# Lecture Notes for Average

## Intro To Averages

You can find out the application of averages across different chapters of aptitude, like Time & Work, Time Speed and Distance, Ratio and Proportion, Percentages etc. So, from the aptitude point of view, the average is an important chapter.

### What is an Average?

Average is a number that measures the central tendency of a set of numbers. In other words, it is an estimate where the centre point of the set of numbers lies. Average is also known as the mean. In mathematics, the average is equal to the sum of the set of numbers divided by numbers of

$$\text{Average} = \frac{\text{Sum of the numbers}}{\text{Number of numbers}}$$

values in the set.
The formula for the average for the set of numbers:-
For Example:

Let x1, x2, x3, … , xn be a set of n numbers and "An" be their average.
So,

An = (x1 + x2 + x3 + … + xn)/n .

**Another meaning of average** is, average is that single number, that can replace each of the given numbers present in the set with the average number and still get the same total.
For Example:

The average of 5 numbers 11, 14, 17, 18 and 20 is:
Average = (11 + 14 + 17 + 18 + 20)/5 = 80/5=16

This means that if you replace all the 5 numbers with 16 (average number), even then the sum will be 80, there would be no change in the total.
The average also refers to one of the 3 central tendencies as we have studied in statistics for any group of numbers. The three central tendencies are: 1. Mean (Average)

2. Median
3. Mode

Mean (Average) we have already discussed above.

**Median:** It is defined as the middle term of the group of numbers arranged in order.
For Example : The numbers 3, 1, 5, 7, 6, 4, 2 are not arranged in order. So for the median, first, you have to arrange these numbers in order.
1, 2, 3, 4, 5, 6, 7 are arranged in order. Thus, Median = 4.(i.e. 4 is the middle number)

**Application :** 1. It is used to measure the distribution of earning.
2. To find the middle age, from the class of students.

**Mode :** In a set of numbers the number that occurs with the greatest frequency(most often) is the mode of that set.
For Example : Let a set of numbers 10, 15, 19, 19, 7, 11, 15, 19, 12, 11, 19, 23
So, Mode = 19( Because it occurs 4 times).

**Application :** 1. It is used to measure the influx of public transport.
2. It is used to measure the number of games succeeded by any team of players.

NOTE : 1. Average of first n natural numbers = (n+1)/2
2. Average of first n even numbers = n+1
3. Average of first n odd numbers = n

# Assumed Average Approach

Assumed average approach is a way to find out the average of a set of numbers by assuming the average.

## What is the assuming average approach?

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We already know that, Average is that one number that can replace each of the numbers in a group of numbers and still keep the same total.

By using this concept the assumed average approach is a bypass for getting the average of the numbers.

Let us say 6, 10, 7 & 5 are the 4 numbers. So, Average is;

Average = (6 + 10 + 7 + 5)/4 = 7

7 can replace all the 4 numbers.

If you see the deviation between the numbers and their average (between left column and right column), the direction should be: left column - right column

| Left column | Right column | Deviation |
|---|---|---|
| 6 | 7 | - 1 |
| 10 | 7 | +3 |
| 7 | 7 | 0 |
| 5 | 7 | - 2 |

The net sum of all these deviations is 0 (-1+3+0-2 = 0). This means the average value is correct.

The following are some steps to calculate correct average from the assumed average:

**Step1.** You have to assume an average.

**Step2.** Calculate how much the given numbers deviate from assumed average.

**Step3.** Calculate the sum of all the deviations (i.e. Total deviation).

**Step4.** Calculate the average deviation with the help of the following formula :

$$\text{Average Deviation} = \frac{\text{Total Deviation}}{\text{Number of numbers}}$$

**Step5.** Now, the correct average will be equal to the sum of assumed average and average deviation. i.e.

Correct average = Assumed average + Average Deviation.

To understand the assumed average approach, consider an example:

Let 37, 75, 83, 94 & 46 are 5 numbers. You don't know the average and you want to find out the average for these numbers without doing the sum of these numbers.

