Eksperimentel Fysik 2 - Experiment 1

The Fresnel relations

Jeppe Lykke Krogh, 202005568
Frederik Rosendal Rytter, 2020005589

Aarhus Universitet

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Introduction

We examine reflection and refraction of a beam of light on the surface of a transparent dielectric material. In doing so, we determine the refractive index of the dielectric material via. Snell's law, and the coefficients of transmission and reflection for both s- and p-polarized light. With these coefficients, we hope to observe the Brewster angle and the total internal reflection angle.

Theoretical background

The following is but a short review of the more detailed notes[1] by Michael Drewsen. Given the refractive indices of two dielectric materials, n_a and n_b, then the angle of incidence θ_1 is related to the angle of refraction θ_2, going from the first dielectric to the second, via. Snell's law:

$$n_2 \sin(\theta_2) = n_1 \sin(\theta_1) \tag{1}$$

This equation does not give any information on intensities or polarizations. The equations providing this information, is the Fresnel Relations for field amplitudes. From these equations it's possible to arrive at the following relationship

$$R + T = 1$$

With R and T being the coefficients of intensity. The former being for reflection and the latter being for transmission. Through the Fresnel Relations one can derive the coefficients for reflection and transmission with regards to the polarisation. These are

$$\begin{aligned}
R_p &= \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \\
T_p &= \frac{\sin(2\theta_1)\sin(2\theta_2)}{\sin^2(\theta_1 + \theta_2)\cos^2(\theta_1 - \theta_2)} \\
R_s &= \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \\
T_p &= \frac{\sin(2\theta_1)\sin(2\theta_2)}{\sin^2(\theta_1 + \theta_2)}
\end{aligned}$$

For s- and p-polarized light two important angles exists. For p-polarized light, this is the Brewster angle, θ_B, at which there is no reflected light, such that $R_p(\theta_B) = 0$. Given the equation for R_p, this is equivalent to $\theta_B + \theta_2 = \frac{\pi}{2}$. From Snell's law it follows that:

$$\begin{aligned}
n_1 \sin(\theta_1) &= n_2 \sin(\theta_2) \\
&= n_2 \sin(\frac{\pi}{2} - \theta_B) \\
&= n_2 \cos(\theta_B)
\end{aligned}$$

It then follows directly that:

$$\theta_B = \arctan(\frac{n_2}{n_1}) \tag{2}$$

Independent of polarization, the critical angle for total internal reflection, θ_C, is found via. Snell's law ($\theta_2 = \frac{\pi}{2}$):

$$n_1 \sin(\theta_1) = n_2 \sin(\frac{\pi}{2}) = n_2$$

It then follows directly that:

$$\theta_C = \arcsin(\frac{n_2}{n_1}) \tag{3}$$

[1] Michael Drewsen. *Exercise 1 - Fresnel Relations.*

Experimental setup

Equipment

For the experiment one needs the following primary equipment

- 1x Laser source
- 1x Aperture
- 1x Adjustable (angle) polariser
- 2x Focusing lenses
- 1x s-/p-polarising filter (analyser filter)
- 1x Turntable with photodetector with angular scales, one small and one large
- 1x Semicircular glass

Setup outline

The equipment can now be arranged such that the laser first passes through the aperture, adjustable polariser, and first focusing lens before reaching the semicircular glass in the middle of the turntable.
The light passing through the glass (reflected or transmitted) should now pass through another focusing lens and the s-/p-polariser placed in front of the photodetector. Figure 1 shows the experimental setup

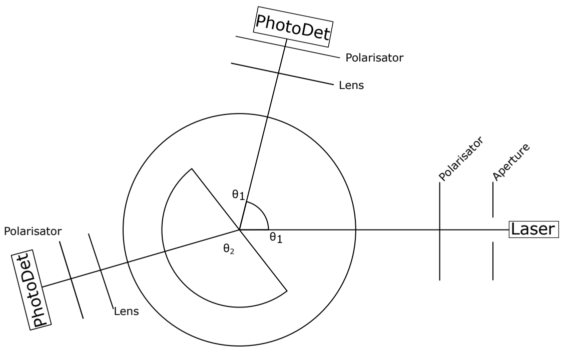

Figure 1: Experimental setup for the case of light passing from air into glass. Note that only one photodetector is needed as it is placed on a turntable.

