COVID-19 pandemic control: balancing detection policy and lockdown intervention under ICU sustainability

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The *SIR* model

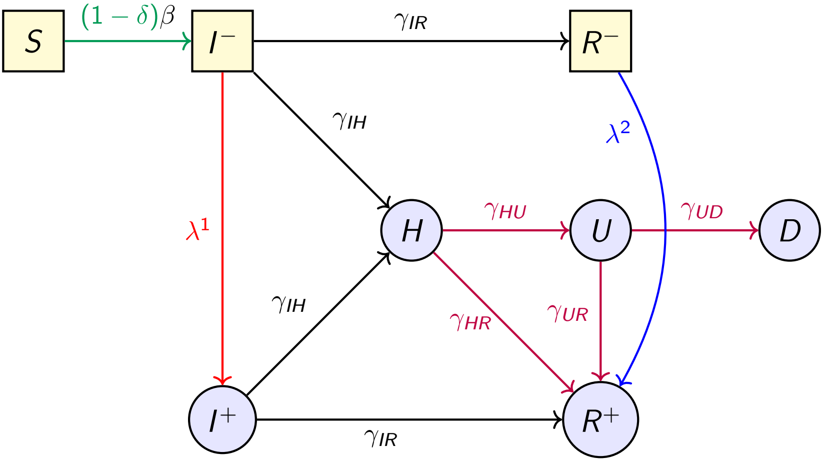
$$\begin{tikzcd} \boxed{S} \arrow[r, "\beta"] & \boxed{I} \arrow[r, "\gamma"] \arrow[l, bend right, dashed] & \boxed{R} \end{tikzcd}$$

$$\frac{\mathrm{d}S_t}{\mathrm{d}t} = -\beta S_t I_t, \quad \frac{\mathrm{d}I_t}{\mathrm{d}t} = \beta S_t I_t - \gamma I_t, \quad \text{and } \frac{\mathrm{d}R_t}{\mathrm{d}t} = \gamma I_t.$$

Important quantity: $\mathfrak{R}_0 = \dfrac{\beta}{\gamma}$ (reproductive ratio).

- lockdown: $S \to (1-\delta)S$
- asymptotic: $I \to (I^+, I^-)$ and $R \to (R^+, R^-)$
- more categories: H, ICU and D
- testing/detection: $I^- \to I^+$ and $R^- \to R^+$

The $SIDUHR^{+/-}$ model



The $SIDUHR^{+/-}$ model

$$\begin{cases}
\mathrm{d}S_t = -(1-\delta_t)\beta_t I^- S_t \mathrm{d}t, & \text{Susceptible}\\
\mathrm{d}I_t^- = (1-\delta_t)\beta I_t^- S_t \mathrm{d}t - \lambda_t^1 I_t^- \mathrm{d}t - (\gamma_{IR}+\gamma_{IH}) I_t^- \mathrm{d}t, & \text{Infected} -\\
\mathrm{d}I_t^+ = \lambda_t^1 I_t^- \mathrm{d}t - (\gamma_{IR}+\gamma_{IH}) I_t^+ \mathrm{d}t, & \text{Infected} +\\
\mathrm{d}R_t^- = \gamma_{IR} I_t^- \mathrm{d}t - \lambda_t^2 R_t^- \mathrm{d}t, & \text{Recovered} -\\
\mathrm{d}R_t^+ = \gamma_{IR} I_t^+ \mathrm{d}t + \lambda_t^2 R_t^- \mathrm{d}t + \gamma_{HR} H_t \mathrm{d}t + \gamma_{UR}(U_t) U_t \mathrm{d}t, & \text{Recovered} +\\
\mathrm{d}H_t = \gamma_{IH}\left(I_t^- + I_t^+\right)\mathrm{d}t - (\gamma_{HR}+\gamma_{HU}) H_t \mathrm{d}t, & \text{Hospitalized}\\
\mathrm{d}U_t = \gamma_{HU} H_t \mathrm{d}t - (\gamma_{UR}(U_t) + \gamma_{UD}(U_t)) U_t \mathrm{d}t, & \text{ICU}\\
\mathrm{d}D_t = \gamma_{UD}(U_t) U_t \mathrm{d}t, & \text{Dead}
\end{cases}$$

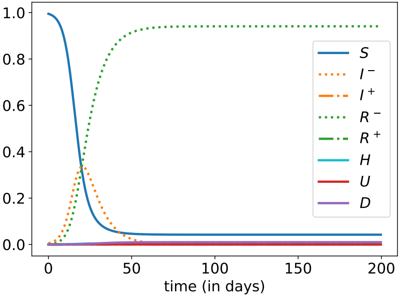
$$\mathfrak{R}_0 = \frac{(1-\delta_0)\beta}{\lambda_0^1 + \gamma_{IR} + \gamma_{IH}} \text{ and } \mathfrak{R}_t = \frac{(1-\delta_t)\beta S_t}{\lambda_t^1 + (\gamma_{IR} + \gamma_{IH})}.$$

