A simple life insurance

LIFE INSURANCE PRODUCTS VALUATION IN R

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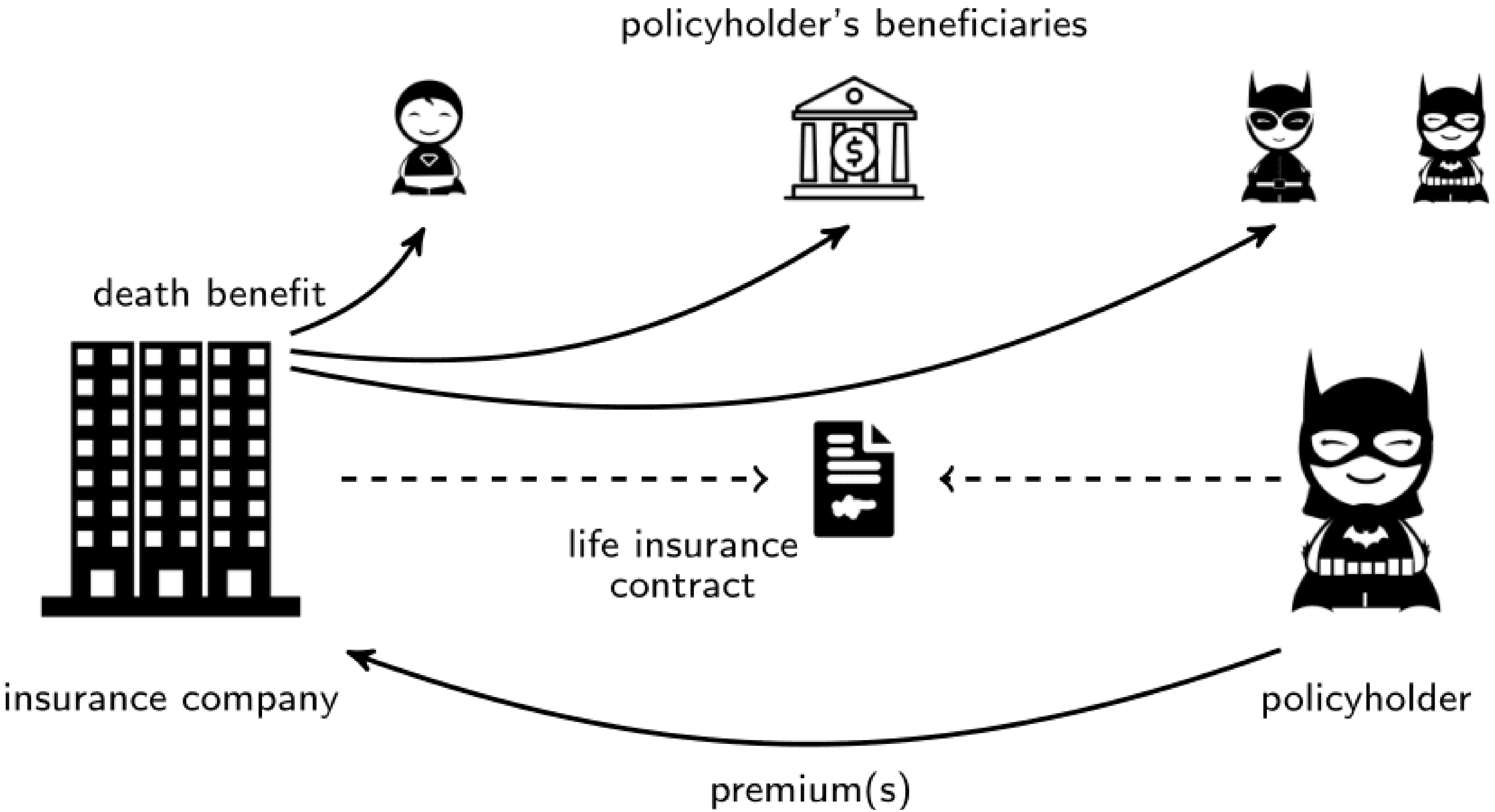
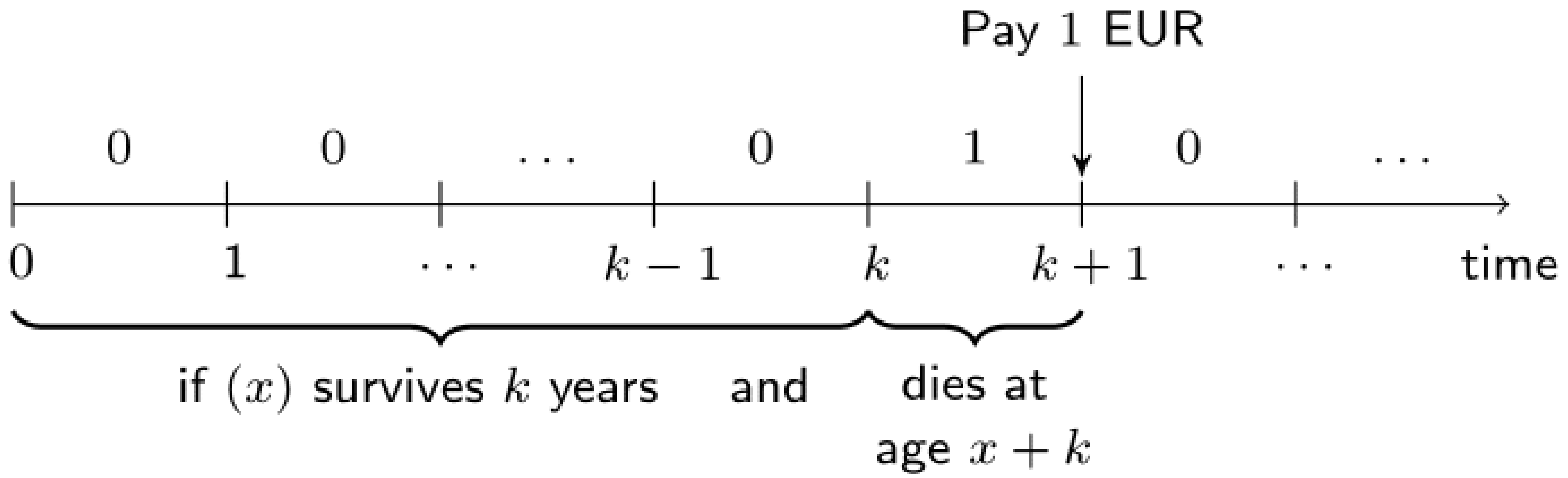
The life insurance



