Exercise 1 - Fresnel Relations

INTRODUCTION

We look at reflection and refraction of a beam of light at the surface of a transparent dielectric. The laws of reflection and refraction (Snell's law) determine the directions of the reflected and refracted beams. However, those directions are only part of the information one might wish to have. The laws do not provide information on intensities and polarizations. These are given by the four Fresnel Relations, which are discussed in Griffiths (Chap. 9.3.3, eq. (9.109), Prob. 9.16), which are also given in the note 'The Fresnel Relations' below. The relations are studied in detail in this exercise.

THE EXPERIMENTAL ARRANGEMENT

The setup is shown in figure 1. The angle of incidence is θ_1, and reflected and refracted beams are seen at θ_1 and θ_2, respectively. The angular scale serves its purpose only when the beam hits vertically over the center of the table.

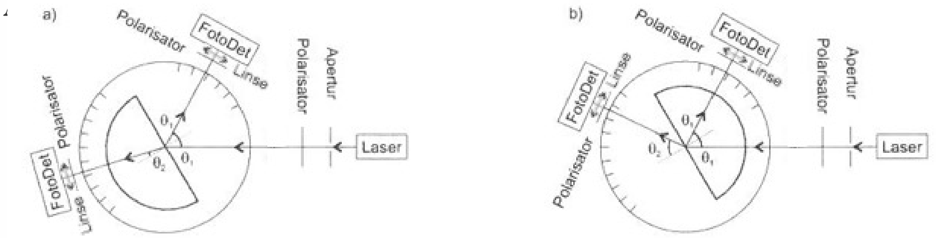

Figure 1: The experimental arrangement. a) Beam from air toward glass. b) Beam from glass toward air. Laser, aperture, polarizer for s- or p-polarized light, lens, semicircular glass, detectors for laser light and in front of those an analyzing polarizer.

The first steps consist of aligning the laser light and the two angular scales.

1. Adjust the large scale so that 0^0 is at the fixed marking.

2. Adjust the scale of the small turntable such that the line near the letter N in $ANGLE$ is at 180^0 on the large scale.

3. Adjust the rotatable polarizer in front of the laser to 45^0, so equal amounts of s- and p-polarized light is transmitted.

4. Turn on the laser. Move the lens to focus the beam at the detector and adjust the beam direction to hit the detector (use 'Horizontal/Vertical Adjust' at the back of the laser). Set Gain=1 at the detector.

5. Maximize the signal from the detector by adjusting the detector angle and 'Vertical Adjust'. The amplifier saturates at about 5 Volts. Choose a smaller slit in case the signal is larger.

PROGRAMME

The four coefficients of transmission and reflection, T_p, T_s, R_p and R_s, are measured as functions of the angle of incidence θ_1.

1. Position the analyzing polarizer in front of the photodetector for horizontal, p, or vertical, s, polarization.

2. Adjust θ_1 by turning the small table with the semicircular glass.

3. Move the detector until a beam is caught (the refracted for $T_{s/p}$ and the reflected for $R_{s/p}$).

4. Fine tune the detector angle for maximum signal $I(\theta_1)$. It may be necessary to adjust also the height of the beam by 'Vertical Adjust'.

- Make a note of $I(\theta_1)$ for the chosen θ_1, polarization (s/p) and beam (T/R). Subtract background. Calculate the appropriate coefficient, which could be $R_s = I(\theta_1)/I(\theta_1 = 90^0)$.

- Make a note also of the detector angle shown on the large scale. From this and θ_1 the angles of refraction θ_2 or reflection may be calculated. The index of refraction n is found from θ_1 and θ_2.

- Compare with theory and discuss potential errors and estimate their sizes.

The Fresnel Relations

THE FOUR FRESNEL RELATIONS

A beam of light moves as shown in Fig. 2 from one dielectric to another. A reflected and a refracted beam are formed at the surface. The angle of refraction follows from Snells law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$, where n_1 og n_2 are indexes of refraction, but there is no information on intensitics or polarizations. This information is found in the Fresnel Relations, which for the the field amplitudes read:

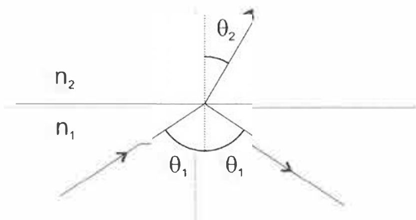

Figure 2: θ_1 is angle of incidence and reflection. θ_2 is angle of refraction.

$$r = \frac{E_1'}{E_1} \quad \text{and} \quad t = \frac{E_2}{E_1} \tag{1}$$

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)} \tag{2}$$

$$t_p = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)} \tag{3}$$

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} \tag{4}$$

$$t_s = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1 + \theta_2)} \tag{5}$$

Here, r and t is amplitude reflection and transmission coefficients, respectively, and the indices p and s stands for parallel and perpendicular (senkrecht) polarizations, respectively.