With the above setup it's but a matter of recording data. One could use picoscope and the build in averaging function. This allows for fast data collection and a resulting uncertainty, and essential eliminating need for tedious data cleanup. This is the method used for this experiment.

Adjustments

Before commencing measurements one should make sure to calibrate the equipment. The main calibration is the adjustable polariser. This should be calibrated such that equals amount of s-polarised and p-polarised light is admitted. Note this can be quite tricky and can likely lead to a systematic error, which can be seen in figure 3, 4, 5 and 6 and will be discussed further in *Errors and issues encountered.*

Experimental data

Note: *Every data set and analysis scripts can be found on the following GitHub repository.*[2]
In total two series of measurements were carried out. The first measurement was letting light pass from air through glass (sketched in figure 1), whereas the second measurement was made with the semicircular glass as shown in figure 2.

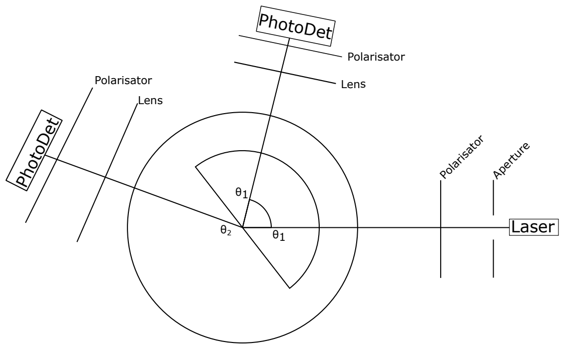

Figure 2: The experimental setup with light traveling through glass initially.

For both series of measurements, the intensities of transmitted and reflected light will be collected, each of which, will be performed for both s- and p-polarized light.

Data - Refractive index

Until further comment, data from the air through glass setup is discussed. By rearranging equation 1

$$n_2 = n_1 \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

where n_1 is the refractive index of air, such that $n_1 = 1$, and n_2 is the refractive index of glass. For every observation, n_2 is calculated, and the corresponding error is propagated according to the calculus approximation of error propagation

[2] Jeppe L. Krogh and Frederik R. Rytter. *https://github.com/fred465f/Forsoeg*$_1$.

for multi-variable functions,[3] where its assumed that, no correlation between the errors occur. The result is:

$$n_2^{(atg)} = 1.47 \pm 0.49$$

Using the data from the glass through air setup, the following result is achieved:

$$n_2^{(gta)} = 1.42 \pm 0.47$$

The mean of these two results give:

$$n_2 = 1.45 \pm 0.34$$

With the same setup the the critical angle for total internal reflection equation 3, θ_C, and the Brewster angle in equation 2, using $n_2 = 1.45$, is found to be:

$$\theta_C = 43.6^\circ \pm 12.8^\circ$$

$$\theta_B^{(gta)} = 34.6^\circ \pm 6.28^\circ$$

Considering the air through glass setup again, and using $n_2 = 1.47$, the Brewster angle in equation 2 is found to be:

$$\theta_B^{(atg)} = 55.4^\circ \pm 6.28^\circ$$

These results will be discussed further in *Comparison of plots and angles.*

Data - Reflection/transmission coefficients

The data collected were essentially measurements of incoming light at various angles. This needed to be converted before meaningful analysis could be carried out. As such the coefficients (Metioned in *Theoretical background*) must be calculated. Chiefly amongst these were R_p (reflected and p-polarised) and R_s (reflected and s-polarised).
This could be calculated simply as

