Assume encryption and decryption use the same key.
Will discuss how to distribute key to all parties later
Symmetric ciphers unusable for authentication of sender

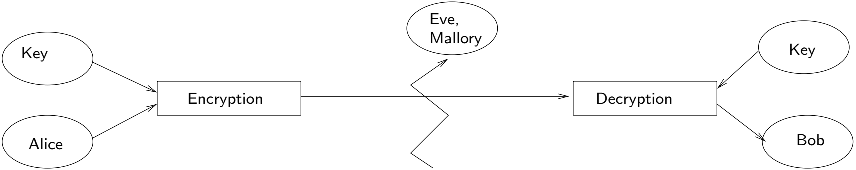
Kinds of symmetric ciphers:

- Block cipher: Symmetric cipher operating on fixed-length groups of bits, called blocks
- Stream cipher Symmetric cipher encrypting plaintext continuously. Bits are encrypted one at a time, differently for each bit.

Players

Have the following main players:

- Alice: sender of an encrypted message
- Bob: intended receiver of encrypted message. Assumed to the key.
- Eve: (Passive) attacker intercepting messages and trying to identify plaintexts or keys
- Mallory: (Active) attacker intercepting and modifying messages to identify plaintexts or keys



Feistel cipher: a way of doing block ciphers

Invented in 1971 at IBM
Important class of ciphers (eg Blowfish, DES, 3DES)
Same encryption scheme applied iteratively for several rounds
Important step: Derive next message state from previous message state via special function called *Feistel function*
Encryption is organised as a series of "rounds".
Each round works as follows:

- Split input in half
- Apply Feistel function to the right half
- Compute xor of result with old left half to be new left half
- Swap old right and new left half, unless we are in the last round

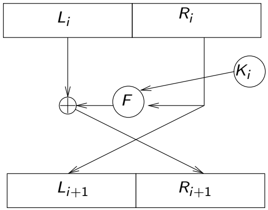
Feistel Cipher, continued

Formal definition:

- Split plaintext block in two equal pieces $M = (L_0, R_0)$
- For each round $i = 0, 1, \ldots, r-1$ compute

$$\begin{aligned} L_{i+1} &= R_i \\ R_{i+1} &= L_i \oplus F(K_i, R_i) \end{aligned}$$

- The ciphertext is $C = (R_r, L_r)$



Decryption

Works as encryption, but with a reversed order of keys

- Split ciphertext block in two equal pieces $C = (R_r, L_r)$
- For each round $i = r, r-1, \ldots, 1$ compute

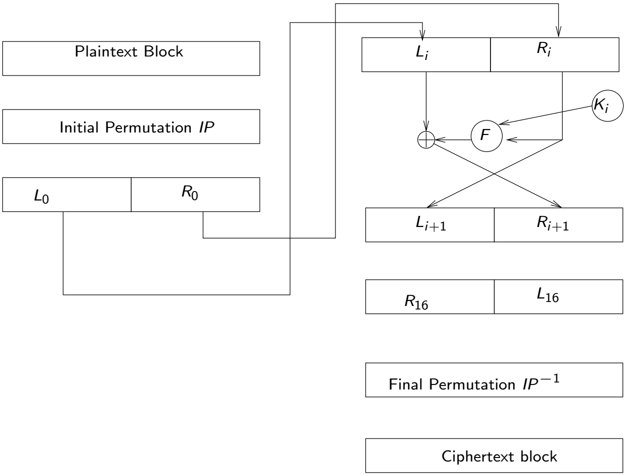
$$\begin{aligned} R_{i-1} &= L_i \\ L_{i-1} &= R_i \oplus F(K_{i-1}, L_i) \end{aligned}$$

- Plaintext is $M = (L_0, R_0)$

DES

Data Encryption Standard (DES) adopted in 1976
Key size (56 bits) is too small for today's computers (can be broken within 10 hours)
Variants still provide good security

