PHY 101

Motion in One Dimension

Terminology

- ***Mechanics***—study of motion and change of position of an object
- ***Kinematics***—description of motion in terms of space-time: how do things move? (x, v, a, t)
- ***Dynamics***—study of the physical causes of motion: What makes things move? (x, v, a, t, & F)

Displacement, Velocity, and Acceleration

2.1.1 Displacement and difference with distance (length)

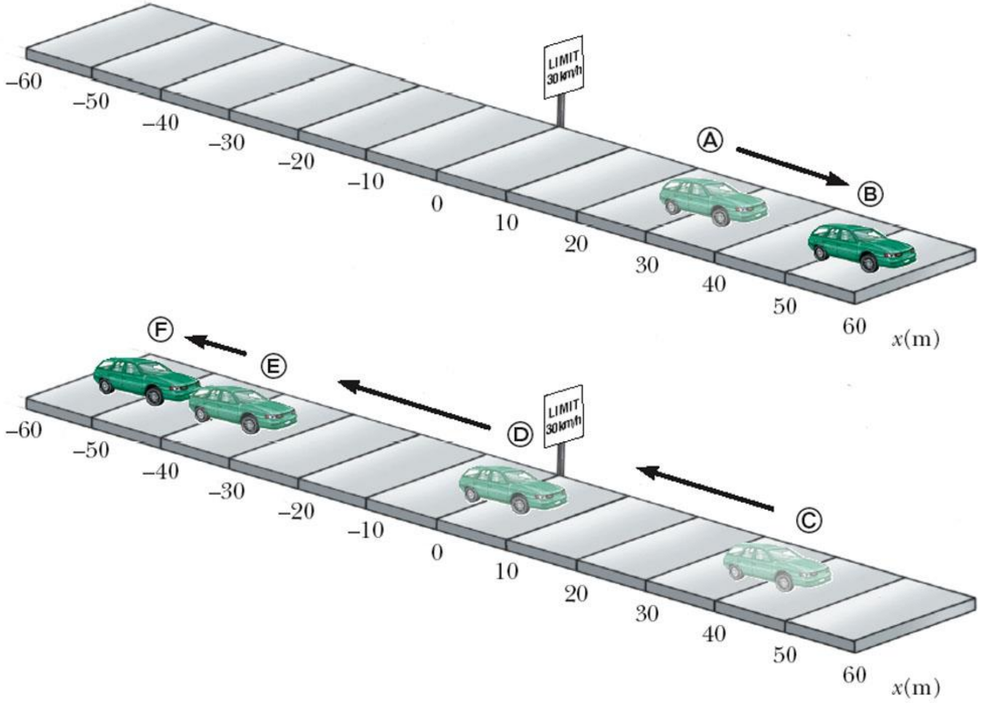
- Distance is the magnitude- Length without regards to direction (scalar)
- The ***displacement*** of an object is the change in its position, direction matters (vector)
- It is defined by coordinates on a straight line:

$$\boxed{\Delta x = x_f - x_i}$$

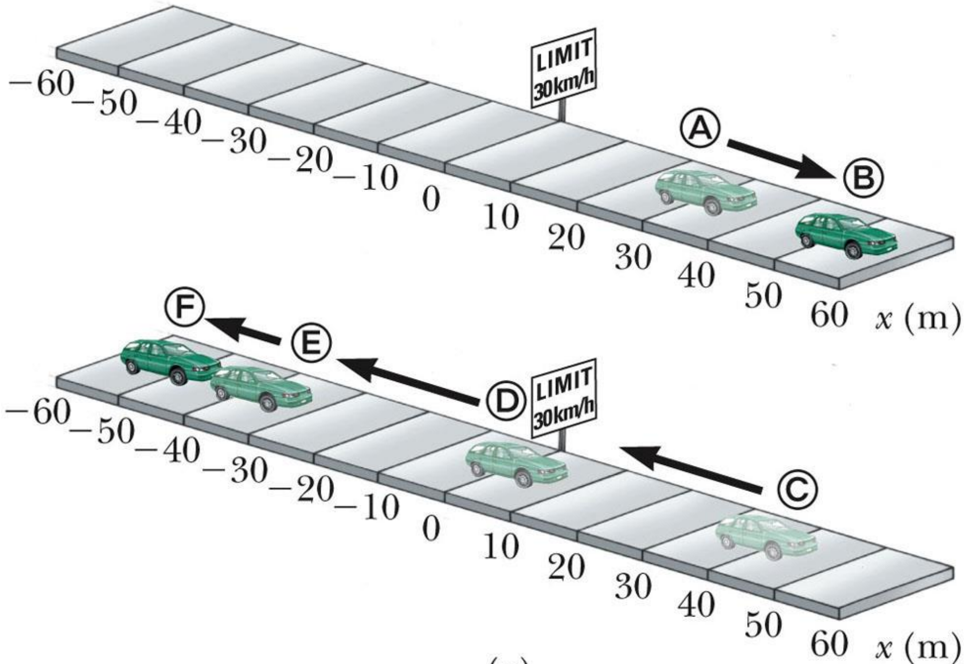
- Displacement can be positive or negative.
- It can also be zero if there is no change in position (i.e. the object begins and ends in the same place).

Displacement Examples

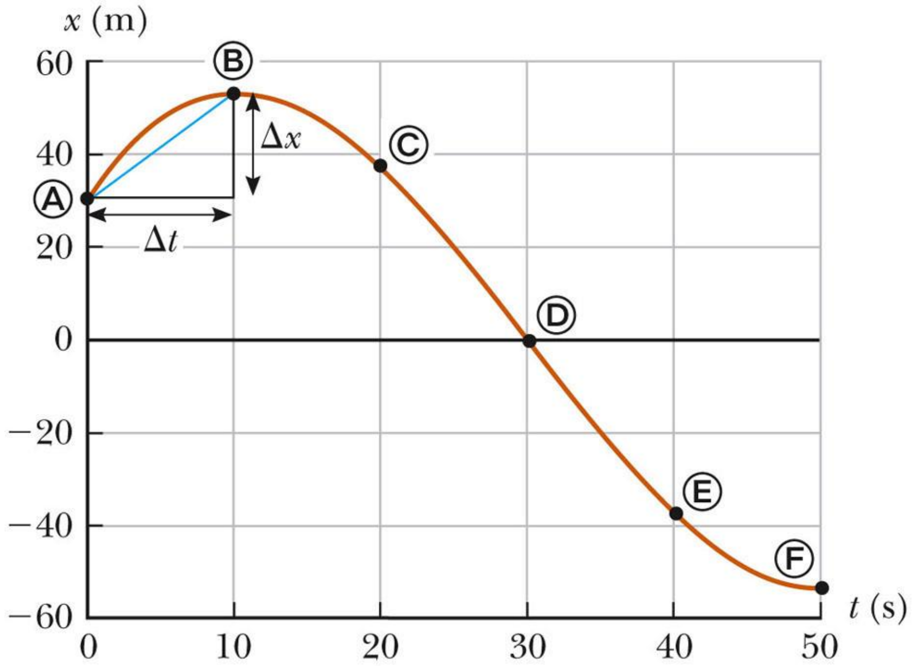
- From A to B
  - $x_i = 30$ m
  - $x_f = 52$ m
  - $\Delta x = 22$ m (52 m – 30 m)
  - The displacement is positive, indicating the motion was in the positive x direction (to the right/East)
- From C to F
  - $x_i = 38$ m
  - $x_f = -53$ m
  - $\Delta x = -91$ m (-53 m – 38 m)
  - The displacement is negative, indicating the motion was in the negative x direction (to the left/West)



Displacement, Graphical



(a)



(b)

Velocity

- Average ***velocity*** is a measure of displacement divided by the time interval over which the displacement occurs:

$$\boxed{\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}}$$

- On a graph of position versus time, velocity equals the slope of the line. Straight lines indicate constant velocity, curving indicates change in slope which means that there is acceleration.

