Unit - 3

Matrices and Determinants

Important Points

- **Matrix :**

  Any rectangular array of numbers is called matrix. A matrix of order $m \times n$ having m rows and n columns. Its element in the i^{th} row and j^{th} column is a_{ij}. We denote matrix by A, B, C etc.

  $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ is a matrix of order 2×2.

  $\begin{bmatrix} -1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ is matrix of order 2×3.

  A matrix of order $m \times n$ is $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

- **Algebra of Matrices**

  (1) Equality : If $A = \left[a_{ij}\right]_{m \times n}$, $B = \left[b_{ij}\right]_{p \times q}$ are said to be equal i.e. A = B if

  (i) $a_{ij} = b_{ij} \quad \forall$ i & j

  (ii) order of A = order of B, i.e. m = p and n = q

- **Types of Matrices :** Let $A = \left[a_{ij}\right]_{m \times n}$

  (1) **Row matrix :** A $1 \times n$ matrix $[a_{11} \ \ a_{12} \ \ a_{13} \ \ ..... \ \ a_{1n}]$ is called a row matrix (row vector)

  (2) **Column matrix :** A $m \times 1$ matrix $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix}$ is called column matrix

  (Column vector)

(3) **Square Matrix :** An n × n matrix is called a square matrix.

(4) **Diagonal matrix :** If in a square matrix $A = [a_{ij}]_{n \times n}$ we have $a_{ij} = 0$ whenever $i \neq j$ then A is called a diagonal matrix.

(5) **Zero (null) matrix :** A matrix with all elements are zero is called zero (null) matrix. It is denoted by $[0]_{m \times n}$ or $O_{m \times n}$ or O.

- **Algebra of Matrices**

(2) Sum and Difference : If A and B are of same oder

i.e. $A = \left[a_{ij}\right]_{m \times n}$, $B = \left[b_{ij}\right]_{m \times n}$, $C = \left[c_{ij}\right]_{m \times n}$

then $A + B = C \Rightarrow \left[a_{ij} + b_{ij}\right]_{m \times n} = \left[c_{ij}\right]_{m \times n}$

$A - B = C \Rightarrow \left[a_{ij} - b_{ij}\right]_{m \times n} = \left[c_{ij}\right]_{m \times n}$

Properties of addition

If matrices A, B, C and O are of same order, then

(i) A + B = B + A (Commutative law)

(ii) A + (B + C) = (A + B) + C (Associative law)

(iii) A + O = O + A (Existence of Identity)

(iv) (-A) + A = A + (-A) = O (Existence of Inverse)

(3) Product of Matrix with a Scalar

If $A = \left[a_{ij}\right]_{m \times n}$ and $k \in R$ then we define product of matrix with a scalar is

$kA = k\left[a_{ij}\right]_{m \times n} = \left[ka_{ij}\right]_{m \times n}$

Properties of Addition of Matrices and Multiplication of a Matrix by a scalar

Let $A = \left[a_{ij}\right]_{m \times n}$, $B = \left[b_{ij}\right]_{m \times n}$, $k, l \in R$

(i) k(A + B) = kA + kB

(ii) $(k + l)$ A = kA + lA

(iii) (kl)A = k(lA)

(iv) 1A = A

(v) (-1) A = -A

(4) Matrix Multiplication :

Let $A = \left[a_{ij}\right]_{m \times n}$, $B = \left[b_{ij}\right]_{n \times p}$. Then AB = C where $C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + ... + a_{in}b_{nj}$

= Scalar product of i^{th} row of A and j^{th} Column of B.

(i) Product AB defined if and only if number of column of A = number of rows of B

(ii) If A is $m \times n$ matrix and B is $n \times p$ matrix then AB is $m \times p$ matrix.

- **Properties of Matrix Multiplication**

Let the matrices A, B, C and O have order compitable for the operations involved.

(i) $A(B + C) = AB + AC$

(ii) $(A + B)C = AC + BC$

(iii) $A(BC) = (AB) C$

(iv) $AO = O = OA$

(v) $AB \neq BA$, generally

(vi) $AB = O$ need not imply $A = O$ or $B = O$

(vii) $AB = AC$ need not imply $B = C$

- **Types of Matrices**

(6) Identity (unit) Matrix : In a diagonal matrix all elements of principal diagonal are 1 is called Identity (unit) Matrix and is denoted by I or I_n or $I_{n \times n}$.

(7) Scalar Matrix : If $k \in R$, then kI called a scalar matrix.

(8) Traspose of a Matrix : If all the rows of matrix $A = [a_{ij}]_{m \times n}$ are converted into corresponding column, the matrix so obtained is called the transpose of A. It is denoted by A^T or A'. $A^T = [a_{ji}]_{n \times m}$

Properties of Transpose

(i) $(A^T)^T = A$

(ii) $(A + B)^T = A^T + B^T$

(iii) $(kA)^T = kA^T$, $k \in R$

(iv) $(AB)^T = B^T A^T$

(9) Symmetric Matrix : For a square matrix A, if $A^T = A$, then A is called a symmetric matrix. Here $a_{ij} = a_{ji}$ for all i and j.

(10) Skew - Symmetric Matrix : For a square matrix A, if $A^T = -A$, then A is called a

Skew - symemtric matrix.

Here $a_{ij} = -a_{ji}$ for all i and j and $a_{ii} = 0 \ \forall \ i$

For square matrix A, $A + A^T$ is symmetric and $A - A^T$ is skew - symmetric matrix.

(11) Triangular Matrices :

(i) Upper Triangular Matrix : A sqare matrix whose element $a_{ij} = 0$ for $i > j$ is called an uppear triangular matrix.

e.g. $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}, \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

(ii) Lower Triangular Matrix : A square matrix whose element $a_{ij} = 0$ for $i < j$ is called a lower triangular matrix.

e.g. $\begin{bmatrix} a & 0 \\ b & c \end{bmatrix}, \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$

Let A be a square matrix of order $n \times n$.

(12) Orthogonal matrix : A is called an orthogonal matrix if and only if $A^TA = I_n = AA^T$

(13) Idempotent Matrix : A is called an idempotgent matrix if $A^2 = A$

(14) Nilpotent Matrix : A is called a nilpotent matrix if $A^m = 0$, $m \in Z^+$

(15) Involutary Matrix : A is called an involutary matrix if $A^2 = I$, i.e. $(I + A)(I - A) = O$

(16) Conjugate of a Matrix : If $A = [a_{ij}]$ is a given matrix, then the matrix obtained on replacing all its elements by their corresponding complex cojugates is called the conjugate of the matrix A and is denoted by $\overline{A} = \left[\overline{a}_{ij}\right]$

Properties :

(i) $\overline{(\overline{A})} = A$

(ii) $\overline{(A + B)} = \overline{A} + \overline{B}$

(iii) $\overline{(kA)} = \overline{k} \cdot \overline{A}$, k being a complex number

(iv) $\overline{AB} = \overline{A} \cdot \overline{B}$

(17) Conjugate Transpose of a matrix : The conjugate of the transpose of a given matrix A is called the conjugate transpoe (Tranjugate) of A and is denoted by A^{θ}.

Properties :

(i) $A^\theta = \overline{(A^T)} = (\overline{A})^T$

(ii) $(A^\theta)^\theta = A$

(iii) $(A + B)^\theta = A^\theta + B^\theta$

(iv) $(kA)^\theta = \overline{k} \cdot A^\theta,$ k being a complex number

(v) $(AB)^\theta = B^\theta \cdot A^\theta$

Let A be a square matrix of oder $n \times n$

(18) Unitary Matrix : A is an unitary matrix if $AA^\theta = I_n = A^\theta A$.

(19) Hermitian Matrix : A is a hermitian matrix if $A^\theta = A$

(20) Skew - Hermitian Matrix : A is a skw-Hermitian matrix if $A^\theta = -A$

- **The determinant of a square matrix :**

If all entries of a square matrix are kept in their respective places and the determinant of this array is taken, then the detrminant so obtained is called the determinant of the given square matrix. If A is a square matrix, then determinant of A is denoted by | A | or det A.

Evaluation of Determinants (Expansion)

Second order determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Third order determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

$= a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$

$= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1$

Some Symbols :

(1) $R_i \to C_i$: To convert every row (column) into corresponding column (row)

(2) R_{ij} $[c_{ij}]$ $(i \neq j)$: Interchange of i^{th} row (column) and j^{th} row (column)

(3) $R_i(k)$ $[c_i(k)]$: multiply i^{th} row (Column) by $k \in R - \{0\}$

(4) $R_{ij}(k)[c_{ij}(k)]$: Multiply i^{th} row (column) by $k \in R$ $(k \neq 0)$ and adding to the corresponding elements of j^{th} row(column)

Properties of Determinants of Matrices

(i) $|A^T| = |A|$

(ii) $|AB| = |A||B|$

$|ABC| = |A|\ |B|\ |C|$

(iii) $|kA| = k^n |A|$ (where A is $n \times n$ matrix)

(iv) $|I| = 1$

Value of some Determinants :

(i) Symmetric Determinant $\begin{vmatrix} x & p & q \\ p & y & r \\ q & r & z \end{vmatrix} = xyz + 2pqr - xp^2 - yq^2 - zr^2$

(ii) Skew - symmetric determinant of odd order : $\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ y & -z & 0 \end{vmatrix} = 0$

(iii) Circular Determinant : $\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = -\left(x^3 + y^3 + z^3 - 3xyz\right)$

Area of a Triangle :

If the vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3) then,

Area of a triangle = $\Delta = \frac{1}{2}|D|$, where $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Shifting of origin does not effect the area of a triangle.

If $D = 0 \Leftrightarrow$ all three points are collinear

Let the sides of the triangle be $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$

$\therefore$ Area of a triangle $= \dfrac{\Delta^2}{2|C_1C_2C_3|}$,

where C_1, C_2, C_3 are respetively the cofactors of c_1, c_2 and c_3 and $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

If A(x_1, y_1) and B(x_2, y_2) are two distinct points of $\overleftrightarrow{AB}$ then the cartesian equation of $\overleftrightarrow{AB}$ is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Propeties of Determiants : (D = value or determinante)

(i) If a row (column) is a zero vector (i.e. all elements of a row or a column are zero), then D = 0

(ii) If two rows (Columns) are identical, then D = 0

(iii) If any two rows (columns) are interchanged, then D becoms - D (additive inverse of D)

(iv) If any two rows (columns) are interchanged, D is unchaged $\Rightarrow |A^T| = |A|$

(v) $\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(vi) $\begin{vmatrix} a_1 + d_1 & b_1 + e_1 & c_1 + f_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & e_1 & f_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(vii) If any rows (columns) is multiplied by $k \in R$ $(k \neq 0)$ and added to another rows (columns), then D is unchanged.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + ka_2 & b_1 + kb_2 & c_1 + kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(vii) All rows of a determinant are converted into corresponding column, D is unchanged.

(viii) Determinants are multiplied in the same way as we multiply matrices.

$$\therefore\ |AB| = |A|\ |B| = |BA| = |AB^T| = |A^TB| = |A^TB^T|$$

(ix) $\Delta = \begin{vmatrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \end{vmatrix}$, where f_r, g_r, h_r are functions of x for $r = 1, 2, 3$.

$$\therefore \frac{d\Delta}{dx} = \begin{vmatrix} f_1' & g_1' & h_1' \\ f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & g_1 & h_1 \\ f_2' & g_2' & h_2' \\ f_3 & g_3 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_3' & g_3' & h_3' \end{vmatrix}$$

(x) Let D(x) be a 3 × 3 determinant whose elements are polynomials.

If D(m) has two identical rows (columns), then x - m is a factor of D(x)

If D(m) has three identical rows (columns), then $(x - m)^2$ is a factor of D(x).

- **Minor and cofactor**

Let $A = \left[a_{ij}\right]_{3 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

The minor of the element a_{ij} (i, j = 1, 2, 3) in A

= M_{ij} = The determiant obtained from A on deleting the row and the column in which a_{ij} occurs.

The cofactor of the element *aij* (*i, j* = 1, 2, 3) in A = $A_{ij} = (-1)^{i+j} M_{ij}$

The value of any third order determinant can be obtained by adding the products of the elements of any of its rows (columns) by their correspdong co-factor.

