Problem 1 - Reionization: Gunn-Peterson Bound on Neutral Hydrogen

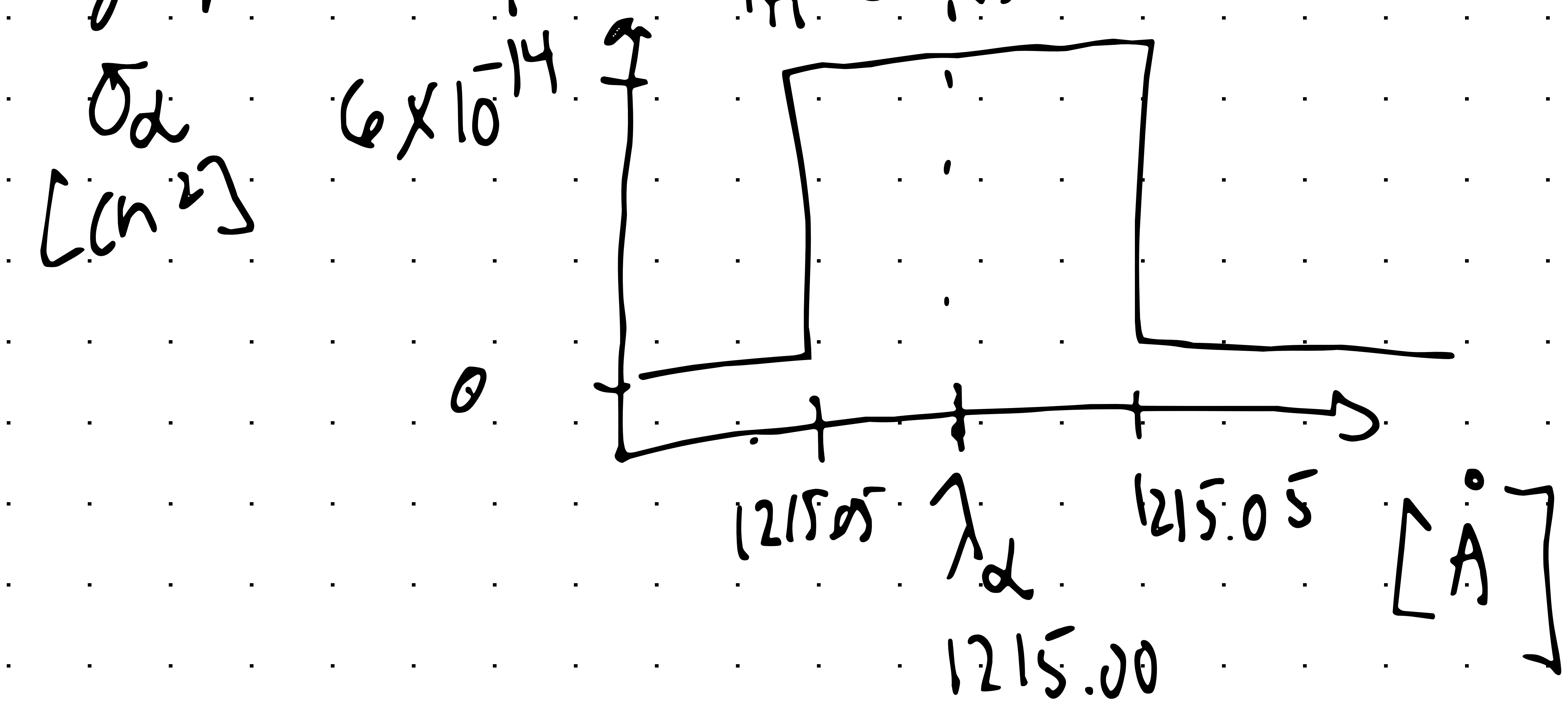
- photon emitted at $\lambda_\alpha = 1200\,\mathring{A}$ at $z=3$
- assume HI all around
- what is the probability τ_α that this photon is absorbed on its way to us?

