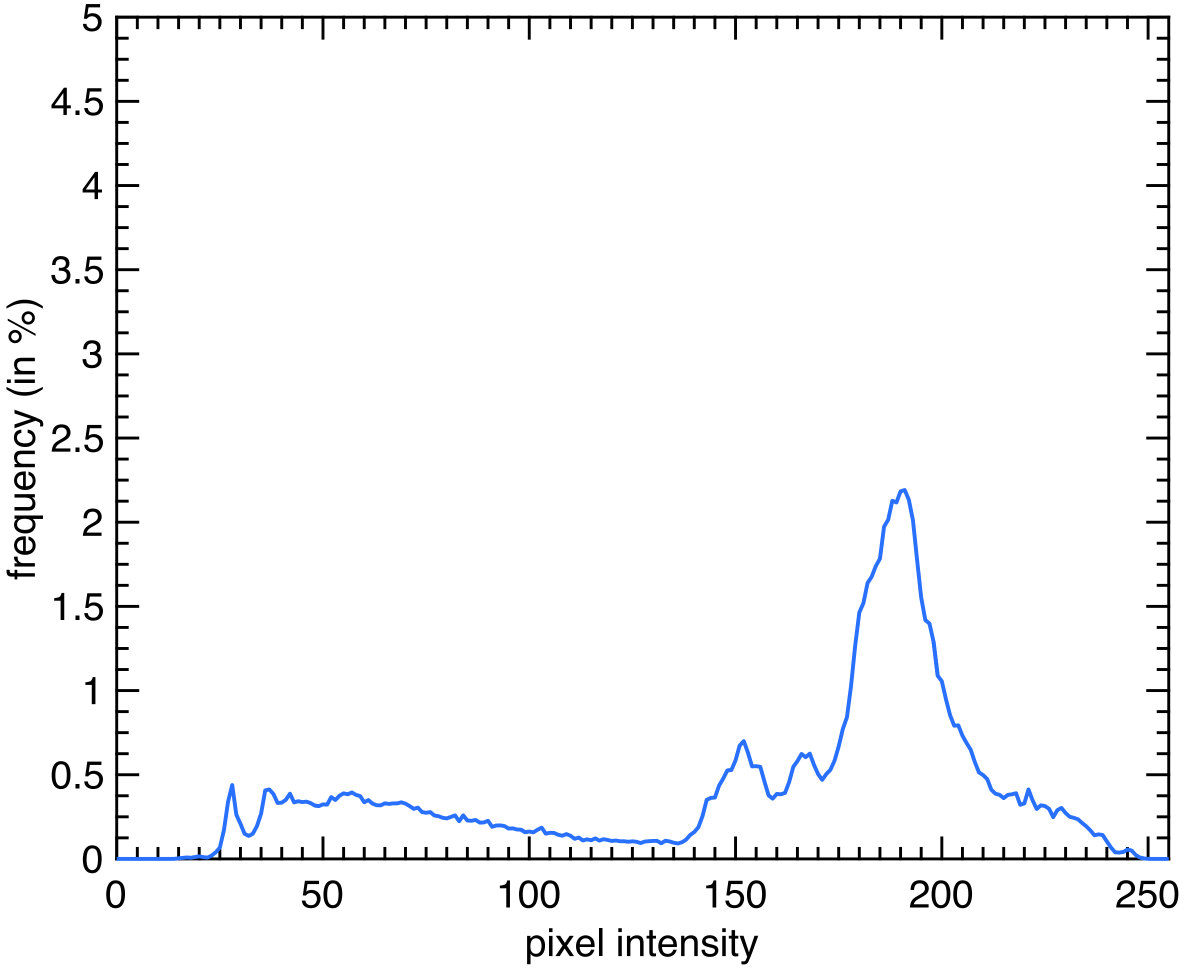
Gamma

The gamma is a non-linear mapping of the colour:

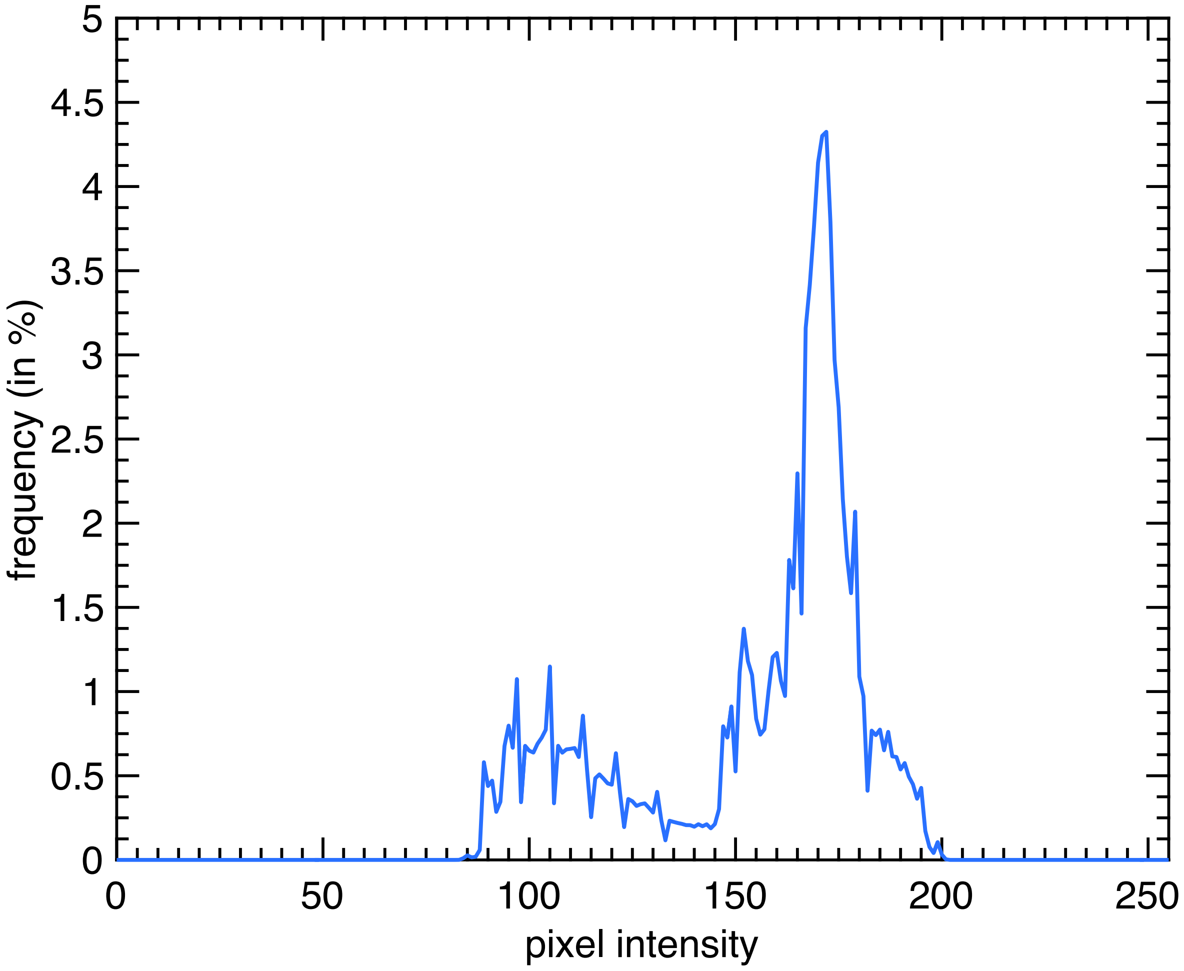
$$G = I^{\gamma}$$

for gamma<1, is a non-linear mapping of the colour:

Colour Mapping and Histogram

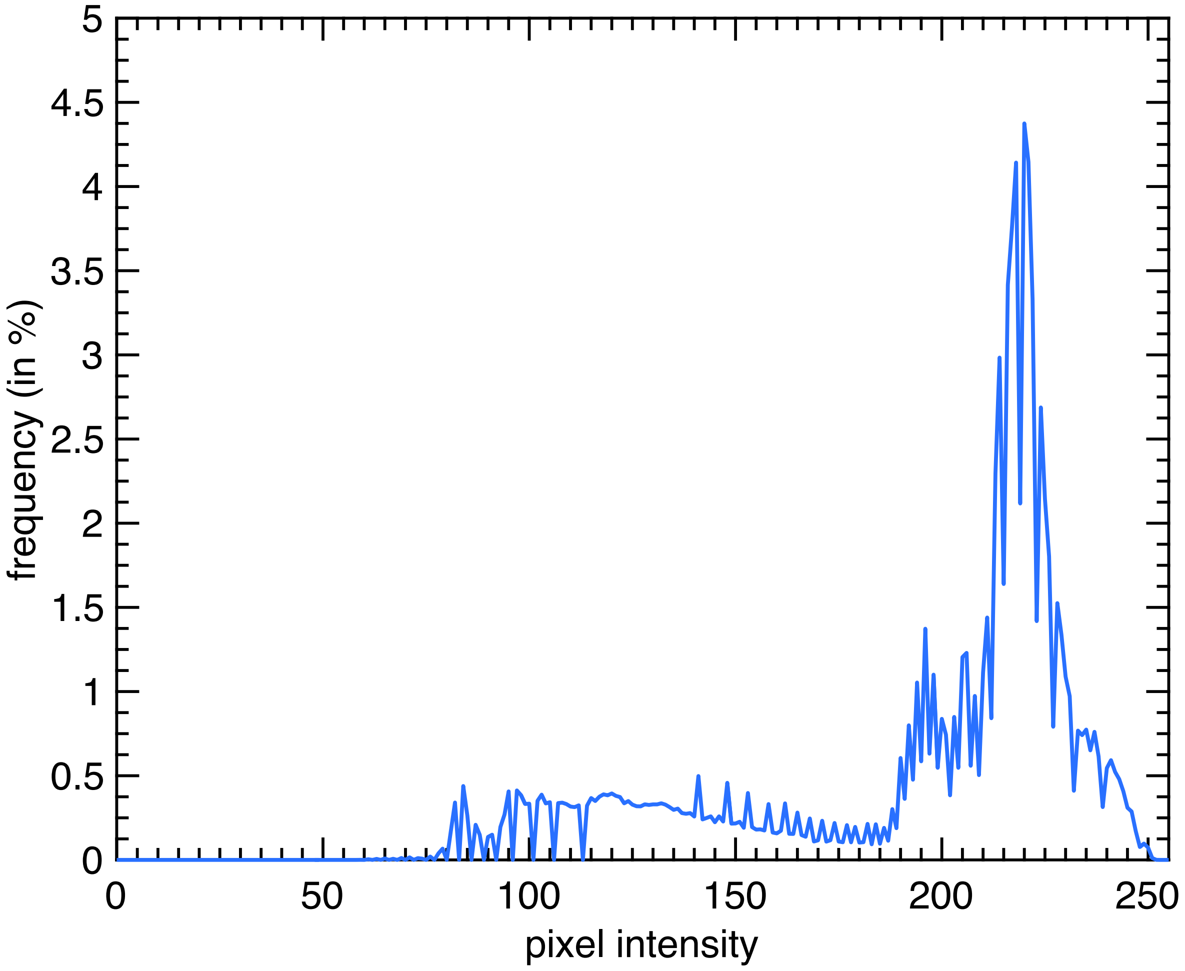


Colour Mapping and Histogram



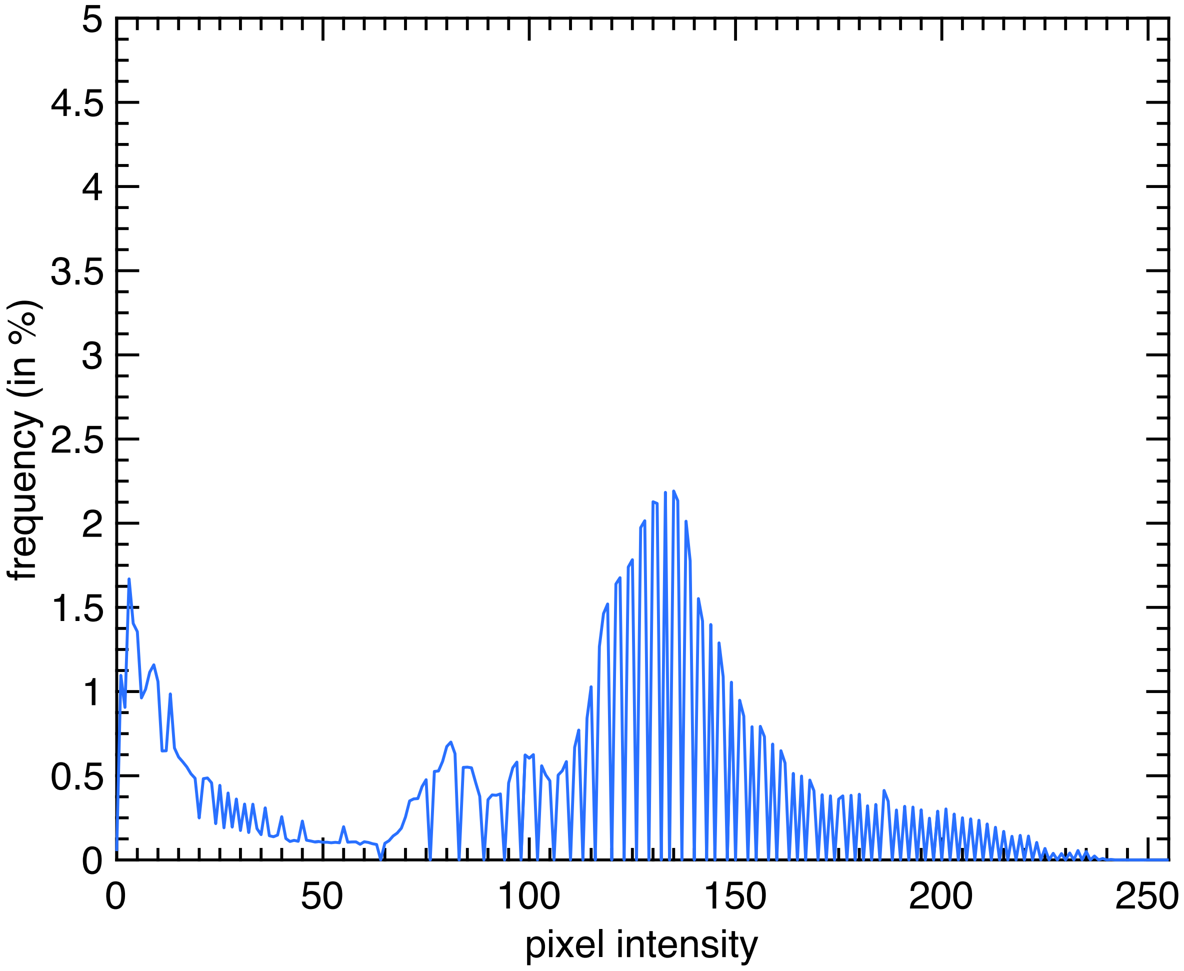
$$G = 0.5I + 76$$

Colour Mapping and Histogram



$$G = I^{1/2}$$

Colour Mapping and Histogram



$$G = I^{2.2}$$

Colour Mappings & Histograms

Colour Mapping and Histogram

What is the relationship between histograms and colour mappings?

We consider here the case where the mapping t is applied to a single channel of an image:

$$
\begin{aligned}
t : &[0, 255] \to [0, 255] \\
&u \mapsto t(u)
\end{aligned}
$$

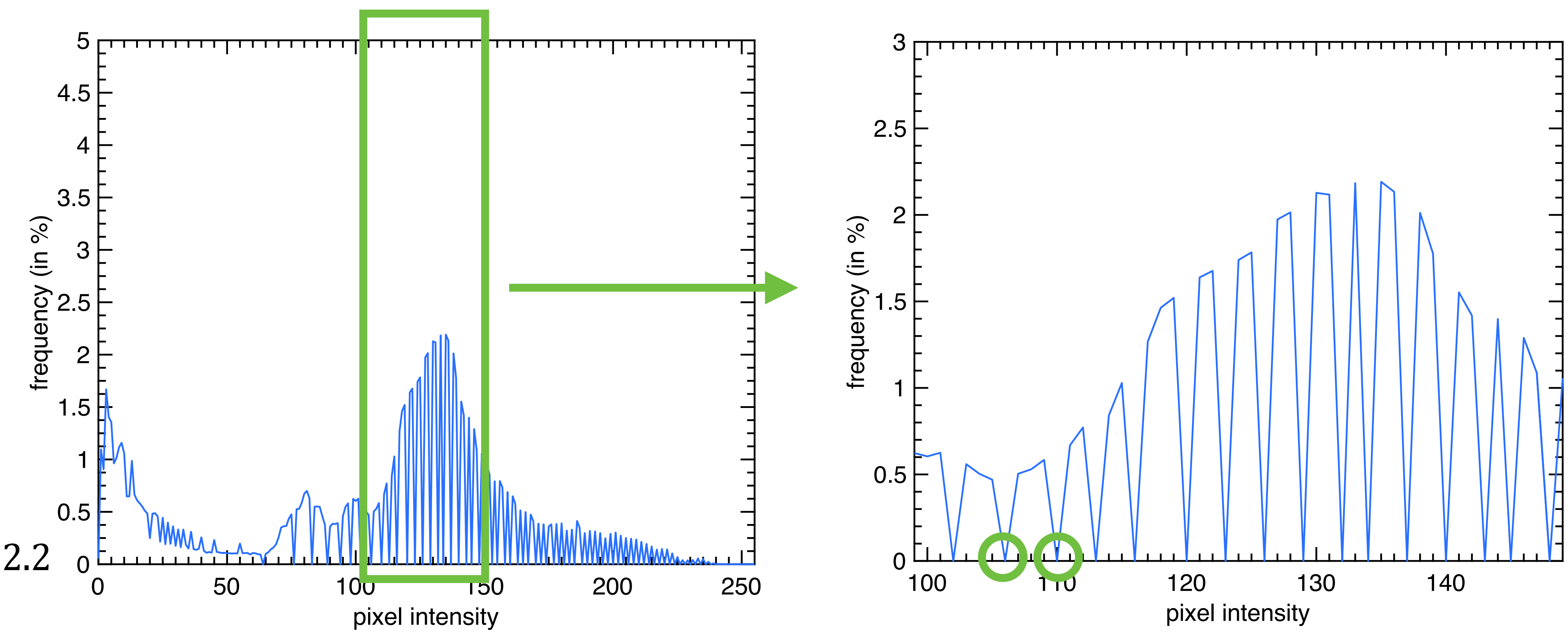
COLOUR MAPPING AND HISTOGRAM (DISCRETE CASE)

Let's look at the discrete case. In the colour graded image, we can easily count the bin for a value v by adding all pixel values $I(x)$ in the original image that map to v:

$$h_{t(I)}(v) = \sum_{u|t(u)=v} h(u)$$

That means that some bins will be empty $\left(h_{t(I)}(v) = 0\right)$ if we can't find an originating pixel value.

Gamma: 2.2



Now consider what happens when the mapping t is assumed to be monotonic increasing. That is, $u_1 < u_2$, iif $t(u_1) < t(u_2)$.

The two assumptions (single channel and monotonicity) cover 99.9% of all use cases. In particular, it covers gamma correction and gain/offset operations, provided that gain>0.