Velocity continued

- Direction will be the same as the direction of the displacement, + or - is sufficient
- Units of velocity are m/s (SI)
  - Other units may be given in a problem, but generally will need to be converted to these
  - In other systems:
    - US Customary: ft/s OR mile/hr
    - cgs: cm/s

Speed

- The **average speed** of an object is defined as the total distance traveled divided by the total time elapsed (different than the speed at a given moment)

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$v = \frac{d}{t}$$

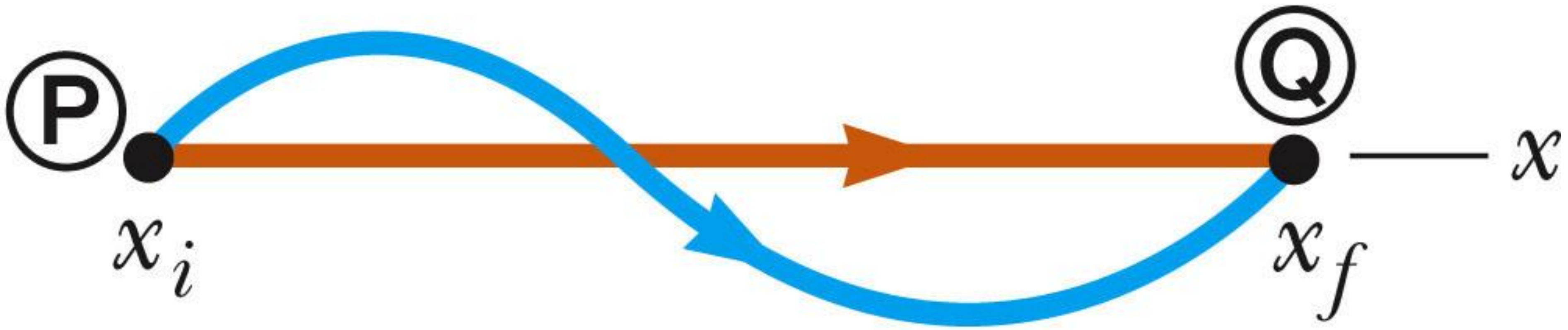
  - Speed is a scalar quantity

-Remarks-

- Average ***speed*** is a measurement of the **total distance traveled** over the time interval.
- Speed = absolute value of velocity=|velocity|

Velocity	Speed
defined by both magnitude and direction	defined by magnitude, only
can be positive or negative	only positive
sign determines direction of the motion	sign is always positive; hence, direction is ignored

Speed vs. Velocity



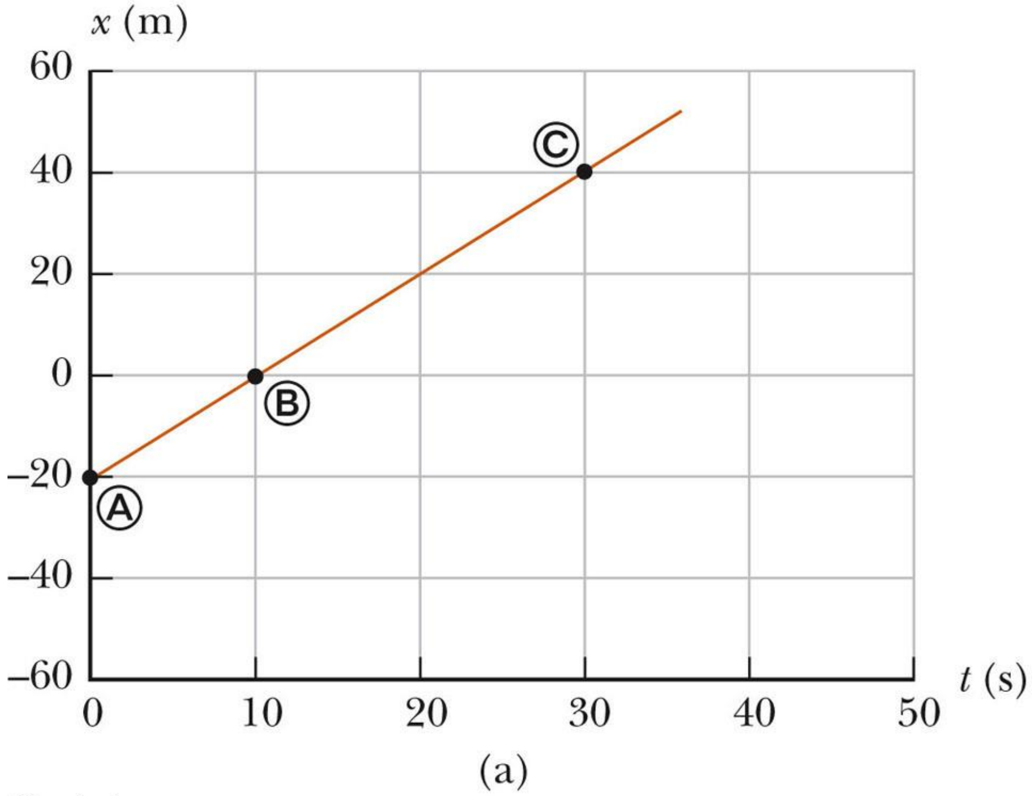
- Cars on both paths have the same average velocity since they had the same displacement in the same time interval
- The car on the blue path will have a greater average speed since the distance it traveled is larger

Graphical Interpretation of Velocity

- Velocity can be determined from a position-time graph
- Average velocity equals the slope of the line joining the initial and final positions
- An object moving with a constant velocity will have a graph that is a straight line

Average Velocity, Constant

- The straight line indicates constant velocity
- The slope of the line is the value of the average velocity



Notes on Slopes

- The general equation for the slope of any line is

$$slope = \frac{change\ in\ vertical\ axis}{change\ in\ horizontal\ axis}$$

- Slope carries units
- The meaning of a specific slope will depend on the physical data being graphed

Velocity, cont…

- ***Instantaneous velocity*** is the velocity at a given time:

$$\boldsymbol{v\ at\ a\ moment\ in\ time}$$

- Instantaneous speed is the absolute value of the instantaneous velocity.
- Consider a traveling car…
  - Its average speed is found between two different points of the trip.
  - Its instantaneous speed is found at one point in time (almost equal to reading the speedometer).

