Dynamical Model of a Pedestrian in a Crowd

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Abstract

The purpose of this study is to propose a mathematical model which describes the walking behavior of a person and analyzes the characteristics of human behavior in the crowd. The fundamental assumption is that the human behavior is not a random process but a deterministic process with several basic mechanisms and each fundamental mechanism is common and only the parameter is different from person to person. The proposed model is based on the servomechanism which drives a person along the planned path from point to point. This model has been applied to simulate the walks of people in a crowd and the simulated results have a good coincidence with actual measurement. After all, this model has been applied to simulate individual movement of people entering abruptly narrow space and it is shown that the so-called grouping phenomenon can be reproduced by this model.

1 INTRODUCTION

There have been several studies for understanding human behavior in a disaster situation. There studies have been experimental, based on an experimental psychology technique and the field researches[9]. Recently some mathematical models have been proposed from the engineering fields. There are several types of the mathematical models for the walking of the person. The one is network type[1], which counts the number of people in networked architectural space. This model is designed to examine the flow of people. The other is mesh type[2][3], which employs meshed spaces for describing human density on meshed space. This model is designed to examine how the people are scattered on the space. However almost all of these models are stochastic and the individuality of each person has been ignored[4][5][6][7]. The proposed model describes not the movement of a crowd itself but individual movement of a person who forms a crowd. In this paper as a starting point of the study, we apply the proposed model to simulate the walking behavior of a person in a crowd and compare simulated results with actual measurement. Then, this model is applied to simulate human behavior entering abruptly narrow space.

2 MODELING

We make the following assumptions from our experience and field date[9]. Based on the following assumptions, the mathematical model of the walking behavior will be derived.

(1) A person plans a path to his destination and walks along the path.
(2) If there are obstacles in walking space, he assigns the scalar values to evaluate them by using their positions and velocities.
(3) A person assigns an obstacle, which has the highest evaluation, a rectangular area to realize whether it disturbs his movements or not. The rectangular area (avoidance area) is characterized by a circle with a radius of r and a relative velocity vector between it and him.
(4) If a person is within the avoidance area given to an obstacle, he changes the present path by using the following fixed algorithm to avoid it.
(5) If a person is within a circle with a radius of r, he stops movement.
(6) If the following condition is satisfied when a person, who walks ahead of him, and he walks to the same direction, he intends to track a person.
(7) The velocity of movement and human density around a person are in inverse proportion. [5]

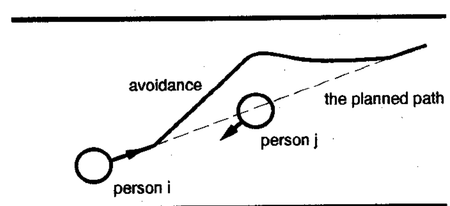

Figure 1: Modeling of the walking behavior

2.1 Dynamical Model of the Walking Behavior

A dynamical model [7], which describes the movement of a person i, is given by

$$m_i\ddot{x}_i + \nu_i\dot{x}_i = u_i \tag{1}$$

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where

x_i	position
m_i	weight
$\ddot{x}_i$	accelerated velocity
$\dot{x}_i$	velocity
ν_i	coefficient of friction
u_i	moving force

2.2 Walking Behavior

Assume that a person decides the path to his destination and moves toward the point on the path. We describes the walking behavior by using servomechanism, which drives a person along the planned path from point to point.

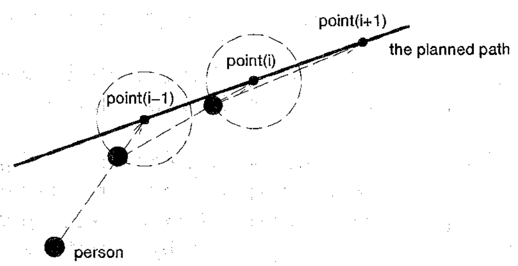

Figure 2: Walking behavior based on servomechanism

If there are people and obstacles in walking space, a person assigns the scalar values to evaluate them by using their positions and velocities. If a distance from a person to an obstacle is shorter, the evaluated values assigned to it should be higher. We assume that this values is calculated by the following equation.

$$evaluated\ values = \frac{1}{|l_{ij}|} \times \cos\theta_{ij} \qquad (2)$$

where

$$l_{ij} = \begin{cases} l_{ij} & l_{ij} \leq l_i \\ \infty & \text{else} \end{cases}$$

$$\cos\theta_{ij} = \begin{cases} \cos\theta_{ij} & |\theta_{ij}| \leq \theta_i \\ 0 & \text{else} \end{cases}$$

l_{ij} is the vector in the direction from a person i to an obstacle. We assumed that if a distance l_{ij} from a person i to the obstacle is larger than l_i, he can not detect it.