Colour Mapping and Cumulative Histogram

Consider a pixel value $I(x)$ and some threshold value u. The monotonicity of t guarantees that

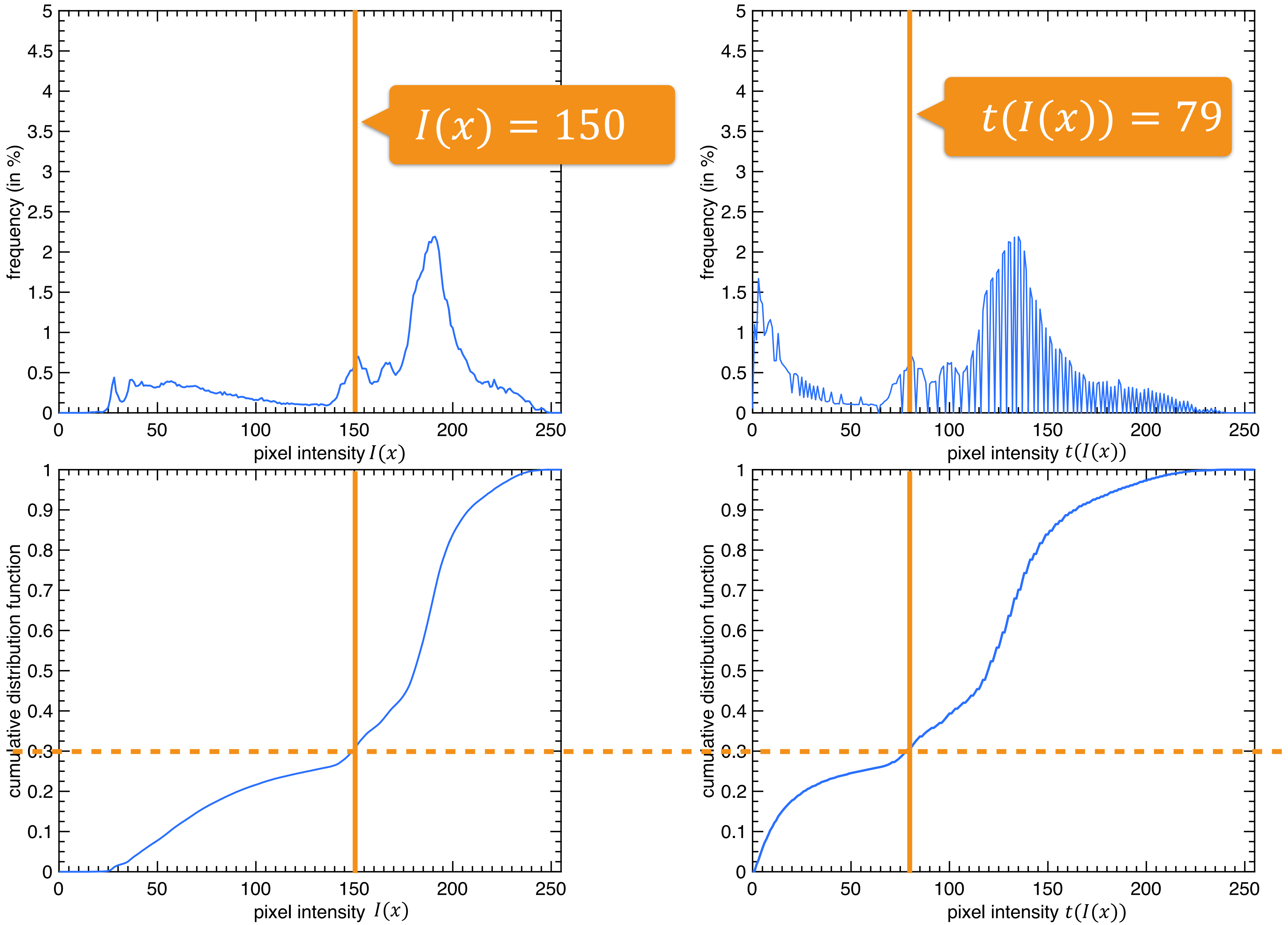
$$I(x) < u \Leftrightarrow t(I(x)) < t(u)$$

Thus we must also have:

$$\boxed{\Pr(I(x) < u) = \Pr(t(I(x)) < t(u))}$$

Note that $u \mapsto \Pr(I(x) < u) = F_I(u)$ is the cumulative distribution function (or cumulative histogram) of the random variable $I(x)$.

Example for $\gamma = 2.2$



$P(I(x) < 150) = P(t(I(x)) < 79) = 0.3$

When looking at the effect of a colour mapping, the cumulative distribution function, or its discreet approximation the cumulative histogram, is the preferred tool.

The mapping has the effect of warping the c.d.f., and there is a simple relationship between the original and mapped c.d.f's.

The relationship is not so simple for the histograms.

In a continuous settings, the random variable $I(x)$ has a continuous probability distribution function (p.d.f.) f_I and the histogram h_I is only a discreet approximation of this continuous p.d.f. By definition we have:

