1a)

$$d\nu n(\nu, T) = \frac{8\pi}{3} \frac{\nu^3 \, d\nu}{e^{h\nu/kT} - 1} \left(\frac{a}{a'}\right)^3 \text{ where } \nu \propto a^{-1}$$

$a \to a'$
$\nu \to \nu'$

$$\nu' \frac{a}{a'} = \nu$$

1b) $$f(v)\,d^3v = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(\frac{-mv^2}{2k_B T}\right) d^3v$$

$\Rightarrow$ it doesn't retain its shape because this not conserved since it is a distribution, V

c.) Photons $T \propto a^x$

Non-Relativistic ideal gas $T \propto a^y \to f(v)\,dv^3$

2.) CMB dipole:

$$\lambda' = \lambda - v\frac{\lambda}{c} \qquad \lambda \propto f^{-1}$$

velocity in the prime frame

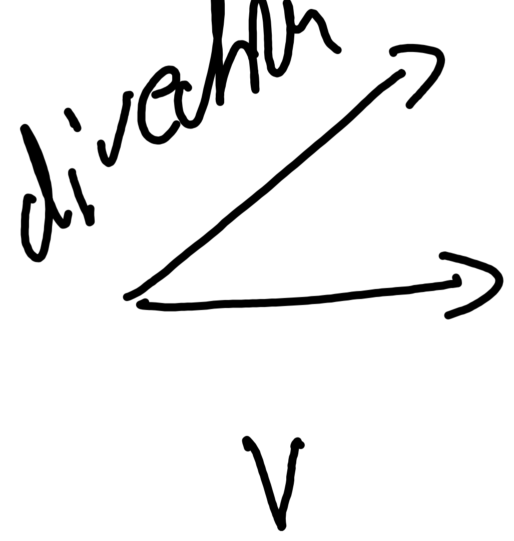
$$f' = \frac{f}{1-\frac{v}{c}}$$

$t' = \gamma t$

$$f' = \frac{f}{\gamma\left(1-\frac{v}{c}\right)} = \frac{f}{\gamma(1-\beta)}$$

β

comes from geometry



$$\beta = \frac{v}{c}$$

$$\gamma = (1-\beta^2)^{-1/2}$$

b.) $\sim 10^4$, noise is $\sqrt{N}$

3.) rotation curves from a sphere versus Thin Disk

from Binney and Tremaine

$$\Phi(R) = -\pi G \Sigma_0 R \left[I_0(y) K_1(y) - I_1(y) K_0(y) \right]$$

Φ → grav. pot

