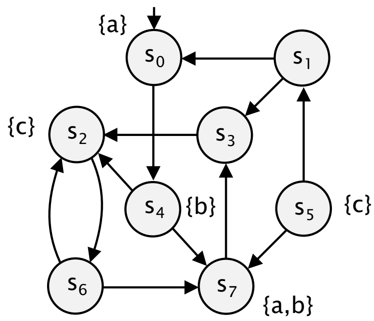
Assignment 2
Temporal Logic

---

1. For which of the two logics LTL and CTL (if any) are these legal (i.e., syntactically correct) formulae? You can assume that a, b and c are atomic propositions.

   (a) $\Box\Diamond\Box a$
   (b) $\exists(a \mathbin{\mathtt{U}} \forall\bigcirc b)$
   (c) $a \wedge b\Box(a \rightarrow \bigcirc b)$
   (d) $\neg((\Box\Box a) \mathbin{\mathtt{U}} \bigcirc (a \vee b))$
   (e) false $\wedge$ true
   (f) $\forall\Box\exists a$ **[6 marks]**

2. Consider the following LTS:



   and the temporal logic formulae below:

   (a) $a \wedge \Diamond b \wedge \Diamond c$
   (b) $\forall\bigcirc (a \vee c) \wedge \exists\Diamond c$
   (c) $\exists\Diamond(\exists\bigcirc (a \wedge b) \wedge \forall\bigcirc \neg c)$
   (d) $\Box\Diamond((a \wedge \neg b) \vee \neg c)$
   (e) $\Box\Diamond\bigcirc \bigcirc (a \wedge b)$

   For each LTL formula, state whether the LTS satisfies it and, for each CTL formula, give the set of states of the LTS that satisfy it.

   [5 marks]

3. Translate the following informally described properties into the specified temporal logic, explaining the meaning of any atomic propositions that you use.

   (a) "servers 1 and 2 are never both down simultaneously" (in CTL)

   (b) "it is always the case that a is true now or in one step's time" (in LTL)

   (c) "zone A is visited infinitely often but zones B and C are visited only finitely often" (in LTL)

   (d) "the robot eventually reaches room 1 and then goes immediately to room 2, all before the alarm goes off" (in LTL)

[8 marks]

4. We saw in lectures that, in addition to standard equivalences for propositional logic, various LTL equivalences exist for temporal operators, for example:

   - $\Box\psi \;\equiv\; \neg\Diamond\neg\psi$
   - $\Diamond\psi \;\equiv\; \neg\Box\neg\psi$

   Using these (where needed), either prove or disprove each of the following proposed LTL equivalences. You can assume that a, b and c are atomic propositions.

   (a) $\neg(\Box\Diamond a \rightarrow \Diamond\Box b) \;\equiv\; \Box\Diamond a \wedge \Box\Diamond\neg b$

   (b) $\Box a \rightarrow \neg\Box(b \wedge c) \;\equiv\; \Diamond\neg a \vee \Diamond(\neg b \vee c)$

[11 marks]