Parameters (Tab. 1)

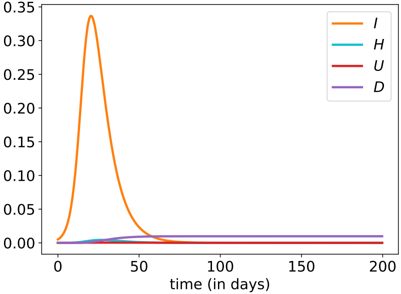
Parameters	Value	Reference
$\mathfrak{R}_0$	3.3	Salje *et al.* (2020)
β	0.436	Salje *et al.* (2020)
γ_{IR}	0.130	Domenico *et al.* (202
γ_{IH}	0.00232	Domenico *et al.* (202
γ_{HR}	0.048	Salje *et al.* (2020)
γ_{HU}	0.091	Salje *et al.* (2020)
$\gamma_{UR}(U) \times U$	$0.078U \wedge 1.564\ 10^{-5}$	Salje *et al.* (2020)
$\gamma_{UD}(U) \times U$	$0.02\ U \wedge U_{\max} + 2(U - 0.0002)^+$	Salje *et al.* (2020)
$U_{\max}$	0.0002	estimated
I_0^-	0.005	estimated

Domenico *et al.* (2020) medrxiv: 2020.04.13.20063933.
Salje *et al.* (2020) Hal Pasteur: 02548181

Dynamics of the $SIDUHR^{+/-}$ model (Fig. 2)



(a) All states



(b) States $I = I^- + I^+, H, U, D$ only

Note: $\max\{I_t\} \sim 33.7\%$ and $D_T \sim 9.8‰$.

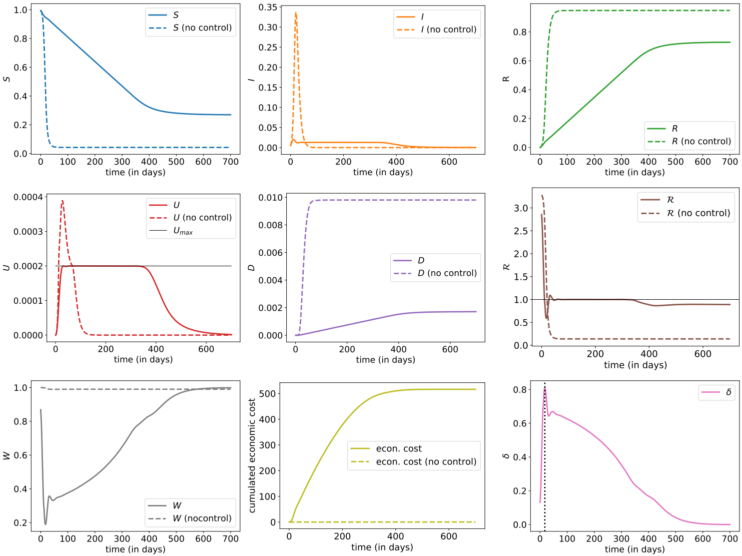
The objective functions

$$\begin{cases} Q_t = R_t^- + I_t^- + S_t & \text{(suceptible to be) quarantined/lockdowned} \\ W_t = (1-\delta_t)Q_t + R_t^+ & \text{work force} \\ N_t^1 = \lambda_t^1 Q_t + \gamma_{IH} I_t^- & \text{virologic tests, type-1 (short term)} \\ N_t^2 = \lambda_t^2 Q_t & \text{antibody tests, type-2 (long term)} \end{cases}$$

