

Introduction to data science & artificial intelligence (INF7100)

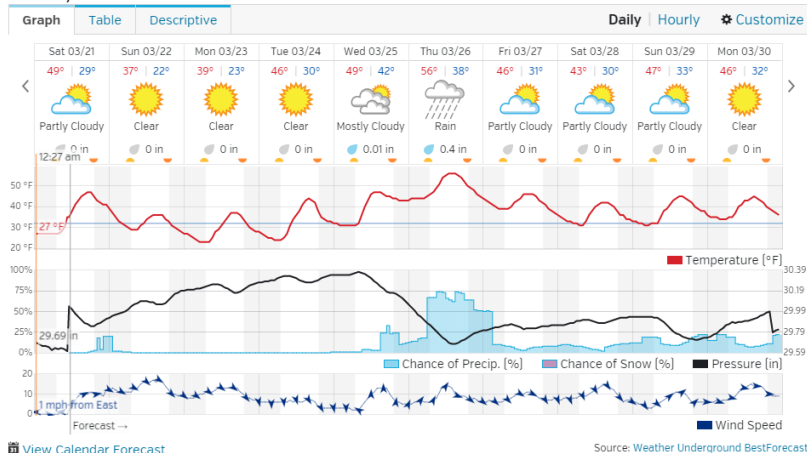
Arthur Charpentier

#291 Time Series

été 2020

Time Series

10-Day Weather Forecast



via <http://stat4701.github.io/>

Time Series

A **time series** is a sequence of observations (Y_t) ordered in time, at regular dates.

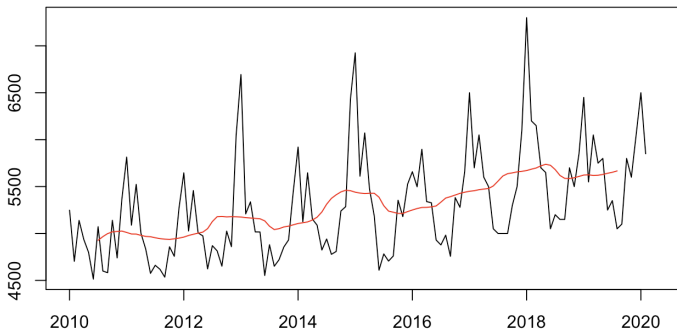
Buys-Ballot (1847, Les changements périodiques de température, dépendants de la nature du soleil et de la lune, mis en rapport avec le pronostic du temps, déduits d'observations néerlandaises de 1729 à 1846) - original probably in Dutch.

TABLEAU REPRÉSENTANT LA MARCHÉ DE LA TEMPÉRATURE PENDANT L'ANNÉE.