Problem 1

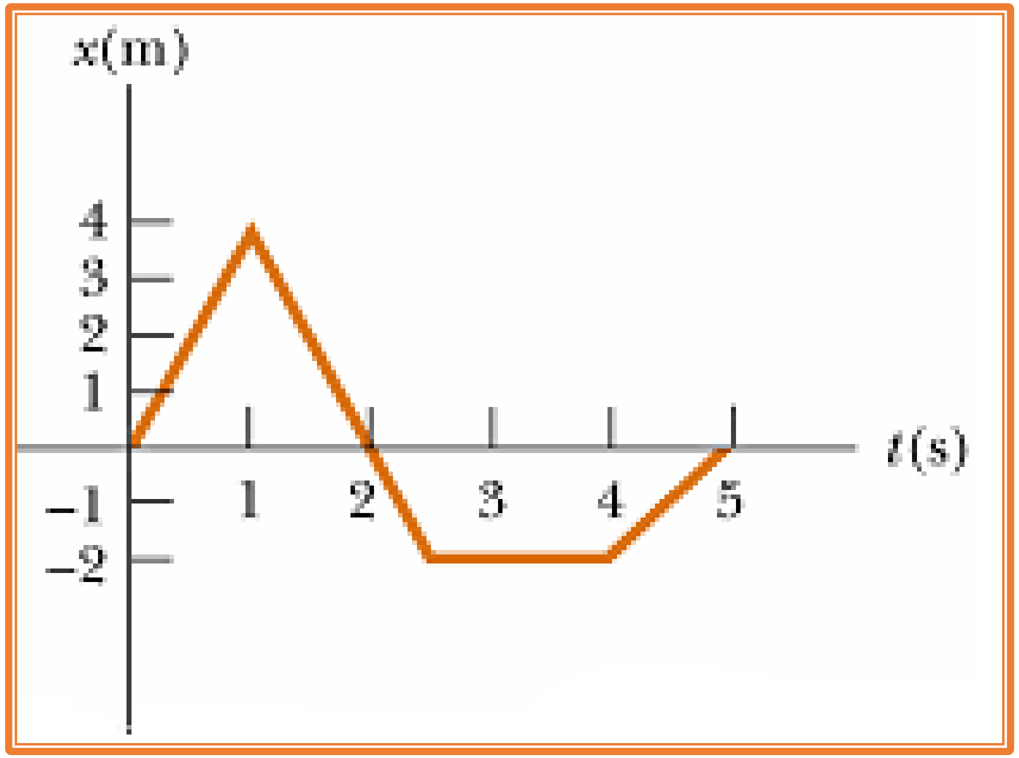
A tennis player moves in a straight-line path as shown in the figure. Find her average velocities and speed values in the time intervals

(a) 0 to 1.0 second,

(b) 0 to 4.0 seconds,

(c) 1.0 to 5.0 seconds, and

(d) 0 to 5.0 seconds.



Problem 1. Solution

(a) $v = (x_1 - x_0)/\Delta t$

= (4.0 m - 0)/1.0 s

= +4.0 m/s, speed = d/t = 4/1 = 4 m/s

(b) $v = (x_4 - x_0)/\Delta t$

= (-2.0 m - 0)/4.0 s

= -0.5 m/s, speed = 10 m/4 s = 2.5 m/s

(c) $v = (x_5 - x_1)/\Delta t$

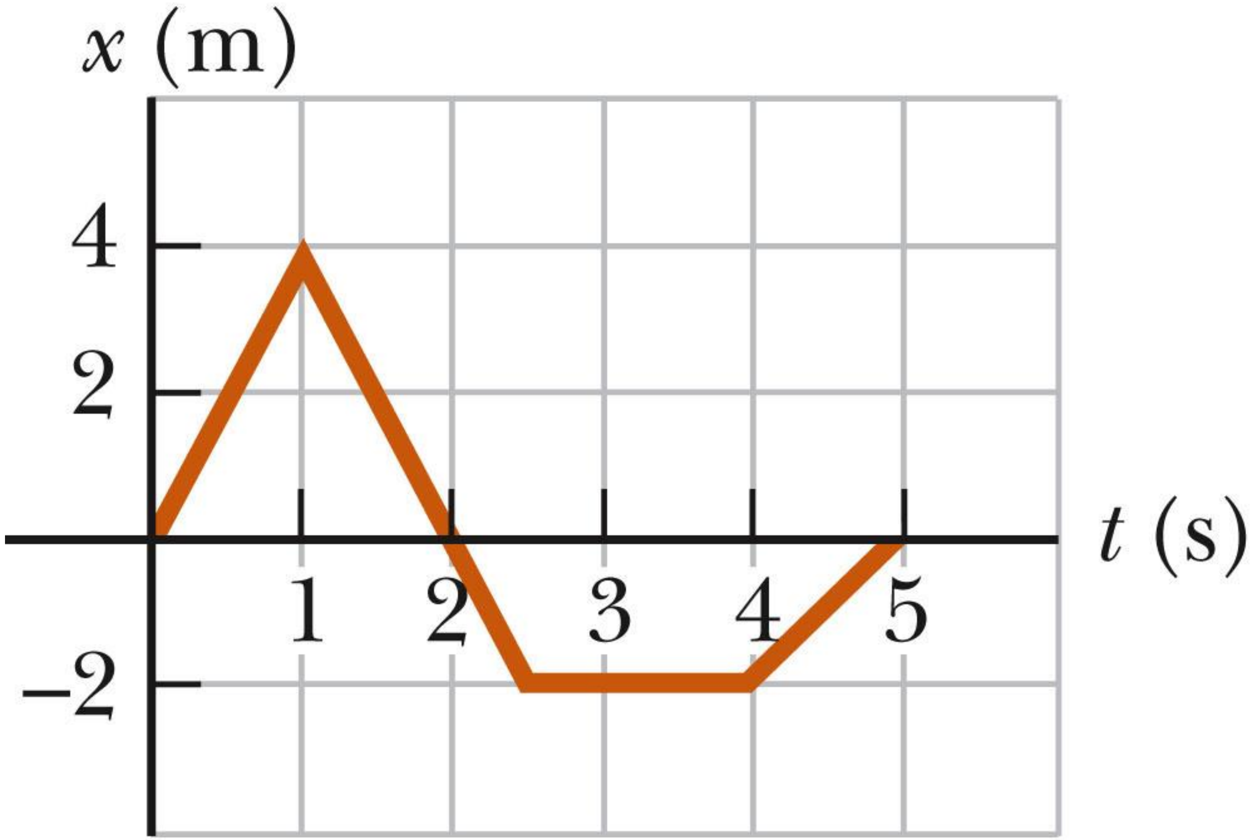
= (0 - 4.0 m)/4.0 s

= -1.0 m/s, speed = 8 m/4 s = 2 m/s

(d) $v = (x_5 - x_0)/\Delta t$

= (0 - 0)/5.0 s

= 0 m/s, speed = 12 m/5 s = 2.4 m/s



Problem 2

An athlete swims the length of a 50.0-meter pool in 20.0 seconds and makes the return trip to the starting position in 22.0 seconds. Determine her average velocities in

(a) the first half of the swim,

(b) the second half of the swim, and

(c) the round trip.