If we multiply all the elements of any rows (columns) of any third order determinant by the cofactos of the corresponding elements of another row (column) and add the products, then the sum is zero.

or in Mathematical notation

$$\sum_{j=1}^{3} a_{ij} A_{kj} = \begin{cases} A & \text{if } i = k = 1, 2, 3 \\ 0 & \text{if } i \neq k = 1, 2, 3 \end{cases} \qquad \sum_{i=1}^{3} a_{ij} A_{ik} = \begin{cases} A & \text{if } j = k = 1, 2, 3 \\ 0 & \text{if } j \neq k = 1, 2, 3 \end{cases}$$

- **Adjoint of Matrix**

Adjoint Matrix of A = *adj* A $= \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

= Tranpose of the matrix of cofactor = $[A_{ji}]_{3 \times 3}$

If $A = [a_{ij}]_{n \times n}$ then *adj* $A = [A_{ji}]_{n \times n}$

To obtain the adjoint of 2 × 2 matrix, interchange the elements on the principal diagonal and change the sign of the elements on the secondary diagonal.

Properties of Adjoint Matrix : If A is square matrix of order n,

(i) A(*adj* A) = (*adj* A) A = $|A| I_n$

(ii) *adj* $I_n = I_n$

(iii) *adj* $(kI_n) = k^{n-1} I_n$, k is a scalar.

(iv) $adj\ A^T = (adj\ A)^T$

(v) $adj\ (kA) = k^{n-1}\ adj A$, k is a scalar.

(vi) $adj(AB) = (adj\ B)(adj\ A)$

(vii) $adj\ (ABC) = (adj\ C)(adj\ B)(adj\ A)$

If A is a non singular matrix of order n, then

(i) $|\ adj\ A\ | = |\ A\ |^{n-1}$

(ii) $adj\ (adj\ A) = |\ A\ |^{n-2} A$

(iii) $|\ adj\ (adj\ A)\ | = |\ A\ |^{(n-1)^2}$

Adjoint of

(i) a diagonal matrix is diagonal

(ii) a triangular matrix is triangular

(iii) a symmetric matrix is symmetric

(iv) a hermitian matrix is thermitian

- **Inverse of a Matrix**

A square matrix A is said to be singular if $|\ A\ | = 0$ and non singular if $|\ A\ | = 0$

- If A is a square matrix of order n, if there exists another square matrix of order n such that
$AB = I_n = BA$
Then B(A) is called inverse of A(B).It is denoted A^{-1}.

$$A^{-1} = \frac{1}{|\ A\ |}(adj\ A)$$

If inverse of matrix A exists, then it is unique.

A square matrix A is non-singular $\Leftrightarrow\ |\ A\ | \neq 0$

$\Leftrightarrow\ A^{-1}$ exists.

Results :

(i) $|\ A^{-1}\ | = |\ A\ |^{-1}$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

(iii) $(A^T)^{-1} = (A^{-1})^T$

(iv) $(A^k)^{-1} = (A^{-1})^k, \quad k \in Z$

(v) $A = \text{diag}\ [a_{11}\ \ a_{22}\ \ a_{33}\ \ \ldots\ \ a_{nn}]$ and $a_{11} \cdot a_{22} \cdot a_{33} \cdot \ldots \cdot a_{nn} \neq 0$ then
$A^{-1} = \text{diag}\ [a_{11}^{-1}\ \ a_{22}^{-1}\ \ a_{33}^{-1} \ldots\ldots\ldots\ a_{nn}^{-1}]$

(vi) Inverse of a symmetric matrix is symmetric.

- **Elementary Transformations (operations) of a matrix**

(i) Interchange of rows (columns)

(ii) The multiplication of the elements of a row (column) by a non - zero scalar.

(iii) The addition (subtraction) to the elements of any row (column) of the scalar multiples of the corresponding elements of any other row (column).

- **Test of Consistency**

If the system of equation possesses atleast one solution set (solution set is not empty) then the equations are said to be consistent.

If the system of equation has no solution they are said to inconsistent.

Solution of simultaneous linear equations in two (three) variables :

Trival solution :

Value of all the variables is zero i.e. $x = 0$, $y = 0$, $z = 0$

Non Triavial Solution :

Value of atleast one variable is non-zero

Homogeneous linear equation :

If constant term is zero, i.e. $ax + by = 0$ or $ax + by + cz = 0$

such equations is called homogenous linear eqaution.

Solutiuon of homogenoeous linear equation

Consider the eqautions

For three variables

$a_{11}x + a_{12}y + a_{13}z = 0$

$a_{21}x + a_{22}y + a_{23}z = 0$

$a_{31}x + a_{32}y + a_{33}z = 0$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = O

For two variables

$a_{11}x + a_{12}y = 0$

$a_{21}x + a_{22}y = 0$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

A X = O

(i) If $|A| \neq 0$ the system is consistent and has only trivial (unique) solution.

(ii) If $|A| = 0$ the system is consistent and has non trivial (infinite number of) Solution.

- **Solution of non-homogeneous linear equation :**

Let three equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Matrix inversion Method

Equations can be expressed as a matrix form AX = B.

Where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

If $|A| \neq 0$ (A is non singular), A^{-1} exists.

The solution is $X = A^{-1}B$

Cramer's Rule

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

Where

$$D_1 = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}, \quad D_2 = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}, \quad D_3 = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}, \quad D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

(i) If $D \neq 0$ then the system has a unique solution and said to be consistent.

(ii) If $D = 0$ and $D_1 = D_2 = D_3 = 0$ then the system has infinite number of solution and said to be consistent.

(iii) If $D = 0$ and atleast one determinants D_1, D_2, D_3 is non-zero, then the system has no solution (solution set is empty) and said to be inconsistent.

Above both method can be used to solve non-homogeneous linear equation in two variables.

- **Characteristic Equation :**

Let $A = [a_{ij}]_{n \times n}$ then $A - \lambda I$ is called characteristic matrix of A.

Equation $|A - \lambda I| = 0$ is called characteristic equation of A.

Homogeneous system of linear equation having non-trival solution if $|A - \lambda I| = 0$

Every square matrix A satisfies its characteristic equation $|A - \lambda I| = 0$

Question Bank

1. If the system of equations x + ky + 3z = 0, 3x + ky - 2z = 0, 2x + 3y - 4z = 0 has non trivial solution, then $\frac{xy}{z^2} = \ldots$

   (a) $\frac{5}{6}$ (b) -$\frac{5}{6}$ (c) $\frac{6}{5}$ (d) -$\frac{6}{5}$

2. $\begin{vmatrix} n & {}_{n}P_{n} & {}_{n}C_{n} \\ n+1 & {}_{n+1}P_{n+1} & {}_{n+1}C_{n+1} \\ n+2 & {}_{n+2}P_{n+2} & {}_{n+2}C_{n+2} \end{vmatrix} = \ldots$

   (a) $(n^2 + n + 1)$ n! (b) n(n + 1)! (c) (n + 1) n! (d) (n + 2) n!

3. Let a, b, c be such that b(a + c) ≠ 0.

   If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$, then n is...

   (a) Zero (b) any even integer (c) any odd interger (d) any integer

4. $\begin{vmatrix} \sin(x+p) & \sin(x+q) & \sin(x+r) \\ \sin(y+p) & \sin(y+q) & \sin(y+r) \\ \sin(z+p) & \sin(z+q) & \sin(z+r) \end{vmatrix} = \ldots$

   (a) sin (x + y + z) (b) sin (p + q + r) (c) 1 (d) 0

5. $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$, then $x = \ldots$

   (a) $\frac{3}{2}, \frac{3}{11}$ (b) $\frac{3}{2}, \frac{11}{3}$ (c) $\frac{2}{3}, \frac{11}{3}$ (d) $\frac{2}{3}, \frac{3}{11}$

6. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k(abc)(a+b+c)^3$, then $k = \ldots$

   (a) 1 (b) -1 (c) -2 (d) 2

A-Z Shiksha

7. $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = kabc,\quad \text{then } k = \ldots$

(a) 4 (b) 3 (c) 2 (d) 1

8. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and the vectors $(1, a, a^2)$, $(1, b, b^2)$, $(1, c, c^2)$ are non-coplanar

then abc = ...

(a) 0 (b) 2 (c) -1 (d) 1

9. $\begin{vmatrix} \sqrt{11}+\sqrt{3} & \sqrt{20} & \sqrt{5} \\ \sqrt{15}+\sqrt{22} & \sqrt{25} & \sqrt{10} \\ 3+\sqrt{55} & \sqrt{15} & \sqrt{25} \end{vmatrix} = \ldots$

(a) $5(5\sqrt{3}-3\sqrt{2})$ (b) $5(3\sqrt{2}+5\sqrt{3})$

(c) $-5(5\sqrt{3}+3\sqrt{2})$ (d) $5(3\sqrt{2}-5\sqrt{3})$

10. If $2s = a + b + c$ and $A = \begin{bmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{bmatrix}$ then det A = ...

(a) $2s^2 (s - a) (s - b) (s - c)$ (b) $2s^3 (s - a) (s - b) (s - c)$

(c) $2s (s - a)^2 (s - b)^2 (s - c)^2$ (d) $2s^2 (s - a)^2 (s - b)^2 (s - c)^2$

11. The homogeneous system of equations

$$\begin{bmatrix} 2 & \alpha+\beta+\gamma+\delta & \alpha\beta+\gamma\delta \\ \alpha+\beta+\gamma+\delta & \alpha(\alpha+\beta)(\gamma+\delta) & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) \\ \alpha\beta+\gamma\delta & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) & 2\alpha\beta\gamma\delta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

has non - trivial solutions only if...

(a) $\alpha + \beta + \gamma + \delta = 0$ (b) for any $\alpha, \beta, \gamma, \delta$

(c) $\alpha\beta + \gamma\delta = 0$ (d) $\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta)$

12. Let $A=\begin{bmatrix}4 & 4k & k\\ 0 & k & 4k\\ 0 & 0 & 4\end{bmatrix}$. If $\det(A^2)=16$ then $|k|$ is ...

(a) 1 (b) $\frac{1}{4}$ (c) 4 (d) 4^2

13. If 1, ω,ω^2 are cube roots of unity, then $\begin{vmatrix}a & a^2 & a^3-1\\ a^{\omega} & a^{2\omega} & a^{3\omega}-1\\ a^{\omega^2} & a^{2\omega^2} & a^{3\omega^2}-1\end{vmatrix}=$ ...

(a) 0 (b) a (c) a^2 (d) a^3

14. If a_1, a_2, a_3 ... are in GP , then

$$\begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}= \ldots$$

(a) 0 (b) 1 (c) 2 (d) 4

15. If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^T$, then $P^TQ^{2013}P=\ldots$

(a) $\begin{bmatrix}1 & 2013\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}4+2013\sqrt{3} & 6039\\ 2012 & 4-2013\sqrt{3}\end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix}2+\sqrt{3} & 1\\ -1 & 2-\sqrt{3}\end{bmatrix}$ (d) $\frac{1}{4}\begin{bmatrix}2012 & 2-\sqrt{3}\\ 2+\sqrt{3} & 2012\end{bmatrix}$

16. $\bar{A}=\begin{bmatrix}-1 & 2-3i & 3+4i\\ 2+3i & 5 & 1+i\\ 3-4i & 1-i & 4\end{bmatrix}$, then det A is ...

(a) Purely real (b) purely imaginary (c) complex number (d) 0

17. The value of $\begin{vmatrix}\log_3 1024 & \log_3 3\\ \log_3 8 & \log_3 9\end{vmatrix}\times\begin{vmatrix}\log_2 3 & \log_4 3\\ \log_3 4 & \log_3 4\end{vmatrix}=\ldots$

(a) 6 (b) 9 (c) 10 (d) 12

18. If $A = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$, $i = \sqrt{-1}$, then $A^n = I$ where I is unit matrix when n = ...

(a) 4p + 1 (b) 4p + 3 (c) 4p (d) 4p + 2

19. If $A = \begin{bmatrix} k & 3 \\ 3 & k \end{bmatrix}$ and $|A^3| = 343$, then find the value of k = ...

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

20. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then $A^n + (n - 1) I = ...$

(a) $2^{n-1}A$ (b) $-nA$ (c) nA (d) $(n + 1) A$

21. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 2x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = 24x + B$ then B = ...