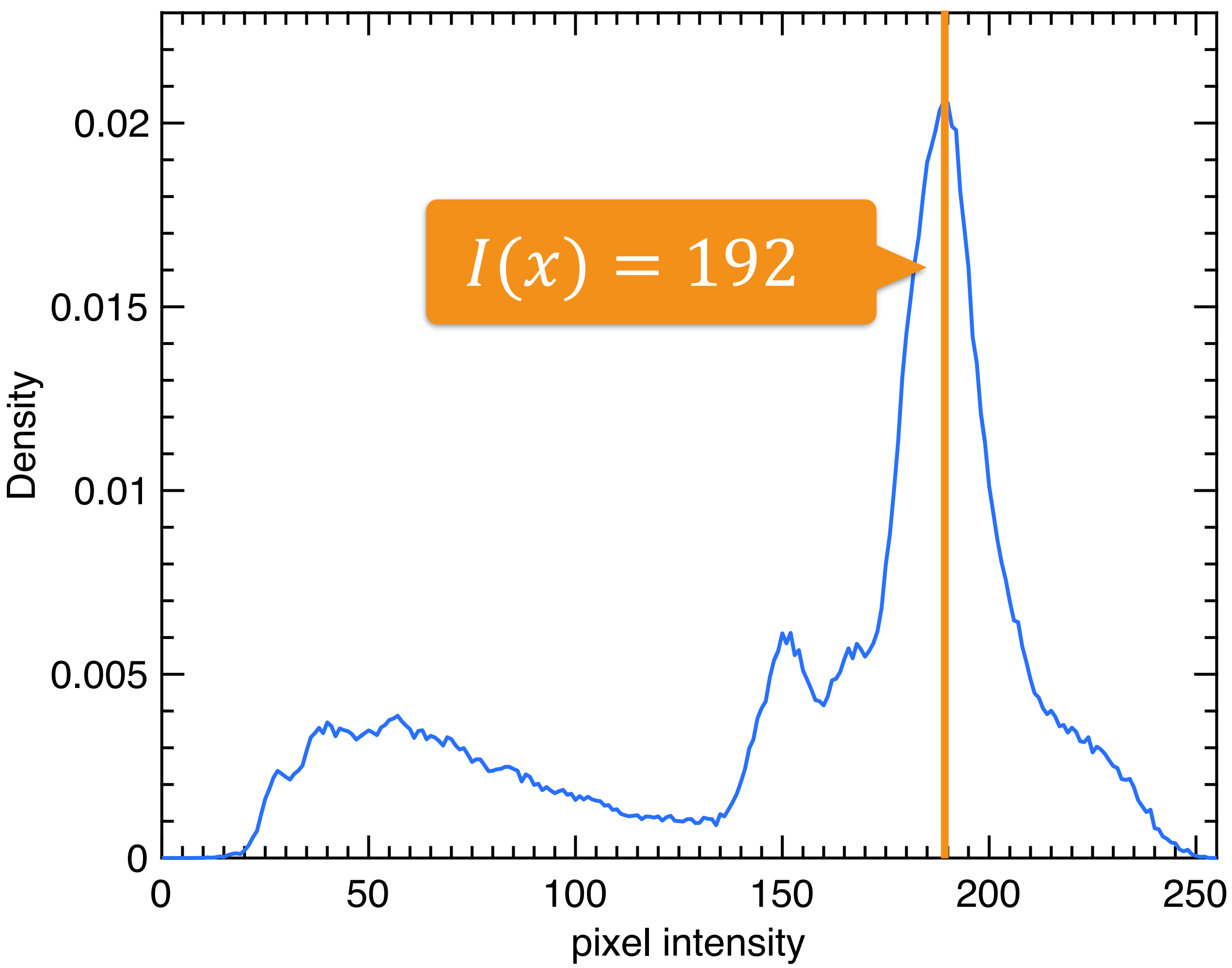
$$F_I(u) = \int_0^u f_I(t)dt$$

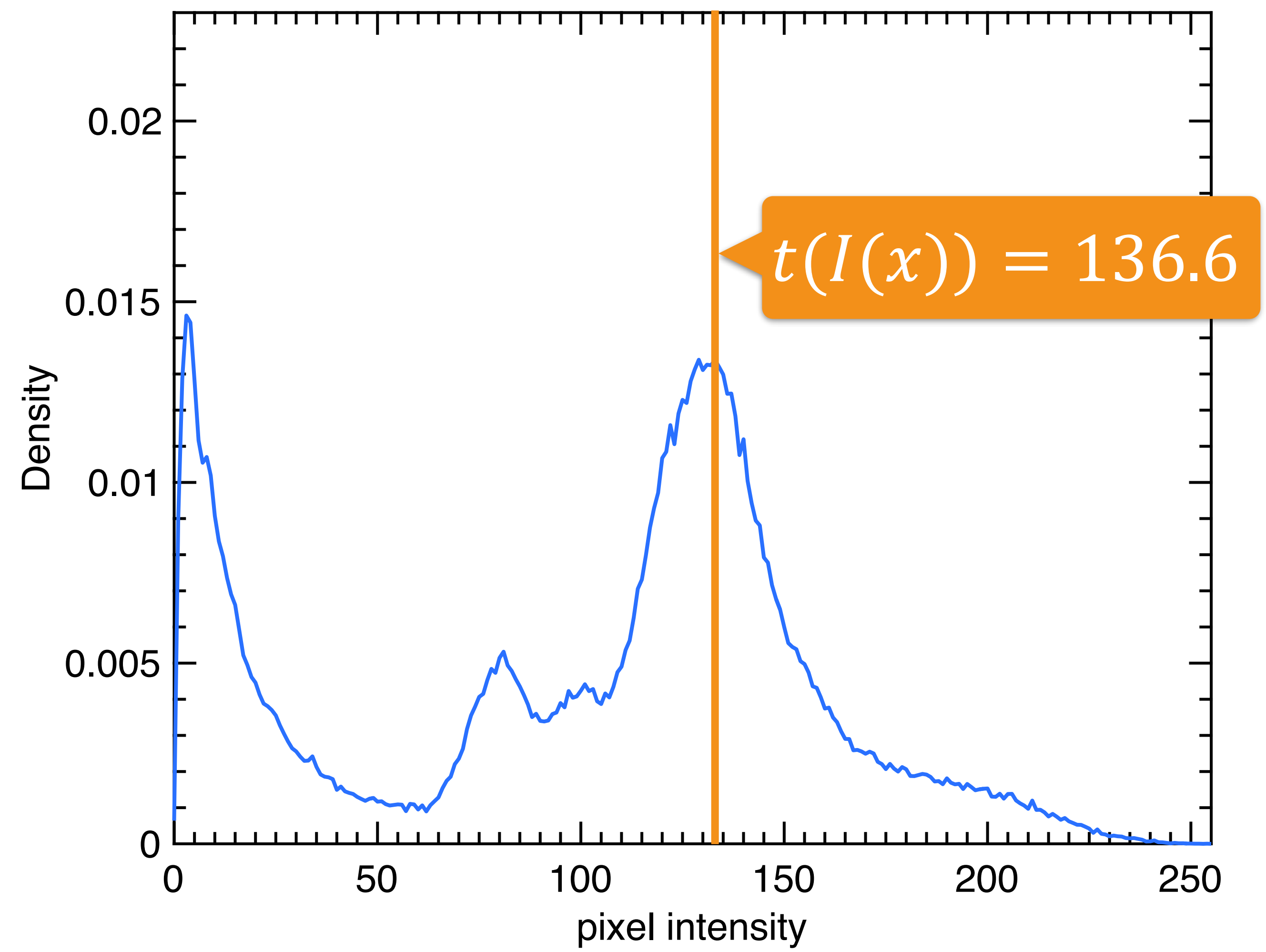
thus from earlier's equality $F_I(u) = F_{t(I)}(t(u))$, we can derive the following relationship between the original and graded continuous p.d.f.'s:

$$\boxed{f_I(u) = f_{t(I)}(t(u))t'(u)}$$

Colour Mapping and Histogram (continuous case)

Here is an example for Gamma:2.2. The estimates of the continuous pdf's are represented (original: left, mapped: right).





$$f_I(u) = f_{t(I)}(t(u))t'(u)$$

$$f_I(192) \approx f_{t(I)}(136.6) \times 1.56$$

Histogram Equalisation

Histogram Equalisation

Recall that for a monotonic mapping, we have:

$$F_I(u) = F_{t(I)}(t(u))$$

To maximise the contrast, we could design a target histogram with maximum colour spread. That histogram is the uniform histogram $h_{t(I)}(v) = 1/M$, with M the number of bins.

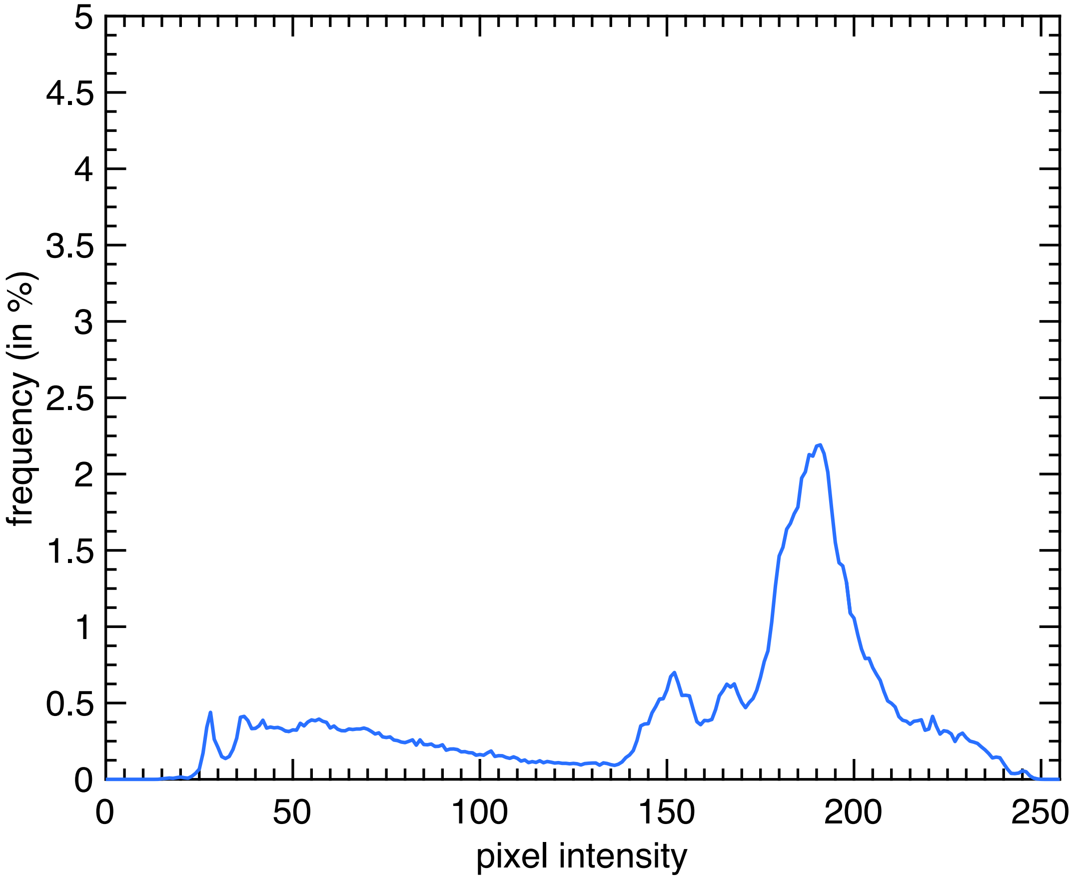
The corresponding c.d.f. is simply $F_{t(I)}(v) = v$ (assuming $v \in [0, 1]$).

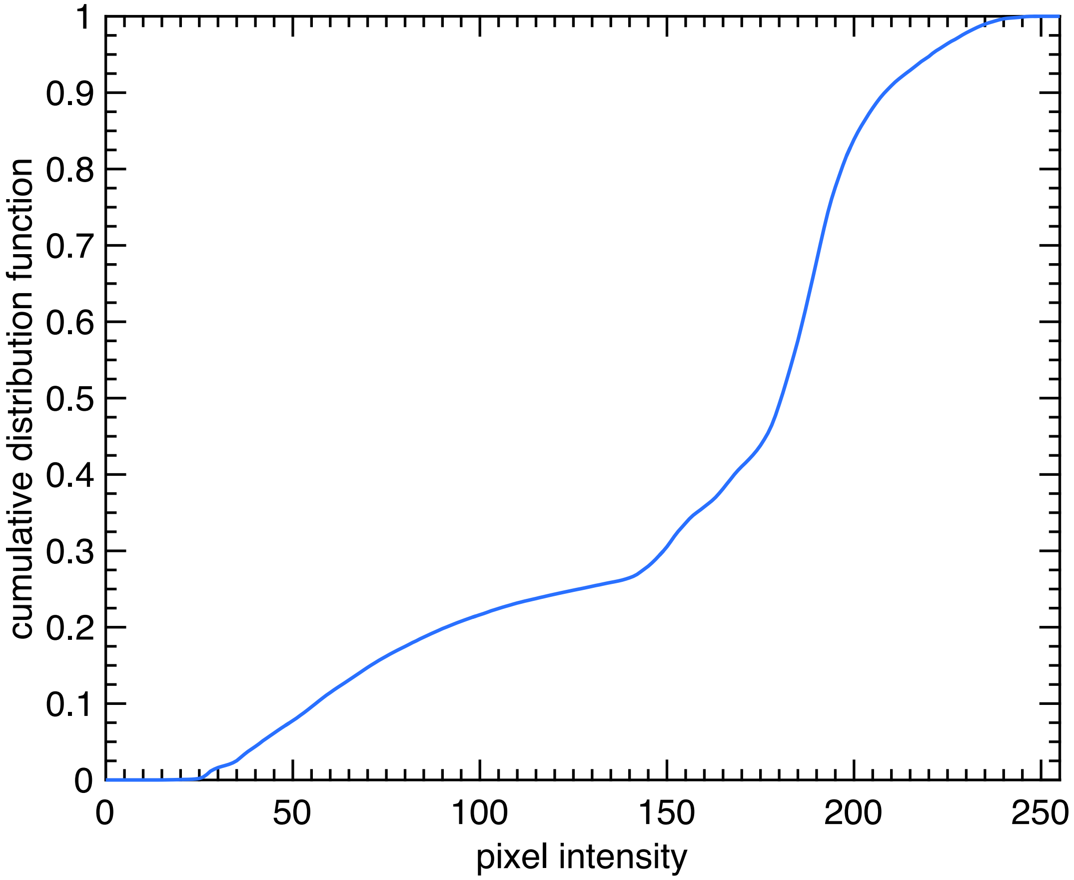
Hence the mapping that transforms the image to a graded image with uniform histogram is:

