The Fresnel relations

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Introduction

We examine reflection and refraction of a beam of light on the surface of a transparent dielectric material. In doing so, we determine the refractive index of the dielectric material via. Snell's law, and the coefficients of transmission and reflection for both s- and p-polarized light. With these coefficients, we hope to observe the Brewster angle and the total internal reflection angle.

Theoretical background

The following is but a short review of the more detailed notes[1] by Michael Drewsen. Given the refractive index's of two dielectric materials, n_a and n_b, then the angle of incidence θ_1 is related to the angle of refraction θ_2, going from the first dielectric to the second, via. Snell's law:

$$n_2 \sin(\theta_2) = n_1 \sin(\theta_1) \tag{1}$$

This equation does not give any information on intensities or polarizations. The equations providing this information, is the Fresnel Relations for field amplitudes. From these equations it's possible to arrive at the following relationship

$$R + T = 1$$

With R and T being the coefficients of intensity. The former being for reflection and the latter being for transmission. Note that this not an exhaustive list of relations, but merely highlights. Through the Fresnel Relations one can derive the coefficients for reflection and transmission with regards to the polarisation. These are

$$R_p = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}$$
$$T_p = \frac{\sin(2\theta_1)\sin(2\theta_2)}{\sin^2(\theta_1 + \theta_2)\cos^2(\theta_1 - \theta_2)}$$
$$R_s = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)}$$
$$T_p = \frac{\sin(2\theta_1)\sin(2\theta_2)}{\sin^2(\theta_1 + \theta_2)}$$

For s- and p-polarized light two important angles exists. For p-polarized light, this is the Brewster angle, θ_B, at which there is no reflected light, such that $R_p(\theta_B) = 0$. Given the equation for R_p, this is equivalent to $\theta_B + \theta_2 = \frac{\pi}{2}$. From Snell's law it follows that:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) = n_2 \sin(\frac{\pi}{2} - \theta_B) = n_2 \cos(\theta_B)$$

It then follows directly that:

$$\theta_B = \arctan(\frac{n_2}{n_1}) \tag{2}$$

Independent of polarization, the critical angle for total internal reflection, θ_C, is found via. Snell's law ($\theta_2 = \frac{\pi}{2}$):

$$n_1 \sin(\theta_1) = n_2 \sin(\frac{\pi}{2}) = n_2$$

It then follows directly that:

$$\theta_C = \arcsin(\frac{n_2}{n_1})$$

[1] Michael Drewsen. *Exercise 1 - Fresnel Relations.*

Experimental setup

Equipment

For the experiment one needs the following primary equipment

- 1x Laser source
- 1x Aperture
- 1x Adjustable (angle) polariser
- 2x Focusing lenses
- 1x s-/p-polarising filter (analyser filter)
- 1x Turntable with photodetector with angular scales, one small and one large
- 1x Semicircular glass

Setup outline

The equipment can now be arranged such that the laser first passes through the aperture, adjustable polariser, and first focusing lens before reaching the semicircular glass in the middle of the turntable.
The light passing through the glass (reflected or transmitted) should now pass through another focusing lens and the s-/p-polariser placed in front of the photodetector. The following figure shows the experimental setup

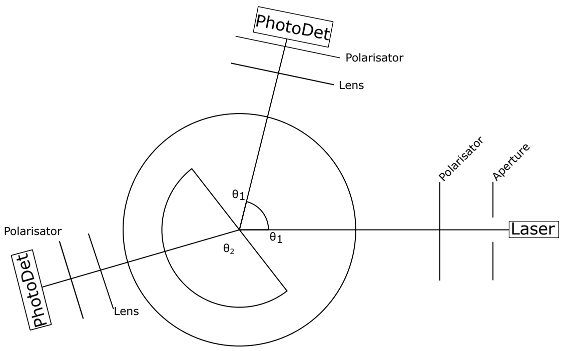

Figure 1: Experimental setup for the case of light passing from air into glass. Note that only one photodetector is needed as it is placed on a turntable.

With the above setup it's but a matter of recording data. One could use picoscope and the build in averaging function. This allows for fast data collection and a resulting uncertainty, and essential eliminating need for tedious data cleanup. This is the method used for this experiment.

Calibrations

Before commencing measurements one should make sure to calibrate the equipment. The main calibration is the adjustable polariser. This should be calibrated such that equals amount of s-polarised and p-polarised light is admitted. Correctly adjustment will allow the analyser filter create equal measurements such that $R + T = 1$ is satisfied. Note this can be quite tricky and can likely lead to a systematic error. This will be discussed further in *Errors and issues encountered*.