(a) -12 (b) 12 (c) 24 (d) -8

22. $\begin{vmatrix} \tan^2 x & -\sec^2 x & 1 \\ -\sec^2 x & \tan^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} = ...$

(a) $12 \tan^2 x - 10 \sec^2 x$ (b) $12 \sec^2 x - 10 \tan^2 x + 2$

(c) 0 (d) $\tan^2 x \cdot \sec^2 x$

23. If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cos x \operatorname{cosec}^2 x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$, then $\int_0^{\frac{\pi}{2}} f(x)\, dx = ...$

(a) $\frac{1}{3} - \frac{\pi}{3}$ (b) $\frac{1}{3} - \frac{\pi}{4}$ (c) $\frac{2}{3} + \frac{\pi}{3}$ (d) $\frac{4}{3} - \frac{\pi}{4}$

24. If $f(x) = \begin{vmatrix} x & e^{x^2} & \sec x \\ \sin x & 2 & \cos x \\ \operatorname{cosec} x & x^2 & 5 \end{vmatrix}$, then the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\, dx = ...$

(a) 0 (b) $5e^{\pi}$ (c) $1-\frac{\pi}{2}$ (d) 34

25. If A = $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c R^+, $a$bc = 1 and | A | > 0, $A^TA = I$, then $a^3 + b^3 + c^3 = ...$

(a) 12 (b) 4 (c) -8 (d) 28

26. If $f(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ p & p^2 & p^3 \end{vmatrix}$,then $\frac{d^n}{dx^n}\left(f(x)\right)$ at $x = 0$ is ... where p is constant.

(a) p (b) p + p^2

(c) p^3 (d) Independent of p.

27. The value of the determinant $\begin{vmatrix} \cos^2\left(\frac{\pi}{2}+x\right) & \cos^2\left(\frac{3\pi}{2}+x\right) & \cos^2\left(\frac{5\pi}{2}+x\right) \\ \cos\left(\frac{\pi}{2}+x\right) & \cos\left(\frac{3\pi}{2}+x\right) & \cos\left(\frac{5\pi}{2}+x\right) \\ \cos\left(\frac{\pi}{2}-x\right) & \cos\left(\frac{3\pi}{2}-x\right) & \cos\left(\frac{5\pi}{2}-x\right) \end{vmatrix}$ is ...

(a) 0 (b) $\cos^2\left(3x-\frac{9\pi}{2}\right)$

(c) $\sin^2\left(\frac{3\pi}{2}+x\right)$ (d) $\cos^2\left(\frac{15\pi}{2}-x\right)$

28. If $\Delta(x) = \begin{vmatrix} x^2-5x+3 & 2x-5 & 3 \\ 3x^2+x+4 & 6x+1 & 9 \\ 7x^2-6x+9 & 14x-6 & 21 \end{vmatrix} = ax^3 + bx^3 + cx + d$, then d = ...

(a) 156 (b) 187 (c) 119 (d) 141

29. If $A = \begin{bmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{bmatrix}$, $a, b, c \in R$, $abc = 1$ and $AA^T = 64I$ and | A | > 0, then $\left(a^3 + b^3 + c^3\right) = ...$

(a) 343 (b) 729 (c) 256 (d) 512

30. Let P be a non-singular matrix and 1 + P + P^2 + ... + P^n = O, (O denotes the null matrix) then P^{-1} = ...

(a) 0 (b) P (c) P^n (d) I

31. The matrix $\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$ is singular if...

(a) $a - b = 0$ (b) $a + b = 0$

(c) $a + b + c = 0$ (d) $a = 0$

32. $\begin{vmatrix} x+1 & x+3 & x+4 \\ x+4 & x+6 & x+8 \\ x+8 & x+10 & x+14 \end{vmatrix} = \ldots$

(a) 2 (b) -2 (c) 4 (d) -4

33. If a, b, c are positive and not all equal, then the value of determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is ...

(a) > 0 (b) ≥ 0 (c) < 0 (d) ≤ 0

34. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = \ldots$

(a) 0 (b) $(a^2 - bc)(b^2 - ca)(c^2 - ab)$

(c) $(a - b)(b - c)(c - a)$ (d) -1

35. If the equations $y + z = -ax$, $z + x = -by$, $x + y = -cz$ have non trivial solutions, then

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = \ldots$$

(a) 1 (b) 2 (c) -1 (d) -2

36. If the equations $a(y + z) = x$, $b(z + x) = y$, $c(x + y) = z$ have non trivial solutions, then

$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = \ldots$$

(a) 1 (b) 2 (c) -1 (d) -2

37. If the equations $x - 2y + 3z = 0$, $-2x + 3y + 2z = 0$, $-8x + \lambda y = 0$ have non-trivial solution then $\lambda = ...$

(a) 18 (b) 13 (c) -10 (d) 4

38. If the equations $x + 3y + z = 0$, $2x - y - z = 0$, $kx + 2y + 3z = 0$ have non-trivial solution then k = ...

(a) $\frac{13}{2}$ (b) $\frac{9}{2}$ (c) $-\frac{15}{2}$ (d) $-\frac{13}{2}$

39. If the equations $ax + by + cz = 0$, $4x + 3y + 2z = 0$, $x + y + z = 0$ have non-trivial solution, then a, b, c are in...

(a) A.P. (b) G.P.

(c) Increasing sequence (d) decreasing sequence.

40. If the system of equations $x + ay = 0$, $az + y = 0$, $ax + z = 0$ has infinite number of solutions then a = ...

(a) 0 (b) 1 (c) -1 (d) -2

41. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is ...

(a) {1, 2} (b) {-1, -2} (c) {1, -2} (d) {-1, 2}

42. The equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$ have....

(a) no solution (b) unique solution

(c) Infinity solutions (d) can not say anything

43. If $A = \begin{bmatrix} 2 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $A^{-1} = ...$

(a) A (b) A^2 (c) A^3 (d) A^4

- Read the following paragraph carefully and answer the following questions No. 44 to 46.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ if U_1, U_2 and U_3 are column matrices satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is 3×3 matrix whose columns are U_1, U_2, U_3, then

44. The value of | U | is ...

(a) 3 (b) -3 (c) $\frac{3}{2}$ (d) 2

45. The sum of the elements of U^{-1} is ...

(a) -1 (b) 0 (c) 1 (d) 3

46. The value of determinant of $[3 \;\; 2 \;\; 0] \; U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is ...

(a) 5 (b) $\frac{5}{2}$ (c) 4 (d) $\frac{3}{2}$

47. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$, then ...

(a) $a = 2$, $c = -\frac{1}{2}$ (b) $a = 1$, $c = -1$

(c) $a = -1$, $c = 1$ (d) $a = \frac{1}{2}$, $c = \frac{1}{2}$

48. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then *adj* A = ...

(a) A (b) A^T (c) 3A (d) $3A^T$

49. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then A^3 = ...

(a) I (b) A^T (c) O (d) A^{-1}

50. If $A = \begin{bmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{bmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$ then f(A) = ...

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\left(\frac{3-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\left(\frac{5-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\left(2+i\sqrt{3}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

51. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$ is an idempotent matrix, then x is equal to ...

(a) - 1 (b) - 5 (c) - 4 (d) - 3

52. Let a, b, c be positive real numbers, the following system of equations in x, y and z

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \quad -\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \quad -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ has ...

(a) unique solutin (b) no solution

(c) finitely many solutions (d) Infinitely many solutions.

53. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = ...$

(a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ (b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$

(c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ (d) $\begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$

54. Suppose a matrix A satisfies $A^2 - 5A + 7I = O$. If $A^5 = aA + bI$ then the value of $2a - 3b$ must be...

(a) 4135 (b) 1435 (c) 1453 (d) 3145

55. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then the value of $64(\sin^2A + \sin^2B + \sin^2C)$ must be...

(a) 64 (b) 144 (c) 128 (d) 0

56. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{2013} = ...$

(a) $2^{2012}A$ (b) $2^{1006}A$ (c) $-2^{2013}A$ (d) I

57. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then $A^{2013} = ...$

(a) $3^{2013}A$ (b) $-3^{2012}I$ (c) $3^{2011}A$ (d) $3^{1006}A$

58. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = (adj\ A)$, $C = 5A$ then $\dfrac{|adj\ B|}{|c|} = ...$

(a) 5 (b) 1 (c) 3 (d) $\frac{1}{5}$

59. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$, then $\left|\left(adj\left(adj\left(adj\left(adj\ A\right)\right)\right)\right)\right| = ...$

(a) 1 (b) 2 (c) 2^4 (d) 2^{12}

60. If $A_r = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix}$, where r is a natureal number then the value of

$\sqrt{\left(\sum_{r=1}^{2013} A_r\right)}$ is ...

(a) 1 (b) 40 (c) 2012 (d) 2013

61. If z is a complex number and a_1, a_2, a_3, b_1, b_2, b_3 are all real, then

$$\begin{vmatrix} a_1 z + b_1 \bar{z} & a_2 z + b_2 \bar{z} & a_3 z + b_3 \bar{z} \\ b_1 z + a_1 \bar{z} & b_2 z + a_2 \bar{z} & b_3 z + a_3 \bar{z} \\ b_1 z + a_1 & b_2 z + a_2 & b_3 z + a_3 \end{vmatrix} = ...$$

(a) $|\bar{z}|^2$ (b) $(a_1 a_2 a_3 + b_1 b_2 b_3)^2\ |z|^2$

(c) 3 (d) 0

62. If $D = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then maximum value of D is...

(a) 9 (b) 1 (c) 10 (d) 16

63. $$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos^2\phi & \cos\theta\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

provided $\theta - \phi = ...$, $n \in Z$

(a) $n\pi$ (b) $(2n+1)\frac{\pi}{2}$ (c) $n\frac{\pi}{2}$ (d) $2n\pi$

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64. If P, Q, R represent the angles of an acute angled triangle, no two of them being equal then the value of $\begin{vmatrix} 1 & 1+\cos P & \cos P\,(1+\cos P) \\ 1 & 1+\cos Q & \cos Q\,(1+\cos Q) \\ 1 & 1+\cos R & \cos R\,(1+\cos R) \end{vmatrix}$ is ...

(a) positive (b) 0 (c) negative (d) can not be determined

65. If $0 \le [x] < 2$, $-1 \le [y] < 1$, $1 \le [z] < 3$ ($[\cdot]$ denotes the greatest integer function) then the maximum value of determinant $D = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is ...

(a) 6 (b) 2 (c) 4 (d) 8

66. If $A = \begin{bmatrix} 1 & \tan\alpha \\ -\tan\alpha & 1 \end{bmatrix}$ then $A^T A^{-1} = ...$

(a) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$ (b) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(c) $\begin{bmatrix} \cos 2x & \sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ (d) $\begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$

67. $f(x) = \begin{bmatrix} \cos x & 0 & \sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix}$, $g(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(i) $f(x)\,.\,g(y) = ...$

(a) $f(xy)$ (b) $f\left(\frac{x}{y}\right)$ (c) $f(x + y)$ (d) $f(x - y)$

(ii) Which of the following is correct ?

(a) $[f(x)]^{-1} = \frac{1}{f(x)}$ (b) $[f(x)]^{-1} = -f(x)$

(c) $[f(x)]^{-1} = f(-x)$ (d) $[f(x)]^{-1} = -f(-x)$

(iii) $[f(x)\,g(y)]^{-1} = ...$

(a) $f(x^{-1})\,g(y^{-1})$ (b) $f(y^{-1})\,g(x^{-1})$

(c) $f(-x)\,g(-y)$ (d) $g(-y)\,f(-x)$

68. If $D_1 = \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}$ and $D_2 = \begin{vmatrix} x & a \\ a & x \end{vmatrix}$, then ...

