You can review the latex source for this assignment-file to learn and use latex to prepare your homework submission. You will see the use of macros (to write uniformly formatted text), different text-styles (emphasized, bold-font), different environments (figures, enumerations).

It is not required that you use exactly this latex source to prepare your submission.

Homework 2 (CTL): ComS/CprE/SE 412, ComS 512

Due-date: Feb 23 at 11:59PM.

Submit online on Canvas two files: the source file in latex format and the pdf file generated from latex. Name your files: ⟨your-net-id⟩-hw2.⟨tex/pdf⟩.

Homework must be individual's original work. Collaborations and discussions of any form with any students or other faculty members or soliciting solutions on online forums are not allowed. Please review the academic dishonesty policy on our syllabus. If you have any questions/doubts/concerns, post your questions/doubts/concerns on Piazza and ask TA/Instructor.

1. We say that a property p holds *infinite often* in a path if for every state in the path, there is a future state in the path where p is satisfied.

   We say that a property p holds *finitely many times* in a path if there exists a state in the path such that in all states appearing after the former (state) $\neg p$ is satisfied.

   Answer the following:

   (a) Prove/disprove that if a state satisfies $\mathtt{AG}(\mathtt{AF}(p))$, then along all paths starting from the state, p holds infinitely often. **Proved**

   $$\begin{aligned} s \in [\![\mathtt{AG}(\mathtt{AF}(p))]\!] &\iff \forall \pi \in \mathit{Paths}(s).\forall i \geq 0, \pi[i] \in [\![\mathtt{AF}(p)]\!] \\ &\iff \forall \pi \in \mathit{Paths}(s).\forall i \geq 0, \forall \pi' \in \mathit{Paths}(\pi[i]).\exists j \geq 0, \pi'[j] \in [\![p]\!] \end{aligned}$$

   Since each path π is infinite, existence of a future state on each state of the infinite path makes the possibility to find such a future state infinite times. Hence, p holds infinitely often.

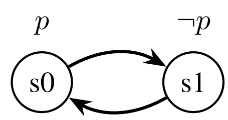
   (b) Prove/disprove that if a state satisfies $\mathtt{AF}(\mathtt{AG}(p))$, then along all paths starting from the state, $\neg p$ holds finitely many times. **Proved**

   $$\begin{aligned} s \in [\![\mathtt{AF}(\mathtt{AG}(p))]\!] &\iff \forall \pi \in \mathit{Paths}(s).\exists i \geq 0, \pi[i][\![\mathtt{AG}(p)]\!] \\ &\iff \forall \pi \in \mathit{Paths}(s).\exists i \geq 0, \forall \pi' \in \mathit{Paths}(\pi[i]).\forall j \geq 0, \pi'[j][\![p]\!] \end{aligned}$$

   On all paths starting from state s, there exist a future state s' that satisfies $\mathtt{AG}(p)$ means that there exist a finite number of states between s and s' where $\neg p$ holds. Hence, $\neg p$ holds finitely many times.

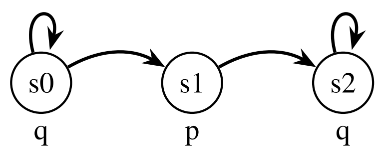
(c) Prove/disprove that if a state satisfies $\texttt{AG}(\texttt{AF}(p))$, then along all paths starting from the state, $\neg p$ holds finitely many times. **Disproved**

Consider the following KS, state $s0$ satisfies $\texttt{AG}(\texttt{AF}(p))$ because on the path from $s0$: $s0, s1, s0, s1, \ldots$:
State $s0$ satisfies $\texttt{AF}(p)$ i.e on itself.
State $s1$ satisfies $\texttt{AF}(p)$ i.e at a future state $s0$.
Also note that on this path $\neg p$ holds infinite times. Hence, disproved.

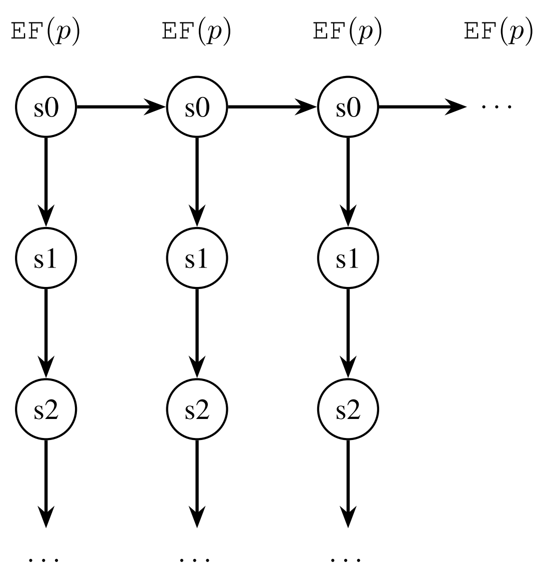


(d) Prove/disprove that if a state satisfies $\texttt{EG}(\texttt{EF}(p))$, then there exists at least one path starting from the state, p holds infinitely many times. **Disproved**

Consider the KS, the path $\pi \in Paths(s0)$ that causes state $s0$ to satisfy $\texttt{EG}(\texttt{EF}(p))$ is : $s0$, $s0$, $s0$, ...
because at every state on this path EFp holds true because $s0$ also branches to $s1$ that satisfies p.
But then it is followed by $s2$ forever and hence p holds only once. In other words, in $\texttt{EG}(\texttt{EF}(p))$, the path considered for EG and EF might be different.

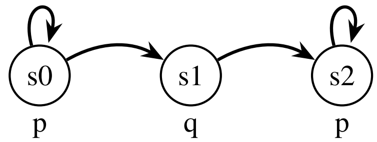


The computation tree for $s0$ looks as such:



(20)

2. Consider the following Kripke structure, with $p \in L(s_0) \cap L(s_2)$ and $q \in L(s_1)$.



Identify the set of states that satisfy each of the following:

(a) $\mathtt{AG}(p)$ $\{s2\}$

(b) $\mathtt{AF}(p)$ $\{s0, s1, s2\}$

(c) $\mathtt{AG}(\mathtt{AF}(p))$ $\{s0, s1, s2\}$

(d) $\mathtt{AF}(\mathtt{AG}(p))$ $\{s1, s2\}$

(12pts)

3. Express the following statements in English.

(a) There exists a execution sequence such that from every configuration in the sequence, it is possible to satisfy property p.
$\mathtt{EG}(\mathtt{EF}(p))$

(b) Along all paths, whenever p is true, it is followed by a state where p is false, and whenever $\neg p$ is true, it is followed by a state where p is true.
$\mathtt{AG}(\ (p \Rightarrow \mathtt{AX}(\mathtt{AF}(\neg p)))\ \wedge\ (\neg p \Rightarrow \mathtt{AX}(\mathtt{AF}(p)))\ )$

(8pts)