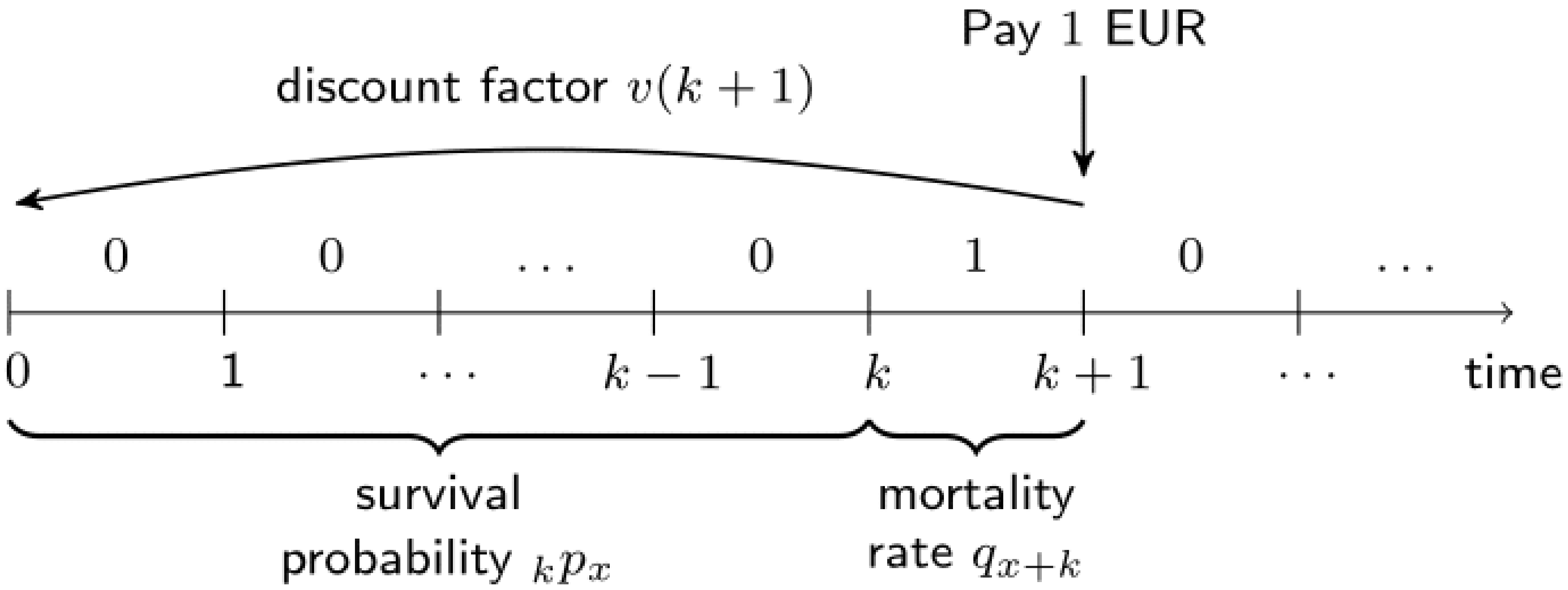
A simple life insurance

- The product is sold to (x) at time 0.



A simple life insurance

- **Expected Present Value:**



The EPV is

$$_{k|1}A_x = 1 \cdot \; v(k+1) \cdot \; _kp_x \cdot \; q_{x+k} = 1 \cdot \; v(k+1) \cdot \; _{k|}q_x \; .$$

A simple life insurance in R

Compute ${}_{5|1}A_{65} = 1 \cdot\ v(6) \cdot\ {}_{5|}q_{65} = 1 \cdot\ v(6) \cdot\ {}_{5}p_{65} \cdot\ q_{70}$ for constant $i = 3\%$.

```
# Mortality rates and one-year survival probabilities
qx <- life_table$qx
px <- 1 - qx
```

```
# 5-year deferred mortality probability of (65)
kpx <- prod(px[(65 + 1):(69 + 1)])
kqx <- kpx * qx[70 + 1]
kqx
```

```
0.02086664
```

A simple life insurance in R (cont.)

```
# Discount factor
discount_factor <- (1 + 0.03) ^ - 6
discount_factor
```

```
0.8374843
```

```
# EPV of the simple life insurance
1 * discount_factor * kqx
```

```
0.01747548
```

Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R

The whole, temporary and deferred life insurance

LIFE INSURANCE PRODUCTS VALUATION IN R

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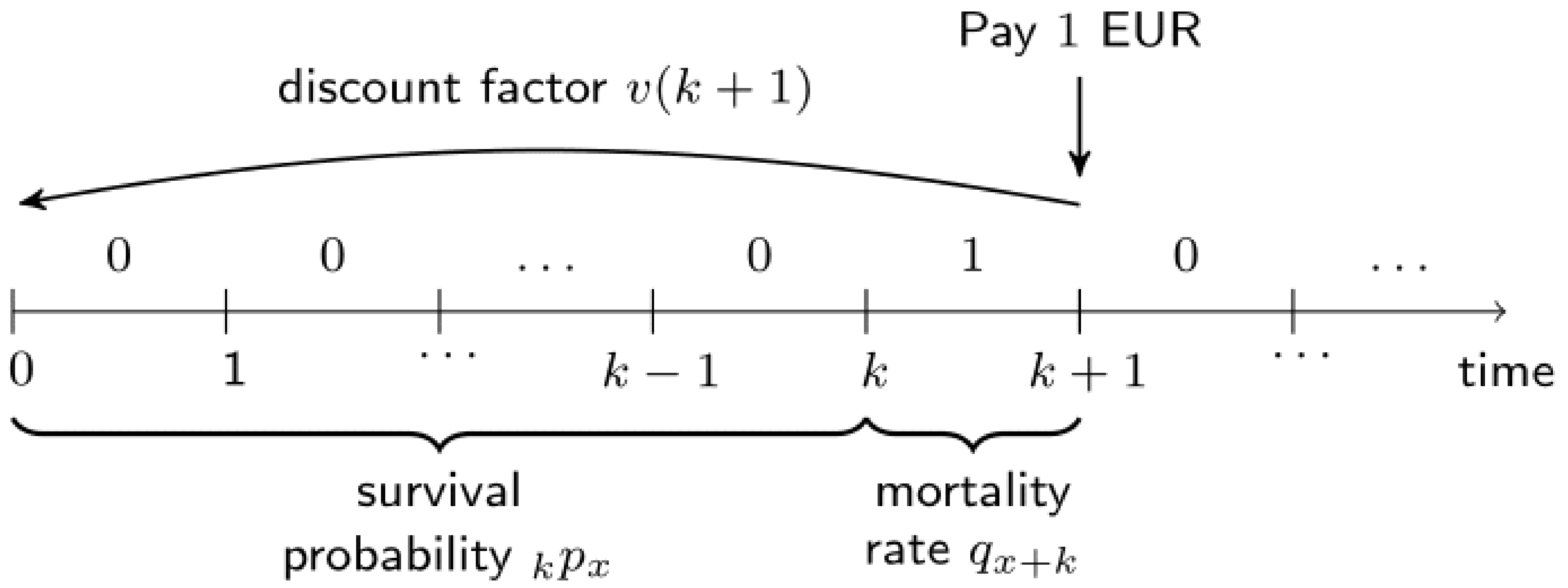
Professor, KU Leuven and University of Amsterdam

A series of one-year contracts



- What if?
  - The benefit is b_k EUR instead of 1 EUR?
  - A series of one-year contracts instead of just one?

General setting

- A life insurance on (x) with **death benefit** vector

$$(b_0, b_1, \ldots, b_k, \ldots)$$

- Series of **one-year** contracts:
  - Each with $b_k \cdot v(k+1) \cdot {}_kp_x \cdot q_{x+k}$ as Expected Present Value (EPV)
  - Together:

$$\sum_{k=0}^{+\infty} b_k \cdot v(k+1) \cdot {}_kp_x \cdot q_{x+k} = \sum_{k=0}^{+\infty} b_k \cdot v(k+1) \cdot {}_{k|}q_x$$

the EPV.

Whole life insurance

Whole life insurance: *lifelong*.

	1	1	1	1	1	...	benefits
0	1	2	3	4	5	...	time
	$v(1)$	$v(2)$	$v(3)$	$v(4)$	$v(5)$	...	discount factors
	q_x	${}_{1\|}q_x$	${}_{2\|}q_x$	${}_{3\|}q_x$	${}_{4\|}q_x$	...	deferred mortality probabilities

A_x for constant benefit of 1 EUR
and constant discount factor v

Temporary life insurance

Temporary (or: **term**) life insurance: maximum of n years.

