Dissertation

Automated optimization of sensitivity in a search for boosted VBF Higgs pair production in the $b\bar{b}b\bar{b}$ quark final state with the ATLAS detector

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Abstract

I am an abstract.

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Chapter 1

Theory

The Standard Model (SM) of Particle Physics is the current theory that describes three of the four fundamental forces, namely the electromagnetic, strong, and weak forces, with the exception of gravity. Over the last decades it has been probed with remarkable precision but although there are observational phenomena that lie beyond its scope.

The SM is based on symmetry principles and is described by a lorentz-invariant Quantum Field Theory (QFT) that is renormalizable and invariant under local gauge transformations. This means that within the non-abelian gauge group

$$G = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{1.0.1}$$

the equations of motions are invariant. $SU(3)_C$ is the special unitary group of rank 3 representing the color symmetry within Quantum Chromodynamics (QCD), the QFT describing the strong interactions. $SU(2)_L \otimes U(1)_Y$ exhibits the unification of the weak and electromagnetic interaction into the electro-weak force of $SU(2)_L$ left-chiral fermions of the weak force and right-handed $U(1)_Y$ fermions with weak hypercharge Y of the electromagnetic force described by Quantum Electrodynamics Quantum Electrodynamics (QED).

The following describes the particles of the SM and gives a brief overview of the QFT's used to describe aforementioned forces. The content of this chapter draws inspiration primarily from [1–5]. Natural units are assumed everywhere $\hbar = c = 1$.

1.1 Particles of the Standard Model

All currently known elementary particles are included in the SM and can be organized as depicted in figure 1.1. This includes 12 fermions, that are particles of half-integer spin, 12 vector bosons with spin 1, and the Higgs boson, a scalar particle with spin 0.

The fermions can be categorized into three generations each consisting of a charged lepton, a neutral neutrino and two quarks. Except for their masses, particles of the different generations have the same quantum numbers. Ordinary matter consists only of particles from the first generation. Moreover each particle has an associated anti-particle with all the quantum numbers inversed.

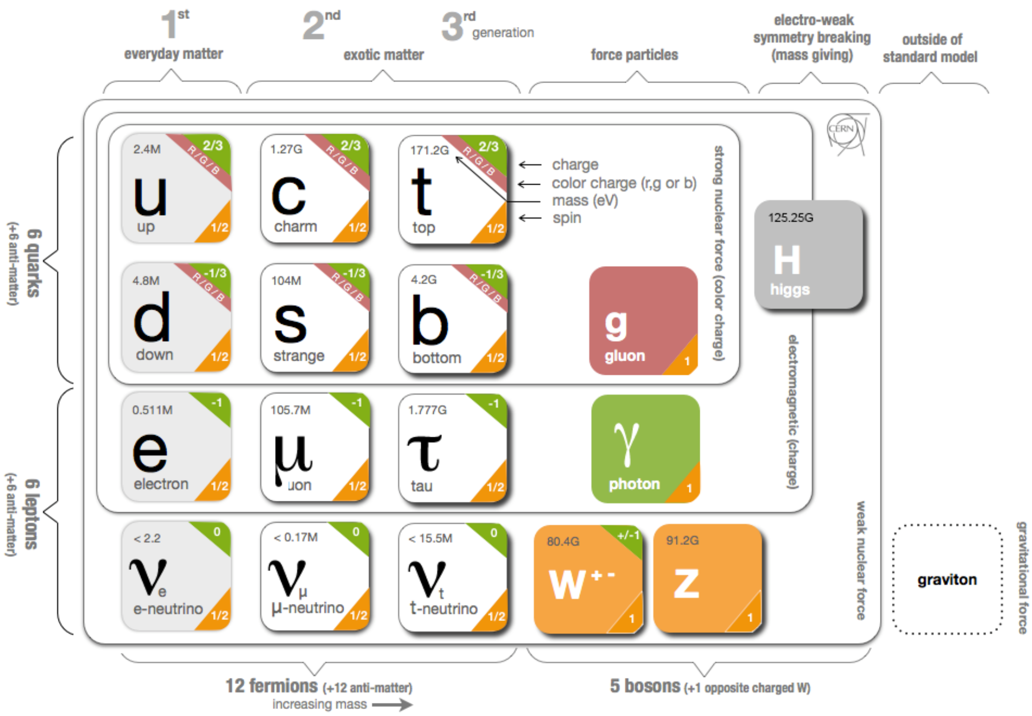


Figure 1.1: Particles in the SM. Adopted from [6]. Higgs Boson mass corrected to the current value [7].

Quarks possess both electric charges and color charges, causing them to interact with each other via weak, electromagnetic, and strong forces. Each generation consists of an up quark (up, charm and top quark) with an electric charge of Q = 2/3 and a down quark (down, strange and bottom quark) with a charge of Q = -1/3. Quarks can only be observed as composite particles - hadrons due to color confinement. This states that if one tries to separate a hadron, it is always energetically favorable to produce a quark-antiquark pair instead. Hadrons constitutes either bound states of 2 quarks - Mesons, e.g. a pion or 3 quarks - Baryons e.g. the proton. To not break Pauli's exclusion principle quarks in a bound state must have different color states.

Leptons in turn do not carry a color charge and encompasses the electron e, muon μ and tau τ and their associated neutrinos ν_e, ν_μ and ν_τ. In the SM neutrinos are considered to be massless. Neutrinos also do not carry a charge and interact solely via the weak force whereas the ones (e, μ, τ) with a charge $Q = -1$ participate also interact electromagnetically.