| Date. | Temp. | Temp. calculée. | Diffé. | Temp. calculée. | Diffé. | Date. | Temp. | Temp. calculée. | Diffé. | Temp. calculée. | Diffé. |
|----------|--------|-----------------|--------|-----------------|--------|-----------|--------|-----------------|--------|-----------------|--------|
| 10 Janv. | 32.50 | 32.5 0 | 0 | 32.58 | 0 | 17 Juill. | 0.58 | + 0.2 4 | + 0.34 | 63.68 | + 0.68 |
| 15 e | + 1.12 | + 0.7 0 | + 0.23 | + 0.8 0 | + 0.13 | 22 e | — 0.36 | 61.33 | — 0.25 | 64.03 | + 0.05 |
| 20 e | 0.89 | 34.14 | + 0.25 | 34.34 | + 0.05 | 27 e | + 0.50 | 64.57 | + 0.01 | 64.27 | + 0.21 |
| 25 e | 0.83 | 34.92 | + 0.30 | 35.22 | 0 | 1 Août | + 0.36 | 64.83 | + 0.02 | 64.71 | + 0.13 |
| 30 e | 1.50 | 35.70 | + 0.70 | + 0.5 9 | + 0.07 | 6 e | + 0.22 | 65.06 | 0 | 65.06 | 0 |
| 4 Fév. | — 0.01 | 36.47 | 0 | 36.40 | + 0.07 | | | | | | |
| 9 e | + 0.39 | + 0.5 5 | — 0.16 | 36.99 | — 0.13 | 11 e | — 0.45 | — 0.4 2 | — 0.03 | — 0.5 0 | + 0.05 |
| 14 e | + 0.36 | 37.57 | — 0.35 | 37.58 | — 0.36 | 16 e | 0.24 | 64.22 | + 0.05 | 64.05 | + 0.22 |
| 19 e | — 0.05 | 38.13 | — 0.06 | 38.17 | — 0.00 | 21 e | 0.42 | 63.80 | + 0.05 | 63.55 | + 0.30 |
| 24 e | + 1.59 | 38.68 | + 0.08 | 38.76 | 0 | 26 e | 0.66 | 63.38 | + 0.19 | 63.04 | + 0.15 |
| 1 Mars | 0.48 | 39.24 | 0 | + 0.6 9 | — 0.35 | 1 Sept. | 0.32 | 62.96 | 0 | 62.54 | + 0.42 |
| 6 e | 0.25 | + 0.7 3 | — 0.40 | 40.14 | — 0.65 | 6 e | 0.30 | — 1.1 3 | + 0.20 | 62.03 | 0 |
| 11 e | 0.11 | 40.69 | — 1.10 | 40.33 | — 1.23 | 11 e | 1.12 | 60.71 | + 0.06 | — 1.2 6 | 0 |
| 17 e | 1.82 | 41.42 | + 0.03 | 41.52 | — 0.09 | 16 e | 0.89 | 59.59 | + 0.43 | 59.51 | + 0.51 |
| 22 e | 0.79 | 42.15 | + 0.07 | 42.23 | 0 | 21 e | 1.37 | 58.46 | + 0.19 | 58.25 | + 0.41 |
| 27 e | 0.07 | 42.89 | 0 | + 1.2 9 | — 0.62 | 26 e | 1.20 | 57.33 | 0 | 56.99 | + 0.34 |
| 1 Avril | 1.24 | + 1.3 0 | — 0.02 | 44.80 | — 0.57 | 1 Oct. | 1.59 | — 1.3 2 | 0.27 | 55.74 | 0 |
| 6 e | 1.90 | 45.81 | + 0.53 | 46.09 | + 0.04 | 6 e | 1.08 | 54.69 | — 0.03 | — 1.4 3 | + 0.35 |
| 11 e | 1.23 | 46.59 | + 0.37 | 47.38 | — 0.02 | 11 e | 1.49 | 53.37 | — 0.20 | 52.88 | + 0.29 |
| 16 e | 0.89 | 48.26 | — 0.11 | 48.67 | — 0.42 | 16 e | 1.22 | 52.05 | — 0.12 | 51.46 | + 0.48 |
| 21 e | 1.45 | 49.70 | 0 | 49.96 | — 0.26 | 21 e | 1.21 | 50.73 | 0 | 50.03 | + 0.70 |
| 26 e | 1.56 | + 1.2 1 | + 0.33 | 51.26 | 0 | 26 e | 2.13 | — 1.6 3 | — 0.50 | 48.60 | 0 |
| 2 Mai | 1.56 | 52.13 | + 0.69 | + 1.1 2 | + 0.44 | 1 Nov. | 1.95 | 47.44 | — 0.49 | — 1.4 1 | — 0.24 |
| 7 e | 1.50 | 53.34 | + 0.90 | 53.50 | + 0.02 | 6 e | 1.71 | 45.92 | — 0.58 | 45.77 | — 0.53 |
| 12 e | 0.29 | 54.56 | — 0.01 | 54.62 | — 0.07 | 11 e | 1.04 | 44.20 | 0 | 44.20 | + 0.16 |
| 17 e | 1.18 | 55.79 | 0 | 55.74 | + 0.05 | 16 e | 0.95 | — 0.7 6 | + 0.19 | 42.95 | + 0.30 |
| 22 e | 1.58 | + 0.9 5 | + 0.34 | 56.87 | + 0.20 | 21 e | 1.72 | 42.68 | — 1.18 | 41.52 | + 0.12 |
| 27 e | 0.82 | 57.69 | + 0.30 | 57.99 | 0 | 26 e | 0.58 | 41.92 | — 0.97 | — 0.7 0 | + 0.12 |
| 1 Juin | 0.26 | 58.64 | — 0.39 | + 0.6 6 | — 0.40 | 1 Déc. | + 0.20 | 41.15 | 0 | 40.13 | + 0.76 |
| 6 e | 1.27 | 59.60 | + 0.02 | 59.32 | + 0.30 | 6 e | 1.55 | — 0.8 5 | — 0.42 | 39.43 | + 0.27 |
| 11 e | 0.93 | 60.65 | 0 | 59.98 | + 0.57 | 11 e | 0.91 | 38.19 | — 0.40 | 38.73 | + 0.06 |
| 16 e | 0.57 | + 0.5 5 | + 0.02 | 60.65 | + 0.47 | 16 e | 0.75 | 38.33 | — 0.29 | 38.04 | 0 |
| 21 e | 0.06 | 61.65 | — 0.47 | 61.31 | — 0.12 | 21 e | 0.64 | 37.40 | 0 | — 1.0 4 | + 0.40 |
| 26 e | 0.77 | 62.20 | — 0.26 | 61.97 | — 0.02 | 26 e | 1.61 | — 1.1 4 | — 0.47 | 35.96 | — 0.17 |
| 3 Juill. | 0.08 | 62.62 | — 0.19 | 62.63 | 0 | 31 e | 1.05 | 35.1 | — 0.35 | 34.52 | + 0.18 |
| 7 e | 0.70 | 63.31 | + 0.02 | + 0.3 5 | + 0.35 | 5 Juin | 1.44 | 33.98 | — 0.68 | 33.88 | — 0.58 |
| 12 e | 0.53 | 63.96 | 0 | 63.33 | + 0.52 | 10 e | 0.72 | 32.58 | 0 | 32.58 | 0 |