Overview of DES



Design parameters

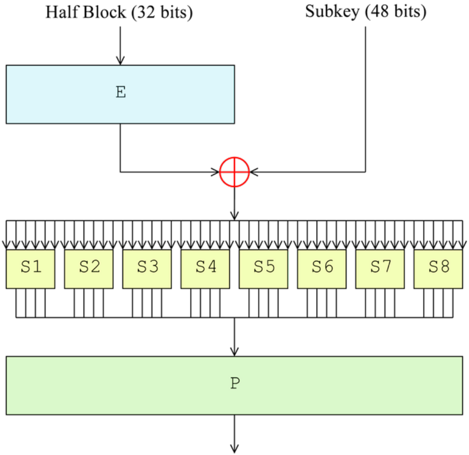
- Block length is 64 bits
- Number of rounds R is 16
- Key length is is 56 bits
- Round key length is 48 bit for each subkey $K_0, \ldots, K_{15}$.
  Subkeys are derived from 56 bit key via special key schedule.

DES Feistel function

Four stage procedure:

- Expansion permutation: Expand 32-bit message half block to 48 bit block by doubling 16 bits and permuting them
- Round key addition: Compute xor of this 48 bit block with round key K_i
- S-Box: Split 48 bit into eight 6-bit blocks. Each of them is given as input to eight substitution boxes, which substitute 6-bit block by 4-bit block.
- P-Box: Combine these eight 4-bit blocks to 32-bit block and apply another permutation.

DES Feistel function, continued



Source: Wikipedia

Notation for DES operations

Have three special operations:

- Cyclic shifts on bitstring blocks: Will denote by $b <<< n$ the move of the bits of block b by n to the left. Bits that would have fallen out are added at the right side of the b. $b >>> n$ is defined similarly
- Permutations on the position of bits: Written down as output order of the input bits.

  Example: the permutation $\boxed{4\,1\,2\,3}$ means that
  - the fourth input bit becomes the first output bit,
  - the first input bit becomes the second output bit,
  - the second input bit becomes the third output bit, and
  - the third input bit becomes the fourth output bit.

  Sometimes, we use the word "permutation" for bit re-arrangements that include duplication or dropping of bits, even though that is not a proper permutation.

S-boxes

- S-boxes: An S-box substitution is a table lookup. Input is 6 bit, output is 4 bit. Works as follows:
  - Strip out outer bits of input and join them. This two-bit number is the row index.
  - Four inner bits indicate column number.
  - Output is corresponding entry in table

Key schedule

Have different keys for each round, computed by so-called *Key schedule*

64-bit key is actually 56-bit key plus 8 parity bits

- First apply a permutation PC-1 which removes the parity bits. This results in 56 bits.
- Split result into half to obtain (C_0, D_0)
- For each round we compute

$$\begin{aligned} C_i &= C_{i-1} <<< p_i \\ D_i &= D_{i-1} <<< p_i \end{aligned}$$

  where

$$p_i = \begin{cases} 1 & \text{if } i = 1, 2, 9, 16 \\ 2 & \text{otherwise} \end{cases}$$

- Now we join C_i and D_i together, and apply a permutation PC-2 which produces a 48-bit output.

Security of block ciphers

To define the security of block ciphers, we look at a more abstract notion: pseudorandom permutations.

Definition

Let $X = \{0,1\}^n$. A *pseudorandom permutation* over (K, X) is a function

$$E \colon K \times X \to X$$

such that

- there exists an efficient deterministic algorithm to compute $E(k, x)$ for any k and x;
- The function $E(k, \_)$ is one-to-one for each k
- There exists a function $D \colon K \times X \to X$ which is efficiently computable, and $D(k, E(k, x)) = x$ for all k and x.

Security of pseudorandom permutations

A pseudorandom permutation is secure if an adversary (who can call it) can't distinguish it from a "genuine" random permutation. Suppose X and K have size N, i.e., $X = K = \{0,1\}^n$.

- There are $N! = 2^n!$ permutations $X \to X$.
- There are $|K| = 2^n$ pseudorandom permutations.

For example, suppose n=64. Then these numbers are, very roughly, $(10^{19})^{10^19}$ and 10^{19}.
So there are much fewer pseudorandom permutations there are permutations in total.

