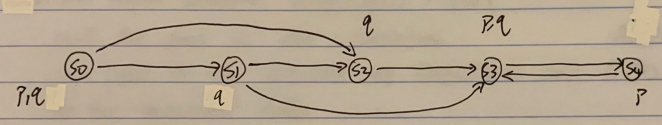
1. (a) firstly, translate $AG(AF(p))$ ~~to~~ to English:

P is always true somewhere along every path starting anywhere,
So. P holds infinitely often along every path.

And also we say p holds infinite often in a path if every state in the path, there is a future state in the path where p is satisfied.
Therefore, the statement (a) holds.

(b) Based on the description, p holds finitely many times when $FG\neg p$ is satisfied.
$AF(AG(p))$ in English: every path $\pi = s_1, s_2, \cdots$ eventually reaches a state which satisfies $AG\,p$, which means a state such that on every path π' starting there, p is always ~~sati~~ satisfied.



In this kripke structure, $p \in L(s_0) \cap L(s_3) \cap L(s_4)$
$q \in L(s_1) \cap L(s_2) \cap L(s_3)$

Now, we see the state ~~is~~ s_0 ~~sta~~ satisfies $AF(AG(p))$
As we say at first, $\neg p$ holds finitely many times if and only if FGp is satisfied.
Also, the state s_0 satisfies FGp, but doesn't satisfy $A(FG(p))$
$\therefore$ cannot satisfy that along all paths starting from the state, $\neg p$ holds finitely many times
Therefore, Based on the definition of $FG\neg p$ and $AF(AG(p))$ and the example (kripke structure), the statement doesn't hold

prove by equation:

$$AF(AG(p)) \rightarrow A(FG(p))$$
$$= EG(\underbrace{EF(\neg p)}_{s_0}) \vee A(FG(p))$$

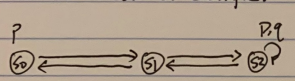
$EF(\neg p) = \{s_0\}$
$EG(EF(\neg p)) = \{\}$

In this kripke structure, the state s_0 satisfies $AF(AG(p))$, but cannot satisfy $EG(EF(\neg p))$ or $A(FG(p))$. $\therefore$ the statement doesn't hold

(c) $\neg p$ holds finitely times means $FG(p)$

$$AG(AF(p)) \rightarrow A(FG(p))$$
$$= \neg(AG(AF(p)) \vee A(FG(p))$$
$$= EF(EG(\neg p)) \vee A(FG(p))$$

There is a counter example.



$p \in L(s_0) \cap L(s_2)$, $q \in L(s_2)$

In this example, $AG(AF(p)) = \{s_0, s_1, s_2\}$
$EF(EG(\neg p))$ cannot be satisfied
also $A(FG(p))$

And $AG(AF(p))$ in English means along all paths in all states, they can reach the state that satisfies $AF(p)$. However, we can see that not all paths starting from the state, $\neg p$ holds ~~many~~ finitely many times, because it couldn't satisfy that all states appear after the state p.
$\therefore$ Based on the definition and the equation, the statement doesn't hold.

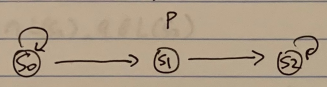
(d) Based on the description, we need to prove/disprove

$$EG(EF(p)) \rightarrow E(FG(\neg p)) \quad = \neg(EG(EF(p))) \vee E(FG(\neg p))$$

$E(FG(\neg p))$: Exists one path, at some point in the future, all the following states will satisfy the property $\neg p$

$EG(EF(p))$: Exists a path, all states are the states which satisfy $EF(p)$

$$EG(EF(p)) \rightarrow E(FG(\neg p)) = AF(AG(\neg p)) \vee E(FG(\neg p))$$



$p \in L(s_1)$

In this kripke structure, $EG(EF(p)) = \{s_0\}$

and the state s_0 also satisfies $AF(AG(\neg p))$

And based on the definition of $EG(EF(p))$ and $E(FG(\neg p))$ and equation the statement holds

2. (a) $AG(p)$: $\{s_2\}$

(b) $AF(p)$: $\{s_0, s_1, s_2\}$

(c) $AG(AF(p))$: $\{s_0\}$

3. (a) $E(AG(EG(p)))$

(b) $A(AG(p \rightarrow EX(\neg p)) \wedge AG(\neg p \rightarrow EX(p)))$