Experimental data

Note: *Every data set and analysis scripts can be found on the following GitHub repository.*[2]
In total two series of measurements were carried out. The first measurement was letting light pass from air through glass (sketched in *Setup Outline*), whereas the second measurement was made with the semicircular glass as shown

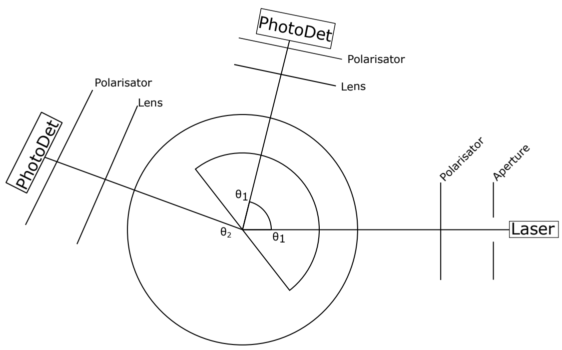

Figure 2: The experimental setup with light traveling through glass initially.

The two series can be measured for two sets of reflected light each with their own characteristics.

Data - Refractive index

Until further comment, data from the air through glass setup is discussed. Via Snell's law,

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

the refractive index n_2 of the dielectric material were found to be,

$$n_2 = n_1 \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

where n_1 is the refractive index of air, such that $n_1 = 1$. For every observation, n_2 is calculated,

[2] Jeppe L. Krogh and Frederik R. Rytter. *https://github.com/fred465f/Forsoeg1*.

and the corresponding error is propagated according to the calculus approximation of error propagation for multi-variable functions,[3] where its assumed that, no correlation between the errors occur. The result is:

$$n_2^{(atg)} = 1.47 \pm 0.49$$

Using the data from the glass through air setup, the following result is achieved:

$$n_2^{(gta)} = 1.42 \pm 0.47$$

The mean of these two results give:

$$n_2 = 1.45 \pm 0.34$$

With the same setup the the critical angle for total internal reflection, θ_C, and the Brewster angle in eq. (2), using $n_2 = 1.45$, is found to be:

$$\theta_C = 43.60° \pm 12.80°$$

$$\theta_B^{(gta)} = 34.60° \pm 6.28°$$

Considering the air through glass setup again, and using $n_2 = 1.47$, the Brewster angle in eq. (2) is found to be:

$$\theta_B^{(atg)} = 55.41° \pm 6.28°$$

These results will be discussed further in *Comparison of plots and angles.*

Data - Coefficients

In the first measurement where light passed from air through glass several points were recorded with the incidence angle ranging from 40° to 80°. Note that one could have certainly carry out measurements with the incidence angle being smaller (In the range of 5° to 35°), but this would hardly yield more useful data as the points do not differ to greatly in this smaller range.
The data collected were essentially measurements of incoming light at various angles. This needed to be converted before meaningful analysis could be carried out. As such the coefficients (Metioned in *Theoretical background*) must be calculated. Chiefly amongst these were R_p (reflected and p-polarised) and R_s (reflected and s-polarised).
This could be calculated simply as

$$R_{p/s} = \frac{I_{p/s}(\theta_1)}{I_{p/s}(\theta_0)}$$

[3]Ifan Hughes and Thomas Hase. *Measurements and their uncertainties: a practical guide to modern error analysis.* OUP Oxford, 2010.

With $I_{p/s}(\theta_0)$ being the intensity measured for p- and s-polarised light, respectively, at an incidence angle of 0. It was here assumed that,

$$I_{p/s}(\theta) \propto V_{p/s}(\theta)$$

where $V_{p/s}(\theta_0)$ is the corresponding voltage measured with PicoScope, such that:

$$R_{p/s} = \frac{I_{p/s}(\theta_1)}{I_{p/s}(\theta_0)} = \frac{V_{p/s}(\theta_1)}{V_{p/s}(\theta_0)}$$

Note that the transmitted coefficients would be calculated in the same manner.
These coefficients plotted the yields the following plots In fig. 3 one finds the estimated Brewster angle being superimposed upon the data points. Qualitatively it's seen that the uncertainties align in such a way, as to suggest that there exists a connection as shown in *Theoretical background*. However it's not immediately clear what value for θ_B is correct. This is considered further in the *Discussion* section.