θ_{ij} is the angle between the velocity vector v_i and the vector l_{ij}. If $|\theta_{ij}|$ is smaller than θ_i, a person i can see it. l_i and θ_i are different from person to person.

If these values given to obstacles or people are larger than 0, a person i can see them and counts the number of them. Human density around him is given by the total number. If every values given to obstacles or people is 0, a person i supposes that they do not disturb movement of him and he moves along the previously planed path.

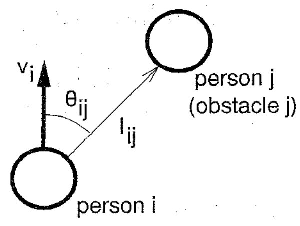

Figure 3: Calculation of the evaluated values

2.3 Tracking Behavior

If the velocity vector of a person j, who is given the most highest evaluation, is almost coincide with that of a person i, a person i intends to track a person j walking ahead of him. If $|\theta_{ij}|$ in eq.(2) is smaller than θ'_i and l_{ij} in eq.(2) is shorter than l'_i, a person i tracks the walker ahead of him. θ'_i and l'_i are parameters, which describe the personality, and are different from person to person.

2.4 Avoidance Behavior

If an obstacle, which is given the highest evaluation, disturbs movement of a person i, he intends to avoid it so that he may not collide with it. We assume that a person j has avoidance area as shown in Figure 4 to a person i because a person j is in an enough large space. When a person i is left-hand side of avoidance area, he veers to the left to avoid a person j. When on the right, he veers to the right.

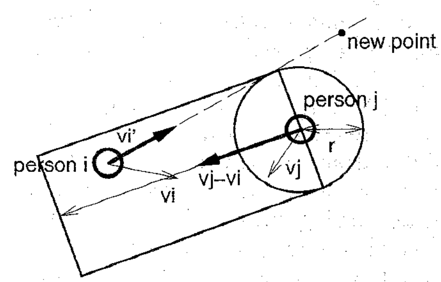

Figure 4: Avoidance area to a person j

The rectangular area shown in Figure 4 characterizes the avoidance area for a person i to a person j and the length of sides are determined by the radius r and the length of vector $v_j - v_i$. The radius r is in proportion to $v_j - v_i$. The size of this rectangular area will be considered to be one aspect of the personality.

2.5 Stopping Movement

If a person i is within the radius r shown in Figure 4, he stops movement. If an obstacle passes across a person, he stops his movement. If a person realizes that he does not collide with obstacle, he starts to move to one's destination again.

2.6 State Equation

To describe the walking behavior, equation (1) is transformed to the following state equation.

$$\begin{bmatrix} \ddot{x}_i \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} -\frac{\nu_i}{m_i} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ x_i \end{bmatrix} + \begin{bmatrix} \frac{1}{m_i} \\ 0 \end{bmatrix} u_i \tag{3}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ x_i \end{bmatrix} \tag{4}$$

2.7 Modeling of Walking Behavior

We replace the model, which describes the walking behavior, with servo system. As shown in Figure 5, this model is given by

$$\begin{bmatrix} \ddot{x}_i \\ \dot{x}_i \\ \dot{z}_1 \end{bmatrix} = \begin{bmatrix} -\frac{\nu_i}{m_i} & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ x_i \\ z_1 \end{bmatrix} + \begin{bmatrix} \frac{1}{m_i} \\ 0 \\ 0 \end{bmatrix} u_i \tag{5}$$

$$u_i = -\begin{bmatrix} \mathbf{f}, -k_1 \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ x_i \\ z_1 \end{bmatrix} \tag{6}$$

$$\dot{\bar{x}} = (\bar{A} - \bar{b}\bar{f})\bar{x} + gr, y = \bar{c}\bar{x} \tag{7}$$

$$\bar{c} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, g = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\bar{A} - \bar{b}\bar{f} = \begin{bmatrix} -\frac{f_1+\nu_i}{m_i} & -\frac{f_2}{m_i} & \frac{k_1}{m_i} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

