STT 1000 - STATISTIQVES

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Loi Beta

$$f_{\alpha,\beta}(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}\mathbf{1}_{[0,1]}(x)$$

avec $\alpha > 0$ et $\beta > 0$. Il s'agit bien d'une densité, car

$$\int_0^1 x^\alpha(1-x)^\beta dx = \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)}$$

$$\begin{cases} E(X) = \dfrac{\alpha}{\alpha+\beta} \\ V(X) = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \end{cases}$$

Loi Beta

$$f_{\alpha,\beta}(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}\mathbf{1}_{[0,1]}(x)$$

$$\mathcal{L}(\alpha,\beta) = \left(\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right)^n \prod_{i=1}^n x_i^\alpha \prod_{i=1}^n (1-x_i)^\beta$$

$$\log\mathcal{L} = n\log\left(\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right) + (\alpha-1)\sum_{i=1}^n \log(x_i) + (\beta-1)\sum_{i=1}^n \log(1-x_i)$$

$$\frac{\partial \log\mathcal{L}}{\partial\alpha} = \frac{n\Gamma'(\alpha+\beta)}{\Gamma(\alpha+\beta)} - \frac{n\Gamma'(\alpha)}{\Gamma(\alpha)} + \sum_{i=1}^n \log(x_i)$$

$$\frac{\partial \log\mathcal{L}}{\partial\beta} = \frac{n\Gamma'(\alpha+\beta)}{\Gamma(\alpha+\beta)} - \frac{n\Gamma'(\beta)}{\Gamma(\beta)} + \sum_{i=1}^n \log(1-x_i)$$

Loi Beta

$$\overline{x} = \frac{\alpha}{\alpha+\beta} \text{ et } s^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$\beta = \frac{\alpha}{\overline{x}} - \alpha$$

$$\widehat{\alpha} = \overline{x}\left(\frac{\overline{x}(1-\overline{x})}{s^2} - 1\right)$$

$$\widehat{\beta} = (1-\overline{x})\left(\frac{\overline{x}(1-\overline{x})}{s^2} - 1\right)$$