Exam 1: COMS/CPRE/SE 412 & COMS 512

March 04, 2021

Time: 75 mins

Learning Outcomes

- Application of knowledge of computing and mathematics
- Ability to understand the implications of mathematical formalisms in computer science
- Ability to apply logic for formalizing requirement specifications.

You can consult lecture materials to answer any questions in this test. You are not allowed to collaborate in any form with anyone.

Question	Points	Score
1 (English to CTL)	10	
2 (CTL Equivalences)	10	
3 (Model Checking)	16	
4 (CTL Semantics/Adequate Set)	4	
5 (Extra Credit)	10	
Total	40	

Questions

1. Express the following in CTL:

   (a) **[412 only]** Along all paths, from any state, it is possible to eventually get a candy.
   $\texttt{AG}(\texttt{EF}(\text{get-candy}))$

   (a) **[512 only]** Along all paths, if there are two states where p is true then there is at least one state in between them where q is true.
   $\neg\texttt{EF}(p \wedge \texttt{EX}(\texttt{E}(\neg q \ \texttt{U} \ p)))$

   (b) There exists a path where the system reaches a configuration from where it inevitably[1] eventually reaches a faulty configuration.
   $\texttt{EF}(\texttt{AF}(\text{faulty-configuration}))$

2. Prove/disprove that the following pairs are equivalent:

   (a) **[412 only]** $\texttt{EG}(\texttt{EX}(q))$ and $\texttt{EX}(\texttt{EG}(q))$
   Not Equivalent. Consider the KS, where (s0,s0), (s0, s1), (s1, s2), (s2, s2) are transition relations, and q is in the label of s1. Then s0 satisfies $\texttt{EG}(\texttt{EX}(q))$ but s0 does not satisfy $\texttt{EX}(\texttt{EG}(q))$.
   If you review the logical definition of the two formulas, you will note that for the pair to be equivalent it would be necessary to imply the claim that ordering of existential and universal quantifiers do not impact the semantics of the definitions (such a claim is invalid).

   (a) **[512 only]** $\texttt{AX}(\texttt{AF}(p))$ and $\texttt{AF}(\texttt{AX}(\text{p}))$
   Review the lecture notes.
   Another way to think about this equivalence is to consider the negation of the pairs. The negation of the pairs are not equivalent (see the proof in the first question).

   (b) $\texttt{E}(\texttt{E}(p \ \texttt{U} \ q) \ \texttt{U} \ r)$ and $\texttt{E}(p \ \texttt{U} \ \texttt{E}(q \ \texttt{U} \ r))$
   Not equivalent. Consider the KS, where (s0, s1), (s1, s1) are transition relations, and p holds in s0 and r holds in s1. Then s0 satisfies $\texttt{E}(p \ \texttt{U} \ \texttt{E}(q \ \texttt{U} \ r))$ because s0 satisfies p and s1 satisfies $\texttt{E}(q \ \texttt{U} \ r)$. But, s0 does not satisfy $\texttt{E}(\texttt{E}(p \ \texttt{U} \ q) \ \texttt{U} \ r)$ because s0 dooes not satisfy r and does not satisfy $\texttt{E}(p \ \texttt{U} \ q)$.

3. Consider the following Kripke structure, with $b \in L(s_3)$, $\{a, b\} \subseteq L(s_2)$, $a \in L(s_1)$ and $b \in L(s_4)$.

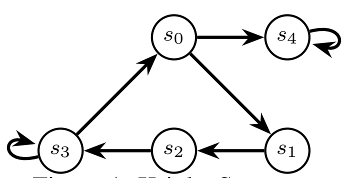

Figure 1: Kripke Structure

   Identify the set of states that satisfy each of the following:

   (a) $\texttt{AG}(b)$ $\{s_4\}$
   (b) $\texttt{A}(b \ \texttt{U} \ \neg b)$ $\{s_0, s_1\}$
   (c) $\texttt{AG}(\texttt{AF}(b))$ $\{s_0, s_1, s_2, s_3, s_4, \}$
   (d) $\texttt{EX}(\texttt{EG}(b))$ $\{s_0, s_1, s_2, s_3, s_4, \}$

[1] inevitable: sure to occur, happen, or come; unalterable. *No matter what future you choose, the exam will end inevitably.*

4. We define a new type of until operator in CTL and describe its semantics as follows: for any state s in any Kripke structure

$$s \in [\![\mathtt{E}(\varphi_1 \;\mathtt{U}^k\; \varphi_2)]\!] \text{ if and only if } \exists\pi \in \text{Paths}(s) : \exists i.(0 \leq i \leq k \;\wedge\; \pi[i] \models \varphi_2) \;\wedge\; \forall j.(j < i \Rightarrow \pi[j] \models \varphi_1)$$

$$s \in [\![\mathtt{A}(\varphi_1 \;\mathtt{U}^k\; \varphi_2)]\!] \text{ if and only if } \forall\pi \in \text{Paths}(s) : \exists i.(0 \leq i \leq k \;\wedge\; \pi[i] \models \varphi_2) \;\wedge\; \forall j.(j < i \Rightarrow \pi[j] \models \varphi_1)$$

Prove or disprove: $\{\neg, \vee, \text{EU}^k, \text{AU}^k\}$ is an adequate set for CTL when k can be specified to be any integer value between 0 and ∞. For instance, $\mathtt{E}(p \;\mathtt{U}^0\; q)$ and $\mathtt{E}(p \;\mathtt{U}^\infty\; q)$ are valid formula using this new until operator.

This is not a adequat set. Consider the semantics of $\mathtt{E}(\varphi_1 \;\mathtt{U}^k\; \varphi_2)$ in terms of the existing CTL.

$\mathtt{E}(\mathit{true} \;\mathtt{U}^k\; p)$ is equivalent to $p \;\vee\; \mathtt{EX}(p) \;\vee\; \mathtt{EX}(\mathtt{EX}(p)) \;\ldots.$

This is weaker than $\mathtt{EX}(p)$ and cannot be used to express $\mathtt{EX}(p)$.

5. **[Extra Credit.]** Consider a function $f : \mathcal{P}(V) \rightarrow \mathcal{P}(V)$, where V denotes the set of vertices in a graph $G = (V, E)$ (E being the set of directed edges). The function is defined as follows:

$$f(X) = X \;\cup\; \{v \mid \forall v'.(v, v') \in E \;\Rightarrow\; v' \in X\}$$

Compute the following for the graph in Figure 1 and justify your answer.

(a) $f^5(\{s_3\})$

$f(\{s_3\}) = \{s_3, s_2\}$
$f^2(\{s_3\}) = \{s_3, s_2, s_1\}$
$f^3(\{s_3\}) = \{s_3, s_2, s_1\}$ No need to compute any further; Not incorrect, if we compute one more step

(b) $f^5(\{s_4\})$