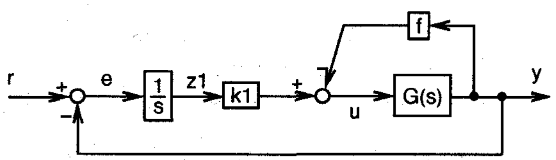

Figure 5: Block diagram of the servo system

The poles of the system are given by the root of following equation :

$$s^3 + \frac{f_1 + \nu_i}{m_i} s^2 + \frac{f_2}{m_i} s + \frac{k_1}{m_i} = 0 \tag{8}$$

By the pole configuration of the system, we can describe the behavior of each person. In case when we assign the poles nearby origin, the person moves slowly. While in case when we assign the poles far from origin, the person moves quickly.

2.8 Velocity of Movement

We assume that velocity of movement is in inverse proportion to a human density around a person.[5] If human density around a person is high, he moves slowly. If it is low, he can walk fast. An inverse proportion is given to the model by the pole configuration of the system. The pole configuration of the system is decided as a velocity of movement and a human density around a person are in inverse proportion to each other. We assume that the relation can be described by the following equation,

$$poles = p_i(1 - D_i\alpha_i)$$

where

- p_i poles of a person i
- D_i human density around a person i
- α_i coefficient.

3 SIMULATION

3.1 Comparison with Actual Measurement

The following simulations are compared with actual measurements to confirm the reality of the proposed model. Actual measurements have been recorded on videotape with a camera mounted on a tall building. The simulation shown in Figure 6 is illustrated to compare movements of each person with that of the other persons. Simulated movements of each person are compared with the actual measurement in Figure 7, 8, and 9.

- ● Simulation result (person)
- ··· Simulation result (locus of person)
- o Actual measurement

3.1.1 Walking Behavior of Three Persons

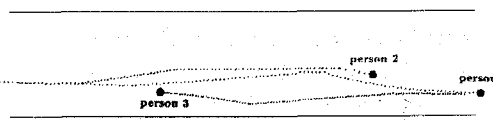

Figure 6: The walking behavior of three persons

3.1.2 Person 1

He walks ahead of Person 2 and opposite to Person 3.

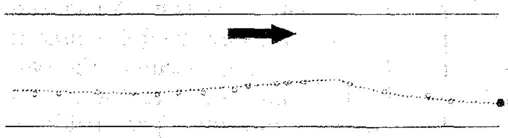

Figure 7: Simulation result of person 1

3.1.3 Person 2

He walks behind Person 1 and opposite to Person 3.

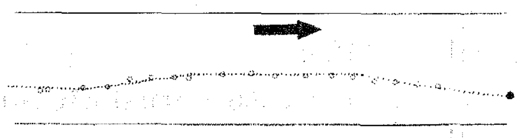

Figure 8: Simulation result of person 2

3.1.4 Person 3

He walks opposite to Person 1 and Person 2.

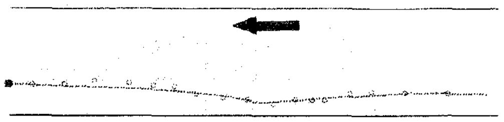

Figure 9: Simulation result of person 3

3.2 Applications of the Model

In this section, we will apply this model to the analysis of the behavior of a crowd.

3.2.1 Passing on Straight Way

The following simulations are carried out to examine a capacity of the proposed model. A slow walker cannot see a fast walker when a person walks slow ahead of a fast walker on straight way. Because the evaluation values assigned from a slow walker to a fast walker is 0, A slow walker cannot see a fast walker. If a fast walker is within the avoidance area given to a slow walker, a fast walker changes the present path by using the fixed algorithm to avoid a slow walker. Passing a slow walker on straight way is simulated as shown in Figure 10. This simulation is good coincident with the date on avoidance behavior.[9]

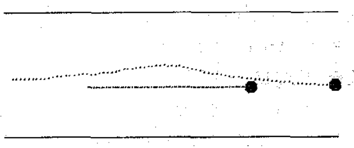

Figure 10: Passing another person at different speed

3.2.2 Decision of the Path at a Corner

When a person arrives at a corner, he has to turn to the direction of one's destination. To describe the behavior of turning at a corner, we assume an algorithm to decide the path to turn at a corner. If a person intends to go to a corner, he walks along the tangent drawn from one's position to a circle centered at a corner and with radius L as shown in Figure 11. If the velocity of a person coming at a corner is higher, radius L is longer. If a person turns left at a corner, the tangent drawn from the left side to a circle with radius L corresponds with the path to leave a corner. A person walks along these paths. These assumptions enable this model to simulate the behavior of turning at a corner.