(a) $D_1 = 3(D_2)^{\frac{3}{2}}$ (b) $D_1 = 3{D_2}^2$

(c) $\frac{d}{dx}(D_1) = 3{D_2}^2$ (d) $\frac{d}{dx}(D_1) = 3D_2$

69. If α, β are the roots of the equation $x^2 + bx + c = 0$, then

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^2+\beta^2 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} = \ldots$$

(a) $(1 + b + c)^2 (b^2 - 4c)$ (b) $(1 - b - c)^2 (b^2 - 4c)$

(c) $(1 - b + c)^2 (b^2 - 4c)$ (d) $(1 + b - c)^2 (b^2 - 4c)$

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70. If $x^a y^b = e^m$, $x^c y^d = e^n$, $D_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $D_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$ and $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then

$x = \ldots$ and $y = \ldots$

(a) $\log\left(\frac{D_1}{D}\right)$, $\log\left(\frac{D_2}{D}\right)$ (b) $\frac{D_1}{D}$, $\frac{D_2}{D}$ (c) $e^{\frac{D_1}{D}}$, $e^{\frac{D_2}{D}}$ (d) $\frac{D_2}{D_1}$, $\frac{D}{D_1}$

71. The value of determinant $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}\,i & \sqrt{27}+2i \end{vmatrix}$ is a

(a) real number (b) rational number

(c) irrational number (d) complex number

72. Match the following columns :

Column I	Column II
1. A is a square matrix such that $A^2 = A$	A. A is a Nilpotent matrix
2. A is a square matreix such that $A^m = O$	B. A is an Involutary matrix
3. A is a square matrix such that $A^2 = I$	C. A is a symmetric matrix
4. A is a square matrix such that $A^T = A$	D. A is an dempotent matrix

(a) 1-D, 2-A, 3-C, 4-B (b) 1-D, 2-A, 3-B, 4-C

(c) 1-A, 2-C, 3-D, 4-B (d) 1-B, 2-D, 3-C, 4-A

73. If $f(x)=\begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then $\int_0^{\frac{\pi}{2}} \left(f(x)+f'(x)\right) dx = ...$

(a) 1 (b) π (c) 0 (d) $\frac{1}{3}-\frac{\pi}{2}$

74. If $f(x)=\begin{vmatrix} x & \sin x & \cos x \\ x^2 & -\tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$, then $\lim_{x\to 0} \frac{f'(x)}{x} = ...$

(a) 0 (b) 1 (c) 2 (d) 4

75. The set of natural numbers N is partitioned into arrays of rows and columns in the form of matrices as $M_1=[1], M_2=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, M_3=\begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}, ..., M_n=[...]$ and so on. Find the sun of the elements of the diagonal in M_6.

(a) 144 (b) 441 (c) 321 (d) 461

76. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5+bx^4+cx^3+dx^2+ex+f$, then match the following columns:

Column I	Column II
1. The Value of f = ...	A . 0
2. The value of e = ...	B. 1
3. The value of a + c = ...	C. -1
4. The value of b + d = ...	D. 3

(a) 1-C, 2-D, 3-A, 4-B (b) 1-A, 2-B, 3-B, 4-C

(c) 1-B, 2-D, 3-C, D-B (d) 1-D, 2-C, 3-D, 4-A

77. If $f(x)=\begin{vmatrix} (1+3x)^m & (1+5x)^m & 1 \\ 1 & (1+3x)^m & (1+5x)^n \\ (1+5x)^n & 1 & (1+3x)^m \end{vmatrix}$, a, b being positive integers, then sum of constant term and coefficient of x is equal to ...

(a) 5 (b) - 8 (c) 1 (d) 0

78. If maximum and minimum value of the determinant

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$$ are M and m respectively, then match the following columns.

Column I	Column II
1. $M^2 + m^{2013} =$	A. always an odd for $k \in N$
2. $M^3 - m^3 =$	B. Being three sides of triangle
3. $M^{2k} - m^{2k} =$	C. 10
4. 2M - 3m, M + m, M + 2m	D. 4
	E. Always an even for $k \in N$
	F. does not being three sides of triangle.
	G. 26

(a) 1-D, 2-G, 3-A, 4-B (b) 1-G, 2-D, 3-A, 4-E
(c) 1-C, 2-G, 3-E, 4-B (d) 1-D, 2-C, 3-E, 4-F

79. If $[x]$ is the greatest integer less than or equal to x, then the determinant's value of the matrix.

$$\begin{bmatrix} [e] & [\pi] & [\pi^2-6] \\ [\pi] & [\pi^2-6] & [e] \\ [\pi^2-6] & [e] & [\pi] \end{bmatrix}$$ is ...

(a) 8 (b) 0 (c) 1 (d) -8

80. When the determinant $f(x) = \begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is expanded in powers of $\sin x$, then the constant term in that expansion is...

(a) 0 (b) -1 (c) 2 (d) 1

81. The value of the determinant $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$ is ...

(a) 0 (b) $2\sin\theta$ (c) $-\sin 2\theta$ (d) $-2\cos\theta$

82. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then $A^2 = B$ for

(a) $\alpha = 4$ (b) $\alpha = 1$ (c) $\alpha = -1$ (d) no α

83. If $A = \begin{bmatrix} \alpha & 0 \\ 2 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 = 9I$ for

(a) $\alpha = 4$ (b) $\alpha = 3$ (c) $\alpha = -3$ (d) no α

84. If $A = \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}$, then $I + 2A + 3A^2 + \ldots \infty = \ldots$

(a) $\begin{bmatrix} 9 & 1 \\ -9 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 1 \\ -9 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 2 \\ -18 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 2 \\ -5 & -18 \end{bmatrix}$

85. If M is a 3×3 matrix, where $M^TM = I$ and det M = 1, then det(M - I) = ...

(a) 0 (b) -1 (c) 4 (d) -3

86. Let λ and α be real. The system of equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$-x + (\sin\alpha)y - (\cos\alpha)z = 0$ has no trivial solution.

(i) The the set of all values of λ is

(a) $\left[-\sqrt{3}, \sqrt{3}\right]$ (b) $\left[-\sqrt{2}, \sqrt{2}\right]$ (c) [-1, 1] (d) $\left[0, \frac{\pi}{2}\right]$

(ii) For $\lambda = 1$, $\alpha = \ldots$

(a) $n\pi, n\pi - \frac{\pi}{4}$ (b) $2n\pi, n\pi - \frac{\pi}{4}$ (c) $n\pi, n\pi + \frac{\pi}{4}$ (d) $-\pi, -\frac{3\pi}{4}$

87. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + kI_2$, then $k = \ldots$

(a) 1 (b) -1 (c) 7 (d) -7

88. The identity element in the group $M = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} / x \in R,\ x \neq 0 \right\}$ with respect to matrix multiplicaiton is ...

(a) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

89. The inverse element of $\begin{bmatrix} y & y & y \\ y & y & y \\ y & y & y \end{bmatrix}$ in group

$$M = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} / x \in R,\ x \neq 0,\ I = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right\} \text{ w.r.t.}$$

matrix multiplication is...

(a) $\begin{bmatrix} \frac{1}{y} & \frac{1}{y} & \frac{1}{y} \\ \frac{1}{y} & \frac{1}{y} & \frac{1}{y} \\ \frac{1}{y} & \frac{1}{y} & \frac{1}{y} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3y} & \frac{1}{3y} & \frac{1}{3y} \\ \frac{1}{3y} & \frac{1}{3y} & \frac{1}{3y} \\ \frac{1}{3y} & \frac{1}{3y} & \frac{1}{3y} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{6y} & \frac{1}{6y} & \frac{1}{6y} \\ \frac{1}{6y} & \frac{1}{6y} & \frac{1}{6y} \\ \frac{1}{6y} & \frac{1}{6y} & \frac{1}{6y} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \end{bmatrix}$

90. If $A = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$ and $\phi(x) = (1+x)(1-x)^{-1}$, then $\phi(A) = \ldots$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

91. Costruct an orthogonal matrix using the skew-symmetric matrix $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

(a) $\begin{bmatrix} -\frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{4}{5} & -\frac{3}{5} \\ -\frac{3}{5} & -\frac{4}{5} \end{bmatrix}$

92. Construct an orthogonal matrix using the skew-symmetric matrix $A = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$

(a) $\frac{1}{31}\begin{bmatrix} 21 & 6 & 22 \\ 14 & 21 & 6 \\ 18 & 14 & 21 \end{bmatrix}$ (b) $\frac{1}{31}\begin{bmatrix} 21 & 6 & 22 \\ 14 & -27 & -6 \\ 18 & 14 & -21 \end{bmatrix}$

(c) $\frac{1}{31}\begin{bmatrix} 21 & 6 & 22 \\ 6 & 14 & 18 \\ 22 & 18 & 7 \end{bmatrix}$ (d) $\frac{1}{31}\begin{bmatrix} 21 & -6 & 22 \\ 6 & 14 & -18 \\ -22 & 18 & 7 \end{bmatrix}$

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93. If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, then $A^3 - 7A^2 + 10A = ...$

(a) $5I - A$ (b) $5I + A$ (c) $A - 5I$ (d) $7I$

94. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $8A^{-4} = ...$

(a) $145 A^{-1} - 27I$ (b) $27I - 145 A^{-1}$

(c) $29A^{-1} + 9I$ (d) $145A^{-1} + 27I$

95. The system of equations

$x_1 + 2x_2 + 3x_3 = \lambda x_1$,

$3x_1 + x_2 + 2x_3 = \lambda x_2$,

$2x_1 + 3x_2 + x_3 = \lambda x_3$ can possess a non-trivial solution then $\lambda = ...$

(a) 1 (b) 2 (c) 3 (d) 6

96. Solution of the system of linear equations (α constant)

$x \sec^2\alpha - y \tan^2\alpha + z = 2.$

$x \cos^2\alpha + y \sin^2\alpha = 1$

$x + z = 2$ is $(x, y, z) = ...$

(a) (1, 1, 1) (b) (1, 2, 2) (c) (2, 1, 2) (d) (1, 0, 1)

97. For what value of k the following system of linear equations

$x + 2y - z = 0$,

$3x + (k + 7)y - 3z = 0$,

$2x + 4y + (k - 3)z = 0$ possesses a non-triuvial solution.

(a) 1 (b) 0 (c) 2 (d) -2

98. The correct match of the following columns is given by

Column I		Column II
1.	Leibnitz	A.	$e^{i\theta}$
2.	Euler	B.	Mathematical logic
3.	Cayley - Hamilton	C.	Calculus
4.	George Boole	D.	$(e^{i\theta})^n = e^{i(n\theta)}$
5.	De-moivre	E.	Theory of Matices

(a) 1-D, 2-A, 3-E, 4-b, 5-A (b) 1-B, 2-D, 3-A, 4-C, 5-E

(c) 1-C, 2-A, 3-D, 4-B, 5-E (d) 1-C, 2-A, 3-E, 4-B, 5-D

99. Let $A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -5 & -2\end{bmatrix}$. If q is the angle between two non-zero column vectors X such that AX = λX for some scalar λ, then tanθ = ...

(a) $\frac{7}{\sqrt{202}}$ (b) $\frac{\sqrt{3}}{19}$ (c) $\sqrt{\frac{3}{202}}$ (d) $\frac{7}{19}$

100. Let the 3-digit numbers A28, 3B9 and 62C, where A, B, C are integers between 0 and 9, be divisible by a fixed interger k, then the determinant $\begin{vmatrix}A & 3 & 6\\ 8 & 9 & C\\ 2 & B & 2\end{vmatrix}$ is divisible by ...

(a) 3k (b) k^3 (c) k (d) $\frac{k}{3}$

101. If $A=\begin{bmatrix}-2 & 3\\ -1 & 1\end{bmatrix}$ then $I + A + A^2 + \dots \infty = \dots$

(a) $\begin{bmatrix}0 & 3\\ 1 & 3\end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix}0 & -3\\ -1 & 3\end{bmatrix}$ (c) $\frac{1}{3}\begin{bmatrix}0 & 3\\ -1 & 3\end{bmatrix}$ (d) undefined.

102. If $A=\begin{bmatrix}-2 & 3\\ -1 & 1\end{bmatrix}$, then $A^3 = \dots$

(a) I (b) O (c) -A (d) A + I

103. If $\begin{vmatrix}\binom{x}{r} & \binom{x+1}{r+1} & \binom{x+2}{r+2}\\ \binom{y}{r} & \binom{y+1}{r+1} & \binom{y+2}{r+2}\\ \binom{z}{r} & \binom{z+1}{r+1} & \binom{z+2}{r+2}\end{vmatrix} = \lambda\begin{vmatrix}\binom{x}{r} & \binom{x}{r+1} & \binom{x}{r+2}\\ \binom{y}{r} & \binom{y}{r+1} & \binom{y}{r+2}\\ \binom{z}{r} & \binom{z}{r+1} & \binom{z}{r+2}\end{vmatrix}$, then λ is ...

(a) 0 (b) 1 (c) -1 (d) 2

104. Investigate the values λ and μ for the system

$x + 2y + 3z = 6$

$x + 3y + 5z = 9$

$2x + 5y + \lambda z = \mu$

and correct match the following columns.