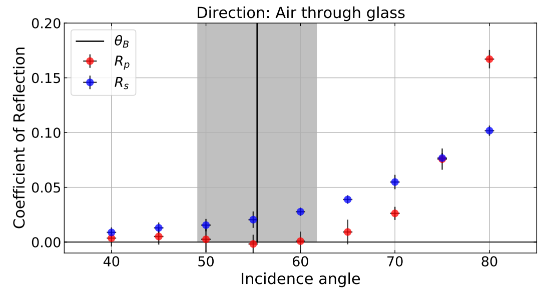

Figure 3: The coefficients of reflected light for p- and s-polarised light passing through air and then glass. Note the interesting Brewster angle θ_B in the range 50° to 60°

For the second measurement (glass through air) the interesting range is set around 15° to 40°. Once again one plots the calculated coefficients

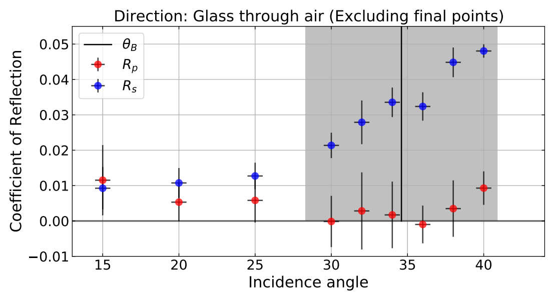

Figure 4: The coefficients of reflected light for p- and s-polarised light passing through glass and then air. The final data point has been omitted.

The complete set of data points would actually give the following plot
In fig.4 one clearly sees the splitting of R_p and R_s as one would expect from the theoretical values, and with the inclusion of fig. 5 a greater

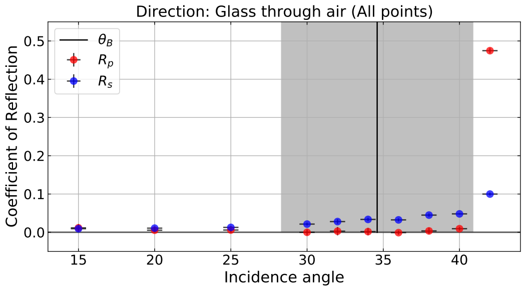

Figure 5: The coefficients of reflected light for p- and s-polarised light passing through glass and then air. The final data point has been included.

picture emerges.
Evidently the sudden rise in R_p is qualitatively indicative of the critical angle of total internal reflection which was calculated in *Data - Coefficients*.

Discussion

From the data collected one cannot state much quantitatively, however it's still possible to do a qualitative analysis and compare the plots with with interesting critical angles.

Comparison of plots and angles

In *Data - Refractive index* several critical angles were calculated. What is worth noting about these is that in the case of the Brewster angle they do appear to be found in the area where one would expect to find them. See the figures *Data - Coefficients*. However, it must be stated clearly that due to uncertainties in the measurements where θ_B is expected, it is exceedingly difficult to accurately pinpoint θ_B. Possible changes to the experiment are elaborated upon in *Errors and issues encountered*. In general it's found that trend one expects to find for the Fresnel Relations does indeed appear. The critical angles found theoretical do also appear in regions where they are expected, but quantitatively these have not been estimated accurately.

Errors and issues encountered

A central issue with the measurement of intensities is the lack of light, or more precisely put the background radiation which masks the very fine measurements. The most clear example is seen in fig. 4. Here it's extremely difficult to assess where θ_B is to be found. Primarily attributed to the relatively far-reaching error bars for R_p.
It's clear from the plot that measurements carried out with values closed to the background radiation will suffer from obscured. In general making it a bit tricky to clearly see the tendencies.
One way of mitigating this could be the isolation of the system to a room with way less outside light sources (lamps, outside light and etc.). This should reduces errors as measurements would more clearly separate from the background.

Conclusion

Given the data, the predicted tendencies, between the coefficients of reflection for p- and s-polarized light respectively, agree with theory, within the errors of the experiment. Specifically the Brewster angle was shown to appear as expected, for p-polarized light, in both experimental setups and estimated to be:

$$\theta_B^{(gta)} = 34.60^\circ \pm 6.28^\circ$$

for the glass through air experimental setup and,

$$\theta_B^{(atg)} = 55.41^\circ \pm 6.28^\circ$$

for the air through glass experimental setup. Likewise, the critical angle for total internal reflection for both polarizations of light, in the glass to air experimental setup, was shown to appear, and estimated to be:

$$\theta_C = 43.60^\circ \pm 12.80^\circ$$

Via the same data, more specifically, the measured angles, it was possible to estimate the refractive index of the dielectric material, that reflected/transmitted the incoming light. The result was:

$$n_2 = 1.45 \pm 0.34$$

Possible improvements to the experiment, are suggested in the Discussion section. Given these improvements, more precise estimations of the above quantities would be possible. This is especially true for the glass through air experimental setup as discussed in Errors and issues encountered.

References

Drewsen, Michael. *Exercise 1 - Fresnel Relations.*

Hughes, Ifan and Thomas Hase. *Measurements and their uncertainties: a practical guide to modern error analysis.* OUP Oxford, 2010.

Krogh, Jeppe L. and Frederik R. Rytter. *https://github.com/fred465f/Forsoeg*$_1$.