assume: $\Omega_m = 1$, flat $\Omega_\Lambda = 0$

$H_0 = 72$ km/s/Mpc $\Omega_b = 0.04$

hydrogen mass fraction $Y_H = 0.76$

→ note $\tau_\alpha < 0.05$ from observations



<u>Recall from Class</u>

the surface of last scattering
→ electron scattering rate Γ
→ prob. for a single photon to scatter b/w t_{lss} and present time t_0

$$\tau_e = \int_{t_{ls}}^{t_0} \Gamma(t)\,dt = 1$$

$$\tau(z_{lss}) = \int_0^{z_{lss}} \frac{\Gamma(z)\,dz}{H(z)(1+z)}$$

where $\Gamma = n_e \sigma_T c$

$$\Gamma = n_H(z)\,\sigma_\alpha\, c$$

where $n_H(z) = n_{H,0}(1+z)^3$

$$n_{H,0} = \frac{\Omega_b \rho_{crit}}{m_H} Y_H$$

$\sigma_\alpha = 6\times10^{-14}$ cm^2

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

$m_H = 1.67\times10^{-24}$ g

$H(z) = H_0(1+z)^{3/2}$ since $\Omega_m = 1$

$$\int \frac{\Gamma(z)\, dz}{H_0 (1+z)^{3/2} (1+z)} = \int_0^3 \frac{n_H(z)\, \sigma_\alpha\, c\, dz}{H_0 (1+z)^{3/2} (1+z)}$$

$$= \int_0^3 \frac{\frac{\Omega_b}{m_H} Y_H\, \rho_{crit}\, (1+z)^{6/2}\, \sigma_\alpha\, c\, dz}{H_0 (1+z)^{3/2} (1+z)}$$

$$= \frac{\Omega_b}{m_H} \frac{Y_H}{H_0} \rho_{crit}\, \sigma_\alpha\, c \int_0^3 (1+z)^{1/2}\, dz$$

$$= \frac{7(0.04)}{1.67\times10^{-24}} \frac{0.76(10^{-29})(6\times10^{-14})}{(2\times10^{-18})} 3\times10^{10}) \;\; \frac{2}{3} \underbrace{\left[1+z^{3/2} \right]_0^3}_{7}$$

$$\tau_\alpha \sim 6 \times 10^{11} \text{ in a matter dominated universe.}$$

$Y_H = 0.76$

$\Omega_b = 0.04$

$\sigma_\alpha = 6\times10^{-14}$

$c = 3\times10^{10}$ cm/s

$H_0 = 72$ km/s/Mpc

$\simeq 2.4\times10^{-15}$ cm/s

$G = 6.67\times10^{-8} \frac{cm^3}{g\, s^2}$

$m_H = 1.67\times10^{-24}$ g

for $\tau_\alpha < 0.05$ to be true Hydrogen should mostly be ionized, so not a lot should be neutral. We made the wrong assumption that it is all matter in the universe

Problem 2 - Recombination H vs. He

we know that hydrogen recombined at $z \sim 1200$, starting at $z \sim 1600$

assuming $\Omega_b = 0.04$

$Y_H = 76\%$ and $H_0 = 70\ \mathrm{km/s/Mpc}$

$Y_{He} = 24\%$ $T_0 = 2.725\ \mathrm{K}$

a.) what was $\frac{n_\gamma}{n_H}$ where n_γ is the number-density of photons with $E \geq 13.6\ \mathrm{eV}$

$$n_\gamma = 0.243 \left(\frac{kT}{hc}\right)^3$$

$$n_H = \frac{\Omega_H \rho_b}{m_H}(1+z)^3$$

$$T_{CMB} = T_0 (1+z)$$

from

$$\frac{1-x}{x^2} = 3.84\, \eta \left(\frac{kT}{hc^2}\right)^{3/2} \exp\left(\frac{Q}{kT}\right)$$

