Introduction to data science & artificial intelligence (IF7100)

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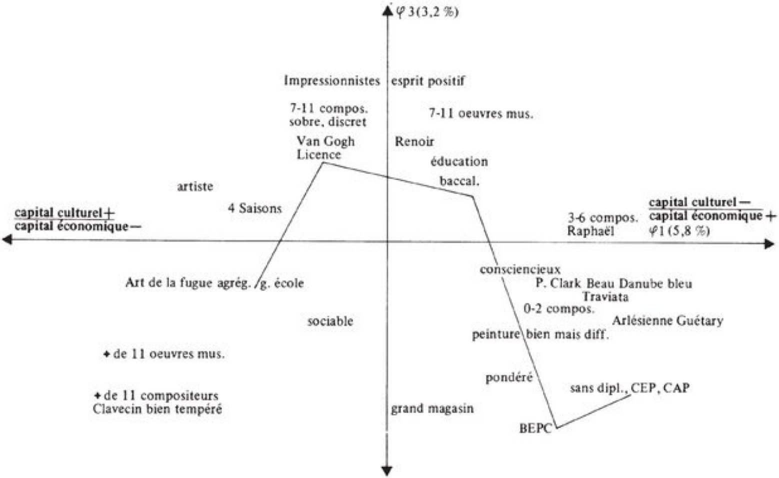
#272 Multivariate Analysis: Clusters

été 2020

Clusters


φ 3(3,2 %)
Impressionnistes
esprit positif
7-11 compos.
sobre, discret
7-11 oeuvres mus.
Van Gogh
Licence
Renoir
éducation
baccal.
artiste
4 Saisons
capital culturel +
capital économique —
capital culturel —
capital économique +
φ1 (5,8 %)
3-6 compos.
Raphaël
consciencieux
Art de la fugue agrég. /g. école
P. Clark Beau Danube bleu
Traviata
0-2 compos.
Arlésienne Guétary
sociable
peinture bien mais diff.
+ de 11 oeuvres mus.
pondéré
sans dipl., CEP, CAP
+ de 11 compositeurs
Clavecin bien tempéré
grand magasin
BEPC

X

k-Means

Consider n observations $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$ in $\mathbb{R}^d$
Given k points $\boldsymbol{\mu}_1, \cdots, \boldsymbol{\mu}_k$ in $\mathbb{R}^d$ (center of clusters), consider the associated Voronoi diagram.

$$C(\boldsymbol{\mu}_1, \cdots, \boldsymbol{\mu}_k) = \sum_{i=1}^{n} \left(\min_{j=1,\cdots,k} \|\boldsymbol{x}_i - \boldsymbol{\mu}_j\| \right)^2$$

But find $\min\{C(\boldsymbol{\mu}_1, \cdots, \boldsymbol{\mu}_k)\}$ this is a (very) difficult problem

k-Means

xxx

Algorithm 1: k-Means (Lloyd's algorithm)

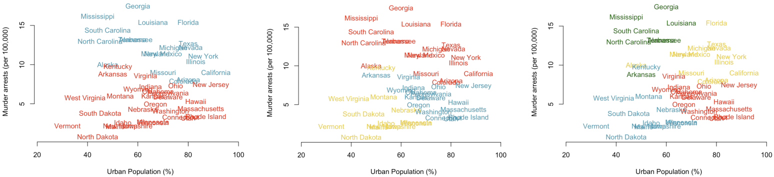
```
initialization : draw $k$ centers $\boldsymbol{\mu}_1, \cdot, \boldsymbol{\mu}_k$;
for $b = 1, 2, \ldots, T$ do
    assign: for $j = 1, 2, \ldots, k$ do
        $C_j \leftarrow \{i : \|\boldsymbol{x}_i - \boldsymbol{\mu}_j\| \leq \|\boldsymbol{x}_i - \boldsymbol{\mu}_{j'}\|, \ \forall j'\}$;
    update: for $j = 1, 2, \ldots, k$ do
        $\boldsymbol{\mu}_j \leftarrow \frac{1}{\#C_j} \sum_{i:\boldsymbol{x}_i \in C_j} \boldsymbol{x}_i$;
```

Comparing US States

Use normalized variables (see Mahalanobis distance, #421)

```
              Murder Assault UrbanPop       Rape
Alabama     1.242564 0.78283 -0.52090 -0.0034164
Alaska      0.507862 1.10682 -1.21176  2.4842029
Arizona     0.071633 1.47880  0.99898  1.0428783
Arkansas    0.232349 0.23086 -1.07359 -0.1849166
California  0.278268 1.26281  1.75892  2.0678202
Colorado    0.025714 0.39885  0.86080  1.8649672
```

$k = 2$, 3 and 4 (see https://uc-r.github.io)

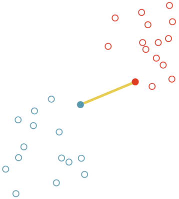


Linkage Methods

Algorithm 2: (Hierarchical) Linkage Algorithm

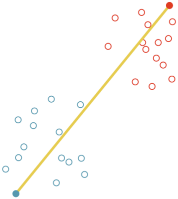
```
initialization : $C_1 = \{x_1\}, \cdots, C_n = \{x_n\}$;
for $i = 1, 2, ..., n-1$ do
    $(j^*, k^*) \leftarrow \text{argmin}\{d(C_j, C_k)\}$;
    $C_{j^*} \leftarrow C_{j^*} \cup C_{k^*}$ (and $C_{k^*} \leftarrow \emptyset$);
```

Problem here: what is the distance between two clusters ?