	1	1	$\ldots$	1	1	0		benefits
0	1	2	$\ldots$	$n-1$	n	$n+1$		time
	$v(1)$	$v(2)$	$\cdots$	$v(n-1)$	$v(n)$			discount factors
	q_x	$_{1\|}q_x$	$\cdots$	$_{n-2\|}q_x$	$_{n-1\|}q_x$			deferred mortality probabilities

$A^{\;1}_{x:\overline{n}|}$ for constant benefit of 1 EUR and constant discount factor v

Deferred whole life insurance

Deferred whole life insurance: no payments in first u years.

0	$\ldots$	0	1	1	$\ldots$		benefits
0	1	$\ldots$	u	$u+1$	$u+2$	$\ldots$	time
				$v(u+1)$	$v(u+2)$	$\cdots$	discount factors
				${}_{u\vert}q_x$	${}_{u+1\vert}q_x$	$\cdots$	deferred mortality probabilities

$${}_{u|}A_x$$

for constant benefit of 1 EUR

and constant discount factor v

Life insurances in R

Compute A_{35} for constant interest rate $i = 3\%$.

```
# Whole-life insurance of (35)
kpx <- c(1, cumprod(px[(35 + 1):(length(px) - 1)
kqx <-  kpx * qx[(35 + 1):length(qx)]
discount_factors <- (1 + 0.03) ^ - (1:length(kqx
benefits <- rep(1, length(kqx))
sum(benefits * discount_factors * kqx)
```

```
0.2880872
```

Now do ${}_{20|}A_{35}$.

```
# Deferred whole-life insurance of (35)
kpx <- c(1, cumprod(px[(35 + 1):(length(px) - 1)
kqx <-  kpx * qx[(35 + 1):length(qx)]
discount_factors <- (1 + 0.03) ^ - (1:length(kqx
benefits <- c(rep(0, 20), rep(1, length(kqx) - 2
sum(benefits * discount_factors * kqx)
```

```
0.2552956
```

Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R

Combined benefits

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Endowment insurance

	1	1	$\cdots$	1	1	death benefits
0	0	0	$\ldots$	0	1	pure endowment
0	1	2	$\ldots$	$n-1$	n	time
	$v(1)$	$v(2)$	$\cdots$	$v(n-1)$	$v(n)$	discount factors
	q_x	$_{1\|}q_x$	$\cdots$	$_{n-2\|}q_x$	$_{n-1\|}q_x$	deferred mortality probabilities
					$_np_x$	survival probability

$A_{x:\overline{n}|}$ for constant benefit of 1 EUR and constant discount factor v

Sending baby Incredible to college

Mrs. Incredible is 35 years old.

She wants to **save money** to send her baby to college. She needs 75,000 EUR when he gets 18.

Given her dangerous lifestyle as a superhero, at the same time **she wants to cover her life**.

The sum insured is 50,000 euro.

Can you design this type of life insurance policy?

Sending baby Incredible to college pictured

35	36	37	...	52	53	age
0	1	2	...	17	18	time
	$50 \cdot 10^3$	$50 \cdot 10^3$	$\cdots$	$50 \cdot 10^3$	$50 \cdot 10^3$	death benefits
0	0	0	$\cdots$	0	$75 \cdot 10^3$	pure endowment
P	P	P	$\cdots$	P	0	premiums

Sending baby Incredible to college in R

- She is 35-years-old, living in Belgium, year 2013.
- Interest rate is 3%.

```
i <- 0.03
```

- Death benefits (using the deferred mortality probabilities q_{35}, ${}_{1|}q_{35}$ to ${}_{17|}q_{35}$)

```
kqx <-  c(1, cumprod(px[(35 + 1):(51 + 1)])) * qx[(35 + 1):(52 + 1)]
discount_factors <- (1 + i) ^ - (1:length(kqx))
benefits <- rep(50000, length(kqx))
EPV_death_benefits <- sum(benefits * discount_factors * kqx)
EPV_death_benefits
```

```
870.8815
```