Problem 2. Solution

(a) In the first half of the trip,

velocity = $(x_2 - x_1)/20.0$ s

= +50.0 m/20.0 s

= +2.50 m/s

(b) On the return leg,

velocity = $(x_3 - x_2)/22.0$ s

= (0 - 50.0 m)/22.0 s

= -2.27 m/s

(c) For the entire trip,

velocity = $(x_3 - x_1)/42.0$ s

= 0/42.0 s

= 0 m/s

Problem 3

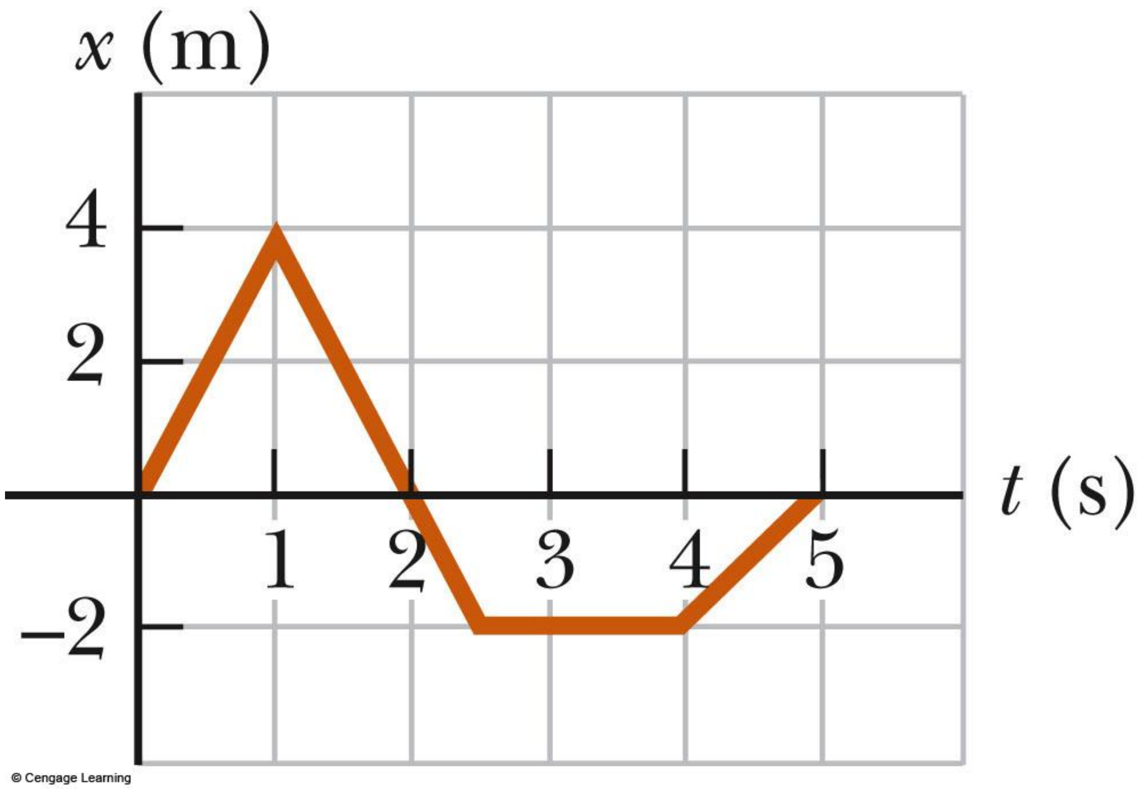
Find the instantaneous velocities of the tennis player in problem 1 at

(a) 0.50 seconds,

(b) 2.0 seconds,

(c) 3.0 seconds, and

(d) 4.5 seconds.



Problem 3. Solution

(a) $v(t = 0.50\text{ s}) = [x(t = 1\text{ s}) - x(t = 0)]\ /(1.00\text{ s})$

$= +4.00\text{m}/1.00\text{s}$

$= +4.00\text{ m/s}$

(b) $v(t = 2\text{ s}) = [x(t = 2.5\text{ s}) - x(t = 1\text{ s})]\ /(2.50\text{ s} - 1.00\text{ s})$

$= (-2.0\text{ m} - 4.0\text{ m})/1.5\text{ s}$

$= -6.0\text{ m}/1.5\text{ s}$

$= -4.0\text{ m/s}$

(c) $v(t=3\text{ s}) = [x(t = 4\text{ s}) - x(t = 2.5\text{ s})]/(4.0\text{s} - 2.5\text{ s})$

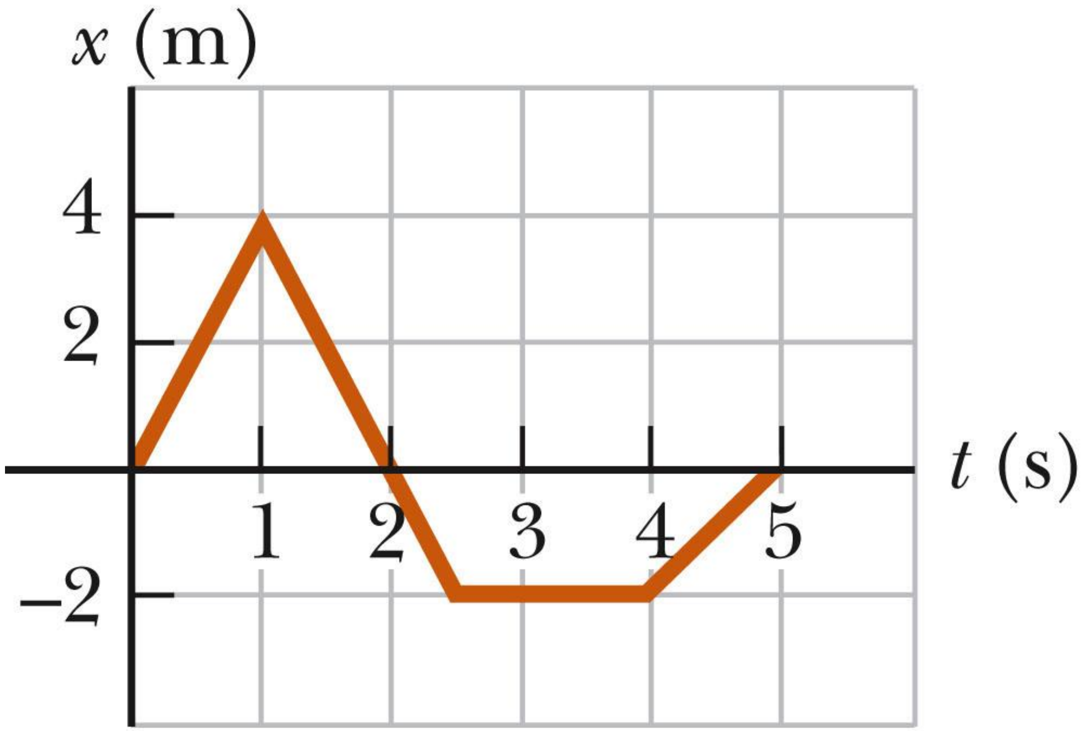
$= (-2.0\text{m} -(-2.0\text{m}))/1.5\text{ s}$

$= 0\text{ m/s}$

(d) $v(t = 4.5\text{ s}) = [x(t = 5\text{ s}) - x(t = 4\text{s})]/(5\text{ s} - 4\text{ s})$

$= (0 - (-2.0\text{ m}))/1.0\text{ s}$

$= +2.0\text{ m/s}$



Problem 4

If the average speed of an orbiting space shuttle is 19,800 mph, determine the time required for it to circle the Earth. Make sure you consider the fact that the shuttle is orbiting about 200 miles above the Earth's surface, and assume that the Earth's radius is 3963 miles.