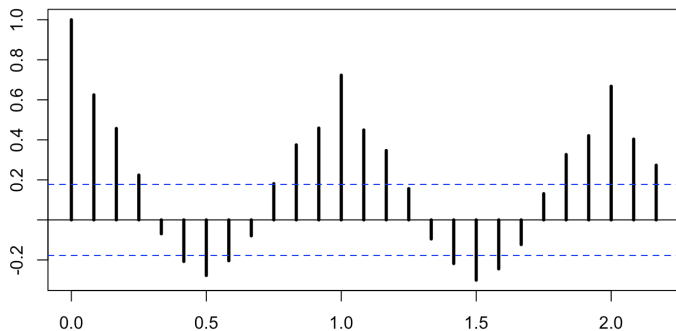
Moving Average

$$\tilde{y}_t = \frac{1}{2h} \left(\frac{y_{t-h}}{2} + \cdots + y_{t-1} + y_t + y_{t+1} + \frac{y_{t+h}}{2} \right)$$



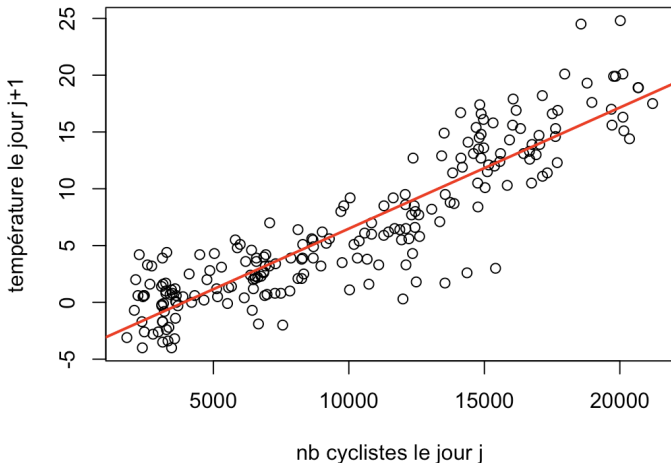
Autocorrelation

$$h \mapsto \text{corr}(y_t, y_{t-h})$$



Time Series & Causality

“an old variable explains 85% of the change in a new variable. So we can talk about causality” Emmanuel Todd (no!!)



Time Series & Causality

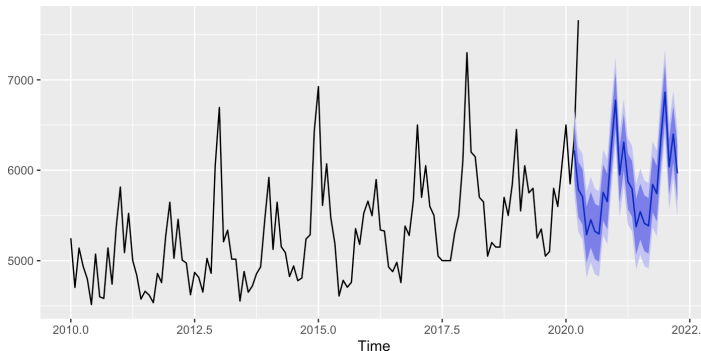
$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix}, \quad \text{Var} \begin{bmatrix} u_t \\ v_t \end{bmatrix} = \begin{bmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}$$

- ▶ $a_{1,2} \neq 0$: causal effect, $y \rightarrow x$
- ▶ $a_{2,1} \neq 0$: causal effect, $x \rightarrow y$
- ▶ $\rho \neq 0$: (instantaneous) causal effect, $x \leftrightarrow y$

see [Tents, Tweets, and Events: The Interplay Between Ongoing Protests and Social Media](#) (on Indignados, Occupy, and Brazilian Vinegar protests)

Forecasting Time Series

See monthly deaths in Québec province



using Exponential Smoothing...