In the interaction picture of QFT, forces are mediated by particles specific to the particular force. These particles are bosons and are mediating as 8 massless gluons g the strong force, as 1 massless photon γ the electromagnetic force and as 3 massive bosons W^+, W^-, Z the weak force.

The scalar Higgs particle has a unique role in the Standard model. A locally gauge invariant QFT requires massless mediators which the $W^\pm, Z$ are not. When unifying the weak force and the electromagnetic force into the electroweak force a new field - the Higgs field - can incorporate mass to these mediators by leaving the qft gauge invariant. This will be discussed in detail in section 1.7. The Higgs field can explain the masses of all fermions as the coupling to each fermion is proportional to its mass. This essentially means that the heavier the particle, the stronger its interaction is with the Higgs field.

If not further specified the following always includes the anti-particles when referred to a species or a particular particle.

particle	field type	Lagrange
spin-0 (scalar)	scalar ϕ	$\mathcal{L}_{\text{Klein-Gordon}} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{m^2}{2}\phi^2$
spin-1/2 (fermion)	spinor ψ	$\mathcal{L}_{\text{Dirac}} = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi$
spin-1 (boson)	vector A_μ	$\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu$

Table 1.1: Quantum fields relevant for the SM. With $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the electromagnetic field strength tensor.

1.2 Quantum Field Theory

Elementary Particles can be created, transformed and vanish in all sorts of particle interactions. Quantum mechanics states that energy can vary greatly on short time scales via the uncertainty principle. Special relativity relates energy with mass allowing energy to manifest as massive particles. Though special relativity lacks a quantum mechanical description and in non-relativistic quantum mechanics the particle number is conserved. Neither of these descriptions is sufficient to fully describe the observations therefore QFT was developed.

For a field description some quantity $\phi(x, y, z, t) = \phi(x)$ is assigned to some region in spacetime x. Similar to the Lagrangian formalism in classical mechanics here a Lagrangian density in spacetime governs the dynamics of the system $\mathcal{L}(\phi_1, \dots, \phi_n)$. The generalized Euler-Lagrange equations of qft then give the according equations of motion for each field component ϕ_i

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i}. \tag{1.2.1}$$

This relation gives the equations of motion for the Lagrangian. Fields which appear in the SM and their associated Lagrangian are summarized in 1.1 table.

In QFT the conventional strategy to describe particle dynamics is to use a perturbation ansatz $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ where one knows the solution of $\mathcal{L}_0$ and adds a small perturbation $\mathcal{L}_1$. Herein the free field/kinetic part of the Lagrangian is $\mathcal{L}_0 = \frac{1}{2}[(\partial_\mu\phi)^2 - m^2\phi^2]$ and a small perturbation/potential term $\mathcal{L}_1 = V(\phi)$ is added as some polyominal in ϕ that governs the interactions of particles. A term

$J(x)\phi(x)$ needs to be added to excite the field or create/destroy particles so that the ansatz reads

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \phi)^2 - m^2\phi^2] - V(\phi) + J(x)\phi(x). \tag{1.2.2}$$

In the path integral formulation of QFT the problem can be reduced to integrals of the form $\int D\phi e^{i\int d^4x \mathcal{L}(\phi(\boldsymbol{x},t))}$. Where $\int D\phi$ is the integral over all possible paths of the field. Usually $V(\phi)$ is just one anharmonic term with $\lambda\phi^4$ with coupling strength λ and is expanded in e to make the integral solvable

$$e^{-V(\phi)} = e^{-\lambda\phi^4} = 1 - \lambda\phi^4 + \frac{1}{2}\lambda^2\phi^8 + \dots \tag{1.2.3}$$

This only works if λ is small. The result is a probability also called the amplitude usually denoted with $\mathcal{M}$. Via this one can derive the Feynman rules and calculate cross sections to a desired order of expansion.

The principle of local gauge invariance generates all the symmetries for the different forces and is inspired by gauge invariance from classical electrodynamics. In the following, this is explained for each of the forces.

1.3 Quantum Electrodynamics

The QFT-description of the electromagnetic interaction QED can be derived from the free fermion field given by the Dirac equation

$$\mathcal{L}_{\text{Dirac}} = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi. \tag{1.3.1}$$

This Lagrangian is invariant under a change of phase α

$$\psi(x) \to e^{-i\alpha}\psi(x). \tag{1.3.2}$$

The requirement that this transformation also holds locally means that α now additionally depends on the point x in spacetime $\alpha \to \alpha(x)$. Since this gives another term because of the derivative, the Lagrangian can be made invariant again by introducing a vector field A_μ with a coupling of the size of the electron charge e

and replacing the derivative ∂_μ by the covariant derivative D_μ

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu. \tag{1.3.3}$$

Thus, the new Lagrangian

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi = \underbrace{\overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi}_{\mathcal{L}_{\text{Dirac}}} + \underbrace{e\overline{\psi}\gamma^\mu \psi A_\mu}_{\mathcal{L}_{\text{int}}} \tag{1.3.4}$$

becomes invariant under the local gauge transformations

$$\psi(x) \to e^{-i\alpha(x)}\psi(x) \tag{1.3.5}$$

$$A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x) \tag{1.3.6}$$

forming the electromagnetic $U(1)_{EM}$ gauge group. The particular scheme of these replacements is also called the minimal substitution rule. This Lagrangian describes a fermion interacting with a vector field A_μ - the photon. A kinetic term for the vector field can be added from the Proca Lagrangian in table 1.1. $F_{\mu\nu}$ is local gauge invariant whereas the $A_\mu A^\mu$ is not, since it picks up a second derivative for α and therefore is required to be massless. The full QED lagrangian is then

$$\mathcal{L}_{\text{QED}} = \underbrace{\overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi}_{\mathcal{L}_{\text{Dirac}}} + \underbrace{e\overline{\psi}\gamma^\mu \psi A_\mu}_{\mathcal{L}_{\text{int}}} \underbrace{- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\mathcal{L}_{\text{Maxwell}}}. \tag{1.3.7}$$