$$\boxed{t(u) = F_I(u)}$$ and $t(u) = 255 \times F_I(u)$ when $v \in [0, 255]$

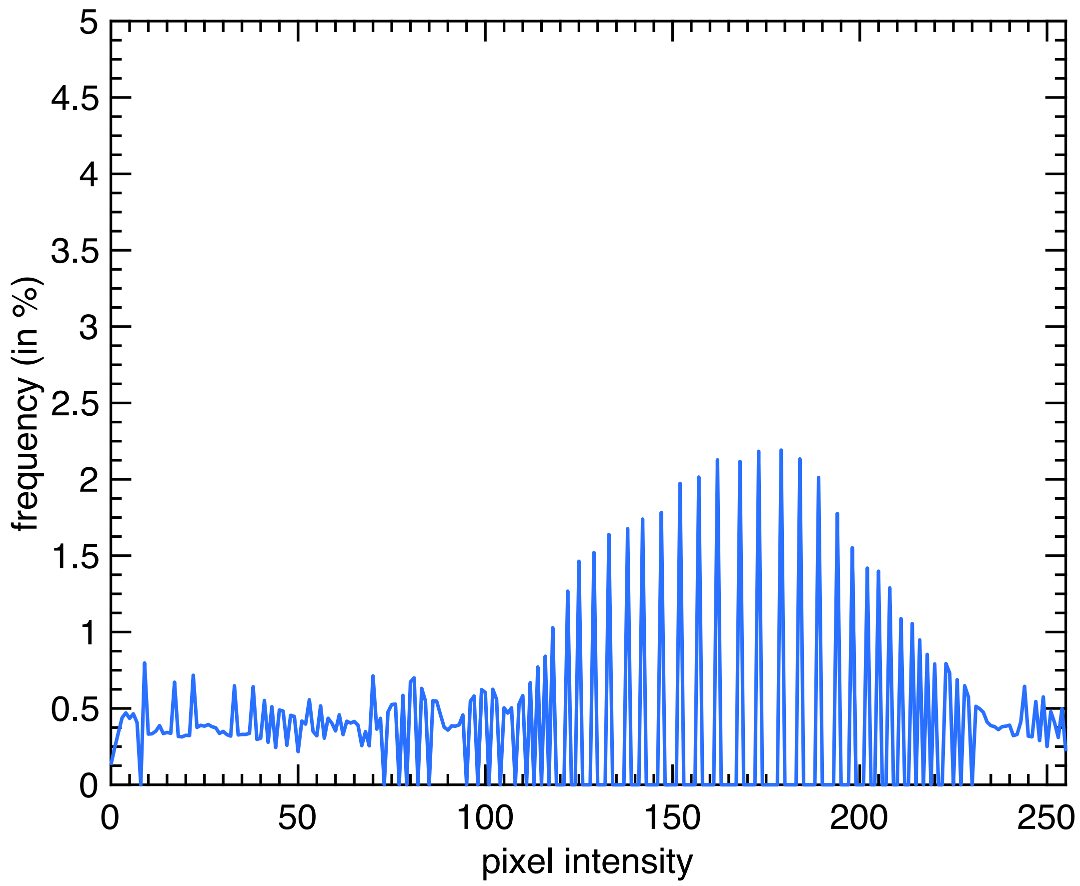
Histogram Equalisation

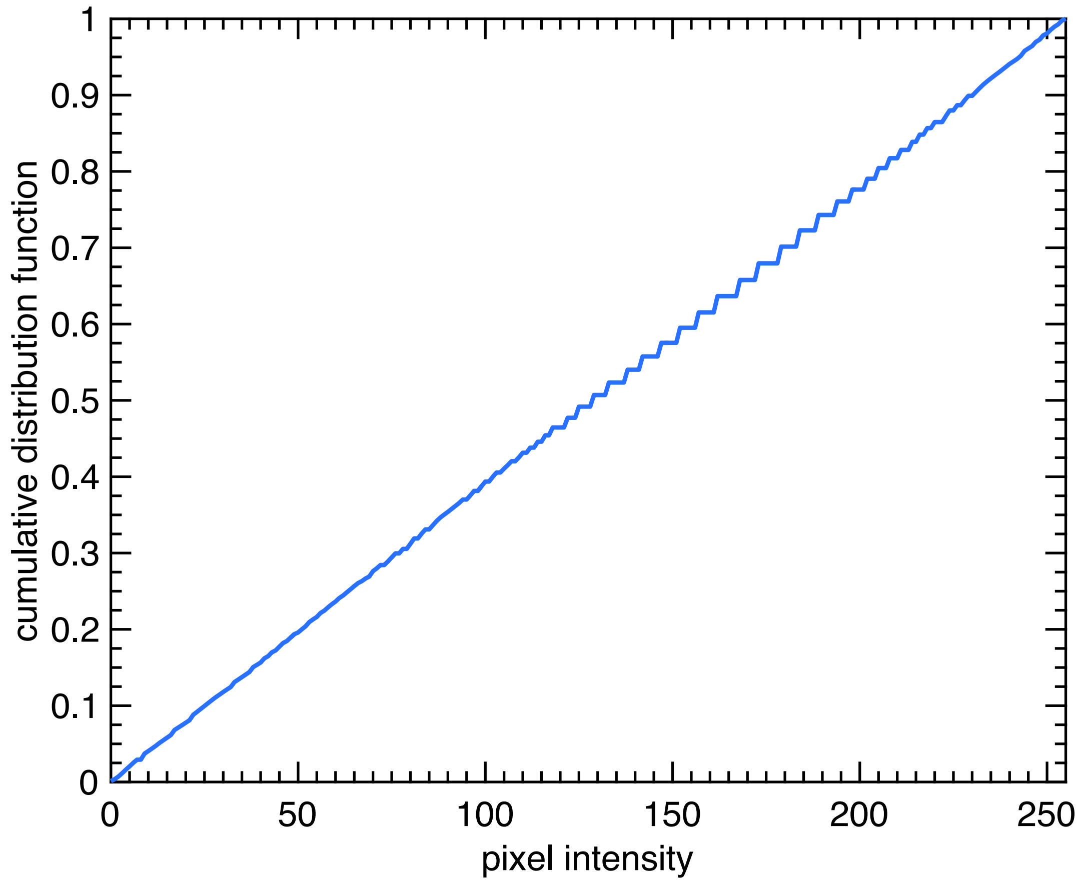
Original Image





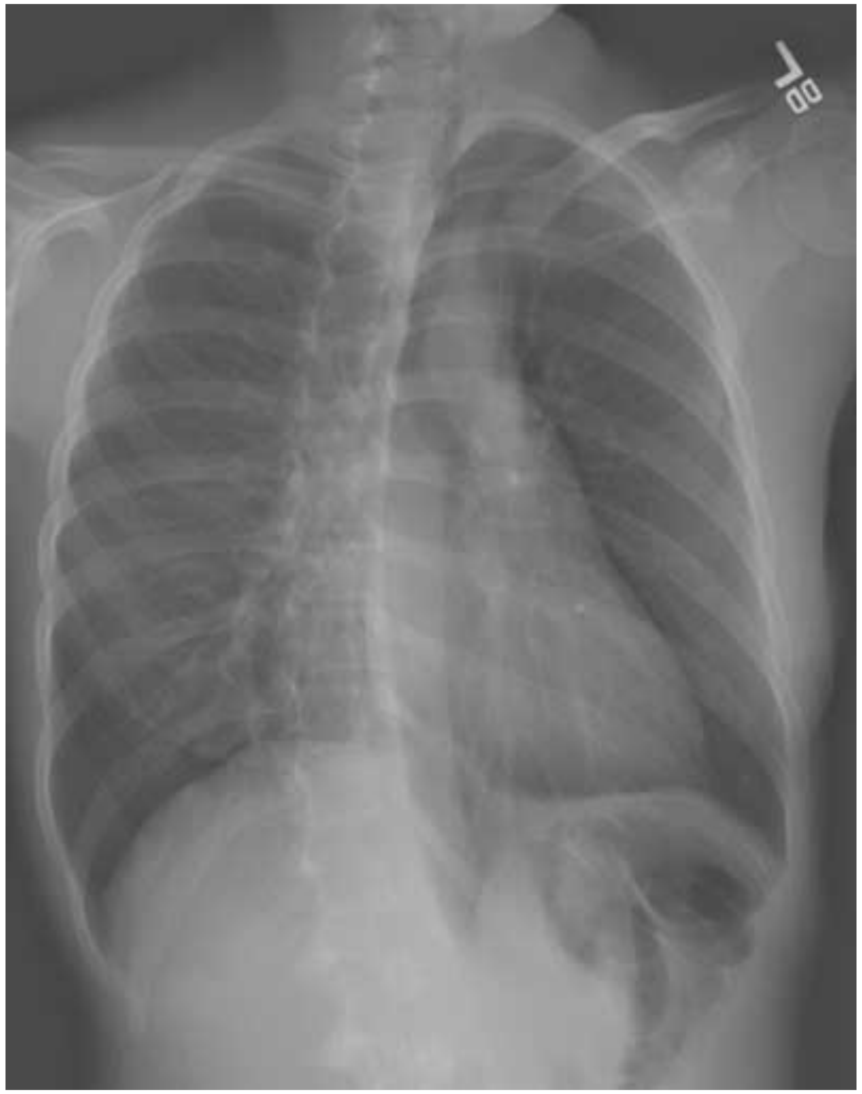
Histogram Equalisation





Histogram Equalised

Histogram Equalisation



Histogram equalisation is commonly applied to improve the contrast of medical images such as x-rays

Colour Mapping & Noise

Colour Mapping & Noise Variance

Applying a colour mapping has a direct impact on the observed noise levels. A good measure of noise on flat areas is the variance:

$$\mathrm{var}(I) = \frac{1}{N}\sum_x \left(I(x) - \bar{I}\right)^2 \quad \text{with} \quad \bar{I} = \frac{1}{N}\sum_x I(x)$$

Typically this measure is taken on a small block (*eg.* 16×16).

