Distributed and Parallel Computing

Lecture 02

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Measuring Parallel Speedup

- **Latency**: the time from initiating to completing a task
  - Units of time
- **Work**: a measure of what the CPU/GPU has to do for a particular task
  - number of floating point operations
  - number of images processed
  - number of pixels processed
  - number of simulation steps
- **Throughput**: work done per unit time

Speedup and Efficiency

- **Speedup$_P$**, S_P: The ratio of the latency for solving a problem with 1 hardware unit to the latency of solving it with P units
  - $S_P = \frac{T_1}{T_P}$
  - Perfect linear speedup: $S_P = P$
- **Efficiency,** E_P: The ratio of the latency for solving a problem with 1 hardware unit to P times the latency of solving it on P units
  - This measures how well the individual hardware units are contributing to the overall solution
  - $E_P = \frac{T_1}{P \times T_P} = \frac{S_P}{P}$
  - Perfect linear efficiency: $E_P = 1$

Interpreting speedup and efficiency

- Sub-linear speedup and efficiency is normal
  - Overhead in parallelizing a problem
- Super-linear speedup is possible, but usually due to special conditions
  - e.g. Serial version does not fit in CPU cache but each of the parallel sub-problems do.
- Important to compare the best serial version of the program with the parallel version
  - Serial algorithm A is fast but hard to parallelize
  - Serial algorithm B is slow but gives Parallel algorithm B
  - To measure speedup/efficiency, compare Serial A to Parallel B
  - Both must solve the same problem, but allow for minor differences
    - e.g. small round-off differences (but be aware of differing precision on host and on GPU!)
    - differences due to different execution order

Strong Scalability

Gene Amdahl, in 1967, argued that the time spent executing a program is composed of the time spent doing non-parallelizable work plus the time spent doing parallelizable work:

$$T_1 = T_{\mathrm{ser}} + T_{\mathrm{par}}$$

Therefore, if the speedup on P units of the parallel part only is s

$$T_P = T_{\mathrm{ser}} + \frac{T_{\mathrm{par}}}{s}$$

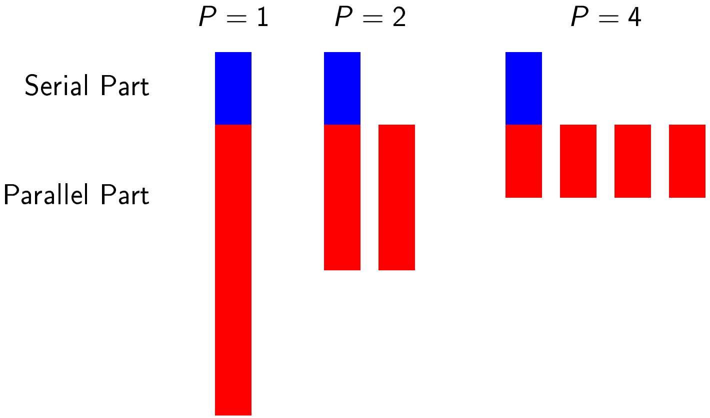
Hence the overall speed up given the speedup of the parallel part is s is:

$$S_P = \frac{T_{\mathrm{ser}} + T_{\mathrm{par}}}{T_{\mathrm{ser}} + \frac{T_{\mathrm{par}}}{s}}$$

If we let f be the fraction of a program that is parallelizable, then $T_{\mathrm{ser}} = (1-f)T_1$ and $T_{\mathrm{par}} = fT_1$. Hence

$$S_P = \frac{(1-f)T_1 + fT_1}{(1-f)T_1 + \frac{fT_1}{s}} = \frac{1}{1-f+\frac{f}{s}} \qquad \text{(Amdahl's Law)}$$

Amdahl's Law Graphically



Interpreting Amdahl's Law

Amdahl's law seems to say that there is a limit to parallel speedup

$$\lim_{s \longrightarrow \infty} S_P = \lim_{s \longrightarrow \infty} \frac{1}{1 - f + \frac{f}{s}} = \frac{1}{1 - f}$$

or, in other words

$$\begin{aligned} &\lim_{s \longrightarrow \infty} \frac{T_1}{T_P} = \frac{1}{1 - f} \\ \Rightarrow &\lim_{P \longrightarrow \infty} T_P = T_{\mathrm{ser}} \qquad \text{assuming } P \rightarrow \infty \Rightarrow s \rightarrow \infty \end{aligned}$$

Weak Scalability

John Gustafson and Edwin Barsis, in 1988, argued that Amdahl's law did not give the full picture

- Amdahl kept the task fixed and considered how much you could shorten the processing time by running in parallel
- Gustafson and Barsis kept the processing time fixed and considered how much larger a task you could handle in that time by running in parallel
- This was motivated by observing that, as computers increase in power, the problems that they are applied to often increase in size

Gustafson-Barsis Law

Assume that W is the workload that can be executed without parallelism in time T. If f is the fraction of the workload that is parallelizable, then

$$W = (1 - f)W + fW$$

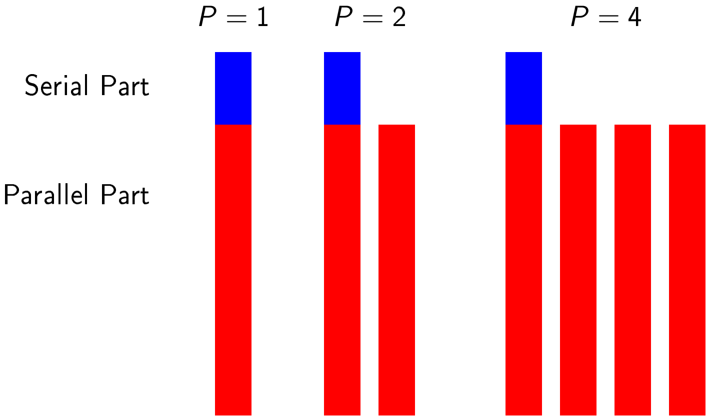
With speedup s, we can run s times the parallelizable part in the same time, although we don't change the amount of work done in the non-parallelizable part:

$$W_s = (1 - f)W + sfW$$

If we do W_s in time T, we are, on average, doing W amount of work in time $\frac{TW}{W_s}$. Hence the total speedup is:

$$S_s = \frac{T}{TW/W_s} = \frac{W_s}{W} = 1 - f + fs \qquad \text{(Gustafson-Barsis)}$$

Gustafson-Barsis Law Graphically



Conclusions

- Both Amdahl (A) and Gustafson-Barsis (GB) are correct
- The two together give guidance on what kinds of problems can benefit from parallelization and how
- You can only go so much faster on a fixed problem by parallelization
- You can avoid Amdahl's limit on speedup if you can increase the size of the parallelizable part of the problem faster than you increase the size of the non-parallelizable part