Column I		Column II
1. $\lambda = 8,\ \mu \neq 15$	A.	Infinity of solutions
2. $\lambda \neq 8,\ \mu \in R$	B.	No solutions
3. $\lambda = 8,\ \mu = 15$	A.	Unique solution

(a) 1-A, 2-B, 3-C (B) 1-B, 2-C, 3-A

(c) 1-C, 2-A, 3-B (d) 1-C, 2-B, 3-A

105. The value of α, β, γ when $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal are ...

(a) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}, \pm\frac{1}{\sqrt{2}}$ (b) $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}$

(c) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}, \pm\frac{1}{\sqrt{3}}$ (d) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}$

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Hint

1. $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow k = \dfrac{33}{2}$

Put value of k in given equations and solved them

$\Rightarrow \dfrac{x}{15} = \dfrac{y}{-2} = \dfrac{z}{6} \Rightarrow \dfrac{xy}{z^2} = \dfrac{-30}{36} = \dfrac{-5}{6}$

2. $D = \begin{vmatrix} n & n! & 1 \\ n+1 & (n+1)! & 1 \\ n+2 & (n+2)! & 1 \end{vmatrix}$

$= n! \begin{vmatrix} n & 1 & 1 \\ n+1 & n+1 & 1 \\ n+2 & (n+2)(n+1) & 1 \end{vmatrix} \quad \because C_2\left(\dfrac{1}{n!}\right)$

$= n! \begin{vmatrix} n & 1 & 1 \\ 1 & n & 0 \\ 1 & (n+1)^2 & 0 \end{vmatrix} \quad \because R_{23}(-1), R_{12}(-1)$

$= \left(n^2+n+1\right) n! \qquad \because$ Expanding along C_3

3. The 2[nd] det $D_2 = \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^n a & -(-1)^n b & (-1)^n c \end{vmatrix}$

$= (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$

$= (-1)^n D_1 \qquad \because R_{13}, R_{23}$ and taking transpose

$\therefore \left(1+(-1)^n\right) D_1 = 0$ For any odd integer

$\because D_1 \neq o$ since $b(a+c) \neq o$

4. Given determinant is a product of two eleminant

$$\begin{vmatrix} \sin x & \cos x & 0 \\ \sin y & \cos y & 0 \\ \sin z & \cos z & 0 \end{vmatrix}\begin{vmatrix} \cos p & \cos q & \cos r \\ \sin p & \sin q & \sin r \\ 0 & 0 & 0 \end{vmatrix} = 0 \times 0 = 0$$

5. $$\begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0 \qquad \because C_{21}(1), C_{31}(1)$$

$$\therefore (3x-2)(3x-11)^2 = 0 \qquad \because C_1\left(\frac{1}{3x-2}\right) \text{ and expanding along } R_1$$

$$\therefore x = \frac{2}{3}, \frac{11}{3}$$

6. Let a = 1, b = - 1, c = 2

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 9 & 1 \\ 4 & 4 & 0 \end{vmatrix} = k(-2)(2)^3$$

$$\therefore -32 = -16k$$

$$\therefore k = 2$$

7. Let a = 1, b = −1, c = 2

$$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 5 & 1 \\ -1 & -1 & -5 \end{vmatrix} = k(1)(-1)(2)$$

$$\therefore -8 = -2k$$

$$\therefore k = 4$$

8. $$\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\therefore (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\therefore abc = -1 \text{ since } \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

9. $$D = (\sqrt{5})(\sqrt{5})\begin{vmatrix} \sqrt{11} + \sqrt{3} & 2 & 1 \\ \sqrt{15} + \sqrt{22} & \sqrt{5} & \sqrt{2} \\ 3 + \sqrt{55} & \sqrt{3} & \sqrt{5} \end{vmatrix} \qquad \because C_2\left(\frac{1}{\sqrt{5}}\right), C_3\left(\frac{1}{\sqrt{5}}\right)$$

$$= 5\begin{vmatrix} -\sqrt{3} & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{5} \end{vmatrix} \qquad \because C_{21}(\sqrt{-3}), C_{31}(-\sqrt{11})$$

$$= 5(3\sqrt{2} - 5\sqrt{3}) \qquad \because \text{ expanding along } R_1$$

10. If $s = 0$, $\det A = a^2b^2c^2\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$\therefore S^2$ is a factor of det A since 3 rows are identical

Let s=a

$$\therefore \det A = \begin{vmatrix} a^2 & 0 & 0 \\ (a-b)^2 & b^2 & (a-b)^2 \\ (a-c)^2 & (a-c)^2 & c^2 \end{vmatrix} = \begin{vmatrix} a^2 & 0 & 0 \\ c^2 & b^2 & c^2 \\ b^2 & b^2 & c^2 \end{vmatrix} = 0 \qquad \because b + c = a$$

$\therefore$ s - a is a factor of det A

Similarly (s - b) and (s - c) are also factors.

but det A is a sixth degree polynomial

$\therefore$ The sixthe factor is of the form k (a+b+c).

$$\therefore \det A = k(a+b+c)s^2(s-a)(s-b)(s-c)$$

Let $a = b = 0,\ c = 2 \Rightarrow s = 1$

$$\therefore \det A = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 4 \end{vmatrix} = -2k \Rightarrow k = 1$$

$$\therefore \det A = 2s^3(s-a)(s-b)(s-c)$$

11. The determinant of coefficient matrix is the product of two determinant.

$$\begin{vmatrix} 1 & 1 & 0 \\ \alpha+\beta & \gamma+\delta & 0 \\ \alpha\beta & \gamma\delta & 0 \end{vmatrix} \begin{vmatrix} 1 & \gamma+\delta & \gamma\delta \\ 1 & \alpha+\beta & \alpha\beta \\ 0 & 0 & 0 \end{vmatrix} = 0 \times 0 = 0$$

for any $\alpha, \beta, \gamma, \delta$

12. $\det(A^2) = (\det A)^2 = \begin{vmatrix} 4 & 4k & k \\ 0 & k & 4k \\ 0 & 0 & 4 \end{vmatrix}^2 = 16$

$$\Rightarrow (16k)^2 = 16$$

$$\Rightarrow k^2 = \frac{1}{16}$$

$$\therefore |k| = \frac{1}{4}$$

13. $D = \begin{vmatrix} a & a^2 & a^3 \\ a^{\omega} & a^{2\omega} & a^{3\omega} \\ a^{\omega^2} & a^{2\omega^2} & a^{3\omega^2} \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ a^{\omega} & a^{2\omega} & 1 \\ a^{\omega^2} & a^{2\omega^2} & 1 \end{vmatrix}$

$$= a\,a^{\omega}\,a^{\omega^2} \begin{vmatrix} 1 & a & a^2 \\ 1 & a^{\omega} & a^{2\omega} \\ 1 & a^{\omega^2} & a^{2\omega^2} \end{vmatrix} + \begin{vmatrix} 1 & a^2 & a \\ 1 & a^{2\omega} & a^{\omega} \\ 1 & a^{2\omega^2} & a^{\omega^2} \end{vmatrix}$$

$=0 \qquad \because 1 + \omega + \omega^2 = 0$

14. Use $a_n = a_1 r^{n-1}$ $\quad \therefore \log a_n = \log a_1 + (n-1)\log r$

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ 3\log r & 3\log r & 3\log r \\ 6\log r & 6\log r & 6\log r \end{vmatrix} = 0$$

$\because R_{12}(-1), R_{13}(-1)$ and then $R_2 = R_3$

15. Clearly P is an orthogonal matrix $\quad \therefore \; PP^T = P^T P = I$

$Q^{2013} = (PAP^T)(PAP^T) \ldots (PAP^T) \quad (2013 \text{ times})$

$= PA^{2013} P^T$

$P^T Q^{2013} P = A^{2013}$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \; A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \ldots, \; A^{2013} = \begin{bmatrix} 1 & 2013 \\ 0 & 1 \end{bmatrix}$$

16. $\det \overline{A} = \det (\overline{A})^T = \det A$

$\Rightarrow \overline{\det A} = \det \overline{A} = \det A$

$\Rightarrow \det A$ is Purely real

17. $$D = \begin{vmatrix} \log_3 2^{10} & \log_{2^3} 3 \\ \log_3 2^3 & \log_{2^2} 3^2 \end{vmatrix} \begin{vmatrix} \log_2 3 & \log_{2^2} 3 \\ \log_3 2^2 & \log_3 2^2 \end{vmatrix}$$

$$= \begin{vmatrix} 10\log_3 2 & \frac{1}{3}\log_2 3 \\ 3\log_3 2 & \frac{2}{2}\log_2 3 \end{vmatrix} \begin{vmatrix} \log_2 3 & \frac{1}{2}\log_2 3 \\ 2\log_3 2 & 2\log_3 2 \end{vmatrix}$$

$= (10-1)(2-1) = 9$

18. $A = iI$

$A^n = i^n I^n = i^n I = I \quad \text{if } n = 4p$

19. $|A^3| = 7^3$

$\Rightarrow |A|^3 = 7^3 \Rightarrow |A| = 7$

$\Rightarrow k^2 - 9 = 7 \Rightarrow k = \pm 4$

20. $$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \ldots, \; A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$$

$$\therefore A^n + (n-1)I = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} = nA$$

21. Putting x=0, we get

$$\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 3 & -1 & -1 \end{vmatrix} = B \Rightarrow B = -12$$

22. $D = \begin{vmatrix} 1 & 1 & 2 \\ -\sec^2 x & \tan^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} \qquad \because R_{21}(1)$

$$= \begin{vmatrix} 1 & 2 & 2 \\ -\sec^2 x & 1 & 1 \\ -10 & 2 & 2 \end{vmatrix} \qquad \because C_{12}(1)$$

$= 0 \qquad \because C_2 = C_3$

23. $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cos x \cdot \operatorname{cosec}^2 x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ \sin^2 x & 0 & 0 \end{vmatrix} \qquad \because C_{23}(-1)$

$= \cos x - \sin^2 x - \cos^3 x$

$= \sin^2 x \cdot \cos x - \sin^2 x$

$$\int_0^{\frac{\pi}{2}} f(x)\,dx = \int_0^{\frac{\pi}{2}} \sin^2 x \cos x \,dx - \int_0^{\frac{\pi}{2}} \sin^2 x \,dx \qquad (\because \text{ taking } \sin x = t \text{ substitution})$$

$$= \left[\frac{t^3}{3}\right]_0^1 - \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2} = \frac{1}{3} - \frac{\pi}{4}$$

24. since f (- x) = - f (x), odd function.

$$\therefore \int_{-\pi/2}^{\pi/2} f(x)\,dx = 0$$

25. since $A^T A = I \Rightarrow A^2 = I \ (\because A^T = A \text{ clearly})$

$$\therefore \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ a & b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A-Z Shiksha

$\therefore a^2+b^2+c^2=1,\ ab+bc+ca=o$

$\therefore (a+b+c)^2=1 \Rightarrow a+b+c=1$

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\therefore a^3+b^3+c^3=3(1)+1=4 \qquad (\because abc=1)$

26. $$\frac{d^n}{dx^n}f(x)=\begin{vmatrix} n! & \sin\left(x+\frac{n\pi}{2}\right) & \cos\left(x+\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ p & p^2 & p^3 \end{vmatrix}$$

$$\therefore \frac{d^n}{dx^n}f(0)=f^n(0)$$

$$=0 \quad \because R_1=R_2$$

27. By $R_{32}(1)$ we get all entries of R_2 is zero

$\therefore D=0$

28. $$\Delta^1(x)=\begin{vmatrix} 2x-5 & 2x-5 & 3 \\ 6x+1 & 6x+1 & 9 \\ 14x-6 & 14x-6 & 21 \end{vmatrix}+\begin{vmatrix} x^2-5x+3 & 2 & 3 \\ 3x^2+x+4 & 6 & 9 \\ 7x^2-6x+9 & 14 & 21 \end{vmatrix}$$