Let's look at an affine map $t(u) = au + b$:

$$\begin{aligned}\mathrm{var}(t(I)) &= \frac{1}{N}\sum_x \left(aI(x) + b - \overline{t(I)}\right)^2 \quad \text{with} \quad \overline{t(I)} = \frac{1}{N}\sum_x aI(x) + b \\ &= a^2\mathrm{var}(I)\end{aligned}$$

Hence the noise increases with the gain of the mapping.

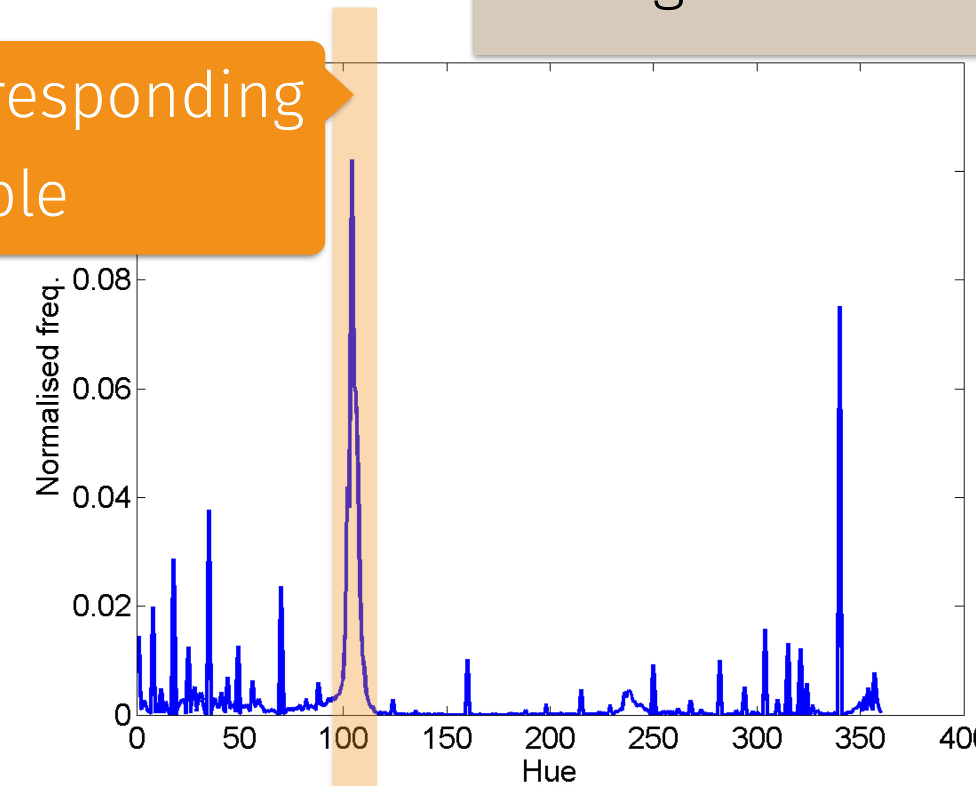
Colour Segmentation

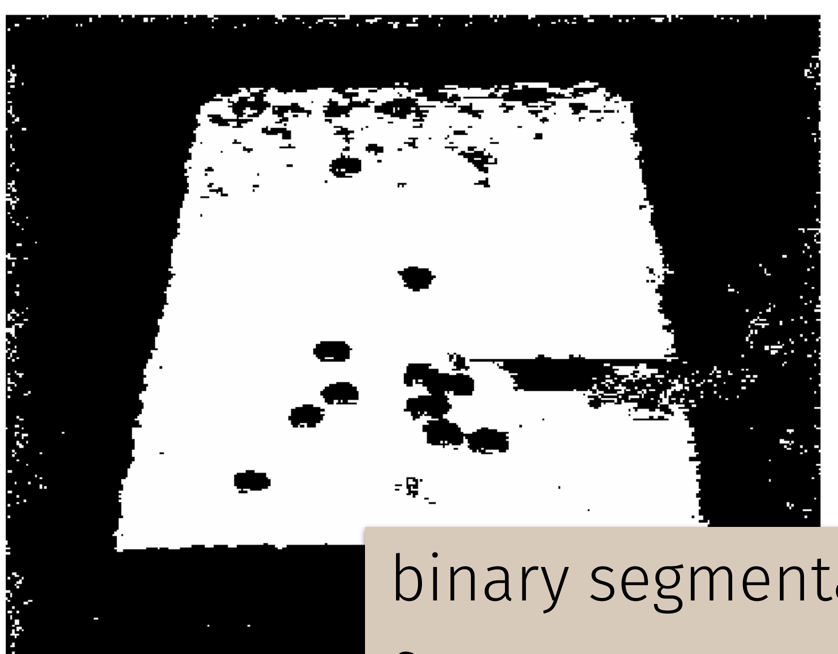
Say we want to isolate the table to do some further analysis.

Colour Segmentation

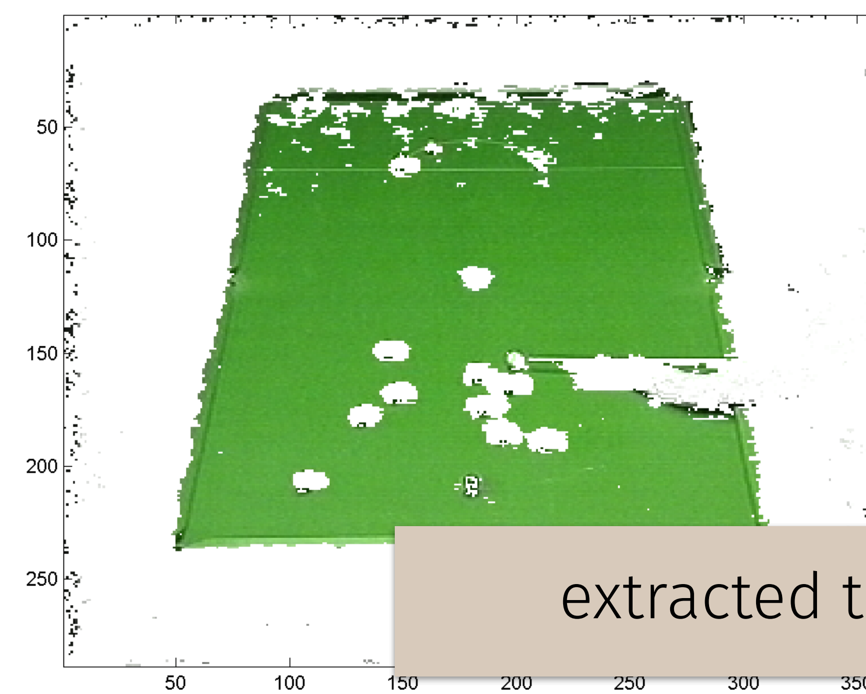
histogram of hues (HSV)