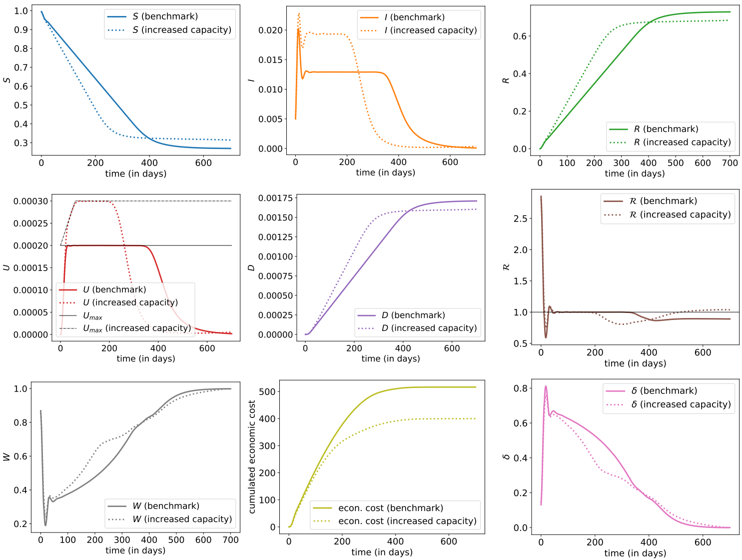
$$\begin{cases} C_{\text{sanitary}} & = \mathbb{E}(D_\tau) = \displaystyle\int_0^\infty e^{-\alpha t}\mathrm{d}D_t \\ C_{\text{econ}} & = \mathbb{E}\left[\displaystyle\int_0^\tau (1-W_t)^2\mathrm{d}t\right] = \displaystyle\int_0^\infty e^{-\alpha t}(1-W_t)^2\mathrm{d}t \\ C_{\text{prevalence}} & = \mathbb{E}\left[\displaystyle\int_0^\tau |N_t^1|^2\mathrm{d}t\right] = \displaystyle\int_0^\infty e^{-\alpha t}|N_t^1|^2\mathrm{d}t \\ C_{\text{immunity}} & = \mathbb{E}\left[\displaystyle\int_0^\tau |N_t^2|^2\mathrm{d}t\right] = \displaystyle\int_0^\infty e^{-\alpha t}|N_t^2|^2\mathrm{d}t \end{cases}$$

Computational issue: $\infty = 700$ days

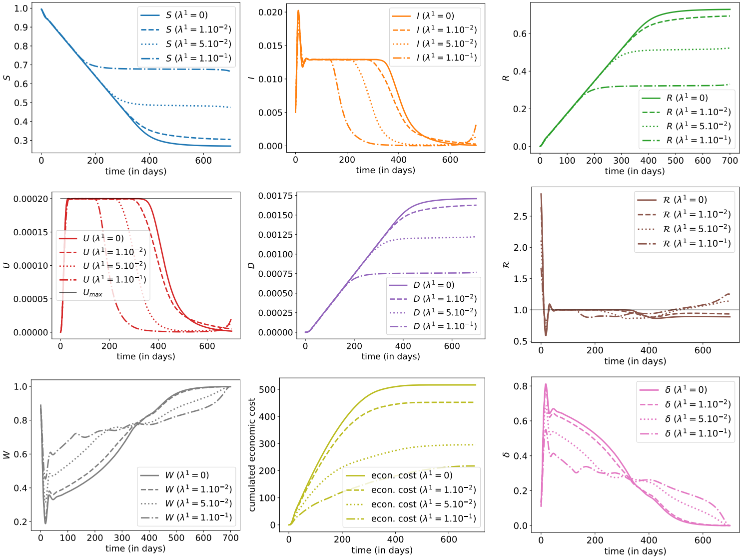
With optimal (δ_t^*) (Fig. 3)



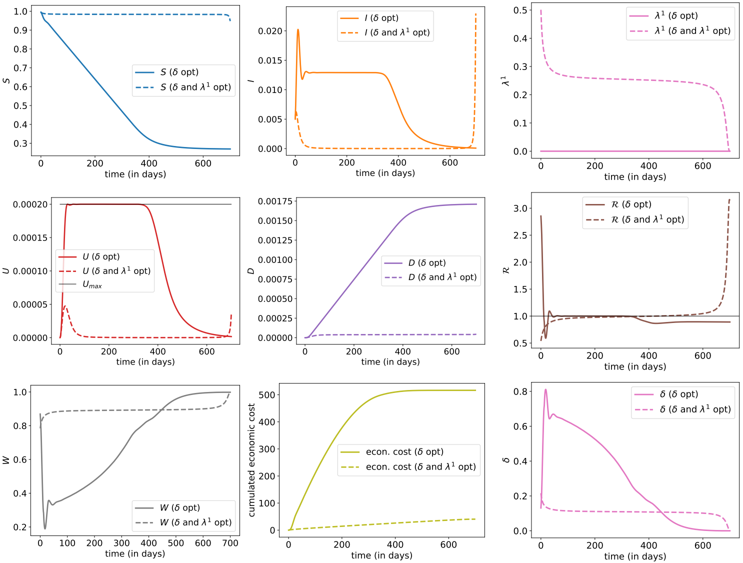
Optimal (δ_t^*) with increase of *ICU* (Fig. 4)



