You can review the latex source for this assignment-file to learn and use latex to prepare your homework submission. You will see the use of macros (to write uniformly formatted text), different text-styles (emphasized, bold-font), different environments (figures, enumerations).

It is not required that you use exactly this latex source to prepare your submission.

Homework 6 (LTL+Büchi & CTL): ComS/CprE/SE 412, ComS 512

Due-date: Apr 28 at 11:59PM.

Submit online on Canvas two files: the source file in latex format and the pdf file generated from latex. Name your files: ⟨your-net-id⟩-hw6.⟨tex/pdf⟩.

Homework must be individual's original work. Collaborations and discussions of any form with any students or other faculty members or soliciting solutions on online forums are not allowed. Please review the academic dishonesty policy on our syllabus. If you have any questions/doubts/concerns, post your questions/doubts/concerns on Piazza and ask TA/Instructor.

1. Prove or disprove the following claims:

   (a) $\mathtt{G}(\mathtt{F}(\mathtt{X}(p)))$ and $\mathtt{G}(\mathtt{X}(\mathtt{F}(p)))$ are equivalent.

   Equivalent:

   The two formulas will be equivalent if $\mathtt{F}(\mathtt{X}(p))$ is equivalent to $\mathtt{X}(\mathtt{F}(p))$. They are equivalent (review class)

   (b) $\mathtt{G}(\mathtt{F}(p)) \Rightarrow \mathtt{G}(\mathtt{F}(q))$ and $\mathtt{F}(\mathtt{G}(\neg p)) \Rightarrow \mathtt{F}(\mathtt{G}(q))$ are equivalent.

   Not equvalent

   Consider a path where $\neg p \wedge \neg q$ is true in all states. The path satisfies the first formula as $\mathtt{G}(\mathtt{F}(p))$ is false. The path does not satisfy the second formula as $\mathtt{F}(\mathtt{G}(\neg p))$ is satisfied but $\mathtt{F}(\mathtt{G}(q))$ is not satisfied.

   (Useful equivalence to recall: $A \Rightarrow B$ is equivalent to $\neg A \vee B$, which, in turn, is equivalent to $\neg B \Rightarrow \neg A$.)

   (c) Given any Kripke structure, one can verify whether a state satisfies the CTL formula $\mathtt{AF}(\mathtt{AG}(p))$ using the LTL formula $\mathtt{G}(\mathtt{F}(\neg p))$. (Note that, the question is not asking whether the two formulas are equivalent or not).

   False:

   A state satisfies $\mathtt{AF}(\mathtt{AG}(p))$ implies that along all paths starting from the state, the proposition p holds true infinitely often. We refer to this as property A.

   Consider the scenario that a state satisfies $\mathtt{G}(\mathtt{F}(\neg p))$. This means that along all paths eventually always p is false. This implies property A is not satisfied. Now consider the case when a state does not satisfy $\mathtt{G}(\mathtt{F}(\neg p))$. This implies, there exists a path that satisfies $\mathtt{F}(\mathtt{G}(p))$, i.e., there exists a path where p occurs infinitely often. These scenarios do not imply the property A.

   (d) A state satisfies $(\mathtt{AG}(p)) \Rightarrow (\mathtt{AF}(q))$ if and only if the state satisfies $(\mathtt{G}(p)) \Rightarrow (\mathtt{F}(q))$.

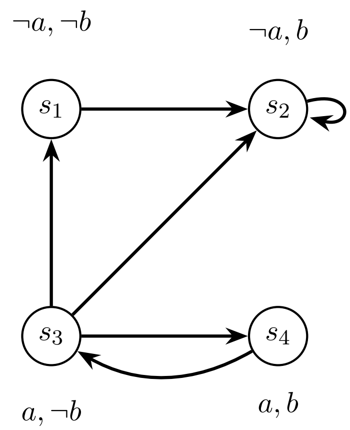
   False:

   Consider the Kripke structure, with $s_0 \to s_1 \to s_1$ and $s_0 \to s_2 \to s_2$. The proposition p is satisfied at states s_0 and s_1. The state s_0 satisfies the CTL formula because $\mathtt{AG}(p)$ is not satisfied by s_0. However, s_0 does not satisfy the LTL formula as there exists a path $s_0 \to s_1 \to s_1 \to \ldots$ that satisfies $\mathtt{G}(p)$ but does not satisfy $\mathtt{F}(q)$.

(e) A state satisfies $((\neg q)\ \mathtt{U}\ (\neg p \wedge \neg q))$ implies that the state satisfies $\neg(p\ \mathtt{U}\ q)$.
True:
$\neg(p\ \mathtt{U}\ q)$ is equivalent to $(\neg q\ \mathtt{U}\ (\neg p \wedge \neg q))\ \vee\ \mathtt{G}(\neg q)$. As $A \Rightarrow A \vee B$, the above claim is true.

(40pts)

2. **Extra Credit** Identify the states in the following Kripke structure that satisfy the given formula.

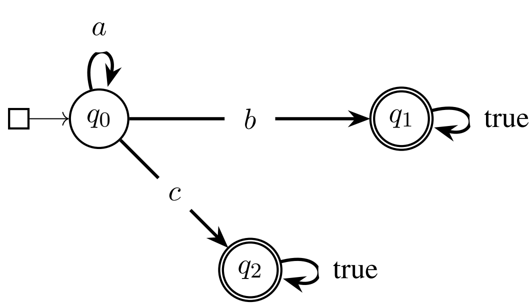


(a) $\mathtt{G}(a)$ $\{\}$: every state has some path that does not satisfy $\mathtt{G}(a)$
(b) $(a\ \mathtt{U}\ b)$ $\{s_2, s_4\}$: other states can lead to states where $\neg a \wedge \neg b$ is true.
(c) $(a\ \mathtt{U}\ \mathtt{X}(a \wedge \neg b))$ $\{s_4\}$: All path from s_4 satisfy $\mathtt{X}(a \wedge \neg b)$.
(d) $\mathtt{X}(a \wedge b)\ \wedge\ \mathtt{F}(\neg a \wedge \neg b)$ $\{\}$: Check each conjunct for each path.

(10pts)

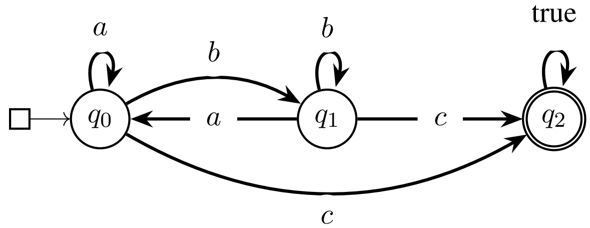
3. **Extra Credit** Identify the LTL formulas whose semantics is represented in the language of the following automata (the edge labels indicate singleton set).

(a) Automata A_1



$(a\ \mathtt{U}\ b)\ \vee\ (a\ \mathtt{U}\ c)$. Exercise: is this equivalent to $(a\ \mathtt{U}\ (b \vee c))$

(b) Automata A_2



Consider the pattern of sequences that can start from q_0 and q_1. From q_0, the sequences can move the automata to q_0 on a, to q_1 on b and to q_2 on c. From q_1, the sequences can move the automata to q_0 on a, to q_1 on b and to q_2 on c. Therefore, the states q_0 and q_1 are roots of strings of same/identical pattern, i.e., these two states can be merged into one. Now, the automata will have two states q_{01} a merged state with self-loops on a and b and an edge on c to q_2. The start state is q_{01} and accepting state is q_2. The LTL formula whose semantics corresponds to the accepting sequences of the automata is $((a \vee b)\ \mathtt{U}\ c)$.

(20pts)