You can review the latex source for this assignment-file to learn and use latex to prepare your homework submission. You will see the use of macros (to write uniformly formatted text), different text-styles (emphasized, bold-font), different environments (figures, enumerations).

It is not required that you use exactly this latex source to prepare your submission.

Homework 5 (LTL & CTL): ComS/CprE/SE 412, ComS 512

Due-date: Apr 19 at 11:59PM.

Submit online on Canvas two files: the source file in latex format and the pdf file generated from latex. Name your files: ⟨your-net-id⟩-hw5.⟨tex/pdf⟩.

Homework must be individual's original work. Collaborations and discussions of any form with any students or other faculty members or soliciting solutions on online forums are not allowed. Please review the academic dishonesty policy on our syllabus. If you have any questions/doubts/concerns, post your questions/doubts/concerns on Piazza and ask TA/Instructor.

1. For each of the following requirements, discuss whether LTL and CTL model checking can be used to verify the requirements. For instance, if your answer for some requirement is: it can be verified using LTL model checking but not using CTL model checking, then justify your answer.

   (a) Along all paths, propositions p and q hold true in alternate states.
   CTL: $\mathrm{AG}((p \Rightarrow \mathrm{AX}(q)) \wedge (q \Rightarrow \mathrm{AX}(p)))$
   LTL: $\mathrm{G}((p \Rightarrow \mathrm{X}(q)) \wedge (q \Rightarrow \mathrm{X}(p)))$

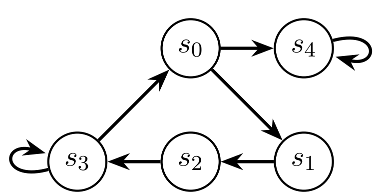
   (b) Along all paths, eventually a state is reached such that all its next states satisfy the proposition p.
   CTL: $\mathrm{AF}(\mathrm{AX}(p))$
   LTL: Not possible

   (c) There exists no path where p holds true until q holds true.
   CTL: $\neg\mathrm{E}(p \ \mathrm{U}\ q)$
   LTL: $\neg(p \ \mathrm{U}\ q)$

   (d) There exists a path where p holds in a state and it is followed by a state where q holds.
   CTL: $\mathrm{EF}(p \wedge \mathrm{EX}(\mathrm{EF}(q)))$
   LTL: Can be verified using $\phi = \mathrm{G}(\neg p \vee \mathrm{X}(\mathrm{G}(\neg q)))$.
   If $s \models \phi$, then s does not satisfy stmt (d).
   If $s \not\models \phi$, then s satisfies stmt (d).

(12pts)

2. Consider the following Kripke structure with $b \in L(s_3)$, $\{a, b\} \subseteq L(s_2)$, $a \in L(s_1)$ and $b \in L(s_4)$:



Identify the states that satisfy the following:

(a) F(X(b))
Every path of every state sees a "b" in future after state itself
Answer: $\{s_0, s_1, s_2, s_3, s_4\}$

(b) (F(G(b)) U a)
All states satisfying a trivially satisfy the until : $s_1, s2$
For other states, need to fulfil F(G(b)) until s_1 or s_2 is reached. Only s_4 satisfies F(G(b)) but it does not reach a-state.
Answer: $\{s_1, s_2\}$

(c) G(F(a)) $\Rightarrow$ G(F(b))
None of the states satisfy G(F(a))
Answer: $\{s_0, s_1, s_2, s_3, s_4\}$

(d) G(F(b)) $\Rightarrow$ G(F(a))
All of the states satisfy G(F(b)), but none satisfy G(F(a)).Hence, implication is false for all states.Draw paths starting from each state and each of them can lead to a b-state.
Answer: $\{\}$

(16pts)

3. Prove or disprove the following claims:

(a) For any path π in a Kripke structure, $\pi \models$ (F(p) U q) and $\pi[0]$ does not satisfy q implies that there must be some state(s) where p holds true before the state where q holds true.
Consider path $\pi = (s_0, s_1, s_2, s_3, ...)$. Let s_2 satisfies q and s_3 satisfies p,
then $\pi[0] \models$ F(p), $\pi[1] \models$ F(p) and $\pi \models$ (F(p) U q).
But on this path, p actually happens after q. (Disprove)

(b) For any state in any Kripke structure, the state satisfies AX(AF(p)) if and only if the same state satisfies F(X(p)).
If a state $s_0 \models$ AX(AF(p)), then all $s_1 \models$ AF(p).
So, every path $(s_0, s_1, \ldots)$ can see a p in future starting from s_1.
If $s_1 \models p$, then $s_0 \models$ X(p) and also, $s_0 \models$ F(X(p)).
$k > 1$: If $s_k \models p$, then $s_{k-1} \models$ X(p) and $s_0 \models$ F(X(p)). (Prove)

(c) The LTL formula F(G(p)) $\Rightarrow$ G(F(p)) is equivalent to propositional constant true.
$\mathrm{F}(\mathrm{G}(p)) \Rightarrow \mathrm{G}(\mathrm{F}(p)) \equiv \neg\mathrm{F}(\mathrm{G}(p)) \vee \mathrm{G}(\mathrm{F}(p)) \equiv \mathrm{G}(\mathrm{F}(\neg p)) \vee \mathrm{G}(\mathrm{F}(p))$
Any infinite path can be represented using a lasso : $s_0 \rightarrow s_1 \rightarrow \ldots \rightarrow s_k \rightarrow \ldots \rightarrow s_k$
If p holds on s_k, then G(F(p)) is true on the path.
If $\neg p$ holds on s_k, then G(F($\neg p$)) is true on the path.
Hence, each path satisfies exactly one of the disjuncts. (Prove)

(12pts)