$=0(\because C_1=C_2)+0(\because C_2=C_3)=0$

$\therefore \Delta(x)$ is a constant

$\therefore a=0,\ b=0,\ c=0$

Put $x=0$ both sides, we get

$$d=\begin{vmatrix} 3 & -5 & 3 \\ 4 & 1 & 9 \\ 9 & -6 & 21 \end{vmatrix}=141$$

29. Clearly $A^T=A \Rightarrow AA^T=64I$

$\Rightarrow A^2=64I$

$\Rightarrow |A|=8$

A-Z Shiksha

$$\Rightarrow \begin{vmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{vmatrix} = 8$$

$$\Rightarrow 29abc - 3\left(a^3 + b^3 + c^3\right) = 8$$

$$\Rightarrow a^3 + b^3 + c^3 = 7 \qquad \because abc = 1$$

$$\Rightarrow \left(a^3 + b^3 + c^3\right)^3 = 343$$

30. $P^{-1}\left(1 + P + P^2 + ... + P^n\right) = P^{-1}.O$

$\therefore P^{-1} + I + IP + ... + IP^{n-1} = O$

$\therefore P^{-1} + I\left(1 + P + ... + P^{n-1}\right) = O$

$\therefore P^{-1} + I\left(-P^n\right) = O$

$\therefore P^{-1} = P^n$

31. determinant $\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ $\because R_{21}(1), R_{31}(1)$

$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$ $\because R_1\left(\frac{1}{a+b+c}\right), C_{12}(-1), C_{13}(-1)$

$= (a+b+c)^3$ $\because C_2\left(\frac{1}{a+b+c}\right), C_3\left(\frac{1}{a+b+c}\right)$ and expanding along R_1

$= 0$ if $a+b+c = 0$

32. $D = \begin{vmatrix} x+1 & x+3 & x+4 \\ 3 & 3 & 4 \\ 4 & 4 & 6 \end{vmatrix}$ $\because R_{23}(-1), R_{12}(-1)$

$= \begin{vmatrix} x+1 & 2 & 1 \\ 3 & 0 & 1 \\ 4 & 0 & 2 \end{vmatrix}$ $\because C_{23}(-1), C_{12}(-1)$

$= -4 \because$ expanding along C_2

A-Z Shiksha

33. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= -\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right] < 0$ $\therefore$ negative

34. $D = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$=(a-b)(b-c)(c-a)-(a-b)(b-c)(c-a)$

$= 0$

35. equations $\begin{matrix} ax+y+z=0 \\ x+by+z=0 \\ x+y+cz=0 \end{matrix}$

$\therefore$ since $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$\therefore \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 0 & 1-b & c-1 \end{vmatrix} = 0$ $\qquad \because R_{23}(-1), R_{12}(-1)$

$\therefore a(b-1)(c-1)-1(1-a)(c-1)+1(1-b)(1-b)=0$

$\therefore \frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$ $\quad \because$ dividing both sides by $(1-a)(1-b)(1-c) \neq 0$

$\therefore \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

36. determinant of coefficient matrix $\begin{vmatrix} -1 & a & a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$

$$\therefore \begin{vmatrix} -1 & 1+a & 1+a \\ b & -1-b & 0 \\ c & 0 & -1-c \end{vmatrix} = 0 \quad \because C_{12}(-1), C_{13}(-1)$$

$$\therefore \begin{vmatrix} \frac{-1}{1+a} & 1 & 1 \\ \frac{b}{1+b} & -1 & 0 \\ \frac{c}{1+c} & 0 & -1 \end{vmatrix} = 0 \qquad \because R_1\left(\frac{1}{1+a}\right), R_2\left(\frac{1}{1+b}\right), R_3\left(\frac{1}{1+c}\right)$$

$$\therefore \frac{-1}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} = 0$$

$$\therefore \frac{-1}{1+a} + 1 - \frac{1}{1+b} + 1 - \frac{1}{1+c} = 0$$

$$\therefore \frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 2$$

37. $$\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & 2 \\ -8 & \lambda & 0 \end{vmatrix} = 0$$

$$\Rightarrow -2\lambda + 32 - 6\lambda + 72 = 0$$

$$\Rightarrow -8\lambda + 104 = 0$$

$$\Rightarrow \lambda = 13$$

38. $$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -1 - 18 - 3k + 4 + k = 0$$

$$\Rightarrow k = -\frac{15}{2}$$

39. $$\begin{vmatrix} a & b & c \\ 4 & 3 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow a - 2b + c = 0 \Rightarrow a, b, c \text{ are in AP}$$

40. $\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 - a\left(-a^2\right) = 0 \Rightarrow a^3 + 1 = 0 \Rightarrow a = -1$

41. clearly if $x = -1 \Rightarrow R_2 = R_3$ and $D = 0$

if $x = 2 \Rightarrow R_1 = R_3$ and $D = 0$

$\therefore$ solution set $\{-1, 2\}$

42. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} \neq 0 \Rightarrow$ unique solution

43. $|A| = 1 \neq 0, A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$ and $A^4 = I$

$\therefore A^4 \cdot A^{-1} = IA^{-1} \Rightarrow A^3 = A^{-1}$

44. Let $U_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ 2x + y \\ 3x + 2y + z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x = 1, y = -2, z = 1$

$\therefore U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, similarly $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}, U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

$\therefore U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & 4 & -3 \end{bmatrix} \Rightarrow |U| = 3$

45. $U^{-1} = \frac{1}{|U|} \, adj \, U = \frac{1}{3} \begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$

Sum of elements of $U^{-1} = \frac{1}{3}[-1 - 2 + 0 - 7 - 5 - 3 + 9 + 6 + 3] = 0$

46. $\begin{bmatrix}3 & 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 2\\ -2 & -1 & -1\\ 1 & 4 & -3\end{bmatrix}\begin{bmatrix}3\\2\\0\end{bmatrix}=\begin{bmatrix}-1 & 4 & 4\end{bmatrix}\begin{bmatrix}3\\2\\0\end{bmatrix}=\begin{bmatrix}5\end{bmatrix}$

det [5] = 5

47. $|A|=8+2(a-6)=2a-4$

cofactor of $a_{12}(=1)$ is 8 in $|A|$

In A^{-1} now $A_{21}=\dfrac{8}{|A|}=-4\Rightarrow\dfrac{8}{2a-4}=-4\Rightarrow\dfrac{2}{2a-4}=-1$

$\Rightarrow 2=-2a+4\Rightarrow 2a=2\Rightarrow a=1 \quad \therefore |A|=-2$

In $A^{-1}, A_{23}=C=\dfrac{\text{cofator a in }|A|}{|A|}=\dfrac{2}{-2}=-1$

48. Matrix of confactors of elements of $A = A^c$

$A^c=\begin{bmatrix}-3 & -6 & -6\\ 6 & 3 & -6\\ 6 & -6 & 3\end{bmatrix}\Rightarrow \mathit{adj}\,A=\left(A^c\right)^T=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}=3A^T$

49. $A^2=\begin{bmatrix}0 & 0 & 0\\ 3 & 3 & 9\\ -1 & -1 & -3\end{bmatrix}$ and $A^3=O$

50. Let $\omega=\dfrac{-1+i\sqrt{3}}{2}$ and $\omega^2=\dfrac{-1-i\sqrt{3}}{2}$

$\therefore \omega^3=1$ and $1+\omega+\omega^2=0$

$\therefore A=\begin{bmatrix}\dfrac{\omega}{i} & \dfrac{\omega^2}{i}\\ \dfrac{-\omega^2}{i} & \dfrac{-\omega}{i}\end{bmatrix}=\dfrac{\omega}{i}\begin{bmatrix}1 & \omega\\ -\omega & -1\end{bmatrix}$

$\therefore A^2=\dfrac{\omega^2}{i^2}\begin{bmatrix}1 & \omega\\ -\omega & -1\end{bmatrix}\begin{bmatrix}1 & \omega\\ -\omega & -1\end{bmatrix}=-\omega^2\begin{bmatrix}1-\omega^2 & 0\\ 0 & 1-\omega^2\end{bmatrix}$

$$=\begin{bmatrix} -\omega^2+\omega^4 & 0 \\ 0 & -\omega^2+\omega^4 \end{bmatrix}=\begin{bmatrix} 1+2\omega & 0 \\ 0 & 1+2\omega \end{bmatrix}$$

$$f(A)=A^2+2I=\begin{bmatrix} 1+2\omega & 0 \\ 0 & 1+2\omega \end{bmatrix}+\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$=(3+2\omega)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\left\{3+2\left(\frac{-1+i\sqrt{3}}{2}\right)\right\}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\left(2+i\sqrt{3}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

51. $A^2 = A$

$$\therefore \begin{bmatrix} 2 & -2 & -16-4x \\ -1 & 3 & 16+4x \\ 4+x & -8-2x & -12+x^2 \end{bmatrix}=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$$

By comparing Compoent, we have $4+x=1 \Rightarrow x=-3$

52. $$\begin{vmatrix} \frac{1}{a^2} & \frac{-1}{b^2} & \frac{-1}{c^2} \\ \frac{-1}{a^2} & \frac{1}{b^2} & \frac{-1}{c^2} \\ \frac{-1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2} \end{vmatrix}=\frac{1}{a^2b^2c^2}\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix}=\frac{-4}{a^2b^2c^2}\neq 0$$

$\therefore$ Unique solution

53. $$A^2=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}=\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

For n = 2 , A^2 = (d) Ans.

54. $A^3=A\cdot A^2=A(5A-7I)=5A^2-7A$

$$=5(5A-7I)-7A=25A-35I-7A=18A-35I$$

Similarly $A^4=A\cdot A^3$

$$=A(18A-35I)$$

$$=18(5A-7I)-35A$$

$$=55A-126I$$

$$A^5=149A-385I=aA+bI$$

$$\therefore a=149, b=-385$$

$$\therefore 2a-3b=2(149)-3(-385)=1453$$

55. On expanding

$$\therefore 1(c^2-ab)-a(c-a)+b(b-c)=0$$

$$\therefore a^2+b^2+c^2-ab-bc-ca=0$$

$$\therefore \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$$

Provided $a=b=c$

$$\Rightarrow A=B=C=\frac{\pi}{3}$$

$$\therefore 64\left(\sin^2\frac{\pi}{3}+\sin^2\frac{\pi}{3}+\sin^2\frac{\pi}{3}\right)=64\left(\frac{3}{4}+\frac{3}{4}+\frac{3}{4}\right)=144$$

56. $A^2=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}=2\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}=2A$

$$A^3=A\cdot A^2=A\cdot 2A=2A^2=2(2A)=2^2\cdot A=2^{3-1}A$$

$$A^{2013}=2^{2012}A$$

57. $A^2=3A$

$$A^3=A^2\cdot A=3A\cdot A=3\cdot 3A=3^2A=3^{3-1}A$$

$$A^{2013}=3^{2012}A$$

58. $|A|=1(0+3)+1(0+6)+1(0-4)=5$

$$\frac{|adjB|}{|c|}=\frac{|adj(adjA)|}{|5A|}=\frac{|A|^{(3-1)^2}}{5^3|A|}=\frac{|A|^4}{5^4}=1$$

59. $|A|=\cos^2\alpha+\sin^2\alpha=1$

$$adj(adjA)=|A|^{n-2}A=(1)^{3-2}A=A$$

$$\therefore \left|\left(adj\left(adj\left(adj\left(adj\,A\right)\right)\right)\right)\right|=|adj(adjA)|$$

$$=|A|^{(n-1)^2}$$

$$=|A|^4=1$$

60. $A_r=r^2-(r-1)^2=2r-1$

$$\sum A_r=\sum(2r-1)=2\frac{r}{2}(r+1)-r=r^2$$

$$\sum_{r=1}^{2013}A_r=(2013)^2\Rightarrow\sqrt{\left(\sum_{r=1}^{2013}A_r\right)}=2013$$

61. Given determinant is a product of two determinant

$$\begin{vmatrix} z & \bar{z} & 0 \\ \bar{z} & z & 0 \\ 1 & z & 0 \end{vmatrix}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 0 & 0 & 0 \end{vmatrix}=0\times0=0$$

62. $D=1(1-3\sin\theta\cos\theta)-3\cos\theta(\sin\theta-3\cos\theta)+1(\sin^2\theta-1)$

$$=1-6\sin\theta\cos\theta+8\cos^2\theta$$

$$=(3\cos\theta-\sin\theta)^2=(a\cos\alpha+b\sin\alpha)^2$$

$$\therefore -\sqrt{a^2+b^2}\le\sqrt{D}\le\sqrt{a^2+b^2}$$

$$\therefore -\sqrt{9+1}\le(3\cos\theta-\sin\theta)\le\sqrt{9+1}$$

$$\therefore 0\le(3\cos\theta-\sin\theta)^2\le10$$

$\therefore$ Range = [0,10]

63. Product =

$$\begin{bmatrix} \cos^2\theta\cos^2\phi+\cos\theta\sin\theta\times\cos\phi\sin\phi & \cos^2\theta\cos\phi\sin\phi+\sin^2\phi\cos\theta\sin\theta \\ \cos\theta\cdot\sin\theta\cos^2\phi+\sin^2\theta\cdot\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi+\sin^2\phi\sin^2\theta \end{bmatrix}$$

$$=\begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\text{ if }\theta-\phi=(2n+1)\frac{\pi}{2}\qquad\because\cos(\theta-\phi)=0$$

64. $$D=\begin{vmatrix}1 & \cos P & \cos^2 P\\ 1 & \cos Q & \cos^2 Q\\ 1 & \cos R & \cos^2 R\end{vmatrix}=-(\cos P-\cos Q)(\cos Q-\cos R)(\cos R-\cos P)$$

if $P<Q<R \quad \Rightarrow D>0$

if $P>Q>R \quad \Rightarrow D<0$ can not be determined.

