Introduction to data science & artificial intelligence (INF7100)

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#311 Correlation

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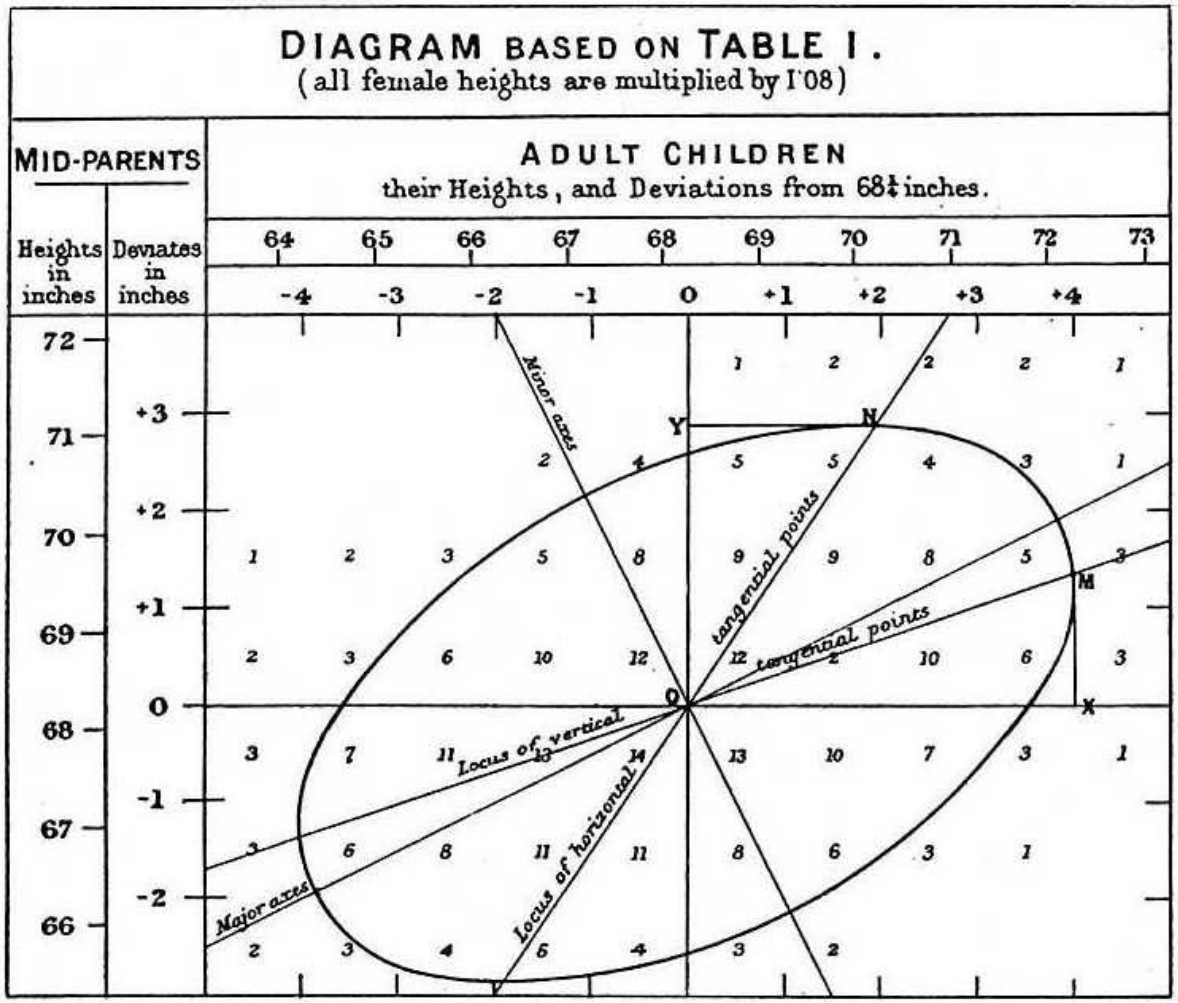
Correlation



Galton regression towards mediocrity in hereditary stature, 1886.

Pearson's Correlation

$$\rho_{XY} = \text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} = \frac{\mathbb{E}[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

$$\rho_{XY} = \frac{\mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)}{\sqrt{\mathbb{E}(X^2) - \mathbb{E}(X)^2} \cdot \sqrt{\mathbb{E}(Y^2) - \mathbb{E}(Y)^2}}$$

Note that $\rho_{XY} \in [-1, +1]$.
The empirical version is

$$r_{xy} \overset{\text{def}}{=} \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}},$$

Pearson's Correlation

- $\rho_{XY} = +1$ means that $X = aY + b$ with $a > 0$
- $\rho_{XY} = -1$ means that $X = aY + b$ with $a < 0$

In python

```
> from scipy.stats import pearsonr
> pearsonr(x, y)
```

and in R

```
> cor(x, y, method "pearson")
```