**Step1.** For this example, assume an average of let us say, 60.
**Step2.** Deviation calculation
60 to 37 there is a deviation of -23.
60 to 75 there is a deviation of +15.
60 to 83 there is a deviation of +23.
60 to 94 there is a deviation of +34.
60 to 46 there is a deviation of -14.
**Step3.** Total deviation = -23+15+23+34-14 = 35.
**Step4.** Average deviation = 35/5 = 7.
**Step5.** Correct average = 60+7 = 67.

You can assume any value of average, but assumed value should be nearly equal to the one of the given value for simple calculation.

In the above example, let us say you assume the average to be 70 instead of 60.
**Step1.** Assumed average = 70.
**Step2.** Deviation calculation
70 to 37 there is a deviation of -33.
70 to 75 there is a deviation of +5.
70 to 83 there is a deviation of +13.
70 to 94 there is a deviation of +24.
70 to 46 there is a deviation of -24.
**Step3.** Total deviation = -33+5+13+24-24 = -15.
**Step4.** Average deviation = -15/5 = -3.
**Step5.** Correct average = 70+(-3) = 67.
So you can see that the answer will be the same irrespective of what average you take.

The benefit of the assumed average method is that it is much faster in the case when numbers are bigger and they are clustered (for example,a group of numbers between the range of 300 to 400), then your calculation is much faster than what you normally do.

## Some questions for practice:

1. Find the average of the following numbers using assumed average approach:
   250, 225, 275, 281, 294
   **Ans : 265.**

2. Find the average of the following numbers using assumed average approach:
   35, 72, 81, 93, 49
   **Ans : 66.**

3. Find the average of the following numbers using assumed average approach:
   792, 775, 724, 765
   **Ans : 764.**

# Standard Language In Average

Every chapter has standard language inside it. You can also observe that there is some standard language inside the Average chapter.

You can understand the standard language on average by the help of some examples. So, here we understand the standard language by the help of following examples:

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## Example 1:

**Statement :** The average of 5 numbers is 12.
When you see this statement two reactions come to mind. The 1st one is that, $5 \times 12 = 60$. and 2nd is that, add 12 five times i.e. 12 + 12 + 12 + 12 + 12 = 60.
So, there are two approaches to tackle this statement.

## Example 2:

**Statement 1:** The average age of 24 students and principle is 15.
**Solution:** When you look at the statement you realise that there are 25 people with an average of 15. Your reaction is $25 \times 15 = 375$, that means the total age of 25 people is 375.

**Statement 2:** The average age of the students is 14.
**Solution:** The total age of the students = $24 \times 14 = 336$.
From these two statements the difference between 375 & 336 (375-336=39) will give you the average age of principle.
This is the total difference approach.
Any question has still not been asked to you, but still you have calculated a lot of things in this question in your mind.

Questions that may be asked from you, is to find the age of principle. So, in this type of question solving while reading, what will happen, you will know the answer much before you actually see what is asked. This is the best kind of solving in the exams. This will definitely increase your solving skills.

## Example 3:

**Statement 1:** The average score of a batsman after 9 innings is x.
**Solution:** Your reaction is 9x i.e. Total run in 9 innings = 9x.
**Statement 2:** In the 10th inning he scored 100 and increased his average by 8 runs.
**Solution:** In the 2nd statement two reactions come in your mind. 1st one is that,
Total run after 10th innings = 9x+100. ………….(1)
And 2nd is, total run after 10th inning = 10(x+8). ………..(2)
**Statement 3:** 1. Find the original average?
2. Find a new average?
3. Find the total run scored by him in 9 innings?

4. Find the total run scored by him in 10 innings?

**Solution:** 1. Equate eq(1) & (2) you will get;

$$9x+100 = 10(x+8)$$

$$x = 20$$, This is the total average.

2. New average = x+8 = 20+8 = 28.

3. Total run scored in 9 innings = 9x = $9 \times 20 = 180$.

4. Total run scored in 10 innings = 9x+100 = $(9 \times 20) + 100 = 280$. Or

= 10(x+8) = 10(20+8) = 280.

As you have deduced the statements before even knowing the questions given to you this makes your solving faster.

