Assignment 2 - Solutions
Temporal Logic

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1. (a) $\Box\Diamond\Box a$ – legal LTL
   (b) $\exists(a\,\mathtt{U}\,\forall\bigcirc b)$ – legal CTL
   (c) $a \wedge b\Box(a \rightarrow \bigcirc b)$ – not legal in either logic
   (d) $\neg((\Box\Box a)\,\mathtt{U}\bigcirc(a \vee b))$ – legal LTL
   (e) false $\wedge$ true – legal in both LTL *and* CTL
   (f) $\forall\Box\exists a$ – not legal in either logic (although it is legal CTL*)

2. (a) $a \wedge \Diamond b \wedge \Diamond c$
   This is an LTL formula. It *is* satisfied by the LTS.
   (Every possible path from the initial state s_0 has a true in the first state, b true in the second, and eventually reaches a state labelled with c.)
   (b) $\forall\bigcirc(a \vee c) \wedge \exists\Diamond c$
   This is a CTL formula. It is satisfied in states s_3, s_4 and s_6 of the LTS.
   (The left-hand side is satisfied in s_3, s_4 and s_6, and the right-hand side is satisfied in all states.)
   (c) $\exists\Diamond(\exists\bigcirc(a \wedge b) \wedge \forall\bigcirc\neg c)$
   This is a CTL formula. It is satisfied only in state s_5 of the LTS.
   (The two $\exists\bigcirc$ formulae are satisfied in states $\{s_4, s_5, s_6\}$ and states $\{s_0, s_1, s_2, s_5, s_7\}$, respectively, so their conjunction is true in s_5 and this state can only be reached from itself.)
   (d) $\Box\Diamond((a \wedge \neg b) \vee \neg c)$
   This is an LTL formula. It *is* satisfied by the LTS.
   (Every path eventually ends up visiting state s_6 infinitely often, which satisfies $\neg c$ and therefore $(a \wedge \neg b) \vee \neg c$.)
   (e) $\Box\Diamond\bigcirc\bigcirc(a \wedge b)$
   This is an LTL formula. It is *not* satisfied by the LTS.
   (Some path (in fact multiple paths) eventually ends up looping between states s_2 and s_6 forever, which never visits state s_7 in two state's time.)

3. (a) "servers 1 and 2 are never both down simultaneously"
$\forall\Box\neg(down_1 \wedge down_2)$, where $down_i$ denotes server i being down
(equivalently, you could write $\neg\exists\Diamond(down_1 \wedge down_2)$)

   (b) "it is always the case that a is true now or in one step's time"
$\Box(a \vee \bigcirc a)$

   (c) "zone A is visited infinitely often but zones B and C are visited only finitely often"
$\Box\Diamond z_A \wedge \Diamond\Box\neg(z_B \vee z_C)$, where proposition names z_X indicate being in zone X.
(equivalently, you could write $\Box\Diamond z_A \wedge \Diamond\Box\neg z_B \wedge \Diamond\Box\neg z_C$ or $\Box\Diamond z_A \wedge \neg\Box\Diamond z_B \wedge \neg\Box\Diamond z_C$)

   (d) "the robot eventually reaches room 1 and then goes immediately to room 2, all before the alarm goes off"
$\neg alarm\, \mathtt{U}\, (\neg alarm \wedge room_1 \wedge \bigcirc\, room_2)$ where $room_X$ indicates being in room X and $alarm$ means that the alarm is going off.
(note this assumes it is ok for the alarm to go off as soon as room 2 is reached; this could be interpreted differently)

4. (a) This *is* an equivalence. We have:

$$\begin{array}{rcll}
\neg(\Box\Diamond a \rightarrow \Diamond\Box b) & \equiv & \neg(\neg\Box\Diamond a \vee \Diamond\Box b) & \text{since } \psi_1 \rightarrow \psi_2 \equiv \neg\psi_1 \vee \psi_2 \\
& \equiv & \neg\neg\Box\Diamond a \wedge \neg\Diamond\Box b & \text{since } \neg(\psi_1 \vee \psi_2) \equiv \neg\psi_1 \wedge \neg\psi_2 \\
& \equiv & \Box\Diamond a \wedge \neg\Diamond\Box b & \text{since } \psi \equiv \neg\neg\psi \\
& \equiv & \Box\Diamond a \wedge \neg\Diamond\neg\neg\Box b & \text{since } \psi \equiv \neg\neg\psi \\
& \equiv & \Box\Diamond a \wedge \Box\neg\Box b & \text{since } \neg\Diamond\neg\psi \equiv \Box\psi \\
& \equiv & \Box\Diamond a \wedge \Box\neg\Box\neg\neg b & \text{since } \psi \equiv \neg\neg\psi \\
& \equiv & \Box\Diamond a \wedge \Box\Diamond\neg b & \text{since } \neg\Box\neg\psi \equiv \Diamond\psi
\end{array}$$

   (b) This is *not* an equivalence, i.e., $\Box a \rightarrow \neg\Box(b \wedge c) \;\not\equiv\; \Diamond\neg a \vee \Diamond(\neg b \vee c)$.
At a glance, it looks like it might be, so we can try to rewrite the left-hand side using similar equivalences to those used above:

$$\begin{array}{rcl}
\Box a \rightarrow \neg\Box(b \wedge c) & \equiv & \neg\Box a \vee \neg\Box(b \wedge c) \\
& \equiv & \neg\Box\neg\neg a \vee \neg\Box\neg\neg(b \wedge c) \\
& \equiv & \Diamond\neg a \vee \Diamond\neg(b \wedge c) \\
& \equiv & \Diamond\neg a \vee \Diamond(\neg b \vee \neg c)
\end{array}$$

This differs from the right-hand side proposed in the question because the c is negated. In other words, it is instead equivalent to $\Box a \rightarrow \neg\Box(b \wedge \neg c)$.

So we can find a trace satisfying the right-hand side but not the left, e.g.: $\{a, b, c\}^\omega$.