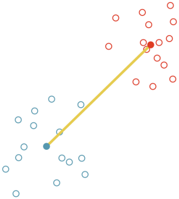
$$\min_{\boldsymbol{x}\in C_1,\boldsymbol{y}\in C_2}\left\{\|\boldsymbol{x}-\boldsymbol{y}\|\right\}$$

(single)



$$\max_{\boldsymbol{x}\in C_1,\boldsymbol{y}\in C_2}\left\{\|\boldsymbol{x}-\boldsymbol{y}\|\right\}$$

(complete)



$$\|\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}}\|$$

Linkage Methods

Average pairwise distance

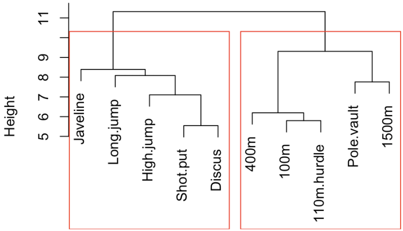
$$d(C_1,C_2)^2=\frac{1}{n_1n_2}\sum_{i,\boldsymbol{x}_i\in C_1}\sum_{j,\boldsymbol{y}_j\in C_2}\|\boldsymbol{x}_i-\boldsymbol{y}_j\|$$

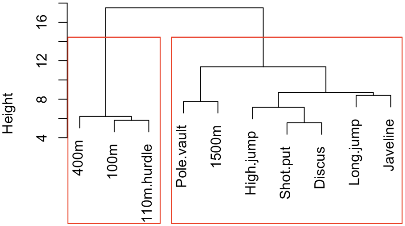
Ward's method: take into account the size of merged groups,

$$d(C_1,C_2)^2=\frac{n_1n_2}{n_1+n_2}\cdot\|\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}}\|^2,\ \overline{\boldsymbol{x}}=\frac{1}{n_1}\sum_{i,\boldsymbol{x}_i\in C_1}\boldsymbol{x}_i,\ \overline{\boldsymbol{y}}=\frac{1}{n_2}\sum_{j,\boldsymbol{y}_j\in C_2}\boldsymbol{y}_j.$$

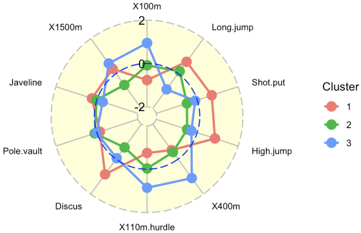
Decathlon

One can look at clusters of sports (Ward and complete)