## Example 4:

**Statement 1:** A boy who has earned an average salary of Rs 4200 per month during his 1st 11 months in India.

**Solution:** Total income in 11 months = $11 \times 4200 = 46200$.

**Statement 2:** He wants to ensure an annual average income of Rs 5000 per month. How much did he earn in 12th month?

**Solution:** Total income in 12 months = $12 \times 5000 = 60000$.

In the 12th month he earns = 60000-46200 = 13800.

So, you can see 3 different questions, each of these questions using the same language. This type of statement is one of the standard statements.

# Standard Situation In Averages

## Standard situation 1:

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This chapter is about identifying those standard situations that are generally asked in exams with the help of some examples. In the above 3 examples (example No. 2, 3 & 4) one thing is common that one new number is entering into the group.
In example 2. Group of 24 students and principle added to it.
In example 3. Group of 9 innings and added 10th innings to it.
In example 4. It has an 11 months income and added 12th month income into it.
Here the situation is entering a new number.

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Let us say you got 5 numbers with an average of 12 and 6th number entered and the average of all 6 numbers becomes 15. What is the 6th number?
**Solution :** There are two ways of solving this type of question.

The 1st way;

6th number = Total of 6 numbers - Total of 5 numbers

$= 6 \times 15 - 5 \times 12 = 30$

The 2nd way;

Addition of a 6th number, increases the average by 3.

$12 + 3 = 15$
$12 + 3 = 15$
$12 + 3 = 15$
$12 + 3 = 15$
$12 + 3 = 15$

The +3 appearing 5 times is due to the 6th number, which is able to maintain the average of 15 first, and then ‘give 3’ to each of the first 5.
Hence, the 6th number in this case = **maintain + contribute**

$= 15 + 3 \times 5 = 30$

This is another way of solving these types of questions. You can solve the example 2,3 & 4 by this approach.

Let's take example 3;
The average score of a batsman after 9 innings is x. In the 10th inning he scored 100 and increased his average by 8 runs. Find the original average?
**Solution :** After 10th inning, increases average by 8 runs.

$x \rightarrow x+8$
$x \rightarrow x+8$
$x \rightarrow x+8$

$$x \rightarrow x+8$$
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$$x \rightarrow x+8$$
$$x \rightarrow x+8$$

In this question, 100 contribute to two things. It maintains an average of (x+8) and +8, nine times. So,

$$100 = (x+8)+(8 \times 9)$$
$$x = 20.$$

## Standard situation 2:

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The 2nd standard situation is about what happens if more than one number enters. This situation can also be explained with the help of examples:

Let us say 8 numbers with an average of 10. Two new numbers enter due to that the average becomes 13. What is the average value of these two numbers?

**Solution:** There are two approaches to solve this question;

1. Total difference approach:
   Total of two numbers = $10 \times 13 - 8 \times 10 = 50$
   Thus, the average of two numbers = 50/2 = 25.

2. 2nd approach;
   Addition of a 2 numbers, increases the average by 3.

| Average of 8 numbers | Average after 2 number entry |
|---|---|
| 10 | 13 |
| 10 | 13 |
| 10 | 13 |
| … | … |
| … | … |
| 8 times… | 10times… |

Average of two number = **maintain + average contribution**

$$= 13 + (3 \times 8)/2$$
$$= 13 + 24/2 = 25$$

The contribution 24 has to be brought by these two together.

You can understand this situation like when you and your friend go to a hotel and you are going to be paid equally. Bill comes out of 24, then you will divide the bill into 2. So, each individual will pay 12.

## Example:

After 120 innings batsman has an average of 55. And he realizes that he is going to play 180 innings more and he wants an average of 100 runs per inning. So what should be the average of the remaining 180 innings?

**Solution:** Average increases by 45runs.

| Average in first 120 innings | Average after 300 innings |
| --- | --- |
| 55 | 100 |
| 55 | 100 |
| 55 | 100 |
| … | 100 |
| … | … |
| 120 times… | 300 times… |

180 new innings maintain the average 100 and make the average contribution in 120 innings.

Average of remaining 180 innings = maintain + average contribution

$= 100 + (45 \times 120)/180$

$= 130$

## Example :

After 83 innings batsman has an average of 48 runs. In the 84th inning he scored 100 runs. What's its new average?

**Solution:**

100 maintained 48 runs average and remaining 52 runs are going to be equally divided in 84 innings. So,

New average = 48 +52/84 = 48.61 runs.