Problem 4. Solution

The distance traveled by the space shuttle in one orbit equals

D = 2π(Earth's radius + 200 miles)

= 2π(3963 + 200)

= 26,156.9 miles.

So the required time is given by

Δt = D/Speed = 26,156.9 miles/19800 mph

= 1.32 hours.

Acceleration

- When velocity changes with time, the object undergoes ***acceleration***.
- If velocity does not change with time, acceleration is equal to zero and the motion is said to be uniform (constant).
- Average acceleration is a measure of change in velocity divided by the time interval over which the change occurs:

$$\boxed{\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}}$$

Acceleration, cont…

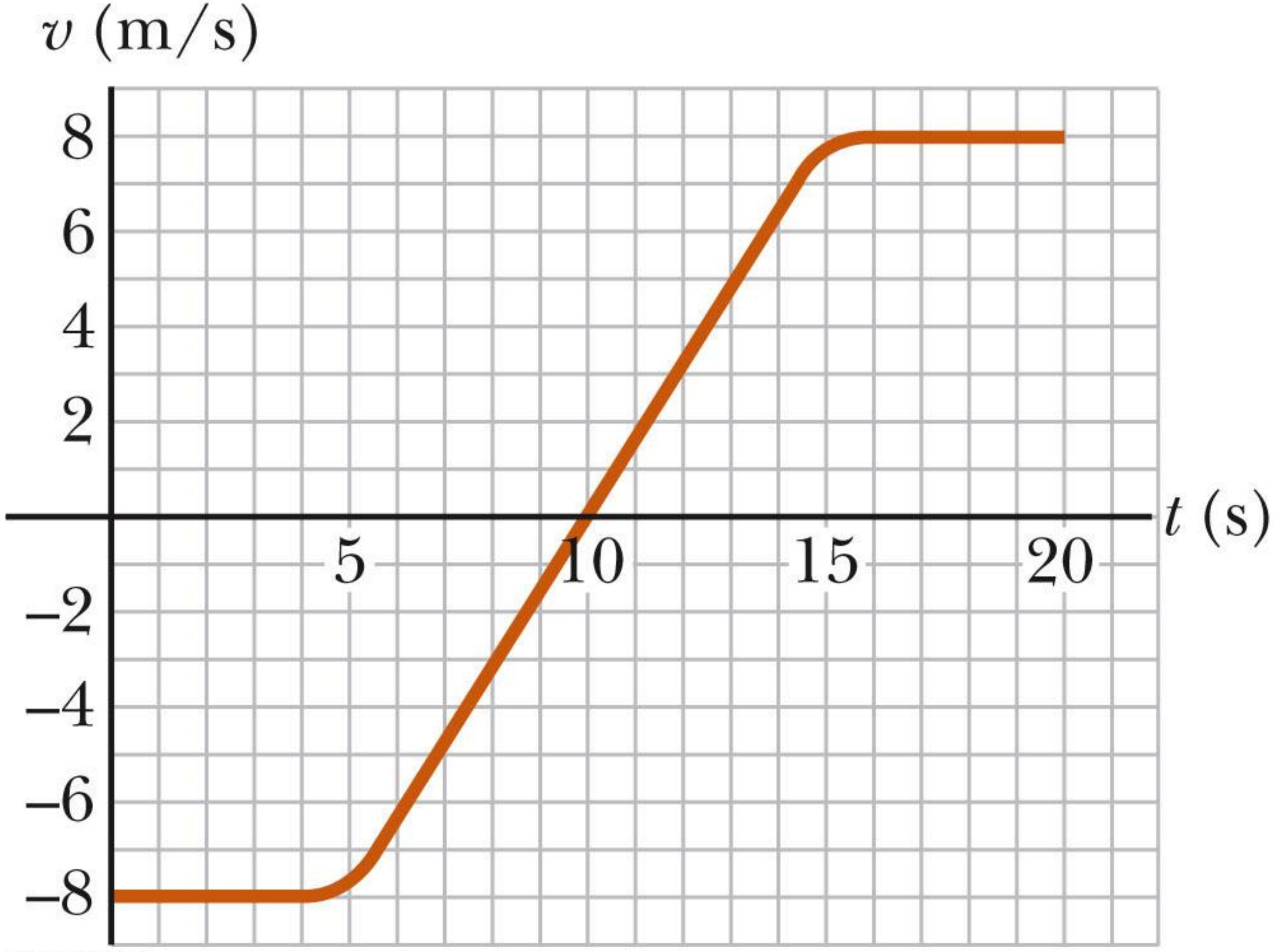
- On a graph of velocity versus time, acceleration equals the slope of the line.

$$\boxed{a = \frac{\Delta v}{\Delta t}}$$

- Acceleration can be positive or negative.
- ***Instantaneous acceleration*** is the acceleration at a given time

-Remark-

- **SI units for displacement, velocity, and acceleration:**
  - $\Delta x \rightarrow m$
  - $v \rightarrow m/s$
  - $a \rightarrow m/s^2$



v (m/s)
8
6
4
2
–2
–4
–6
–8
t (s)
5
10
15
20

Problem 5

A certain car is capable of accelerating at a rate of +0.60 m/s^2. How long does it take for this car to go from a speed of 55 mph to a speed of 60 mph?

Problem 5. Solution

Begin by converting initial and final velocities to SI units:

v_i = 55 mph = 24.58 m/s

v_f = 60 mph = 26.82 m/s.