From these amplitude relations, one finds the corresponding intensity relations for reflection R and transmission T:

$$R = r^2 \quad , \quad T = \frac{\cos\theta_2}{\cos\theta_1}\frac{n_2}{n_1} t^2 \quad \text{and} \quad R + T = 1 \tag{6}$$

$$R_p = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \tag{7}$$

$$T_p = \frac{\sin 2\theta_1 \sin 2\theta_2}{\sin^2(\theta_1 + \theta_2)\cos^2(\theta_1 - \theta_2)} \tag{8}$$

$$R_s = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \tag{9}$$

$$T_s = \frac{\sin 2\theta_1 \sin 2\theta_2}{\sin^2(\theta_1 + \theta_2)} \tag{10}$$

For p-polarized light an important angle exists. It is the Brewster angle at which there is no reflected light. The Brewster angle is given by $R_p(\theta_B) = 0$ or (see (7))

$$\Theta_B + \Theta_2 = \frac{\pi}{2} \ , \tag{11}$$

from which

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) \ . \tag{12}$$

θ_B is marked in Fig. 3.

Another importent angle is the critical angle for total internal reflecsion. It follows from Snell's law and $\theta_2 = \pi/2$ (independent of polarization)

$$\theta_C = \arcsin\left(\frac{1}{n}\right) \ . \tag{13}$$

θ_C is also market in Fig. 3.

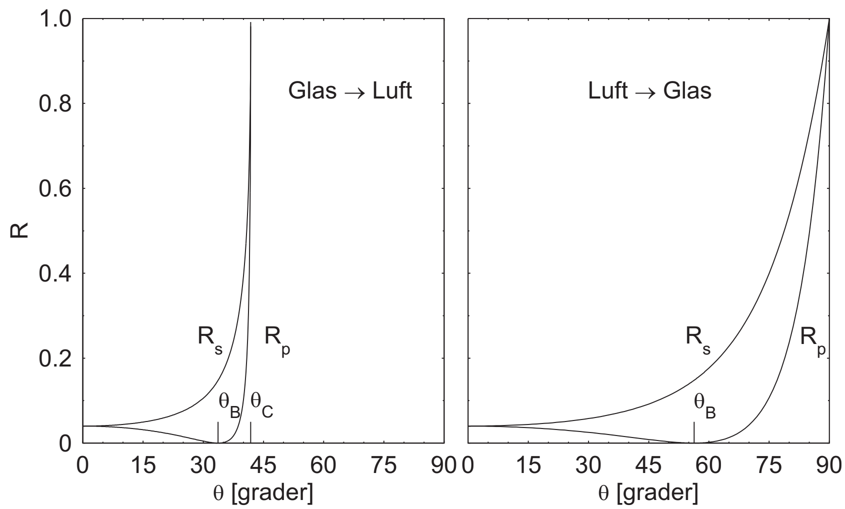

Figure 3: Coefficients of reflection for glass ($n = 1.5$) in air, formulas (7) and (9). Coefficients of transmission follow from $T + R = 1$.

EXAMPLE

Light through glass
$n = 1.5$, s-polarization, I_0 =4mW and $\theta_1 = 30^0$

- Into the glass

$$\begin{array}{ll} \text{Snell's law} : \theta_2 = 19.47^0 \ , & \\ \text{from (10)} \ : T_s = \sin 60^0 \sin 38.94^0 / \sin^2 49.47^0 & = 0.942 \ , \\ \text{from (9)} \ \ \ : R_s = 1 - T_s & = 0.058 \ . \end{array}$$

- Out again

$$\begin{array}{ll} \theta_1 = \theta_2 = 0^0 \ , & \\ \text{from (5)} \ : t_s = 2 \cdot 1.5/(1.5+1) & = 1.20 \ , \\ \text{from (6)} \ : T_s = 1.2^2/1.5 & = 0.96 \ , \\ \text{from (9)} \ : R_s = 1 - T_s & = 0.04 \ . \end{array}$$

- Result

$$\begin{array}{lcl} T_s = 0.942 \cdot 0.96 = 0.905 & : & I_T = 3.62\text{mW}. \\ R_s = 0.058 & : & I_R = 0.23\text{mW}. \\ T_s^{'} = 0.942 \cdot 0.04 \cdot 0.058 \cdot 0.96 = 0.0021 & : & I_T^{'} = 8.40\mu\text{W}. \end{array}$$

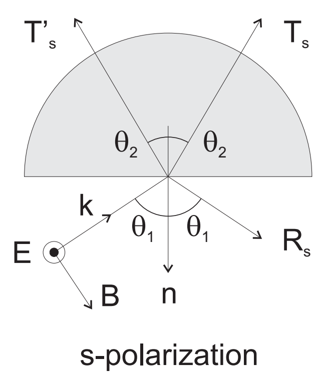

Figure 4: Reflection and transmission.