Sending baby Incredible to college in R

- Pure endowment (using the survival probability ${}_{18}p_{35}$)

```
EPV_pure_endowment <- 75000 * (1 + i) ^ - 18 * prod(px[(35 + 1):(52 + 1)])
EPV_pure_endowment
```

```
42975.86
```

- Premium pattern rho (using the survival probabilities ${}_{0}p_{35}$ to ${}_{17}p_{35}$)

```
# Premium pattern rho
kpx <- c(1, cumprod(px[(35 + 1):(51 + 1)]))
discount_factors <- (1 + i) ^ - (0:(length(kpx) - 1))
rho <- rep(1, length(kpx))
EPV_rho <- sum(rho * discount_factors * kpx)
EPV_rho
```

```
14.06193
```

Sending baby Incredible to college in R

- Actuarial equivalence

$$P = \frac{\text{EPV}(\text{death benefits}) + \text{EPV}(\text{pure endowment})}{\text{EPV}(\text{rho})}.$$

```
# Premium level
(EPV_death_benefits + EPV_pure_endowment) / EPV_rho
```

```
3118.116
```

Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R

Congratulations!

LIFE INSURANCE PRODUCTS VALUATION IN R

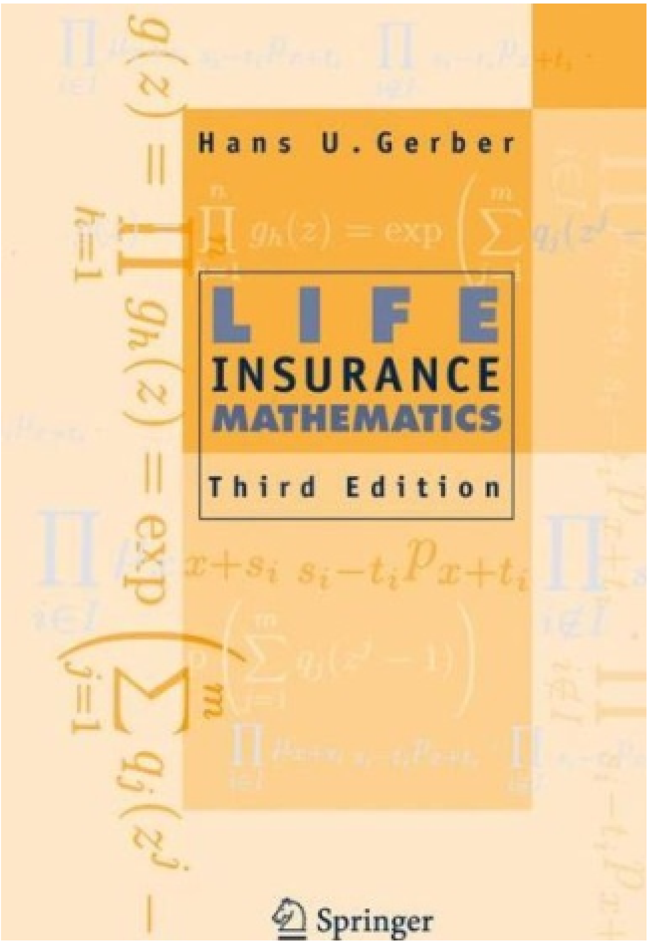
Katrien Antonio and Roel Verbelen

Professor, KU Leuven and University of Amsterdam Postdoctoral researcher,

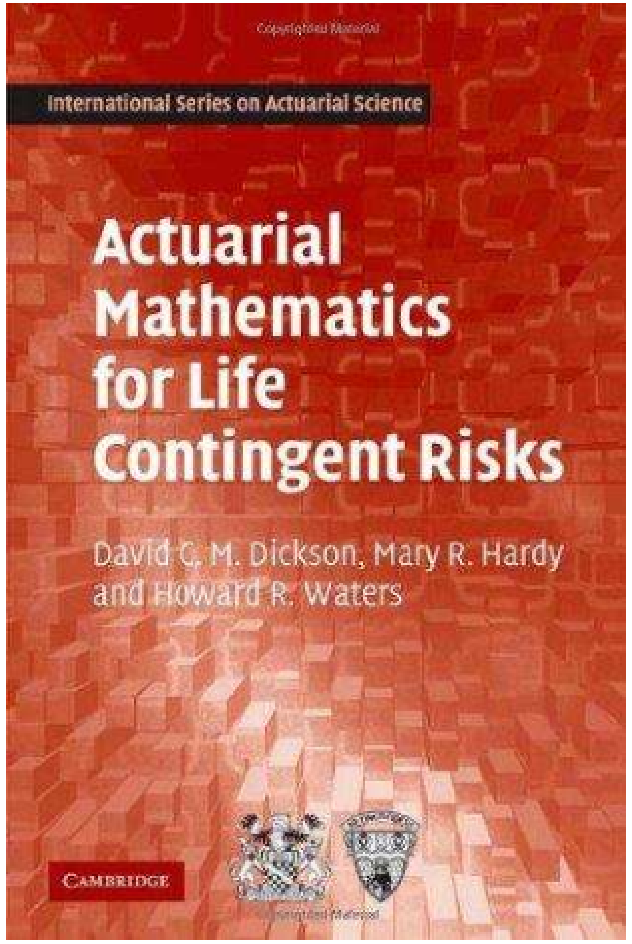
What you've learned

- Valuation of cash flows
- Life tables
- Life annuities
- Life insurances

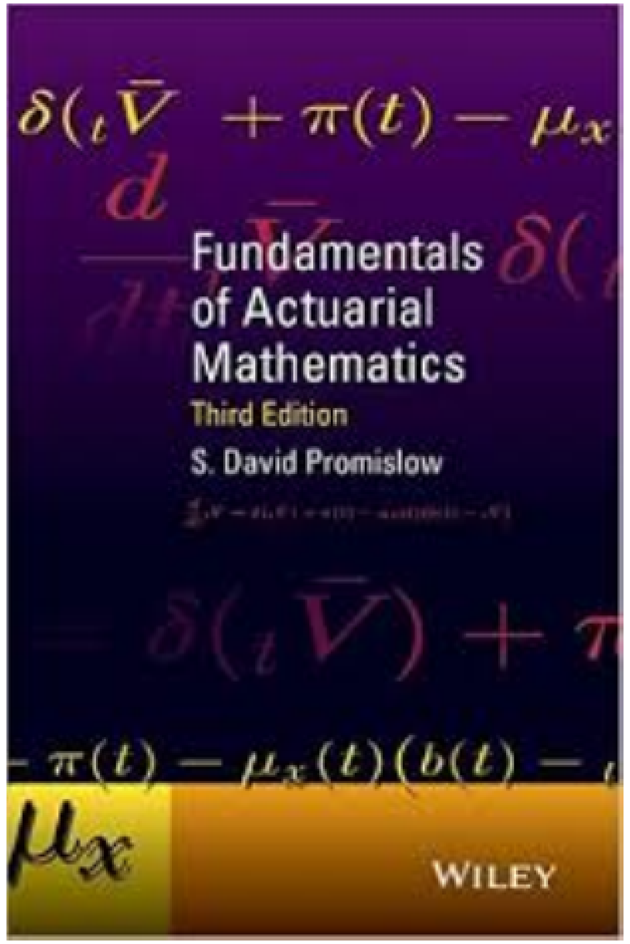
Want to know more?



Gerber
(1997, Springer)



Dickson, Hardy & Waters
(2013, Cambridge University Press)



Promislow
(2015, Wiley)

What else is there?

- More advanced life insurance products.
- Loss models for frequencies and severities.
- Data science in insurance.

Enjoy your journey as an actuary!

LIFE INSURANCE PRODUCTS VALUATION IN R