Saying that this symmetry for $\alpha(x)$ holds locally for all unitary 1×1 matrices $U(1)$ is a bit extravagant, but the formalism is extendable to higher orders as for the electroweak theory and QCD case. It is called an abelian gauge group as any 1×1 matrix also commutes with itself.

1.4 Quantum Chromodynamics

Along the same lines as QED is derived in section 1.3, the theory of the strong interactions QCD is now a non-abelian gauge theory of the symmetry group $SU(3)$. The latter is generated by the 3×3 Gell-Mann matrices λ_a with $a \in \{1, \ldots, 8\}$.

The fundamental charge is now color and each quark is a triplet of the three color fermion fields $\Psi_k = (\psi_r, \psi_g, \psi_b)^T$ for all quark flavors k. Local gauge invariance of the Lagrangian

$$\mathcal{L} = \sum_k \begin{pmatrix} \overline{\psi}_r & \overline{\psi}_g & \overline{\psi}_b \end{pmatrix} (i\gamma^\mu \partial_\mu - m) \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix} = \sum_k \overline{\Psi}_k (i\gamma^\mu \partial_\mu - m)\Psi_k \tag{1.4.1}$$

means that the spinors are required to be invariant under the transformation

$$\Psi_k(x) \to e^{i\alpha_a(x)\lambda_a/2}\Psi_k(x), \qquad \alpha \in \mathbb{R}, \quad a \in \{1, \dots, 8\}, \tag{1.4.2}$$

with $\alpha_a(x)$ a local phase and the index a for the 8 gluons. Here and in the following summation over equal indices $\alpha_a(x)\lambda_a = \sum_i \alpha_a(x)\lambda_a$ is assumed. As in QED a covariant derivative is introduced

$$D_\mu = \partial_\mu - ig_s \frac{\lambda_a}{2} G^a_\mu, \tag{1.4.3}$$

involving the eight gluon vector fields G^a_μ and the coupling strength g_s, which is related to the strong coupling constant as

$$\alpha_s = \frac{g_s^2}{4\pi}. \tag{1.4.4}$$

Again self coupling terms are added

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^a_{\beta\gamma} G^\beta_\mu G^\gamma_\nu, \qquad \text{with } [\lambda_a, \lambda_b] = i f^c_{ab}\lambda_c, \tag{1.4.5}$$

to get the gauge invariant QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \sum_k \overline{\Psi}_k \left(i\gamma^\mu D_\mu - m_k\right) \Psi_k - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \tag{1.4.6}$$

$$= \sum_k \overline{\Psi}_k \left(i\gamma^\mu \partial_\mu - m_k\right) \Psi_k + g_s \overline{\Psi}_k \gamma^\mu \frac{\lambda_a}{2} \Psi_k G^a_\mu - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}. \tag{1.4.7}$$

This Lagrange consists of a kinetic term for each quark, an interaction term of the quarks with the gluons and gluon-gluon interactions giving vertices shown in 1.2.

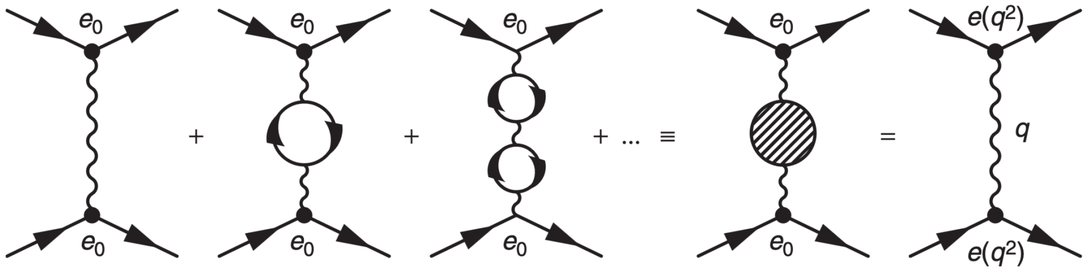


Figure 1.3: Higher order loop corrections in QED schematically treated as one effective diagram. Adopted from [3].

This becomes clear when $G^a_{\mu\nu}$ is squared and also leads to cubic and quartic terms for the fields.

