ComS 412/512 HW6

(a) $GFXp$ and $GXFp$ are equivalent

Since FXp and XFp represent the same thing $\Rightarrow$ $FXp \equiv XFp$

then $GFXp \equiv GXFp$

(b) $GFp \to GFq$ and $FG\neg p \to FGq$ are not equivalent

$!GFp \vee GFq \equiv !FG\neg p \vee FGq$

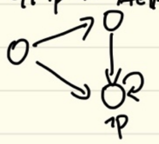
$GF\neg p \vee GFq \equiv FGp \vee FGq$

the left hand : in all paths they will eventually reach $\neg p$ or they will eventually reach p.

the right hand : in the future, p is true in all suffix or q is true in all suffix

(c) using $GF\neg p_{(LTL)}$ to verify $AFAGp_{(CTL)}$ is not true.

$GF\neg p \equiv AGAF(\neg p)$



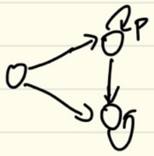
In this case, $AGAF\neg p$ satisfies, but $AFAGp$ is not

And if $AGAF\neg p$ satisfies, then $AFAGp$ must be false

So, we can know $AFAGp$ cannot be satisfied if $AGAF\neg p$ is true

if the situation that $AGAF\neg p$ is not satisfied can determine $AFAGp$ is true

then, the statement is true.

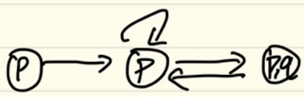


$\neg AGAF\neg p \equiv EFEGp$, (negation of $GF\neg p$)

In this case, $EFEGp$ is satisfied, but $AFAGp$ still is not satisfied.

So, the statement is false

(d) $AGp \to AFq$ iff $Gp \to Fq$ is false. Consider the following KS.



This KS satisfies $Gp \to Fp$, but not $AGp \to AFq$. So. $AGp \to AFq$ iff $Gp \to Fp$ doesn't hold.

(e) $((\neg q) \,U\, (\neg p \wedge \neg q))$ implies $\neg(p \,U\, q)$ is true

$\{p, \neg q\} \to \{p, \neg q\} \to \cdots \{\neg p, \neg q\} \to \{q\}$

we can see, even if q is satisfied after $\{\neg p, \neg q\}$

$p \,U\, q$ never comes up, because there must be $\neg q$ before q. so $p \,U\, q$ is not true. then

$\neg(p \,U\, q)$ is satisfied. the statement is true.

2. (a) $G(a) = \{\}$

(b) $a \,U\, b = \{s_2, s_3, s_4\}$

(c) $a \,U\, X(a \wedge \neg b) = \{s_4\}$

(d) $X(a \wedge b) \wedge F(G(a \wedge \neg b)) = \{s_3\}$

3. (a) $(a \,U\, b) \vee (a \,U\, c)$

(b) $(a \,U\, b) \to ((b \,U\, a) \vee (b \,U\, c)) \vee (a \,U\, c)$