Assignment 3 - Solutions
Automata & Model Checking

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1. First, we convert the formula $\phi = \forall\Diamond(a \vee \forall\bigcirc c)$ to ENF: $\phi' = \neg\exists\Box\neg(a \vee \neg\exists\bigcirc \neg c)$.

   We compute $Sat(\phi')$ recursively.

   First, we have:

   - $Sat(c) = \{s_2, s_4\}$
   - $Sat(\neg c) = S\backslash\{s_2, s_4\} = \{s_0, s_1, s_3, s_5, s_6, s_7, s_8\}$

   and looking at the predecessors of these, we have:

   - $Sat(\exists\bigcirc \neg c) = \{s_0, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$
   - $Sat(\neg\exists\bigcirc \neg c) = S\backslash\{s_0, s_2, s_3, s_4, s_5, s_6, s_7, s_8\} = \{s_1\}$

   Then

   - $Sat(a) = \{s_6\}$
   - $Sat(a \vee \neg\exists\bigcirc \neg c) = \{s_1, s_6\}$
   - $Sat(\neg(a \vee \neg\exists\bigcirc \neg c)) = S\backslash\{s_1, s_6\} = \{s_0, s_2, s_3, s_4, s_5, s_7, s_8\}$

   For $Sat(\exists\Box\neg(a \vee \neg\exists\bigcirc \neg c))$, the algorithm for $\exists\Box$ gives the sequence of sets:

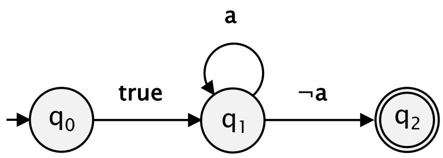
   - $\{s_0, s_2, s_3, s_4, s_5, s_7, s_8\}$
   - $\{s_0, s_4, s_5, s_7, s_8\}$
   - $\{s_4, s_5, s_7, s_8\}$
   - $\{s_4, s_5, s_7, s_8\}$

   which yields $Sat(\exists\Box\neg(a \vee \neg\exists\bigcirc \neg c)) = \{s_4, s_5, s_7, s_8\}$.

   Finally, $Sat(\neg\exists\Box\neg(a \vee \neg\exists\bigcirc \neg c)) = S\backslash\{s_4, s_5, s_7, s_8\} = \{\mathbf{s_0}, \mathbf{s_1}, \mathbf{s_2}, \mathbf{s_3}, \mathbf{s_6}\}$.

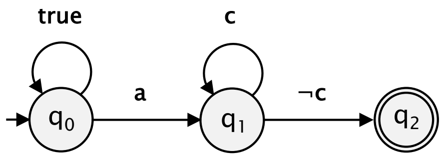
   So the LTS *does* satisfy the CTL formula ϕ.

2. (i) An appropriate NFA for $\square \bigcirc a$ (which is equivalent to $\bigcirc \square a$) is:



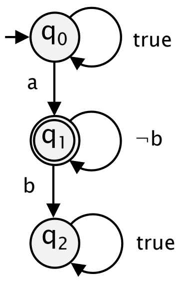
(The a-self-loop on q_1 could equally be labelled with true.)

(ii) An appropriate NFA for $\square(a \rightarrow \bigcirc \square c)$ is:

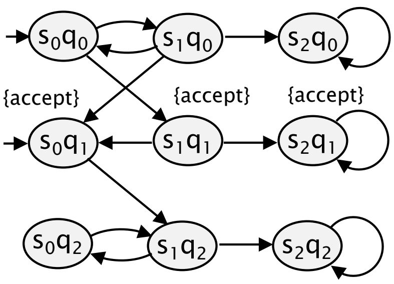


Note: In this case, the left self-loop could have been labelled with $\neg a$. This is because $\square(a \rightarrow \bigcirc \square c)$ is equivalent to saying that the *first* instance of a must trigger $\square c$. This is not true for the formula $\square(a \rightarrow \bigcirc c)$, as mentioned in lectures.

3. (i) The NBA is not non-blocking, so first we make it so:



The LTS-NBA $\mathcal{M} \otimes \mathcal{A}_{\neg\psi}$ product is:



There is a path to an accepting cycle $(s_0, q_0), (s_1, q_1), (s_2, q_1)^\omega$ so the formula ψ is *not* satisfied in the original LTS $\mathcal{M}$.

(ii) A suitable LTL formula ψ is $\square(a \rightarrow \bigcirc \Diamond b)$.

We can see that this is not satisfied because there is a path (that can be extracted from the product above) $s_0 s_1 s_2^\omega$ which does not satisfy ψ. For the first occurrence of a (in s_0), there is a subsequent occurrence of b; however, for the second occurrence of a (in s_1), there is none. The fact that b is already true in s_1 does not help because of the "next" in ψ.