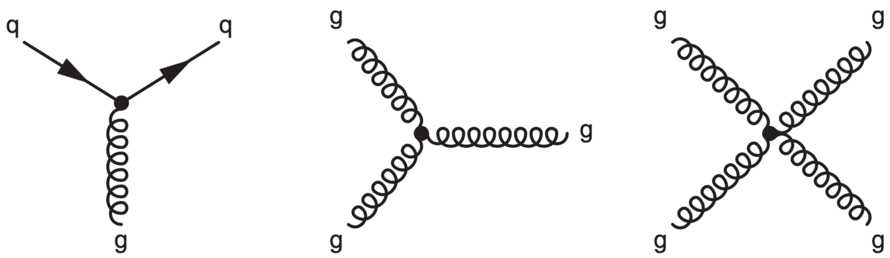
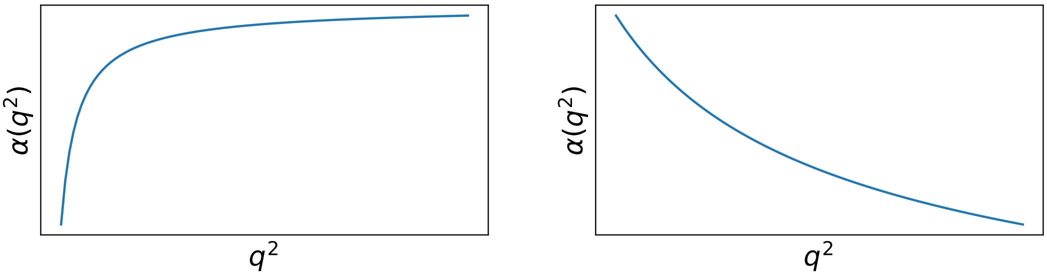


Figure 1.2: (left) Quarks interacting with a gluon. (middle) triplet and (right) quartic self coupling of gluons. Adopted from [3].

1.5 Renormalization

When trying to calculate amplitudes $\mathcal{M}$ of higher order diagrams in QED like the second or third one in figure 1.5 it results in diverging integrals. These diagrams are also referred to as vacuum polarization as virtual particle-antiparticle pairs screen the actual charge of the electron e_0 like a dielectric medium in classical electrodynamics. The situation can be fixed by absorbing the appearing infinities into an effective charge/coupling $e(q^2)$ which is now a function of the squared four momentum q^2 at the virtual photon vertex shown schematically in figure 1.5. For the the second diagram in figure 1.5 involving only one loop correction it can be shown that for some measured coupling $e(q^2 = \mu^2)$ the actual coupling $e(q^2)$ follows a scaling behavior that holds if q^2 and μ^2 are larger than the electron mass [3].



(a) (b)

Figure 1.4: Qualitative behavior of the running couplings for (**a**) QED as of equation 1.5.1 and (**b**) QCD as of equation 1.5.2.

The coupling constant is now a running coupling $e(q^2)$ and reads in terms of the fine structure constant $\alpha(q^2) = e^2(q^2)/4\pi$,

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \alpha(\mu)\frac{1}{3\pi}\ln\left(\frac{q^2}{\mu^2}\right)}. \tag{1.5.1}$$

Therefore with increasing momentum transfer or closer approach in a collision the coupling at the virtual photon vertex increases as can be seen qualitatively in figure 1.4(a).

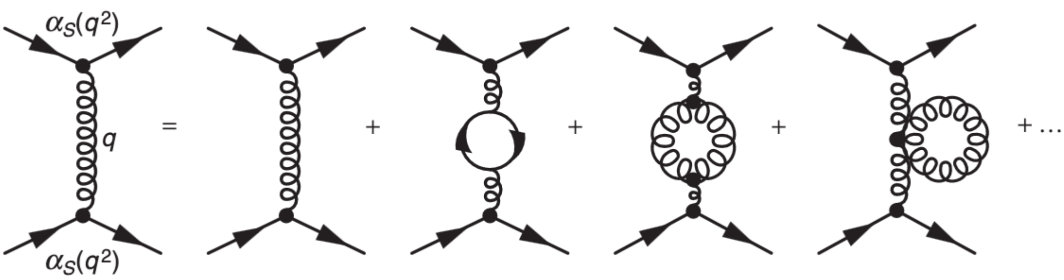
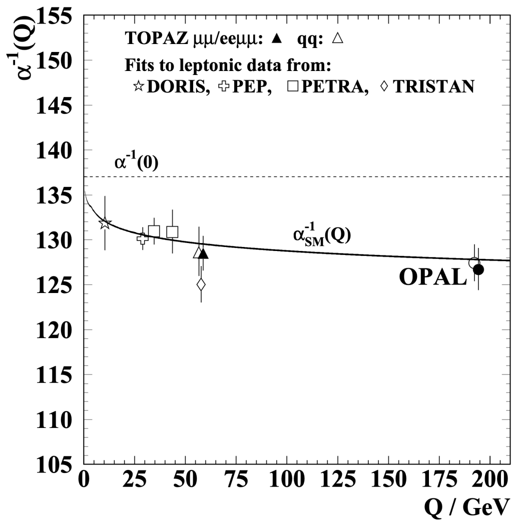
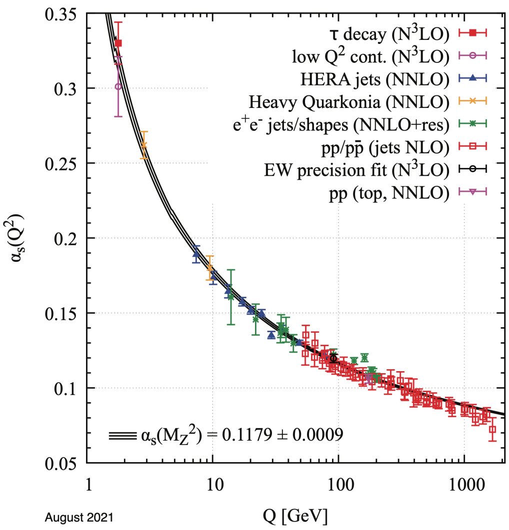


Figure 1.5: Some higher order loop corrections in QCD. Adopted from [3].