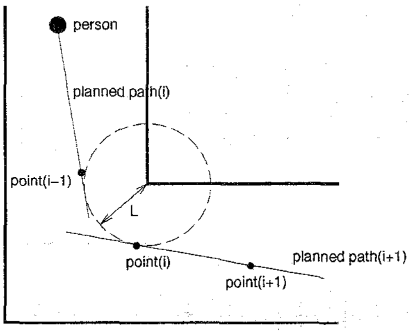

Figure 11: Decision of the path at corner

3.2.3 Passing at Corner

The following simulation is carried out to examine the algorithm to decide the path at a corner. In case a person passes a person walking slowly ahead of him at corner, we examine how the model realizes the behavior of two persons. This simulation is shown in Figure 12. The algorithm to decide a path at a corner is useful to simulate movements of persons entering abruptly narrow space.

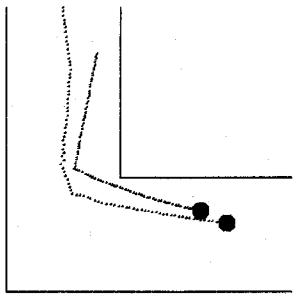

Figure 12: Passing another person at corner

3.2.4 Movement of Persons in a Crowd

In case this model is applied to simulate movement of a crowd, It is examined what the simulation bring about. At the beginning of the simulation, each position of the thirteen people is chosen at random. They intend to enter a space that abruptly narrows and form two groups. This model realizes a formation of group which has been reported by [2][8]. The simulation is shown in Figure 13.

4 CONCLUSION

In this paper, the mathematical model of the walking behavior of a person is proposed. As shown in Figure 7, 8 and 9, simulation of each person shows a good coincidence with actual measurement. These results can be regarded to show that the proposed model is effective in simulating the walking behavior of a person. There have been models which describe a formation of group based on especial assumptions. In this paper, the model does not describe an assumption of a formation of group especially. However the model shows the grouping phenomenon naturally. Improvements will lead this model to simulate the human behavior in a complex architecture or city and enable us to analyze the human behavior during rush hour or of evacuation, and fire.

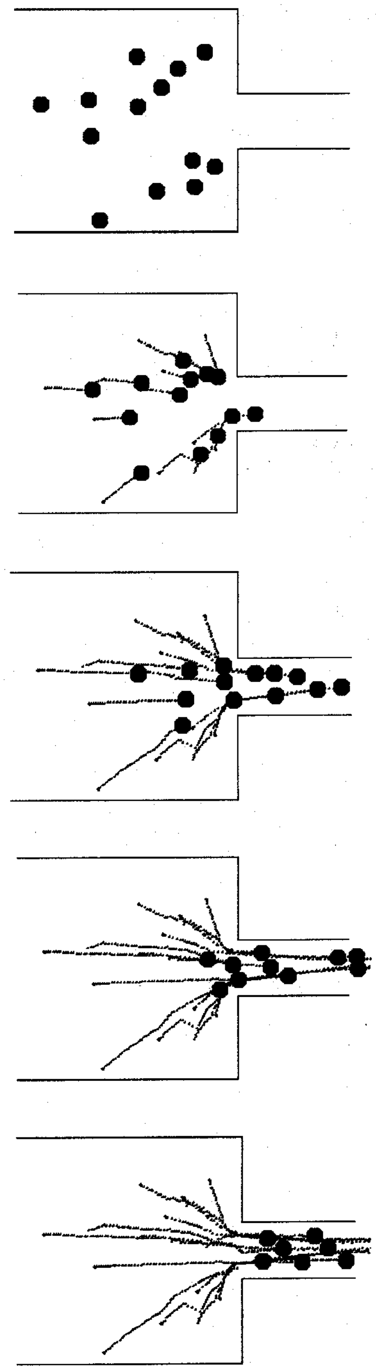

Figure 13: Persons entering narrow space

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