peak corresponding to the table

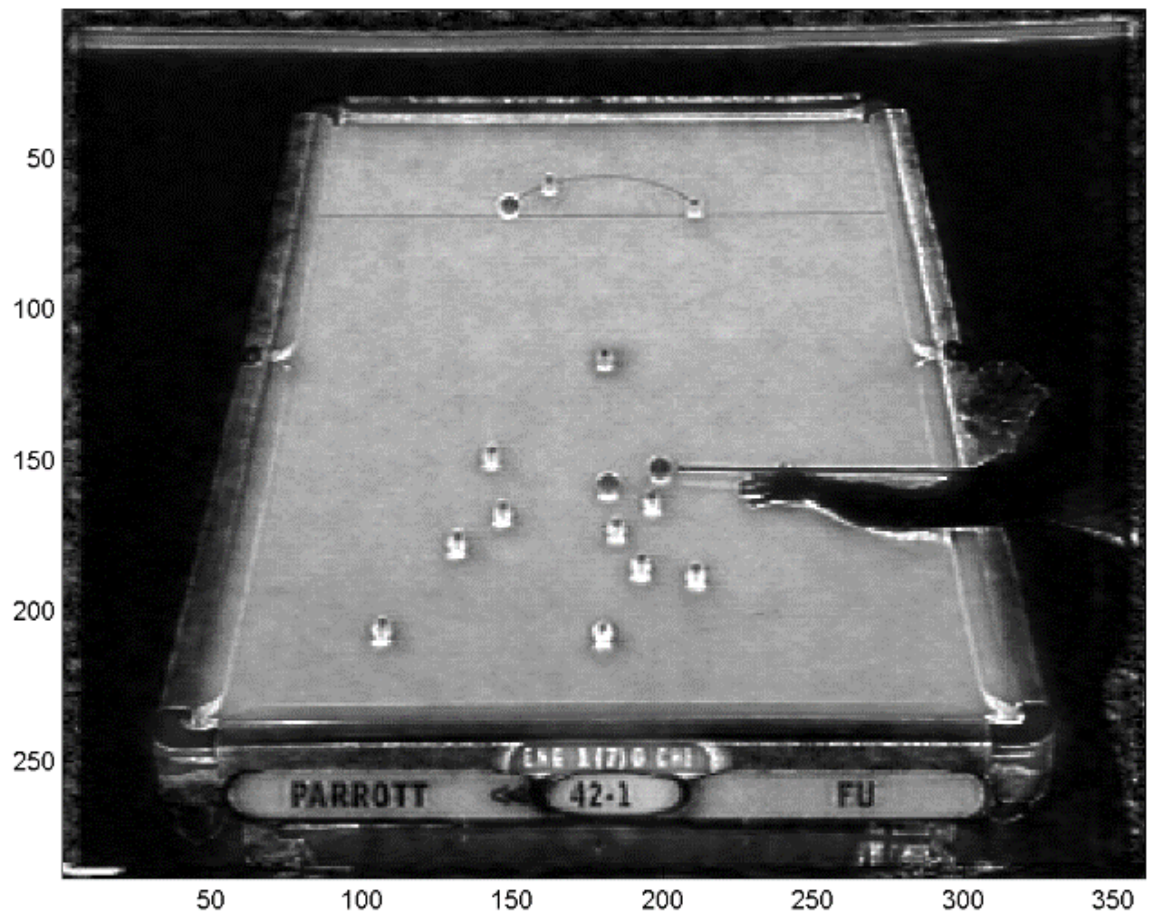


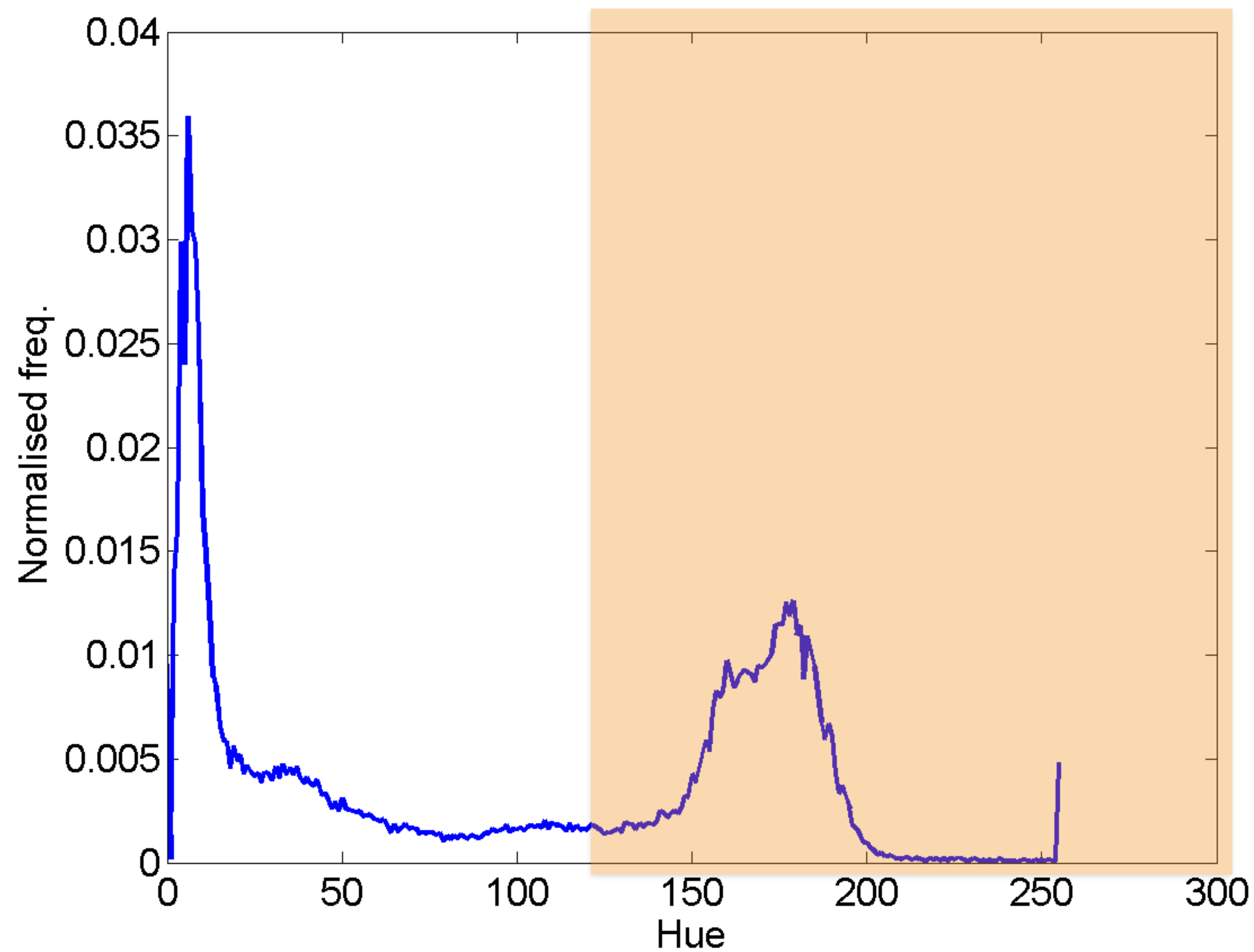


binary segmentation for t1=100, t2=110



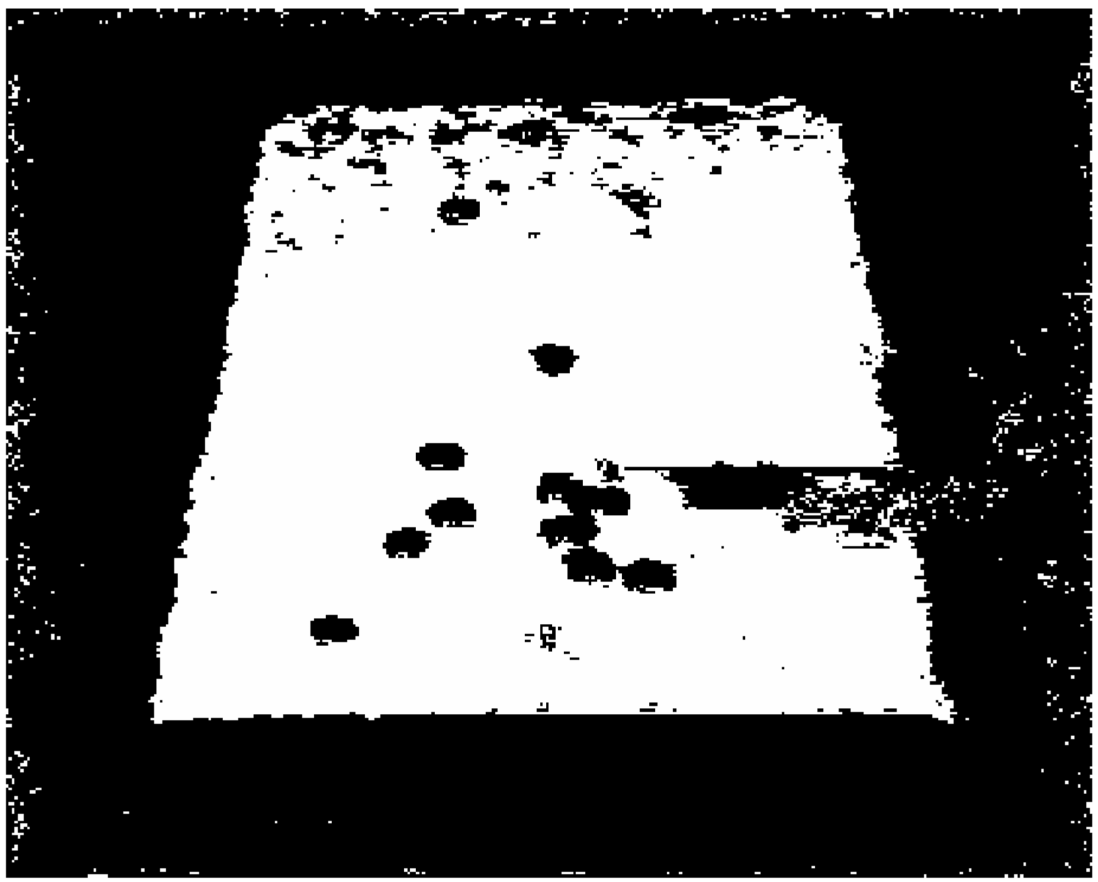
extracted table



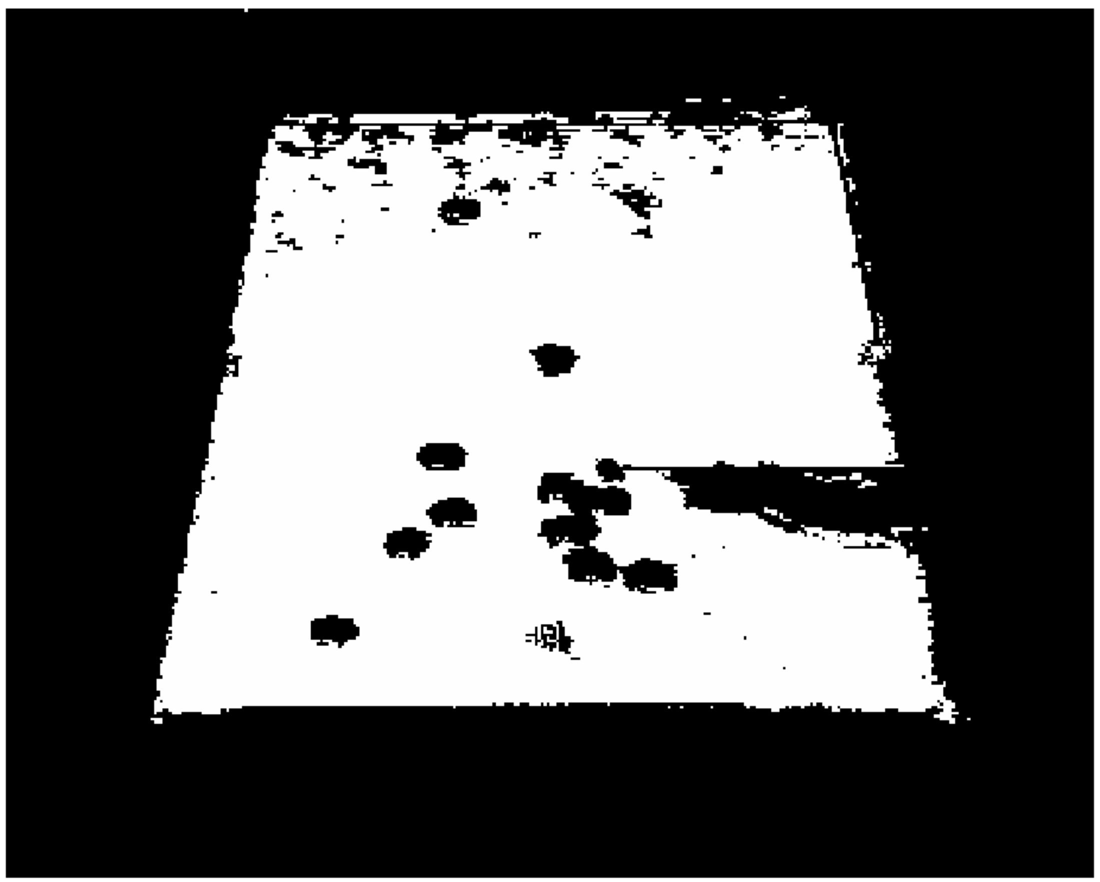


We can further refine the segmentation by discriminating the saturation values (saturation > 128).