Renormalization in QCD can be derived similarly but also the quartic and triplet couplings exemplified in figure 1.5 need to be considered that result in a



(a)



(b)

Figure 1.6: Measurements of the running couplings for (**a**) QED (note the inverted coupling on the y-axis) adopted from [8] and (**b**) QCD adopted from [7].

scaling for the strong coupling

$$\alpha_S(q^2) = \frac{\alpha_S(\mu^2)}{1 + B\alpha_S(\mu)\ln\left(\frac{q^2}{\mu^2}\right)}, \qquad \text{with } B = \frac{11N_c - 2N_f}{12\pi}. \tag{1.5.2}$$

For 3 color charges N_c and 6 fermions N_f in the SM, B is positive and the coupling becomes weaker for shorter scales or higher momentum transfer as can be seen in figure 1.4(b).

The fine structure constant of QED $\alpha(q^2 \approx 0) \approx 1/137$ does not vary dramatically over the energy ranges of matter for particle physics as shown in figure 1.6(a). Most importantly the running coupling of QED does not disturb the perturbation ansatz sketched out in section 1.2 since $\alpha \ll 1$. This is not the case for QCD where α_S at $q \approx 1\,\text{GeV}$ is of $\mathcal{O}(1)$ and perturbation theory breaks down for calculations on bound hadronic states and latter processes in hadronization. While perturbation theory for QCD remains valid for $\alpha_S \approx 0.1$ which corresponds to $q \approx 100\,\text{GeV}$ in basically all processes that are of interest at the Large Hadron Collider (LHC) higher order corrections must be considered in QCD calculations.

The behavior of the running coupling in QCD is called asymptotic freedom, since the theory is free of asymptotics with increasing energy scale or decreasing distance. In turn, since the coupling increases with larger distances, this leads to color confinement, which means that colored particles can only be observed in bound states.

1.6 Electroweak Unification

The weak force can be added to the gauge invariant formalism with a $SU(2)$ symmetry and can be combined with the electromagnetic force so that both forces originate from one electroweak force by requiring a symmetry $SU(2)_L \otimes U(1)_Y$. The weak force couples to left handed chiral particle states only, e.g. for some fermion ψ_L. Fermions can be grouped by their characteristics into left handed doublets

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L, \begin{pmatrix}u \\ d\end{pmatrix}_L, \begin{pmatrix}c \\ s\end{pmatrix}_L, \begin{pmatrix}t \\ b\end{pmatrix}_L, \tag{1.6.1}$$

with weak isospin $I = 1/2$, with the third component $I_3 = \pm 1/2$ for the upper and lower doublet particle respectively, whereas the weak hypercharge Y is associated to right handed singlets

$$e_R, \quad \mu_R, \quad \tau_R, \quad u_R, \quad d_R, \quad c_R, \quad s_R, \quad t_R, \quad b_R, \tag{1.6.2}$$

with $I = 0$. The relation between the electric charge of the particle and these quantum numbers is governed by the Gell-Mann-Nishijima Formula

$$Q = I_3 + Y/2. \tag{1.6.3}$$

The electroweak Lagrangian is then composed of four basic terms

$$\mathcal{L}_{\mathrm{EW}} = \mathcal{L}_{\mathrm{fermions}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Yukawa}}. \tag{1.6.4}$$

Following the same steps as for QCD and QED the Lagrangian can be rendered gauge invariant by introducing a covariant derivative and gauge fields that are dictated by the group symmetry.

$SU(2)$ is generated through the three Pauli matrices $\boldsymbol{\sigma}$ requiring 3 vector gauge fields W_μ^a, $a = \{1, 2, 3\}$ whereas the $U(1)$ symmetry of the vector gauge field B_μ is generated by the weak hypercharge Y so the Lagrangian needs to be invariant under the transformation

$$\psi_L \to e^{i\alpha_a(x)\sigma_a/2} e^{iY/2} \psi_L, \qquad a \in \{1, \dots, 3\}, \quad \alpha \in \mathbb{R} \tag{1.6.5}$$

$$\psi_R \to e^{i\beta(x)Y/2} \psi_R, \qquad \beta \in \mathbb{R} \tag{1.6.6}$$

At this stage the new vector fields are still massless and give the fermionic and gauge parts of the Lagrangian and will be explained below. The masses for the fermions and bosons can be incorporated via the Higgs mechanism that is described in section 1.7 yielding the Higgs and Yukawa parts of the Lagrangian.

Fermion term

To distinguish left- and right handed particle states the according spinors can be written as

$$\psi_L = \frac{1 - \gamma^5}{2} \psi, \quad \psi_R = \frac{1 + \gamma^5}{2} \psi. \tag{1.6.7}$$

These are not helicity eigenstates but rather $\psi_{L,R}$ become ψ if ψ has the corresponding helicity and vanish otherwise, e.g. for a left-handed state that carries helicity -1

$$\frac{1}{2}(1 - y^5)\psi = \begin{cases} 0 & \text{if } \psi \text{ has helicity } +1, \\ \psi & \text{if } \psi \text{ has helicity -1.} \end{cases} \tag{1.6.8}$$