Thus, Δv = 2.24 m/s, and

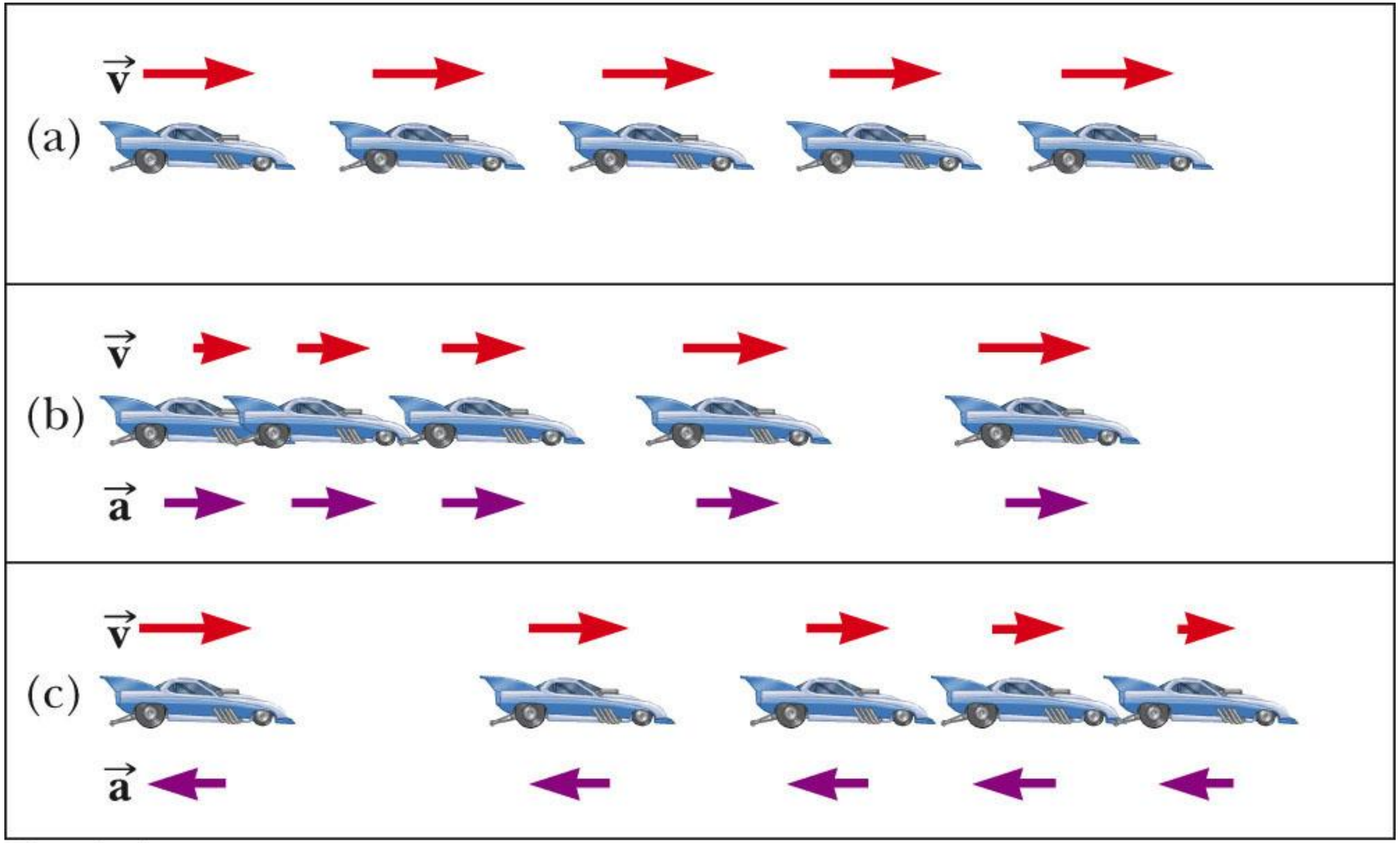
$\Delta t = \Delta v/a$

= (2.24 m/s)/(0.6 m/s2)

= 3.73 s.

Motion Diagrams

- If $a = 0$, velocity is constant (uniform motion).
- An object is speeding up whenever its velocity and acceleration have the same sign.
- If the velocity and acceleration have different signs, the object is slowing down.
- Four scenarios are possible:
  - $v > 0$ and $a > 0$
    $\rightarrow$ object is moving to the right, speeding up
  - $v > 0$ and $a < 0$
    $\rightarrow$ object is moving to the right, slowing down
  - $v < 0$ and $a > 0$
    $\rightarrow$ object is moving to the left, slowing down
  - $v < 0$ and $a < 0$
    $\rightarrow$ object is moving to the left, speeding up



$\vec{v}$
(a)
(b)
$\vec{a}$
(c)

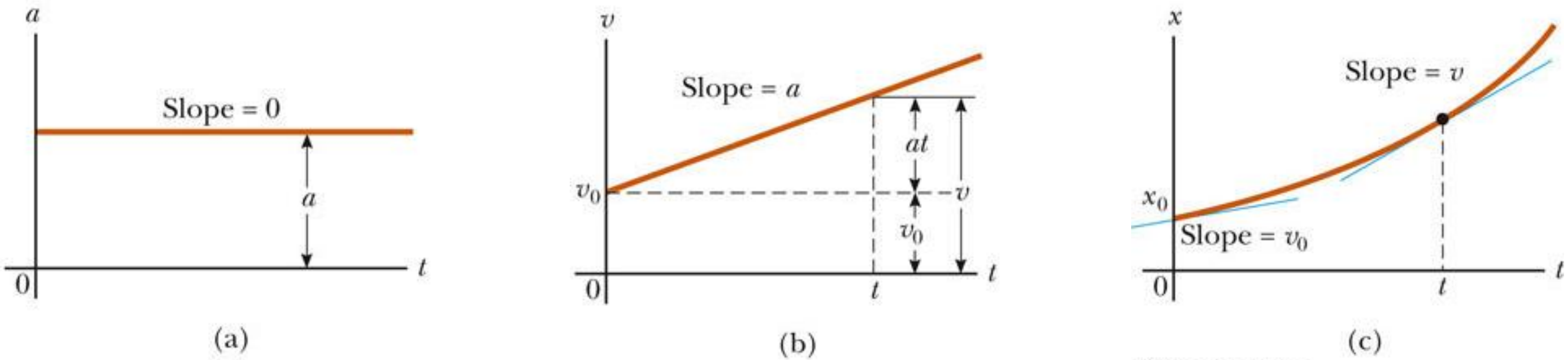
One-Dimensional Motion with Constant Acceleration

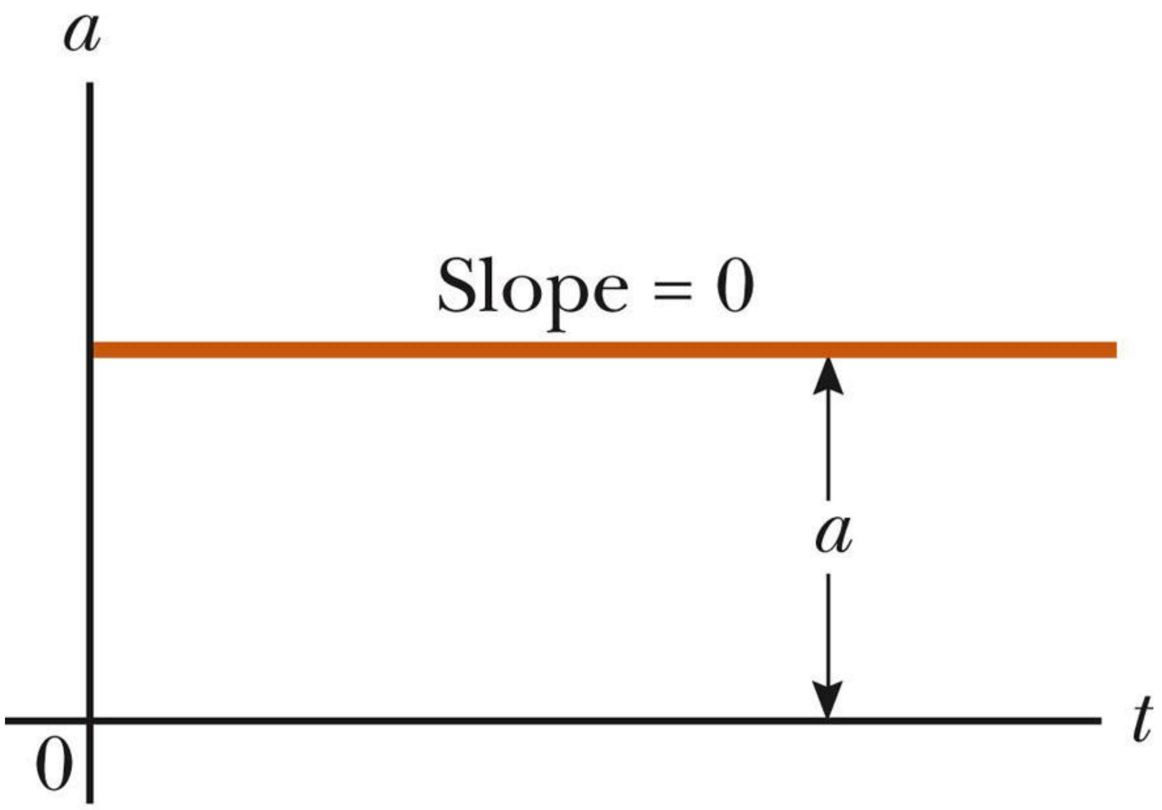
- When *a* is constant (not equal to zero), four relations can be derived between position, velocity, acceleration, and time.
- Originally derived by Galileo, these are the equations of kinematics.
- Any kinematical problem can be solved using one or more of these.