65. $$D=\begin{vmatrix}1 & [y] & [z]\\ 1 & [y]+1 & [z]\\ 1 & [y] & [z]+1\end{vmatrix}([x]+[y]+[z]+1) \because C_{21}(1), C_{31}(1) \text{ and } C_1\left(\frac{1}{[x]+[y]+[z]+1}\right)$$

$$\begin{vmatrix}1 & [y] & [z]\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}([x]+[y]+[z]+1) \because R_{12}(-1), R_{13}(-1)$$

$$=([x]+[y]+[z]+1)$$

$\therefore$ Maximum value of D= 1+0+2+1=4

66. $$A^T A^{-1}=\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}\left(\frac{1}{1+\tan^2 x}\right)\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}$$

$$=\frac{1}{1+\tan^2 x}\begin{bmatrix}1-\tan^2 x & -2\tan x\\ 2\tan x & 1-\tan^2 x\end{bmatrix}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$$

67. (i) $$f(x)f(y)=\begin{bmatrix}\cos x & 0 & \sin x\\ 0 & 1 & 0\\ -\sin x & 0 & \cos x\end{bmatrix}\begin{bmatrix}\cos y & 0 & \sin y\\ 0 & 1 & 0\\ -\sin y & 0 & \cos y\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & 0 & \sin(x+y)\\ 0 & 1 & 0\\ -\sin(x+y) & 0 & \cos(x+y)\end{bmatrix}=f(x+y)$$

(ii) $$|f(x)|=1\neq 0, \operatorname{adj}(f(x))=\begin{bmatrix}\cos x & 0 & -\sin x\\ 0 & 1 & 0\\ \sin x & 0 & \cos x\end{bmatrix}=f(-x)$$

$$\therefore [f(x)]^{-1}=\frac{1}{|f(x)|}\operatorname{adj}(f(x))=\frac{1}{1}f(-x)=f(-x)$$

(iii) $\left[f(x)g(y)\right]^{-1} = \left[g(y)\right]^{-1}\left[f(x)\right]^{-1} = g(-y)f(-x)$ $\quad\because$ (ii)

68. $D_1 = x^3 - 3a^2x + 2a^3,\ D_2 = x^2 - a^2$

$$\frac{d}{dx}(D_1) = 3x^2 - 3a^2 = 3\left(x^2 - a^2\right) = 3D_2$$

69. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$ $\quad\because$ D is a product of two deferminant

$= \{(1-\alpha)(\alpha-\beta)(\beta-1)\}\{(1-\alpha)(\alpha-\beta)(\beta-1)\}$

$= (1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

since $x^2 + bx + c = (x-\alpha)(x-\beta)$

$\therefore 1 + b + c = (1-\alpha)(1-\beta)$

& $\alpha + \beta = -b, \alpha\beta = c$

$\therefore (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (-b)^2 - 4c = b^2 - 4c$

$\therefore D = (1+b+c)^2\left(b^2 - 4c\right)$

70. Linear equations

$a \log x + b \log y = m$

$c \log x + d \log y = n$

$\therefore$ By cramer's rule : $\log x = \frac{D_1}{D}, \log y \frac{D_2}{D}$

$\therefore x = e^{\frac{D_1}{D}}, y = e^{\frac{D_2}{D}}$

71. $D = \sqrt{6}\begin{vmatrix} 1 & 2i & 3+\sqrt{6} \\ \sqrt{2} & \sqrt{3}+2\sqrt{2}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{3} & \sqrt{2}+2\sqrt{3}i & 3\sqrt{3}+2i \end{vmatrix}$ $\quad\because C_1\left(\frac{1}{\sqrt{6}}\right)$

$= \sqrt{6}\begin{vmatrix} 1 & 0 & \sqrt{6} \\ \sqrt{2} & \sqrt{3} & \sqrt{6}i \\ \sqrt{3} & \sqrt{2} & 2i \end{vmatrix}$ $\quad\because C_{12}(-2i), C_{13}(-3)$

$$=\sqrt{6}\begin{vmatrix} 1 & 0 & \sqrt{6} \\ \sqrt{2} & \sqrt{3} & 0 \\ \sqrt{3} & \sqrt{2} & 0 \end{vmatrix} \quad \because C_{23}\left(-\sqrt{2}i\right)=0-0+\sqrt{6}.\sqrt{6}(2-3)=-6 \text{ a real number.}$$

72. Match the following colums.

Column I		Column II	
1.	A is a square matrix such that $A^2 = A$	D.	A is a an Idempotent matrix
2.	A is a square matrix such that $A^m = O$	A.	A is a nilpotent matrix
3.	A is a square matrix such that $A^2 = I$	B.	A is an Involutanry matrix
4.	A is a square matrix such that $A^T = A$	C.	A is a symmetric materix

73. $$f(x)=\begin{vmatrix} 2 & 0 & -\sin x \\ 0 & 2 & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix} \quad \because C_{31}(-2\sin x),\ C_{32}(2\cos x)$$

$$=2\left(0+\cos^2 x\right)-0-\sin x\left(0-2\sin x\right)=2$$

$$f(x)=2,\quad f^1(x)=2 \qquad \therefore I=\int_0^{\pi/2}\left(f(x)+f^1(x)\right)dx$$

$$=2[x]_0^{\pi/2}=\pi$$

74. $$\frac{f^1(x)}{x}=\begin{vmatrix} 1 & \cos x & -\sin x \\ x & \frac{-\tan x}{x} & -x^2 \\ 2x & \sin 2x & 5x \end{vmatrix}+\begin{vmatrix} x & \sin x & \cos x \\ 2x & -\sec^2 x & -3x^2 \\ 2 & \frac{\sin 2x}{x} & 5 \end{vmatrix}+\begin{vmatrix} x & \sin x & \cos x \\ x & \frac{-\tan x}{x} & -x^2 \\ 2 & 2\cos 2x & 5 \end{vmatrix}$$

$$\therefore \lim_{x\to 0}\frac{f^1(x)}{x}=\begin{vmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 2 & 2 & 5 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 2 & 2 & 5 \end{vmatrix}$$

$$=0+2+2=4$$

75. Consider $S_n = 1+2+6+15+\ldots+t_{n-1}+t_n$,

Where $t_n =$ element a_{11} of M_n

1st We find a_{11} of M_n

$$S_n = 1 + 2 + 6 + 15 + \ldots + t_{n-1} + t_n$$

$$S_n = \quad 1 + 2 + 6 + \ldots + t_{n-2} + t_{n-1} + t_n$$

$$- \quad - \quad - \quad - \quad - \quad - \quad - \quad -$$

$$0 = 1 + 1 + 4 + 9 + \ldots + \left(t_n - t_{n-1}\right) - t_n$$

$$\therefore t_n = 1 + \left(1 + 2^2 + 3^2 + 4^2 + \ldots + (n-1)^2\right)$$

$$= 1 + \frac{n-1}{6}\left((n-1)+1\right)\left(2(n-1)+1\right)$$

$$t_n = 1 + \frac{n(n-1)(2n-1)}{6}$$

It is clear in nxn matrix that distance of consecutive diagonal element is $(n+1)$

$\therefore$ First term $= 1 + \dfrac{n(n-1)(2n-1)}{6}$, difference $= n + 1$

$\therefore$ Sum of diagonal element of M_n $= \dfrac{n}{6}\left[2\left(1 + \dfrac{n(n-1)(2n-1)}{6}\right) + (n-1)(n+1)\right]$

$$= \frac{n}{6}\left[2n^3 + n + 3\right]$$

For n=6, Sum of diagonal element =441

76. Let x=0 both sides $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ =0+0+0+0+0+f $\quad \therefore$ f = 1

Differentiate both sides and Put x = 0.

$$\therefore \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = e \qquad \therefore e = 3$$

Put x = 1 both sides

$$\therefore \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = a + b + c + d + e + f$$

Put x = - 1 both sides

$$\therefore \begin{vmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} = -a + b - c + d - e + f$$

$\therefore 4 = a + b + c + d + 3 + 1$

$\therefore a + b + c + d = 0$ ...... (1)

$\therefore 200 = -a + b - c + d - 3 + 1$

$\therefore -a + b - c + d = 2$ .... (ii)

by (i) & (ii)

$a + c = -1, b + d = 1$

77. Put x = 0, constant term $= f(0) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Differentiate and Put x = 0,

$$\therefore \text{ coefficient of } x = f'(0) = \begin{vmatrix} 3m & 5n & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$+\begin{vmatrix} 1 & 1 & 1 \\ 0 & 3m & 5n \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 5n & 0 & 3m \end{vmatrix}$$

$\therefore f'(0) = 0.$

$\therefore$ Required sum = $f(0) + f'(0) = 0 + 0 = 0$

78. $D = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ $\quad \because R_{31}(-1), R_{32}(-1)$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \sin^2 x & \cos^2 x & 2+\sin 2x \end{vmatrix}$ $\quad \because C_{13}(1),\ C_{23}(1)$

$= 2 + \sin 2x$

Now $-1 \le \sin 2x \le 1$

$1 \le 2 + \sin 2x \le 3$ $\quad \therefore M = 3, m = 1$

1. $M^2 + m^{2013} = 10$,

2. $M^3 - m^3 = 26$

3. $M^{2k} - m^{2k} = \text{odd} - 1 = \text{even}$ always

4. 2 M - 3 m = 3, M + m = 4 , M + 2 m = 5 are become three sides of triangle.

79. Since

$2 < e < 3 \Rightarrow [e] = 2$

$3 < \pi < 4 \Rightarrow [\pi] = 3$

$3<\pi^2-6<4 \Rightarrow [\pi^2-6]=3$

$$\therefore \det\begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix} = -8$$

80. Constant term $= f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$

81. $$D=\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} \quad \because R_{32}(1).$$

$= 0 \quad \because R_1 = R_2$

since $\sin\left(\theta+\frac{2\pi}{3}\right)+\sin\left(\theta-\frac{2\pi}{3}\right) = 2\sin\theta\cos\frac{2\pi}{3} = 2\sin\theta\left(\frac{-1}{2}\right) = -\sin\theta$

similarly $\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta-\frac{2\pi}{3}\right) = -\cos\theta$ and

$\sin\left(2\theta+\frac{4\pi}{3}\right)+\sin\left(2\theta-\frac{4\pi}{3}\right) = -\sin 2\theta$

82. $A^2 = B$

$$\therefore \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\therefore \alpha^2 = 1, \alpha+1 = 5$

There is no α given in option satisfies the obtain equation.

$\therefore$ no $= \alpha$

83. $A^2 = 9I \Rightarrow \begin{bmatrix} \alpha & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 2 & 3 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

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$$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ 2\alpha+6 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\Rightarrow \alpha^2 = 9, 2\alpha + 6 = 0$$

$\Rightarrow \alpha = -3$ satisfies above both aquations.