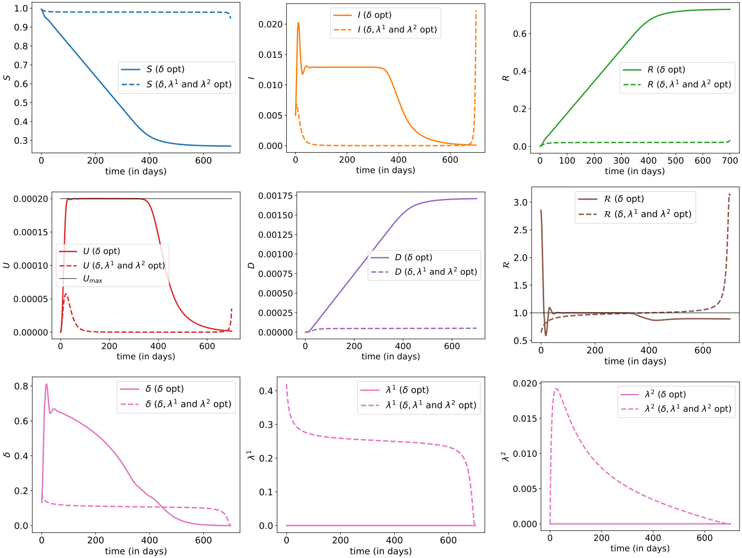
Impact of (λ^1_t) (constant, Fig. 10, B5)



Impact of (λ_t^{1*}) (Fig. 12, B7)



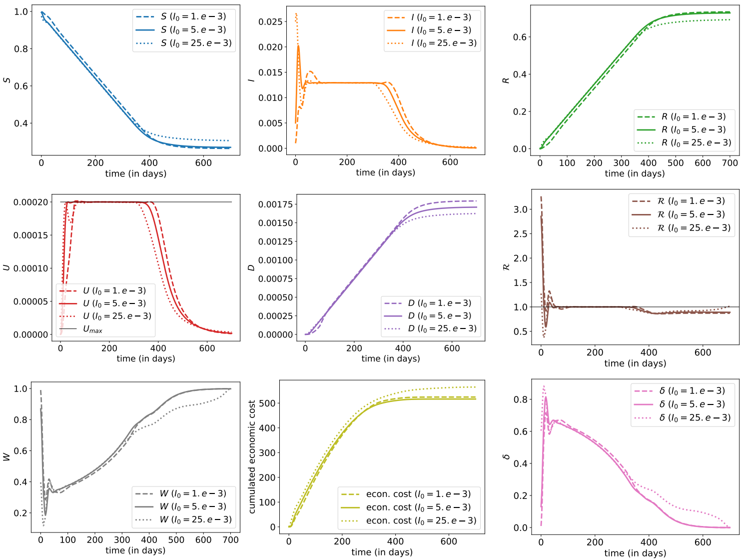
Impact of (λ_t^{2*}) (Fig. 16, B11)



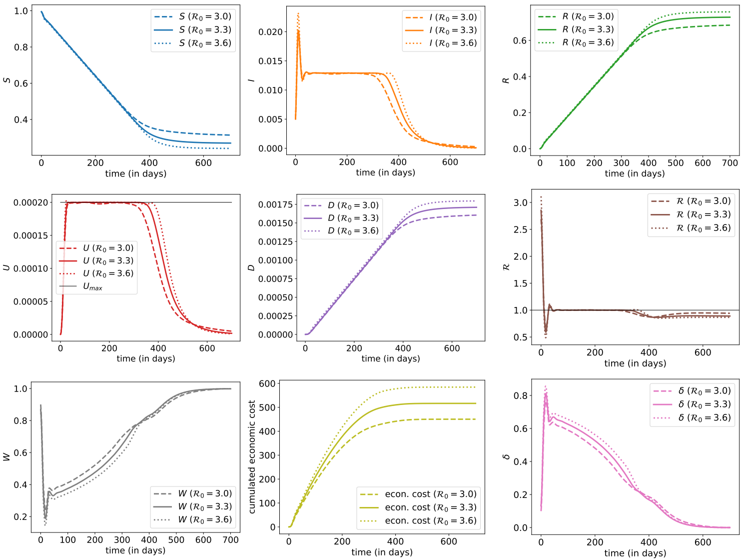
Wrap-Up

- initial objective : realistic model for the COVID-19 pandemic
- "*flatten the curve*" : ICU sustainability
- virologic tests, type-1 (short term) : identify I^- ($\rightarrow I^+$)
- antibody tests, type-2 (long term) : identify R^- ($\rightarrow R^+$)
- need to get accurate cost function (trade-off sanitary / economic & welfare)
- never relax the control before reaching herd immunity
- quite robust on various parameters
- more on https://doi.org/10.1101/2020.05.13.20100842
- can go further...
  - take into account age structure (contact matrices)
  - impact of lockdown duration
  - trace, test and isolate

Sensitivity in I_0 (Fig. 6, B1)



Sensitivity in $\mathfrak{R}_0$ (Fig. 7, B2)



Sensitivity in α (Fig. 9, B4)