$$R_{p/s} = \frac{I_{p/s}(\theta_1)}{I_{p/s}(\theta_0)}$$

With $I_{p/s}(\theta_0)$ being the intensity measured for p- and s-polarised light, respectively, at an incidence angle of 0. It was here assumed that,

$$I_{p/s}(\theta) \propto V_{p/s}(\theta)$$

where $V_{p/s}(\theta_0)$ is the corresponding voltage measured with PicoScope, such that:

$$R_{p/s} = \frac{I_{p/s}(\theta_1)}{I_{p/s}(\theta_0)} = \frac{V_{p/s}(\theta_1)}{V_{p/s}(\theta_0)}$$

[3] Ifan Hughes and Thomas Hase. *Measurements and their uncertainties: a practical guide to modern error analysis.* OUP Oxford, 2010.

Note that the transmitted coefficients would be calculated in the same manner.

In figure 3 and figure 4 the reflection coefficients for s- and p-polarized light has been plotted against the theoretical reflection coefficients. Note that the theoretical R_ss and R_ps has been calculated with a refractive index of $n = 1.5$ as is expected for glass. From figure

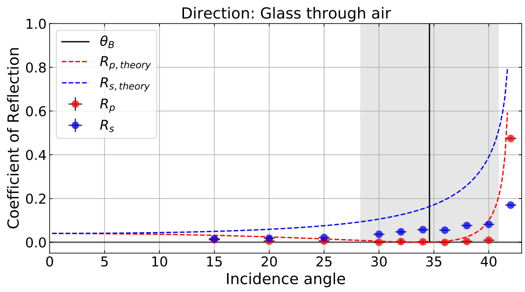

Figure 3: Plot showing reflection coefficients of "Glass-through-air"-setup superimposed on the theoretical values via Fresnel relations. Note also the Brewster angle with it's uncertainty (shaded area).

4 one finds a disconnect the measurements of R_s and it's theoretical values. This cannot solely be due to a difference in refractive indices but indicates a systematic error. This will be examined further in *Discussion.*

In figure 4 on sees that the trends for both R_p and R_s seem to follow the theoretical trend. However, there is also a clear, systematic offset. This will discussed further in *Discussion*

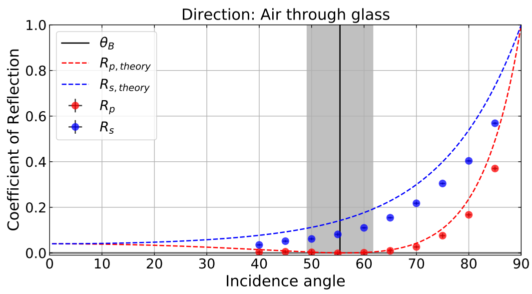

Figure 4: Plot showing reflection coefficients of "Air-through-glass"-setup superimposed on the theoretical values via Fresnel relations. Note also the Brewster angle with it's uncertainty (shaded area).

In figure 5 and figure 6 transmission coefficients has similarly been plotted.
In figure 5 the measured coefficients do not match the theoretical values, and one would be hardpressed to say that they even follow the theoretical trend. This will be examined further in *Discussion*

In figure 6 the measured transmission coefficients do not agree with the theoretical values

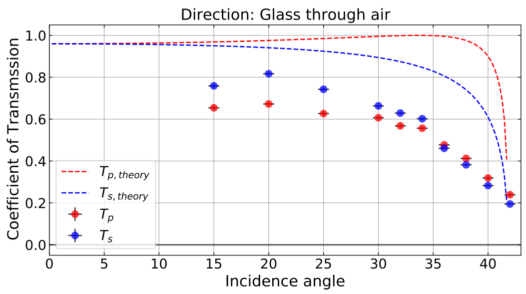

Figure 5: Plot showing transmission coefficients of "Glass-through-air"-setup superimposed on the theoretical values via Fresnel relations.

at all. This disagreement must stem from an error made during measurements. This will be examined further in *Discussion*