I_0 → Bessel $I(0, y)$

$K(0, y)$

$$y = \frac{R}{2R_d}$$

$$\Sigma = \Sigma_0 \exp\left(\frac{R}{R_d}\right)$$

$$\boxed{\frac{V^2}{R} = \frac{d\Phi}{dR}}$$

$$10^{12} = M_T = \int_0^{R_{max} = 50\,kpc} \Sigma(R)\, 2\pi R\, dR$$

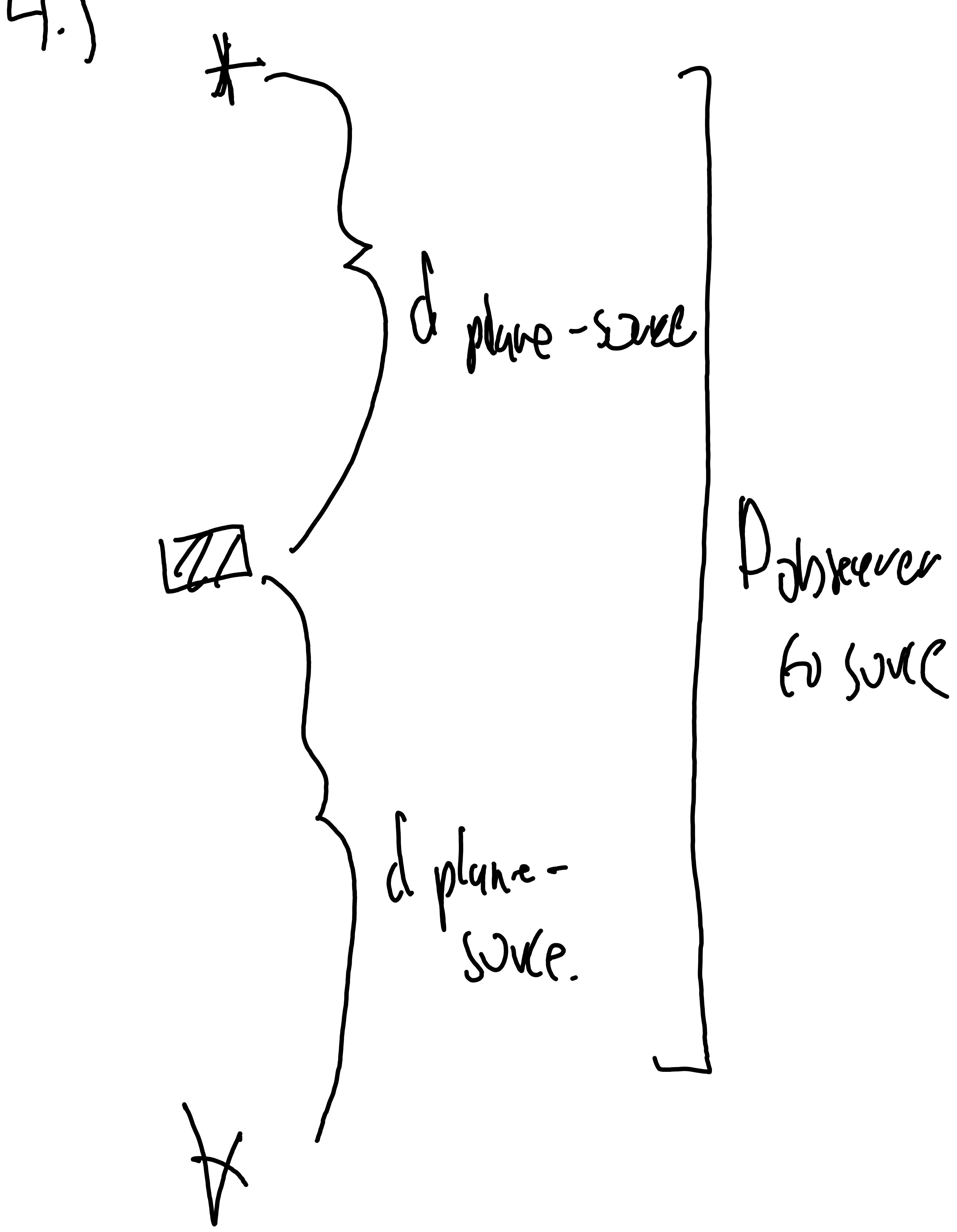
$$\Sigma_0(M_T)$$

$$M_T = \int_0^{50\,kpc} \Sigma_0\, e^{-R/R_d} \cdot 2\pi R\, dR$$

$$M = f(\Sigma_0)$$

(b) $\frac{V^2(R)}{R} = \frac{G M(R)}{R^2} \rightarrow M(R) = \int \ldots$

4.)



$\underbrace{M_{GOG}}_{\text{total mass of machos.}} \sim 10^{12} M_\odot$

$m \sim 1 M_\odot$

macho mass

$D_{op} \sim 50 \text{kpc}$

$D_{os} \sim 700 \text{kpc}$

Lensing probability = optical depth τ

$= \int \sigma n \, dz$

where σ is the crossection for interaction

n = # density

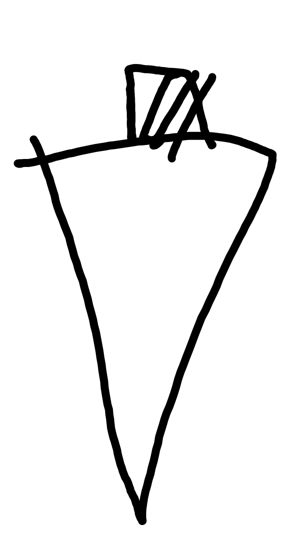
dz = length = $D_{\text{observer to plane}}$.

mean free path

$\lambda = \frac{1}{n\sigma}$

$n \to \frac{\#}{c^3}$, $\sigma \to \rho^2 = p^2$

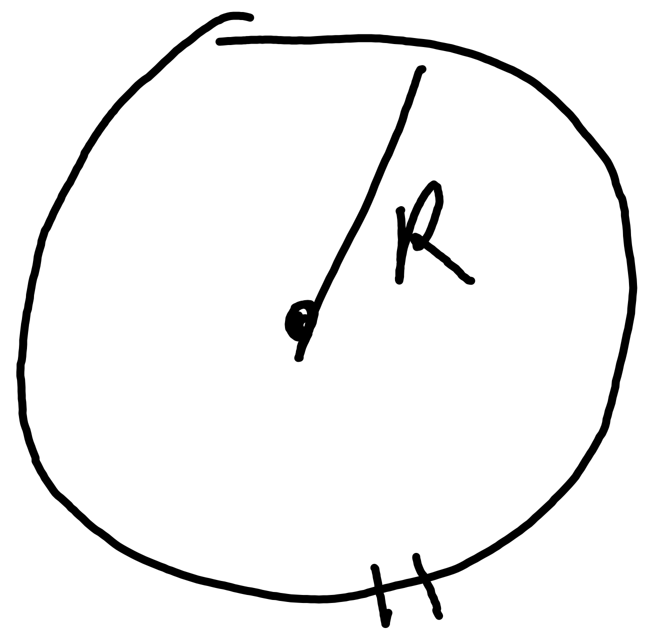
$\tau = \int \frac{dp}{\lambda}$



Zoltan's way

$\frac{\text{lens area}}{\text{observing area}} = p$

5.)