Definition

Let $X = \{0, 1\}^n$, and $\mathcal{F}$ be the set of all permutations on X, and E a pseudorandom permutation over (K, X). Define the following game between the attacker and the challenger:

- The challenger chooses a random bit $b \in \{0, 1\}$.
- If $b = 0$, the challenger chooses a $k \in K$ at random, and if $b = 1$, the challenger chooses a permutation f on X at random.
- The attacker does arbitrary computations.
- The attacker has access to a black box, which is a function from X to X operated by the challenger. He can ask the challenger for the values $g(x_1), \ldots, g(x_n)$ during his computation.
- If $b = 0$, the challenger answers the query $g(x_i)$ by returning $E(k, x_i)$, and if $b = 1$, the answer is $f(x_i)$.
- Eventually the attacker outputs a bit $b' \in \{0, 1\}$.

The attacker wins this game if $b = b'$.

The attacker's power in security games

In security games, attacker can only do efficient operations, and only "efficiently" many of them

Formally: attacker is *probabilistic polynomial-time Turing machine* (PPT)

Importantly: attacker cannot search through all keys, as the number of possible keys increases exponentially with the length of the key

Definition

A function $\epsilon : \mathbb{N} \rightarrow \mathbb{R}^+$ is called *negligible* if for all d there exists a x_d such that for all $x \geq x_d$,

$$\epsilon(x) \leq \frac{1}{x^d}$$

Definition

A pseudorandom permutation $E\colon K \times X \rightarrow X$ is *secure* if for all PPT attackers A,

$$\left| Pr[b = b'] - \frac{1}{2} \right|$$

is negligible in the size of K.

Note that $\left| Pr[b = b'] - \frac{1}{2} \right|$ is a function of the size of K.

Example

1. Let $X = \{0,1\}^n$ and $K = \{1,\ldots,n\}$.

   Let $E(k,x)$ be computed as follows:
   Apply the Rail Fence cipher bitwise to x with key k.

   Is that a secure pseudorandom permutation?

Example

2. Let $X = \{A, B, \ldots Z\}^n$ and $K = \{$the set of permutations on $\{A, B, \ldots, Z\}\}$.

   Let $E(k, x)$ be computed as follows: apply the permutation k to each of the characters x in turn.

   Is that a secure pseudorandom permutation?

What we know about DES

DES a good design, but as it only has 56 bit keys, it has only approximately 2^{56} security. (There are some cryptanalytic attacks on DES, but not very serious ones, so let's say its security is about 2^{56}.)

How about using DES twice? Take a 112-bit key, split it into two keys K_1 and K_2 and encrypt M like this:

$$\mathsf{Enc}_{K_1}(\mathsf{Enc}_{K_2}(M))$$

Would that give us 2^{112} security?

"2DES" is not significantly more secure than DES

Suppose we have a pair (M, C) consisting of a valid plaintext-ciphertext pair. With approximately 2^{57} work, we can find the 112-bit key $K_1 K_2$ used in 2DES. Here is how to do it.

- Try all 2^{56} possible keys K_2, and store all the results $\mathrm{Enc}_{K_2}(M)$. Sort them in order. This is 2^{56} work for the encryption, and $2^{56} \log(2^{56})$ for the sorting.
- Try all the 2^{56} possible keys K_1, computing $\mathrm{Dec}_{K_1}(C)$. For each such value, check if it is one of the stored $\mathrm{Enc}_{K_2}(M)$. That is 2^{56} work for the Dec, and $\log(2^{56})$ work for the checking.

The total work is not much more than 2^{57}.

3DES is good, but slow

3DES takes the same idea, but uses DES three times. That gives us a 168-bit key. Take the 168-bit key, split it into three keys K_1, K_2 and K_3, and encrypt M like this:

$$\mathrm{Enc}_{K_1}(\mathrm{Dec}_{K_2}(\mathrm{Enc}_{K_3}(M)))$$

- Why Enc-Dec-Enc instead of Enc-Enc-Enc?
  Enc-Dec-Enc gives us an option of setting $K_1 = K_2 = K_3$, which is then equivalent to DES. So if you have 3DES, you can make it do DES. This could be useful in some circumstances.
- How much security does 3DES give us? It doesn't give us 2^{168} of security, because the same meet-in-the-middle attack as we had for "2DES" is possible. It is said to give us 2^{118} of security.

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