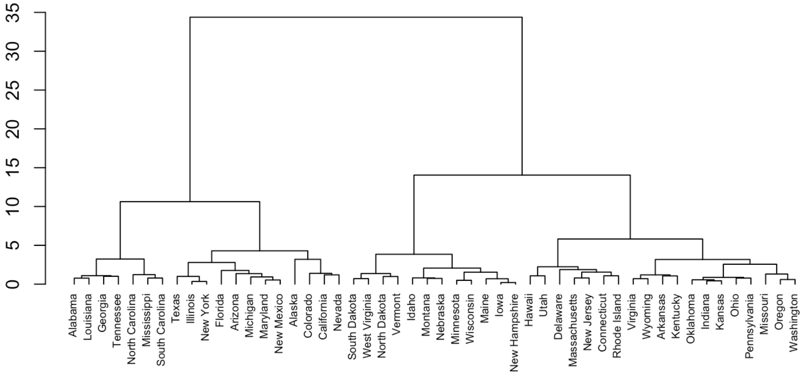


or clusters of sportmen



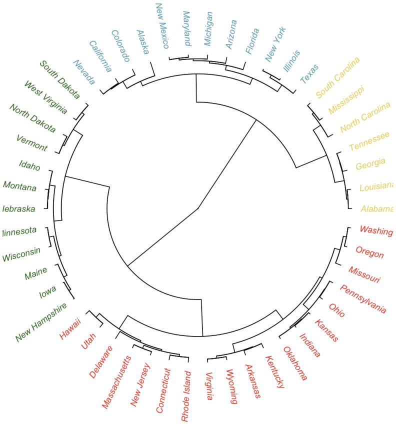
Comparing US States

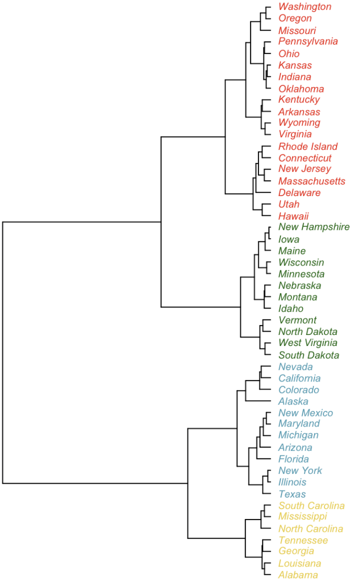
Using Ward's method,



Comparing US States

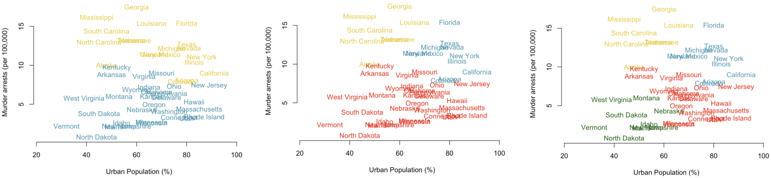
One can use a phylogenetic tree



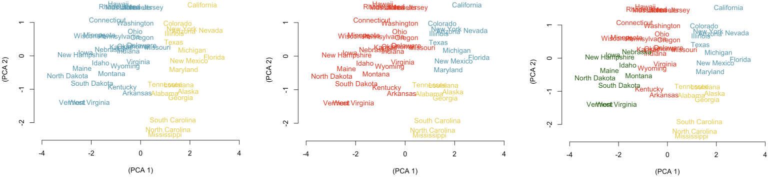


Comparing US States

On the two dimensional representation (urban population, murder rate), when obtain, as *optimal* 2, 3 or 4 classes

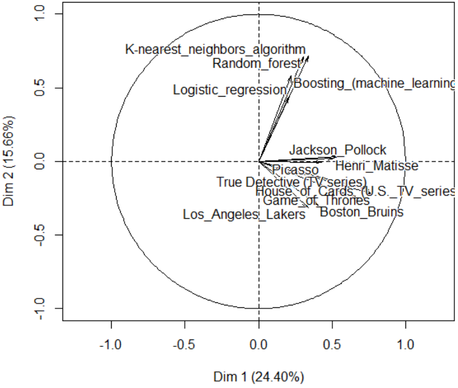


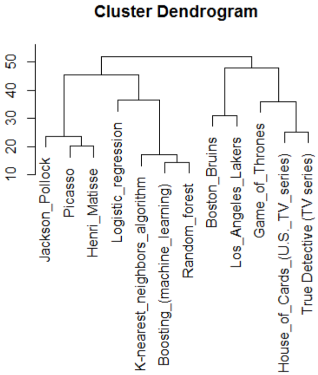
.2cm
or on the first 2 principal components (PCA projection)



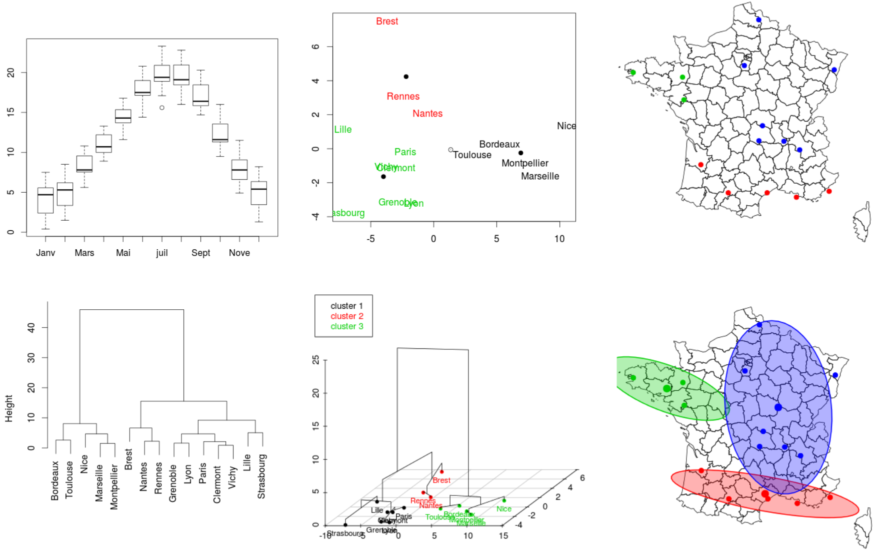
Comparing Wikipedia Pages

Bag of works from some pages Boosting_(machine_learning), Random_forest, K-nearest_neighbors, Logistic_regression, Boston_Bruins, Los_Angeles_Lakers, Game_of_Thrones, House_of_Cards, True_Detective, Picasso, Henri_Matisse, Jackson_Pollock.





Comparing 15 Cities Temperatures (in France)


Janv
Mars
Mai
juil
Sept
Nove
Brest
Rennes
Nantes
Lille
Nice
Bordeaux
Toulouse
Montpellier
Marseille
Paris
Vichy
Clermont
Grenoble
Lyon
Strasbourg
Height
Bordeaux
Toulouse
Nice
Marseille
Montpellier
Brest
Nantes
Rennes
Grenoble
Lyon
Paris
Clermont
Vichy
Lille
Strasbourg
cluster 1
cluster 2
cluster 3