$\lambda = R\theta$

(i) $\lambda = R\theta$ R not changing

$x^\mu = (r, \theta)$

contravariant

$$A^\mu = \frac{dx^\mu}{d\lambda}$$

$$\lambda / R = \theta$$

$$\frac{d\lambda}{R} = d\theta$$

$$d\lambda = dR\, d\theta$$

$$= \left[\frac{R}{d\lambda}, \frac{\theta}{d\lambda}\right]$$

$$= \left[0, \frac{\cancel{\theta}}{R\,\cancel{d\theta}}\right]$$

$$= \left[0, \frac{1}{R}\right]$$

$\Lambda^{\prime\mu}{}_{\nu}$

$$x^{\prime\mu} = (\overset{x}{R\cos\theta}, \overset{y}{R\sin\theta})$$

$$x^\nu = (R, \theta)$$

$$\Lambda^{\prime\mu}{}_{\nu} = \begin{pmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{pmatrix}$$

$$g_{m\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & R^2 \end{pmatrix}$$

covaricaf vecto r

$$\phi = R = \sqrt{x^2 + y^2}$$

$$\frac{\partial \phi}{\partial x^\mu}$$

· Can we transform the special relativity metric where the primed frame is moving w/ some velocity V?

$$g_{ik} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\downarrow$ time $\quad$ $\searrow x$

equivalently $x^i = (t, x)$

$x'^i = (t', x')$

where we can transform $\beta = \frac{V}{c}$

$$x^i = \left[\gamma (t' + \beta x'), \gamma (\beta t' + x')\right]$$

$$\underbrace{g'_{mn}}_{\text{metric in primed frame}} = g_{ik} \frac{\partial x^i}{\partial x'^m} \frac{\partial x^k}{\partial x'^n}$$

$$g'_{00} = g_{00} \frac{\partial x^0 \, \partial x^0}{\partial x'^0 \, \partial x'^0} + g_{11} \frac{\partial x^1}{\partial x'^0} \frac{\partial x^1}{\partial x'^0}$$

$$= -1 (\gamma^2) + 1 (\gamma^2 \beta^2)$$

$$= -\gamma^2 (1 - \beta^2) = -1$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

* If you do the same thing for $g'_{11} = 1$

the metric stays the same regardless of metric.

$$\eta_{nm} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\boxed{\begin{array}{c}\text{Covariant derivative} \\ \hline \text{of a covariant vector}\end{array}}$$

$$B_{i;k} = B_{i,k} - \Gamma^{l}_{ik} B_l$$

take the derivative of a vector field

$$\frac{\partial B'_k}{\partial x'^m} = \frac{\partial}{\partial x'^m}\underbrace{\left(\frac{\partial x^i}{\partial x'^k} B_i\right)}_{B'_k} = \frac{\partial x^i}{\partial x'^k}\overbrace{\left(\frac{\partial x^n}{\partial x'^m}\frac{\partial B_i}{\partial x^n}\right)}^{\text{regular derivative}} + \underbrace{\frac{\partial^2 x^i}{\partial x'^k \partial x'^m}}_{\text{term that accounts for curvature}} B_i$$

$$\frac{\partial^2 x^i}{\partial x'^k \partial x'^m} \rightarrow \Gamma^{i}_{km}$$

$$\Gamma'^{\rho}_{\mu\nu} = \frac{\partial x'^\rho}{\partial x^\alpha}\frac{\partial x^\beta}{\partial x'^\mu}\frac{\partial x^\gamma}{\partial x'^\nu}\Gamma^{\alpha}_{\beta\gamma} + \frac{\partial x'^\rho}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x'^\mu \partial x'^\nu}$$

Christophal symbols lets you know how to take a derivative.

$$\nabla_\nu (A_\mu A^\mu) = A^\mu \nabla_\nu A_\mu + A_\mu \nabla_\nu A^\mu$$

semi-colon derivative

$$\nabla_\nu = \partial_\nu A_\mu - \Gamma^\rho_{\nu\mu} A_\rho$$

definition of the covariant derivative

$$\nabla_\nu (A_\mu A^\mu) = A^\mu (\partial_\nu A_\mu - \Gamma^\rho_{\nu\mu} A_\rho) + A_\mu \underline{\nabla_\nu A^\mu}$$

diye tensor

$$\partial_\nu (A_\mu A^\mu)$$

$$\Rightarrow A^\mu \partial_\nu A_\mu + A_\mu \partial_\nu A^\mu = A^\mu \partial_\nu A_\mu - A^\mu \Gamma^\rho_{\nu\mu} A_\rho + A_\mu \nabla_\nu A^\mu$$

$$A_\mu \partial_\nu A^\mu = A_\mu \nabla_\nu A^\mu - A^\mu \Gamma^\rho_{i\mu} \cancel{A_\rho} \; \rightarrow \mu$$

$$\boxed{\nabla_\nu A^\mu = \partial_\nu A^\mu + A^\mu \Gamma^\rho_{\nu\mu}}$$