TABLE 2.3 **Equations for Motion in a Straight Line Under Constant Acceleration**

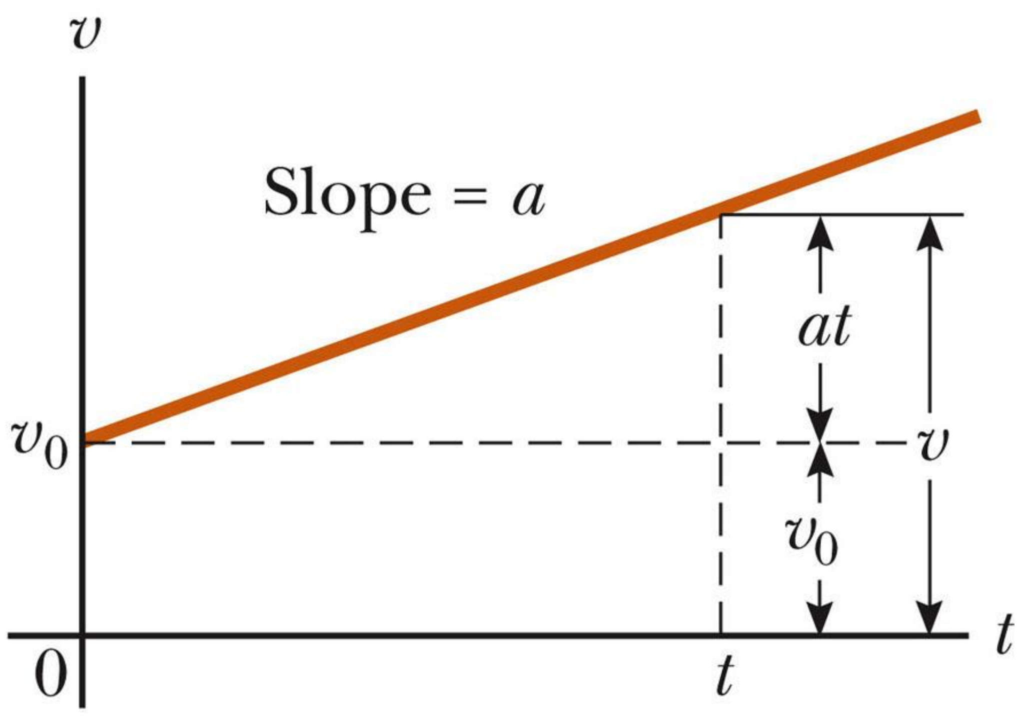
Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$\Delta x = \frac{1}{2}(v_0 + v)t$	Displacement as a function of velocity and time
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0{}^2 + 2a\,\Delta x$	Velocity as a function of displacement

Note: Motion is along the x axis. At $t = 0$, the velocity of the particle is v_0.

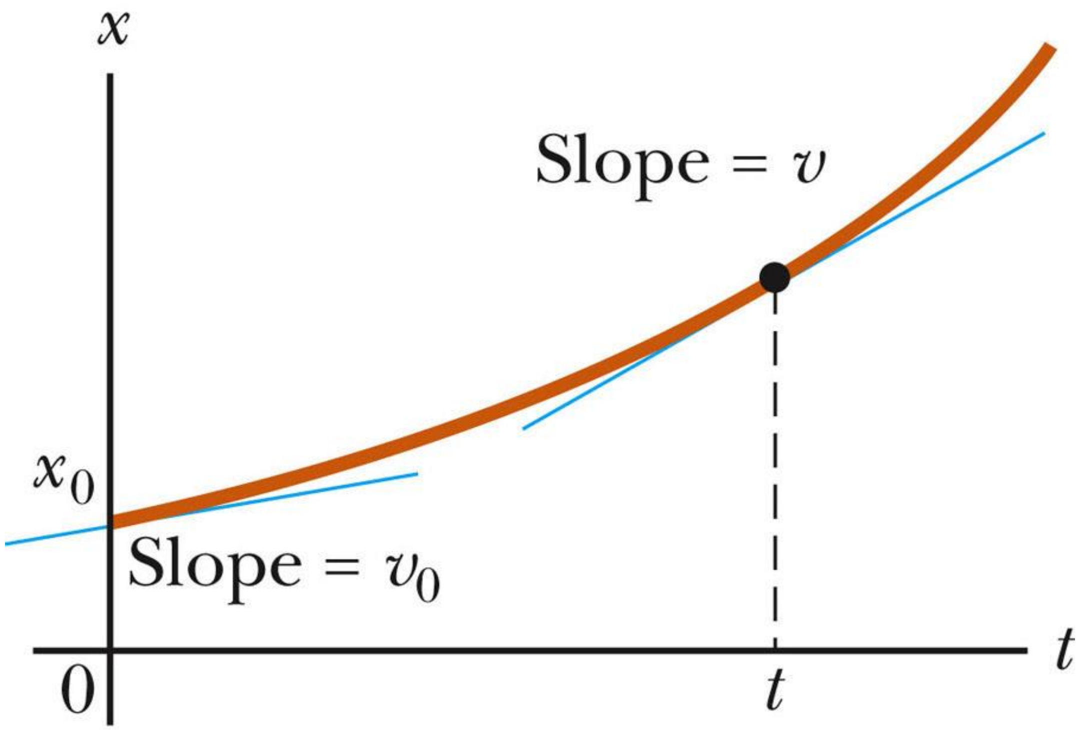

a
Slope = 0
a
t
0
(a)
v
Slope = a
at
v_0
v
v_0
t
0
t
(b)
x
Slope = v
x_0
Slope = v_0
t
0
t
(c)



(a)



(b)



(c)

Derivation of kinematics equations

- ***First Equation:***
- *Take ti=0, tf=t, vi=* V_0 *, vf=f*
- $a = (V - V_0)/t$
- we have
- $\boldsymbol{V = V_0 + at}$ **,** **(1)**

Third Equation

- *Inserting* $V = V_0 + at$ *into Eq(2), we can write the displacement as*
- $x = (V+V_0)t/2 = [(V_0 + at) + V_0]\,t/2$
- ***Or***
- $$x = V_0 t + at^2/2 \qquad (3)$$

Fourth Equation

- Now, since $t = (V\text{-}V0)/a$

- Eq. (2) leads to
- $\boldsymbol{x = (V+V_0)t/2 = (V+V_0)(V-V_0)/2a = (V^2-V_0^2)/2a}$

- which in turn leads to
- $\boldsymbol{V^2-V_0^2 = 2ax}$ (4)

Kinematics Equations

- *If acceleration:* $a = constant$, *then*

- $V = V_0 + at,$ (1)
- $x = (V+V_0)t/2,$ (2)
- $x = V_0 t + at^2/2,$ (3)
- $V^2 - V_0^2 = 2ax,$ (4)

Problem 6

A Cessna aircraft has a lift-off speed of 120 km/h.