84. $$A^2 = \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$\therefore$ I+2A+3A^2+... ∞ =I+2A+0+... ∞

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ -18 & -6 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ -18 & -5 \end{bmatrix}$$

85. $\det(\text{M-I}) = \det(M - I)^T$

$= \det(M^T - I)$

$= \det(M^T - M^T M)$

$= \det(M^T(I - M)$

$= \det M^T \cdot \det(I - M)$

$= \det M \cdot \det(-(M - I))$

$= (-1)^3 \det(M - I)$

det (M-I) = - det (M-I)

$\therefore$ det (M-I) = 0

86. $$\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0 \quad \text{on expanding along } C_1$$

$$\therefore \lambda = \cos 2\alpha + \sin 2\alpha$$

Compare with $f(\alpha) = a\cos\alpha + b\sin\alpha$

whose range is $[-\sqrt{a^2+b^2}, \sqrt{a^2+b}\ ]$

(i) $\therefore$ Range of λ is $[-\sqrt{2}, \sqrt{2}]$ ($\because$ Here a = 1, b = 1)

(ii) For $\lambda = 1$

$\cos 2\alpha + \sin 2\alpha = 1$

Dividing both sides by $\sqrt{2}$

$$\therefore \frac{1}{\sqrt{2}}\cos 2\alpha + \frac{1}{\sqrt{2}}\sin 2\alpha = \frac{1}{\sqrt{2}}$$

$$\therefore \cos\left(2\alpha - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\therefore 2\alpha - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\Rightarrow \alpha = n\pi, n\pi + \frac{\pi}{4}$$

87. $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 1-\lambda & O \\ -1 & 7-\lambda \end{vmatrix} = 0 \qquad \Rightarrow \lambda^2 - 8\lambda + 7 = O$$

$$\Rightarrow A^2 - 8A + 7I = O$$

$$\Rightarrow A^2 = 8A - 7I$$

$$\Rightarrow k = -7$$

88. Let $\begin{bmatrix} k & k & k \\ k & k & k \\ k & k & k \end{bmatrix}$ be the identity element, then

$$\therefore \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}\begin{bmatrix} k & k & k \\ k & k & k \\ k & k & k \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3kx & 3kx & 3kx \\ 3kx & 3kx & 3kx \\ 3kx & 3kx & 3kx \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

$$\therefore 3kx = x$$

$$\therefore (3k-1)x = 0$$

$$k = \frac{1}{3} \; (\because x \mp 0)$$

$\therefore$ Required identity elenent $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

89. $AB = I \Rightarrow \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}\begin{bmatrix} y & y & y \\ y & y & y \\ y & y & y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3xy & 3xy & 3xy \\ 3xy & 3xy & 3xy \\ 3xy & 3xy & 3xy \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$\Rightarrow 3xy = \frac{1}{3}$$

$$\Rightarrow y = \frac{1}{9x} \quad \text{or} \quad x = \frac{1}{9y}$$

The required inverse of $\begin{bmatrix} y & y & y \\ y & y & y \\ y & y & y \end{bmatrix}$ is $\begin{bmatrix} \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \\ \frac{1}{9y} & \frac{1}{9y} & \frac{1}{9y} \end{bmatrix}$

90. $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix}$

$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}, \quad |I - A| = 4 \neq 0$

$\therefore (I - A)^{-1} = \frac{1}{|I - A|} adj(I - A) = \frac{1}{4}\begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix}$

$\therefore \phi(A) = (I + A)(I - A)^{-1}$

$= \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix} \frac{1}{4}\begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 0 & 4 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$

91. Construct an orthogonal matrix using the skew-symmetric matrix $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, |I - A| = 5 \neq 0$

$(I - A)^{-1} = \frac{1}{|I - A|} adj(I - A) = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

$\therefore$ Orthogonal matrix $\phi(A) = (I + A)(I - A)^{-1}$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix}$$

92. $I + A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -5 \\ -1 & 5 & 1 \end{bmatrix}, I - A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 5 \\ 1 & -5 & 1 \end{bmatrix}, |I - A| = 31 \neq 0$

$$(I - A)^{-1} = \frac{1}{|I - A|}\left(\operatorname{adj}(I - A)\right) = \frac{1}{31} \begin{bmatrix} 26 & 3 & 11 \\ 7 & 2 & -3 \\ 9 & 7 & 5 \end{bmatrix}$$

Required Orthogonal matrix= $\phi(A) = (I + A)(I - A)^{-1}$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -5 \\ -1 & 5 & 1 \end{bmatrix} \frac{1}{31} \begin{bmatrix} 26 & 3 & 11 \\ 7 & 2 & -3 \\ 9 & 7 & 5 \end{bmatrix} = \frac{1}{31} \begin{bmatrix} 21 & 6 & 22 \\ 14 & -27 & -6 \\ 18 & 14 & -21 \end{bmatrix}$$

93. The Charateristic eqution is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0 \qquad \Rightarrow \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

$$\Rightarrow A^3 - 7A^2 + 11A - 5I = 0$$

$$\Rightarrow A^3 - 7A^2 + 10A = 5I - A$$

94. The Characteristic equation of A is $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \qquad \Rightarrow \lambda^2 - 5\lambda - 2 = 0$$

$$\Rightarrow A^2 - 5A - 2I = O$$

$$\Rightarrow I - 5A^{-1} - 2A^{-2} = O$$

$$\Rightarrow A^{-2} = \frac{1}{2}\left[I - 5A^{-1}\right]$$

$$\Rightarrow A^{-4} = \frac{1}{4}\left(I - 5A^{-1}\right)^2$$

$$\Rightarrow A^{-4} = \frac{1}{4}\left(I - 10A^{-1} + 25A^{-2}\right)$$

$$\Rightarrow A^{-4}=\frac{1}{4}\left[I-10A^{-1}+\frac{25}{2}\left(I-5A^{-1}\right)\right]$$

$$\Rightarrow 8A^{-4}=27I-145A^{-1}$$

95. $(1-\lambda)x_1+2x_2+3x_3=0$

$3x_1+(1-\lambda)x_2+2x_3=0$

$2x_1+3x_2+(1-\lambda)x_3=0$

$$\therefore \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix}=0$$

$$\Rightarrow (1-\lambda)\left((1-\lambda)^2-6\right)-2\left(3-3\lambda-4\right)+3(9-2+2\lambda)=0$$

$$\Rightarrow (1-\lambda)(\lambda^2-2\lambda-5)-2(-3\lambda-1)+3(7+2\lambda)=0$$

$$\Rightarrow \lambda^2-2\lambda-5-\lambda^3+2\lambda^2+5\lambda+6\lambda+2+21+6\lambda=0$$

$$\Rightarrow -\lambda^3+3\lambda^2+15\lambda+18=0$$

$\Rightarrow \lambda=6$ Satisfies the equation.

96. determinant of Coefficient matrix is

$$D=\begin{vmatrix} \sec^2\alpha & -\tan^2\alpha & 1 \\ \cos^2\alpha & \sin^2\alpha & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$=\sec^2\alpha\sin^2\alpha+\tan^2\alpha\cdot\cos^2\alpha-\sin^2\alpha$$

$=\tan^2\alpha\neq 0$ unique solution exists

$$D_x=D_y=D_z=\tan^2\alpha$$

$$\therefore x=\frac{D_x}{D}=1, y=\frac{D_y}{D}=1, Z=\frac{D_z}{D}=1$$

$$\therefore (x,y,z)=(1,1,1)$$

97. The determinant of the coefficient matrix is

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & k+7 & -3 \\ 3 & 4 & k-3 \end{vmatrix}=0\Rightarrow k^2-1=0 \qquad (\because \text{By expanding along } C_1)$$

$$\Rightarrow k=\pm 1$$

98. clearly (d).

99. $$\begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -5 & -2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (\lambda-1)(\lambda-2)^2 = 0 \Rightarrow \lambda = 1, 2$

For $\lambda = 1 \;\therefore 3x + 6y + 6z = 0$

$x + 2y + 2z = 0$

$x - 5y - 3z = 0$

By cross multiplication $\dfrac{x}{4} = \dfrac{y}{1} = \dfrac{z}{-3} \quad \therefore x = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}$......(1)

For $\lambda = 2 \;\therefore 2x + 6y + 6z = 0$

$x + y + 2z = 0$

$-x - 5y - 4z = 0$

$\therefore \dfrac{x}{6} = \dfrac{y}{2} = \dfrac{z}{-4} \;\therefore x = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$.........(2)

By (i) and (ii), $\cos\theta = \dfrac{(4,1,-3)}{\sqrt{16+1+9}} \cdot \dfrac{(3,1,-2)}{\sqrt{9+1+4}} = \dfrac{19}{\sqrt{364}}$

$\therefore \tan\theta = \sqrt{\sec^2\theta - 1} = \dfrac{\sqrt{3}}{19}$

100. $$\begin{vmatrix} A & 3 & 6 \\ 100A+8+20 & 300+9+10B & 600+C+20 \\ 2 & B & 2 \end{vmatrix} = \begin{vmatrix} A & 3 & 6 \\ A28 & 3B9 & 62C \\ 2 & B & 2 \end{vmatrix} = km$$

is divisible by k since the 2nd row is divisible by k.

101. $I + A + A^2 + \ldots \infty = (I - A)^{-1} = \begin{bmatrix} 3 & -3 \\ 1 & 0 \end{bmatrix}^{-1} = \dfrac{1}{3}\begin{bmatrix} 0 & 3 \\ -1 & 3 \end{bmatrix}$

102. $\begin{vmatrix} -2-\lambda & 3 \\ -1 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + \lambda + 1 = 0$

$\Rightarrow A^2 + A + I = O$

$\Rightarrow A^2 = - A - I$

$\Rightarrow A^3 = -A^2 - A$

$$= -(-A - I) - A$$

$$\Rightarrow A^3 = I$$

103. By the pascal rule $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

$$\therefore \binom{n+1}{r+1} - \binom{n}{r} = \binom{n}{r+1}$$

$$\therefore \text{LHS}= \begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x+1}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y+1}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z+1}{r+2} \end{vmatrix} \quad \because C_{23}(-1) \text{ and } C_{12}(-1)$$

$$\begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z}{r+2} \end{vmatrix} \quad \because C_{23}(-1)$$

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$$\therefore \lambda = 1$$

104. Say $x + 2y + 3z = 6$ .... (1)

$x + 3y + 5z = 9$ .... (2)

$2x + 5y + \lambda z = \mu$........(3)

By (3) - 2(1) ,(3) - 2(2) we get

$y + (\lambda - 6)z = \mu - 12$.......(4)

$-y + (\lambda - 10)z = \mu - 18$......(5)

By (4) +(5) we get $(\lambda - 8)z = \mu - 15$

$\lambda \neq 8, \mu \in R$ unique solution

$\lambda = 8, \mu \neq 15$ no solution

$\lambda = 8, \mu = 15$ Infinity of solutions.

105 $AA^T = I$ since A is orthogonal

$$\therefore \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix} \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore 4\beta^2 + \gamma^2 = 1......(i)$

$2\beta^2 - \gamma^2 = 0......(ii)$

$\alpha^2 + \beta^2 + \gamma^2 = 1......(iii)$

$\alpha^2 - \beta^2 - \gamma^2 = 0......(iv)$

By solving (i) ,(ii) ,(iii) ,(iv) we get

$\alpha = \pm\dfrac{1}{\sqrt{2}}, \beta = \pm\dfrac{1}{\sqrt{6}}, \gamma = \pm\dfrac{1}{\sqrt{3}}$

ANSWERS

1	(b)	22	(c)	43	(c)	64	(d)	83	(c)
2	(a)	23	(b)	44	(a)	65	(c)	84	(c)
3	(c)	24	(a)	45	(b)	66	(b)	85	(a)
4	(d)	25	(b)	46	(a)	67 (i)	(d)	86 (i)	(b)
5	(c)	26	(d)	47	(b)	67 (ii)	(c)	86 (ii)	(c)
6	(d)	27	(a)	48	(d)	67 (iii)	(c)	87	(d)
7	(a)	28	(d)	49	(c)	68	(d)	88	(a)
8	(c)	29	(a)	50	(d)	69	(a)	89	(d)
9	(d)	30	(c)	51	(d)	70	(c)	90	(b)
10	(b)	31	(c)	52	(a)	71	(a)	91	(a)
11	(b)	32	(d)	53	(d)	72	(b)	92	(b)
12	(b)	33	(c)	54	(c)	73	(b)	93	(a)
13	(a)	34	(a)	55	(b)	74	(d)	94	(b)
14	(a)	35	(a)	56	(a)	75	(b)	95	(d)
15	(a)	36	(b)	57	(c)	76	(c)	96	(a)
16	(a)	37	(b)	58	(b)	77	(d)	97	(a)
17	(b)	38	(c)	59	(a)	78	(c)	98	(d)
18	(c)	39	(a)	60	(d)	79	(d)	99	(b)
19	(d)	40	(c)	61	(d)	80	(b)	100	(c)
20	(c)	41	(d)	62	(c)	81	(a)	101	(c)
21	(a)	42	(b)	63	(b)	82	(d)	102	(a)
								103	(b)
								104	(b)
								105	(c)

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