The aforementioned doublets and singlets are then represented by

$$\psi_L^j = \begin{pmatrix} \psi_{L+}^j \\ \psi_{L-}^j \end{pmatrix}, \quad \psi_{R\xi}^j, \tag{1.6.9}$$

with j running over the doublets from equation 1.6.1 and $\xi = +$ for u-type fermions and $\xi = -$ for d-type fermions. The covariant derivative is

$$D_\mu^L = \partial_\mu - ig_2 \frac{\sigma_a}{2} W_\mu^a + ig_1 \frac{Y}{2} B_\mu, \tag{1.6.10}$$

$$D_\mu^R = \partial_\mu + ig_1 \frac{Y}{2} B_\mu, \tag{1.6.11}$$

with coupling g_2 and g_1 to the vector fields and σ_a for the corresponding Pauli matrix, so that the fermionic part of the Lagrangian becomes

$$\mathcal{L}_{\text{fermions}} = \sum_j \overline{\psi}_L^j i\gamma^\mu D_\mu^L \psi_L^j + \sum_{j,\xi} \overline{\psi}_{R\xi}^j i\gamma^\mu D_\mu^R \psi_{R\xi}^j. \tag{1.6.12}$$

Gauge term

The gauge field self interaction terms are

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon_{abc} W_\mu^b W_\nu^c, \tag{1.6.13}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{1.6.14}$$

with g_2 the weak coupling constant and ϵ_{abc} the totally asymmetric Levi-Civita tensor yielding the gauge field part of the Lagrangian

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}. \tag{1.6.15}$$

So far this Lagrangian describes massless fermions and gauge fields W_μ^a because the gauge invariance would be broken otherwise. A Proca mass term $\frac{1}{2} m^2 B_\mu B^\mu$ is not invariant as discussed in section 1.3. For a Dirac term like $-m\overline{\psi}\psi$ this can be seen by rewriting it in terms of the left-handed and right-handed fields by inserting a 1 for e.g. the electron and exploiting the transformation of chiral states as e.g. from equation 1.6.8

$$-m\bar{e}e = -m\bar{e} \left[\frac{1}{2}(1 - y^5) + \frac{1}{2}(1 + y^5) \right] e = -m(\bar{e}_R e_L + \bar{e}_L e_R). \tag{1.6.16}$$

$\bar{e}_R$ is a $SU(2)$ singlet and e_L is one component of a $SU(2)$ doublet and therefore such a term is not gauge invariant. Masses will be incorporated into the Lagrangian with the Higgs mechanism explained in section 1.7.

1.7 Higgs mechanism

In the previous sections, it was shown that the principle of local gauge invariance applied to the free Dirac Lagrangian can generate all the dynamics for a given interaction. However, this assumes that the accompanying gauge boson vector fields are massless, which is not the case for the weak interactions. The Higgs mechanism adds a field to the Lagrangian that provides mass terms for the vector bosons and fermions while preserving the principle of local gauge invariance. For a $U(1)$ symmetry a way to achieve this is to introduce a complex scalar field

$$\phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x)) \tag{1.7.1}$$

so that a Lagrangian with a kinetic term $T(\phi)$ and a potential $V(\phi)$ with parameters μ and λ can be constructed as

$$\mathcal{L} = T(\phi) - V(\phi) = [(\partial_\mu \phi)^* (\partial^\mu \phi)] - \left[-\mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2\right]. \tag{1.7.2}$$

To make this Lagrangian locally gauge invariant the transformation steps from section 1.3 on QED can be followed. Again by replacing the derivative with the covariant derivative with some vector field B_μ and coupling g

$$\partial_\mu \to D_\mu = \partial_\mu + igB_\mu. \tag{1.7.3}$$

The resulting Lagrangian can be made locally gauge invariant with the transformations

$$\phi(x) \to e^{-i\alpha(x)}\phi(x), \tag{1.7.4}$$

$$B_\mu(x) \to B_\mu(x) - \partial_\mu \alpha(x). \tag{1.7.5}$$

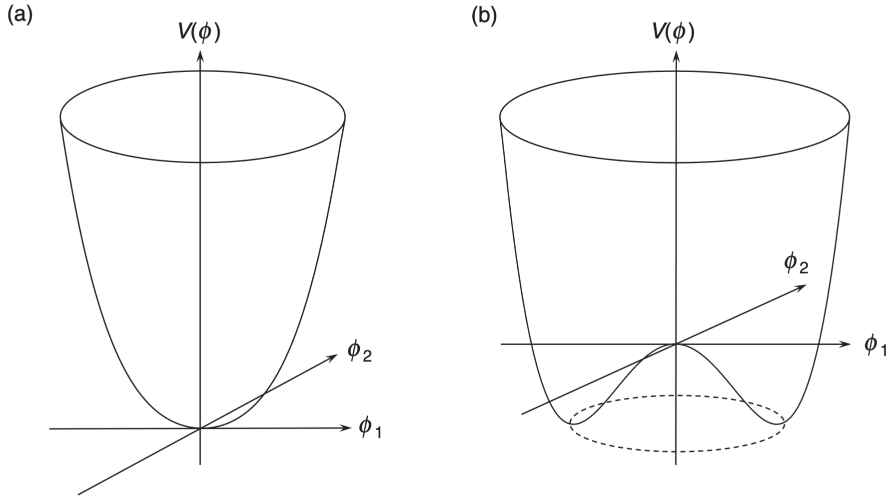


Figure 1.7: Potential $V(\phi)$ (a) for $\lambda > 0$ and $\mu^2 < 0$ and (b) for values $\lambda > 0$ and $\mu^2 > 0$. Adopted from [3].

The lowest energy state of a free field theory is the vacuum state and therefore its eigenvalue is also called vacuum expectation value (VEV). It is the one that minimizes the potential $V(\phi)$

$$v = \begin{cases} 0 & \lambda > 0, \mu^2 < 0 \\ \sqrt{\phi_1^2 + \phi_2^2} = \sqrt{\frac{\mu^2}{\lambda}} & \lambda > 0, \mu^2 > 0, \end{cases} \tag{1.7.6}$$

and is either 0 or forms an infinite set of minima as illustrated in figure 1.7 by the dashed circle.