(a) What minimum constant acceleration does this require if the aircraft is to be airborne after a takeoff run of 240 meters?

(b) How long does it take the aircraft to become airborne?

Problem 6. Solution

(a) With 120 km/h = 33.33 m/s,

$$v^2 = v_0^2 + 2ax$$

$(33.33\ \text{m/s})^2 = 0 + 2a(240\ \text{m})$.

From this, $a = 2.32\ \text{m/s}^2$.

(b) Using $v = v_0 + at$,

$33.33\ \text{m/s} = 0 + (2.32\ \text{m/s}^2)t$,

or $t = 14.4$ s.

Problem 7

A jet plane lands with a velocity of +100 m/s and can accelerate at a maximum rate of -5.0 m/s^2 as it comes to rest.

(a) From the instant it touches the runway, what is the minimum time needed before it can come to rest?

(b) Can this plane land on a small island airport where the runway is 0.80 km long?

Problem 7. Solution

(a) $t = (v - v_0)/a$

$= (0 - 100\ \text{m/s})/{-5\ \text{m/s}^2}$

$= 20\ \text{s}$

(b) $x = vt$

$= 50\ \text{m/s} * 20\ \text{s}$

$= 1000\ \text{m}$

Note that the minimum distance to stop is 1 km, which exceeds the length of the runway. So, the plane cannot land safely.

Freely Falling Objects

- When an object is moving under the influence of gravity alone—regardless of its initial conditions—it is experiencing free fall motion.
- An object thrown upward or downward will experience the same acceleration as an object released from rest.
- Once in free fall, all objects experience a downward acceleration equal to the acceleration due to gravity:

$$a = -g_e = -9.8\ m/s^2 = -32\ ft/s^2.$$

- The equations of kinematics can be rewritten to describe free fall motion by replacing –*g* for *a* and *y* for *x*.

Freely Falling Objects

- Consider two identical objects: Object 1 is released from rest from a certain height ***h***, while Object 2 is thrown downwards from the same height with an initial speed v_0. Which object has a higher acceleration? 1 or 2?
- Now, Object 2 is thrown upwards with an initial speed *v0.* Which object has a higher acceleration? 1 or 2?

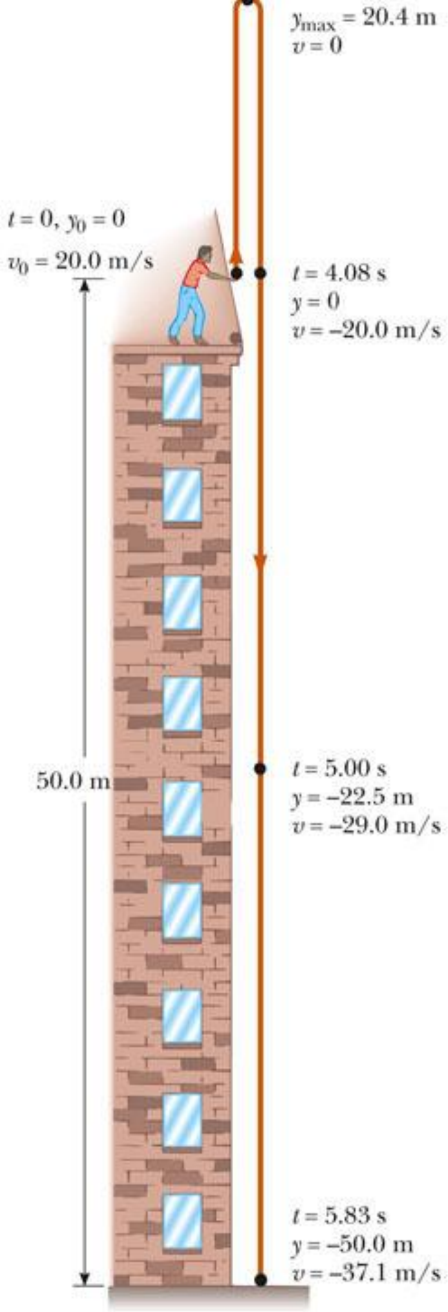
Free Fall Equations

- When air resistance is neglected, we have
- $V = v_0 - gt$ (1)
- $y = (V+v_0)t/2$ (2)
- $y = v_0 t - gt^2/2$ (3)
- $v^2 - v_0^2 = -2gy$ (4)

- On earth: $g = 9.8\ m/s2$
- Note: v_0 can be (+) or (-) depending on the motion being upward or downward.

-Remarks-

- When an object is free-falling, its speed increases rapidly with every passing second.
- Acceleration due to gravity varies on different stellar bodies:
  - g(moon) = 1.6 m/s^2
  - g(sun) = 270 m/s^2
  - g(black hole) = ∞
- Neglecting air resistance, all objects fall at the same rate. (i.e. a feather and a hammer will have the same speed when falling in a vacuum).
- Free fall motion is symmetrical for an object thrown up that then falls back to Earth.


$y_{max} = 20.4$ m
$v = 0$
$t = 0$, $y_0 = 0$
$v_0 = 20.0$ m/s
$t = 4.08$ s
$y = 0$
$v = -20.0$ m/s
50.0 m
$t = 5.00$ s
$y = -22.5$ m
$v = -29.0$ m/s
$t = 5.83$ s
$y = -50.0$ m
$v = -37.1$ m/s

Problem 7

A ball thrown vertically upward is caught by the thrower after 2.00 seconds. Find

(a) the initial velocity of the ball and

(b) the maximum height it reaches.

Problem 7. Solution

(a) Using a = -g and placing the origin at the thrower yields $\Delta y = v_0 t - gt^2/2$.

When $\Delta y = 0$, we get $[v_0 - gt/2]t = 0$, which has two solutions, $t = 0$, and $t = 2v_0/g$. The $t = 0$ solution corresponds to when the ball is thrown, while the $t = 2v_0/g$ solution corresponds to when the ball is caught. Therefore, the initial velocity must be $v_0 = gt/2 = (9.80)(2.00)/2 =$ 9.80 m/s.

(b) Using $v^2 = v_0^2 - 2gy$, we have, at the maximum height, $v = 0$, so that

$y(\max) = v_0^2/2g = (9.80)^2/19.6 =$ 4.90 m.

Problem 8

A parachutist with a camera, both descending at a speed of 10 m/s, releases that camera at an altitude of 50 meters.

(a) What is the velocity of the camera just before it hits the ground?

(b) How long does it take the camera to reach the ground?

Problem 8. Solution

(a) Choose the origin of the coordinate system at the location of the parachutist. We use $v^2 = v_0^2 + 2ay$, which gives

$v^2 = (-10\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-50\ \text{m})$, from which $v = -32.9$ m/s.

(b) The time to reach the ground is found from $v = v_0 + at$. This yields $t = 2.33$ s.