Colour Segmentation



hue alone



hue & saturation

3D Histograms

3D Histogram

So far, we only considered histograms as 1D histograms, ie. histogram of the grayscale conversion (Y component of YUV), each colour channel independently.

In reality, histograms of colour images should be 3D histograms. Many colour segmentation algorithms account for this (eg. k-means, meanshift, etc.).

3D Histogram

What happens when the histograms are not 1D? For instance, we might consider a 2D histogram for measuring joint frequency of (U,V) chrominance pairs.

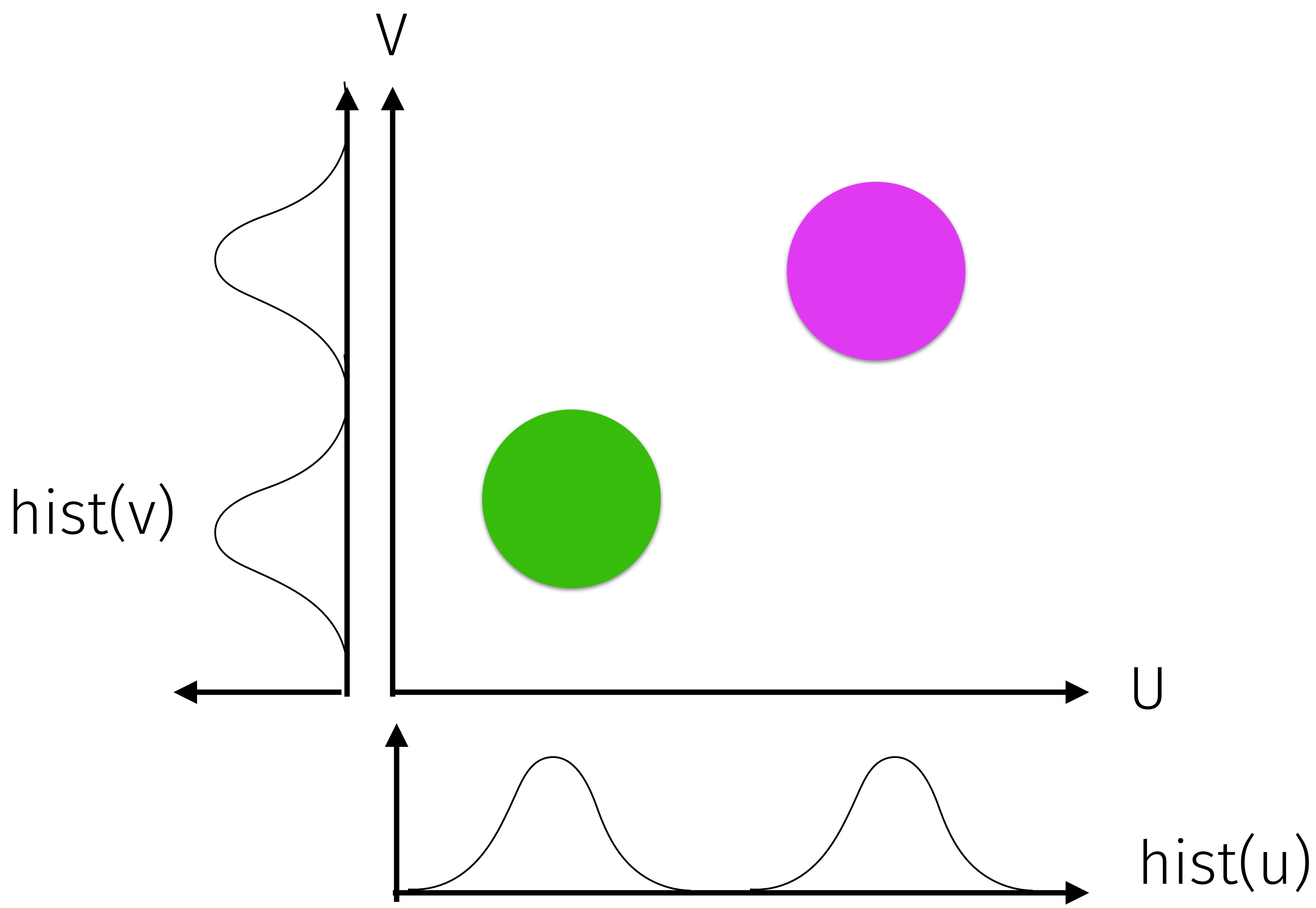
Well, things start to get more complicated. For instance, when looking at the relationship between histograms and colour mappings, the main problem is that the concept of monotonicity breaks down as you cannot rank multichannel data.

3D Histogram

Also, a common mistake is to believe that marginal 1D histograms, i.e. histograms taken separately on each channel, give a complete picture of the colour distribution of an image.

Here is why this is not the case...

Two pictures with very different colour palettes, can still have identical marginal 1D histograms.



3D Histogram

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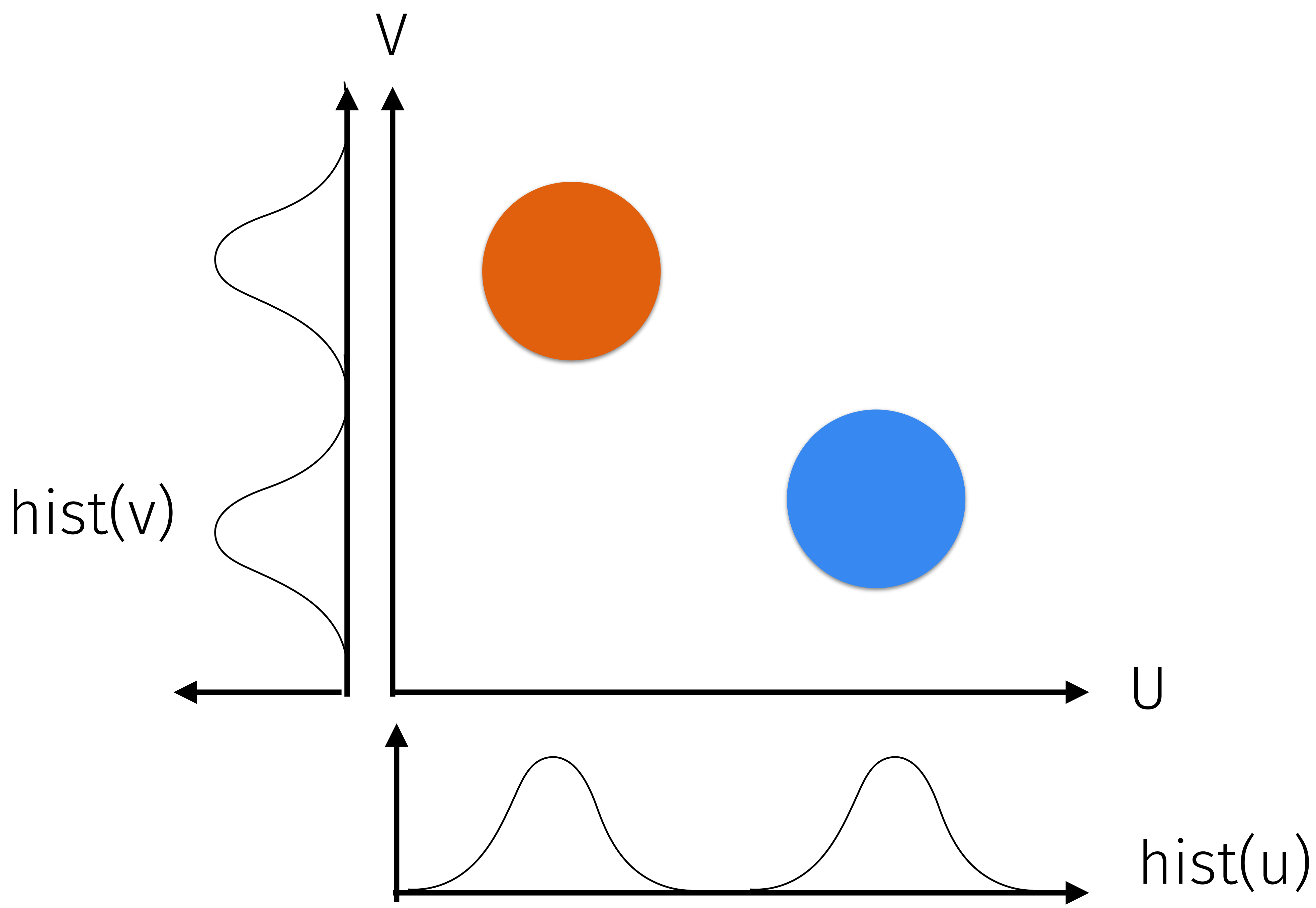

Conclusion

TAKE OUT

Applying a colour transformation has an impact on the histogram. The change manifests as a warp in the cumulative histogram.

Colour mapping can change the perceived contrast of an image. Maximum contrast can be achieved using histogram equalisation.

Applying a colour mapping has an impact on the noise levels.

Peaks in histograms usually correspond to semantic objects in a picture. Localising these peaks helps segmenting objects.

In colour images, histograms should really be 3D arrays, but these are hard to manipulate. Sometimes we use one 1D histogram per channel instead, but this is ambiguous.