The QFT perturbation ansatz sketched out in section 1.2 starts from perturbations around the ground state. Thus for the ansatz to work with the $\mu^2 > 0$ case new field variables $\eta(x)$ and ξ can be introduced so the perturbation calculus can be applied about the ground state

$$\phi_1(x) = v + \eta(x), \quad \phi_2 = \xi. \tag{1.7.7}$$

The physical vacuum state spontaneously breaks the symmetry of the Lagrangian. Furthermore the physical predictions of the Lagrangian do not depend on the choice of the gauge it can be chosen in a way that it eliminates the field ξ. In particular

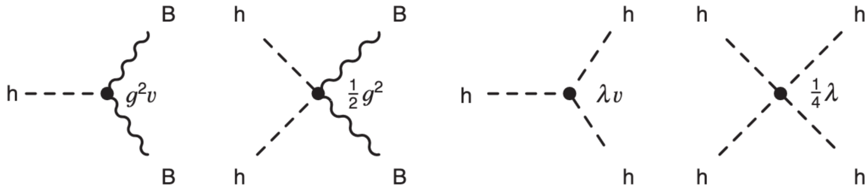


Figure 1.8: Higgs self interactions for a local $U(1)$ gauge symmetry. Adopted from [3].

if $\alpha(x) = -\xi(x)/(gv)$ the transformation from equation 1.7.4 can be made unitary ($UU^\dagger = 1$) when expressed to first order in the fields. Moreover $\eta(x)$ can then be reinterpreted as the physical Higgs field $h(x)$

$$\phi(x) \to e^{-i\alpha(x)}\frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)) \approx \frac{1}{\sqrt{2}}e^{-i\frac{\xi(x)}{gv}}[v + \eta(x)]e^{i\frac{\xi(x)}{gv}} = \frac{1}{\sqrt{2}}(v + h(x)). \tag{1.7.8}$$

The written out Lagrangian except for constants is then

$$\begin{aligned}\mathcal{L} = &\underbrace{\frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2}_{\text{massive h scalar}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}g^2v^2B_\mu B^\mu}_{\text{massive boson}} \\ &+ \underbrace{g^2vB_\mu B^\mu h + \frac{1}{2}g^2B_\mu B^\mu h^2}_{\text{h,B interactions}} - \underbrace{\lambda v h^3 - \frac{1}{4}\lambda h^4}_{\text{h self-interactions}}\end{aligned} \tag{1.7.9}$$

and describes a massive scalar particle, a massive boson, interactions between the scalar and boson and as well interactions of the scalar itself depicted in figure 1.8.

The Standard Model Higgs

For the electroweak Lagrangian a Higgs mechanism needs to break down the $SU(2)_L \otimes U(1)_Y$ symmetry while maintaining the electromagnetic $U(1)_{EM}$ symmetry which is also called electroweak symmetry breaking (EWSB). This can be done with a single isospin doublet of complex scalar fields

$$\phi(x) = \begin{pmatrix}\phi^+(x) \\ \phi^0(x)\end{pmatrix} = \begin{pmatrix}\phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x)\end{pmatrix}. \tag{1.7.10}$$

with weak hyperchage $Y = 1$. With the Gell-mann Nishijima formula from equation 1.6.3, the upper state ϕ^+ with $I_3 = 1/2$ is electrically charged and the lower state ϕ^0 with $I_3 = -1/2$ is electrically neutral.

Higgs term

The covariant derivative from equation 1.6.10 then becomes

$$D_\mu = \partial_\mu - ig_2 \frac{\sigma_a}{2} W_\mu^a + ig_1 \frac{1}{2} B_\mu. \tag{1.7.11}$$

and the Higgs term for the electroweak Lagrangian of equation 1.6.4 is

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi), \tag{1.7.12}$$

with the Higgs potential

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{4} \left(\phi^\dagger \phi\right)^2. \tag{1.7.13}$$

In total analogy to the steps for the $U(1)$ Higgs mechanism from section 1.7 there is a set of degenerate minima for $\mu^2, \lambda > 0$

$$\phi^\dagger \phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = \frac{4\mu^2}{\lambda}. \tag{1.7.14}$$

The VEV is chosen in a way that the vacuum shares the symmetry of the electromagnetic gauge group $U(1)_{EM}$ which is $\phi_1 = \phi_2 = \phi_4 = 0$ and $\phi_3 = v$

$$\langle 0|\phi|0\rangle = \phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{1.7.15}$$

The reason for this is that the photon needs to remain massless and any broken symmetry creates gauge bosons with mass as shown in section 1.7. The generator of the electromagnetic subgroup is $Q = I_3 + Y/2$. ϕ_0 breaks $SU(2)_L$ and $U(1)_Y$ but since it is electrically neutral $Q = 0$ it remains locally gauge invariant under