2nd method:

Total runs in 83 innings = $83 \times 48 = 3984$.

Total runs in 84 innings = $3984 + 100 = 4084$

New average = $4084/84 = 48.61$

## Standard situation 3:

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The 3rd standard situation that you will see in the average chapter is replacement of a number.

## Example 1:

A set of 5 numbers with an average of 13 and one number is replaced. Average is increased by 4. The outgoing number is 32, then find the replaced number?

**Solution:**

In this situation there is an outgoing number and an incoming number and average changes by 4 for 5 numbers. The difference in the total = $(5 \times 17) - (5 \times 13) = 20$.

**Incoming number how much larger = (change in average) × (number of numbers)**

$$= 4 \times 5 = 20$$

If the average increases then it is obvious that the incoming number is larger.

Incoming number - outgoing number = difference in total

Incoming number = 20 + 32 = 52.

NOTE: If average increases then; Ni - No = Difference in total. ( Ni = incoming number, No = outgoing number), If average decreases then, No - Ni = Difference in total.

## Example 2:

The average age of an office is 32 with 10 people in office. One person is replaced and due to this, the average age decreases by 3. Replaced person age was 52. What is the age of the incoming person?

**Solution:** Average age decreases means the incoming person's age is smaller than outgoing person's age. So,

$$No - Ni = \text{Difference in total}$$
$$52 - Ni = (10 \times 32) - (10 \times 29)$$
$$Ni = 52 - 30 = 22$$

## Example 3:

Average temperature on Monday, Tuesday and Wednesday was 37 degrees. And the average temperature of Tuesday, Wednesday and Thursday was 39 degrees. What is the temperature on thursday?

**Solution :**

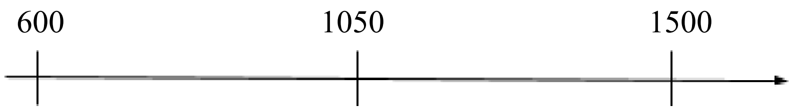
Monday is replaced by Thursday and the average increases by 2. So,

Temperature on thursday = (change in average) $\times$ (number of days)

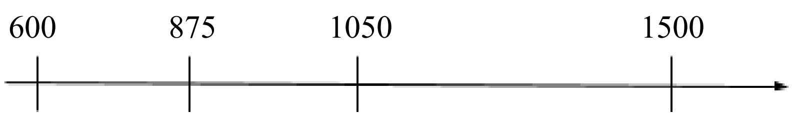
$= 2 \times 3 = 6$ degrees more than monday.

# Concept Of Weighted Average

Concept of a weighted average can be understood with the help of an example.
Suppose I had to buy a T-shirt and jeans and let us say that the average cost of a T-shirt was 600, while that of jeans was 1500.
In such a case, the average cost of a T-shirt and jeans would be given by (600 + 1500)/2 = 1050.
This can be observed on the number line as:
(midpoint) = answer.




From the figure it is observed that the average occurs at the midpoint of the two numbers.
Now, let us try to modify the situation:
Suppose I had to buy 3 T-shirts and 1jeans. In such a case I would end up spending (600 + 600 + 600 +1500) = 3300 in buying a total of 4 items. So,
Average = 3300/4 = 825. Clearly, the average has shifted.
On the number line :




It is clearly visible that the average has shifted towards 600 (which was the cost price of the T-shirts, the larger purchased item.)
In a way, this shift is similar to the way a two pan weighing balance shifts when the weights are put on it. The balance shifts towards the pan containing the larger weight.
Similarly, in this case, the correct average (875) is closer to 600 than it is to 1500. Since, this is very similar to the system of weights, we call this a weighted average situation.

## Formula for weighted average:

Let say, we have k groups with averages A1, A2 ... Ak and having n1, n2 ... nk elements then the weighted average is;

$$A_w = \frac{n1A1+n2A2+n3A3+..................+nkAk}{n1+n2+n3+..........+nk}$$

## Situation involving weighted average

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**Situation 1:** Purchasing two kinds or k varieties of something and mixing them together, to form composite.