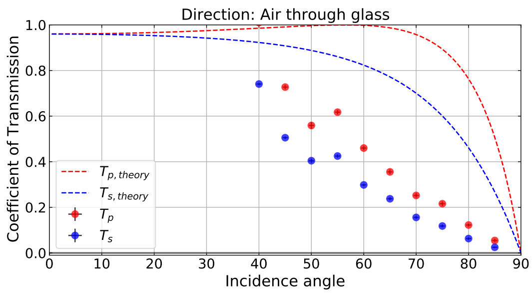

Figure 6: Plot showing transmission coefficients of "Air-through-glass"-setup superimposed on the theoretical values via Fresnel relations.

Discussion

From the data collected one cannot state much quantitatively, however it's still possible to do a qualitative analysis and compare the plots with with interesting critical angles.

Comparison of plots and angles

In *Data - Refractive index* several critical angles were calculated. What is worth noting about these is that in the case of the Brewster angle they do appear to be found in the area where one would expect to find them. See the figures 3 and 4. However, it must be stated clearly that due to uncertainties in the measurements where θ_B is expected, it is exceedingly difficult to accurately pinpoint θ_B. Possible changes to the experiment are elaborated upon in *Errors and issues encountered.*
In *Data - Reflection/transmission coefficients* one finds that the data points to a lesser extend agrees with the theoretical values. However, there is also a clear offset for both s- and p-polarized light. This is is attributable to an error made in adjustments. When adjustments was done the measurements of intensity at the angle of incidence being $\theta_1 = 0$, from here on $I(\theta_0)$, gave the following:

$$I(\theta_0)_p = 1.98 \pm 0.02V$$
$$I(\theta_0)_s = 3.53 \pm 0.01V$$

Clearly show that not equal amounts of light were being shone at the detector. This seems to be the culprit of the disagreement between measurements and theory.

Errors and issues encountered

In light of the discussion order to improve the measurements one should be more careful in carrying out the adjustments. Had one been careful to make sure that $I(\theta_0)_p = I(\theta_0)_s$ then the coefficients of reflection would have aligned better with the theoretical values. Especially the "Glass-through-air" would have benefited from this considerably.
Both setups for the coefficients of transmission could also have been improved upon with this change.
In short this has most likely been the greatest error in the experiment.
Another issue with the measurement of intensities is the lack of light, or more precisely put the background radiation which masks the very fine measurements. One way of mitigating this could be the isolation of the system to a room with way less outside light sources (lamps, outside light and etc.). This should reduces errors as measurements would more clearly separate from the background.

Conclusion

Given the data, the predicted tendencies, between the coefficients of reflection/transmission for p- and s-polarized light respectively, it can be concluded that systematic errors have occurred, because of the fundamental error in adjusting the polarizer such that, equal amounts of p- and s-polarized light passes through the dielectric, as specified in *Errors and issues encountered.*
The Brewster angle was shown to appear as expected, for p-polarized light, in both experimental setups and estimated to be:

$$\theta_B^{(gta)} = 34.60° \pm 6.28°$$

for the glass through air experimental setup and,

$$\theta_B^{(atg)} = 55.41° \pm 6.28°$$

for the air through glass experimental setup. Likewise, the critical angle for total internal reflection for both polarizations of light, in the glass to air experimental setup, was shown to appear, and estimated to be:

$$\theta_C = 43.60° \pm 12.80°$$

Via the same data, more specifically, the measured angles, it was possible to estimate the refractive index of the dielectric material, that reflected/transmitted the incoming light. The result was:

$$n_2 = 1.45 \pm 0.34$$

References

Drewsen, Michael. *Exercise 1 - Fresnel Relations.*

Hughes, Ifan and Thomas Hase. *Measurements and their uncertainties: a practical guide to modern error analysis.* OUP Oxford, 2010.

Krogh, Jeppe L. and Frederik R. Rytter. $https://github.com/fred465f/Forsoeg_1$.