$U(1)_{EM}$ as can be seen from

$$\phi_0 \to e^{i\alpha(x)Q}\phi_0 = \phi_0. \tag{1.7.16}$$

Expanding the field about the VEV

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{pmatrix}, \tag{1.7.17}$$

becomes in the unitary gauge with the physical SM Higgs field

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{1.7.18}$$

In this form the potential then becomes in terms of μ and v through equation 1.7.14

$$V = \mu^2 h^2 + \frac{\mu^2}{v}h^3 + \frac{\mu^2}{4v^2}h^4 = \frac{m_h^2}{2}h^2 + \frac{m_h^2}{2v}h^3 + \frac{m_h^2}{8v^2}h^4. \tag{1.7.19}$$

with triplet and quartic self-couplings proportional to the Higgs mass $m_h = \mu\sqrt{2}$. The kinematic term of the Higgs Lagrangian can be brought into a form that the gauge fields $W_\mu^1, W_\mu^2, W_\mu^3, B_\mu$ represent the physical fields. The charged massive $W^\pm$ gauge bosons then are

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \tag{1.7.20}$$

and the neutral massive Z boson and the massless photon A_μ are mixed states of

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \tag{1.7.21}$$

rotated by the weak mixing angle θ_W. This yields the kinematic Higgs term

$$(D_\mu\phi)^\dagger (D^\mu\phi) = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{g_2^2}{4}W_\mu^- W^{+\mu}(v+h)^2 + \frac{g_1^2 + g_2^2}{4}Z_\mu Z^\mu (v+h)^2. \tag{1.7.22}$$

with trilinear HWW, HZZ and quadrilinear terms $HHWW, HHZZ$ vertices. The photon becomes naturally massless in this form and the masses of the massive gauge bosons masses can be read off

$$m_W = \frac{1}{2} g_2 v, \quad m_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}. \tag{1.7.23}$$

Furthermore a relation to the weak mixing angle and the electric charge follow

$$\cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{m_W}{m_Z}, \quad e = g \sin\theta_W. \tag{1.7.24}$$

The $\cos\theta_W$ was verifiable before the Higgs discovery and a compelling argument for the Higgs to exist.

Yukawa term

Fermion masses can be incorporated gauge invariant into the electroweak Lagrangian of equation 1.6.4 with the help of the Higgs mechanism in the form of Yukawa couplings. For one generation of leptons and quarks gauge invariant terms are introduced via couplings between the left-handed doublet states, the Higgs field and right handed singlet states

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}}^{\text{one gen}} = & - G_e \begin{pmatrix} \overline{\nu}_e & \overline{e} \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R && (1.7.25)\\
& - G_d \begin{pmatrix} \overline{u} & \overline{d} \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R && (1.7.26)\\
& - G_u \begin{pmatrix} \overline{u} & \overline{d} \end{pmatrix}_L \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} u_R && (1.7.27)\\
& + h.c. && (1.7.28)\\
= & -G_e \overline{L}_L \phi e_R - G_d \overline{Q}_L \phi d_R - G_u \overline{L}_L \phi_c e_R + h.c. && (1.7.29)
\end{aligned}$$

with Yukawa coupling constant G_e, G_d, G_u and the charge conjugated Higgs field $\phi_c = i\sigma_2\phi$. The latter can be used because it transforms just like the Higgs field and therefore leaves the Lagrangian gauge invariant. In the unitary gauge of equation 1.7.18 the terms become interpretable as mass terms that have the desired form of equation 1.6.16 but are now introduced gauge invariant and read e.g. for the electron

$$\mathcal{L}_{\mathrm{Y}}^{e} = \frac{-G_e}{\sqrt{2}} v(\bar{e}_L e_R + \bar{e}_R e_L) + \frac{-G_e}{\sqrt{2}} h(\bar{e}_L e_R + \bar{e}_R e_L). \tag{1.7.30}$$

The mass of fermion f is therefore

$$m_f = G_f \frac{v}{\sqrt{2}} \tag{1.7.31}$$

so that the full Yukawa Lagrangian can be written as

$$\mathcal{L}_{\mathrm{Yukawa}} = -\sum_f m_f \overline{\psi}_F \psi_f - \sum_f \frac{m_f}{v} \overline{\psi}_f \psi_f h. \tag{1.7.32}$$

Consequently the Higgs couples to fermions proportional to their mass.

Appendix A

Acronyms

CERN Organisation européenne pour la recherche nucléaire

ATLAS A Toroidal LHC Apparatus

SM Standard Model

QFT Quantum Field Theory

QCD Quantum Chromodynamics

QED Quantum Electrodynamics

EWSB electroweak symmetry breaking

VEV vacuum expectation value

LHC Large Hadron Collider

HL-LHC High Luminosity LHC

ID Inner Detector

SCT semiconductor tracker

TRT transition radiation tracker

ITk Inner Tracker

IBL insertable *b*-layer

EM electromagnetic

LAr liquid argon

MS muon spectrometer

RPCs resistive plate chambers

TGCs thin gap chambers

MDTs monitored drift tubes

CSCs cathod strip chambers

HLT high level trigger

RoI region of interest

L1 Level-1

PDF Parton Distribution Function

DGLAP Dokshitzer–Gribov–Lipatov–Altarelli–Parisi

MC Monte Carlo

MPI multi-parton interaction

PS parton shower

ME matrix element

ISR initial state radiation

FSR final state radiation

4FS four-flavour scheme

5FS five-flavour scheme

NLO next-to-leading order

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I declare that I have authored this thesis independently, that I have not used other than the declared sources/ resources and that I have explicitly marked all materials which has been quoted either literally or by content form the used sources.

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig verfasst, andere als die angegebenen Quellen/Hilfsmittel nicht benutzt und die den benutzten Quellen wörtlich und inhaltlich entnommenen Stellen als solche kenntlich gemacht habe.

Berlin, 30.10.2023

Frederic Renner