Suppose I purchase 30Rs/kg rice and 70Rs/kg rice in the ratio 2:3. What is the average price of rice?

**Solution :** Average price = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30 , A2 = 70 , n1 = 2 , n2 = 3

Average price = (2 × 30 +3 × 70)/(2 + 3)

= 270/5 = 54Rs/kg.

### Situation 2:

Let's say you drive a car 30km/hr and 70km/hr and drive it for 2hr and 3 hr respectively. Find the average speed?

**Solution :** Average speed = (total distance)/(total time)

= (2 × 30 +3 × 70)/(2 + 3)

= 270/5 = 54 km/hr.

Situation 1 and 2 are the same but the story is different.

### Situation 3:

Let say you invest 2 lac and give 30% return. Investment of 3 lac rupees, give70% return. What is the average % return?

**Solution :** Average % return = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30% , A2 = 70% , n1 = 2 Lac , n2 = 3 Lac

Average % return = (2 × 30 +3 × 70)/(2 + 3)

= 270/5 = 54 %.

### Important thing in weighted average :

Let us say 30Rs/kg rice: 2kg and 70Rs/kg rice: 3kg.

And for average we did = (2 × 30 +3 × 70)/(2 + 3) = 54 Rs/kg …………..**(1)**

Suppose I changed the numbers in the question, let say 30Rs/kg rice: 12kg and 70Rs/kg rice: 18kg. Find the average price.

So, using formula $(n1A1 + n2A2)/(n1+n2)$

Average price = $(12 \times 30 + 18 \times 70)/(12 + 18)$

$= (360+1260)/30$

$= 1620/30 = 54$Rs/kg.

Same average price as i obtained with the above numbers.

From this, we observed that, in the given formula we don't need to insert the exact value of n1 & n2. Instead of 12 and 18 kg, If we simply use 2 & 3 and calculate it by equation **(1)**, we would definitely get the same answer. So, in the formula you should always use the ratio of the quantity.

## Situation 3:

There are two sections, in section 1 there are 20 students who scored 30 marks on an average in exam, while in section 2 there are 30 students who scored 70 marks on an average in exam. What is the average marks of both the sections?

**Solution :** Ratio of the quantities 20:30 = 2:3

So, Average marks = $(2 \times 30 + 3 \times 70)/(2 + 3)$

$= 270/5 = 54.$

This situation can be modified into Boys and Girls in a class with ratio 2:3 and Boys average marks is 30 and Girls average marks is 70. So what are the average marks of the class?

Average marks of the class will be 54.

## Situation 4: Alloys and Mixture

Let say two water and milk solutions of 2L and 3L, In one solution milk is 30% and other solution milk is 70% respectively. Mix both the solutions then what is the % of milk in the mixture?

**Solution :** % of milk in the mixture = $(2 \times 30 + 3 \times 70)/(2 + 3)$

$= 270/5 = 54\%.$

Instead of water milk solution, we can take gold and copper alloy, 2kg gold and copper alloy with 30% of gold & 3kg gold and copper alloy with 70% of gold. If both the alloys are mixed and a new alloy is formed, then what is the % of gold in the new alloy?

**Solution :** % of gold in the new alloy = $(2 \times 30 + 3 \times 70)/(2 + 3)$

$= 270/5 = 54\%.$

These are some important situations that are used in weighted averages.

## **Some questions for practice**:

1. The average of a batsman after 25 innings was 56 runs per innings. If after the 26th inning his average increased by 2 runs, then what was his score in the 26th inning?
   **Ans : 108.**

2. The average age of a class of 30 students and a teacher reduces by 0.5 years if we exclude the teacher. If the initial average is 14 years, find the age of the class teacher.
   **Ans : 29 years.**
3. The average marks of a group of 20 students on a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student have?
   **Ans : 10.**

4. The mean temperature of Monday to Wednesday was 27 °C and of Tuesday to Thursday was 24 °C. If the temperature on Thursday was 2/3rd of the temperature on Monday, what was the temperature on Thursday?
   **Ans : 18.**

5. A school has only 3 classes that contain 10, 20 and 30 students respectively. The pass percentage of these classes are 20% , 30% and 40% respectively. Find the pass % of the entire school?
   **Ans : 33.33%.**