Q: 13.6 eV

baryon density

$$\eta = \frac{n_b}{n_\gamma} = \frac{n_p}{x\, n_\gamma}$$

reduce

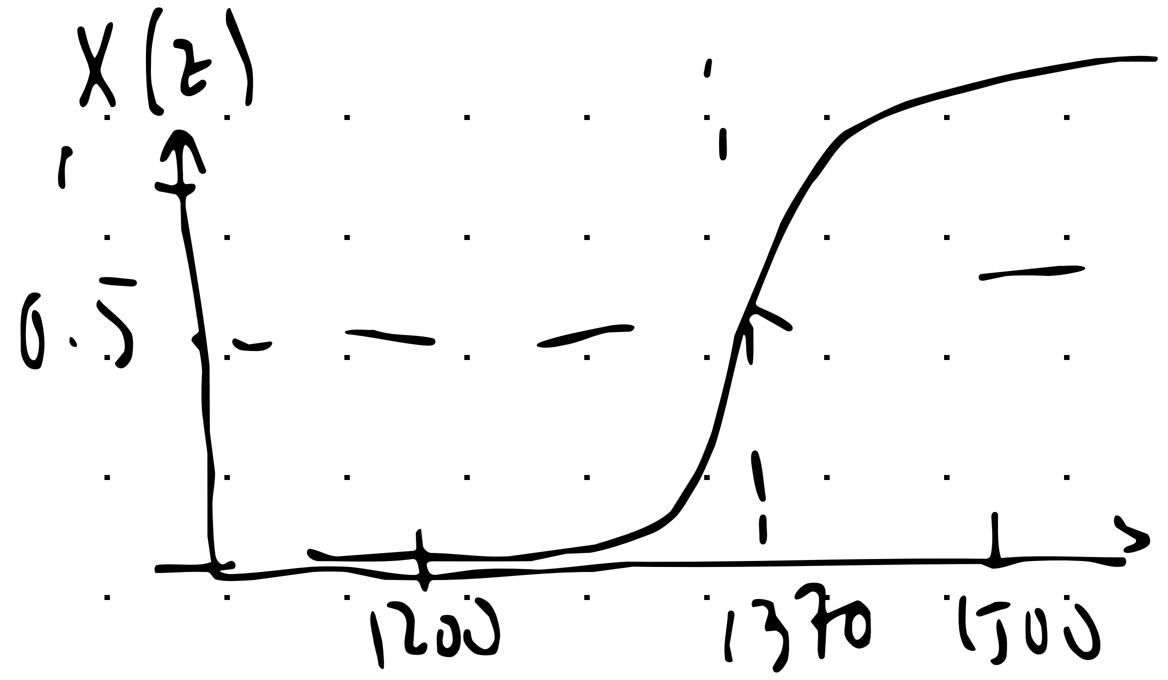
$$n_\gamma \sim 0.243 \left(\frac{kT}{\hbar c}\right)^3$$

$$\rho_b = \Omega_b \, \rho_{crit} = 0.04 \left(\frac{3H_0^2}{8\pi G}\right), \quad \Omega_H = 0.04\,(Y_H)$$

$10^{-29}\ \mathrm{g/cm^3}$

$$n_\gamma \sim 7 \times 10^{11}\ \mathrm{cm^{-3}}$$

$$n_H = \frac{\Omega_b (Y_H)(10^{-29})}{m_H}(1+z)^3 \Big|_{z=1200} = 362\ \mathrm{cm^{-3}}$$



$\eta = \frac{n_H}{n_\gamma} \sim 5.11 \times 10^{-10}$, note η need not be 1 since recombination involves electrons binding to H^+ due to the universe expanding and the mean free path increasing. Direct recombination to the H ground state are insufficient, H atoms generally form at higher energy state, transition to lower energy state then release photons.

From the 2p state after recombining, Ly-α photon is released to get to ground state, this emitted photon gets reabsorbed by another Hydrogen atom in its ground state.

(b) $z = 1600$

$E \geq 24.6\,eV$

$T_{CMB} = T_0 (1+z)$

$$n_{He} = \frac{\Omega_b (Y_{He}) (10^{-29})}{m_{He}} (1+z)^3 \Big|_{z=1600} = 68 \ cm^{-3}$$

$m_{He} \to 4 \cdot m_p$

$$n_\gamma = 0.243 \left(\frac{kT}{\hbar c} \right)^3 \sim 1.6 \times 10^{12} \ cm^{-3}$$

$$\frac{n_{He}}{n_\gamma} \sim 4 \times 10^{-11}$$

$\to$ it implies that it happens before, since this ratio is less

$\hookrightarrow$ higher-z