Exercise 2: The Michelson interferometer

Aims:

1. Set up a Michelson interferometer.

2. Determination of the expansion coefficient of a piezoelectric element

3. Study the effect of intensity differences of the interferometer arms

Basic theory of the Michelson interferometer

Before building the interferometer, we are first going to examine the underlying basic theory. The generic situation is schematically presented in Fig. 1.

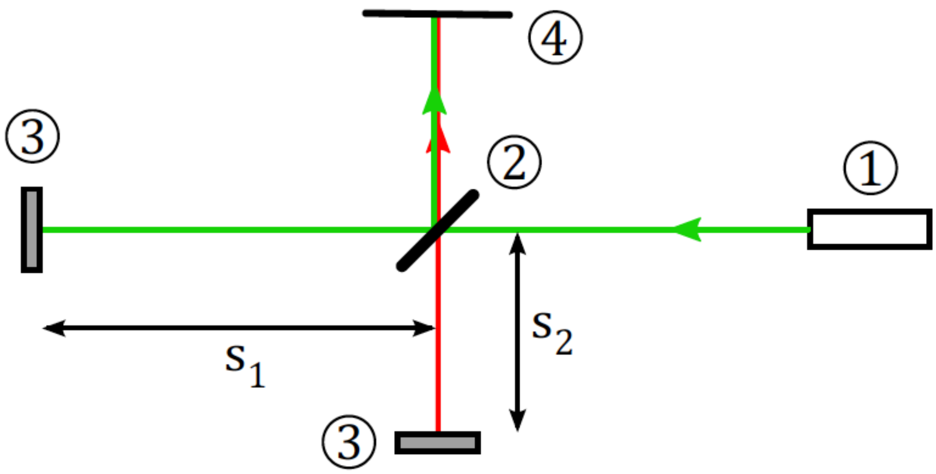

Figure 1: Sketch of a Michelson interferometer. The beam from the laser (1) is aimed at the beam splitter (2) which divides the beam into two partial beams. The two beams are reflected by the mirrors (3). An interference pattern can be observed on the screen/detector (4).

The laser beam is divided by a typically 50:50 beam splitter (2), and the partial beams reflected by the mirrors (3) overlap again at the beam splitter, where half the light travels back in the direction of the laser and the other half towards a screen/detector (4). Mathematically, we can describe how the light intensity on the screen/detector depends on the effective path length difference Δs between the two paths S_1 and S_2. We limit ourselves to examining an incident plane wave along the optical axis:

$$\boldsymbol{E_i} = \boldsymbol{E_0} \cos(\omega t - kx) \tag{1}$$

Here ω is the angular frequency, t the time, k the wave number (i.e., $2\pi/\lambda$) and x the local spatial variable. In the following example, we represent the transmission of the beamsplitter with T and the reflection with R. Now let us examine the amplitude of the partial wave of one interferometer arm at the location of the screen/detector:

$$|\boldsymbol{E_1}| = \sqrt{R \cdot T} \cdot E_0 \cdot \cos(\omega t + \varphi_1) \tag{2}$$

Here φ_1 is the phase, the value of which is established by the actual optical path. The factor $\sqrt{R \cdot T}$ is therefore explained because the beam in path 1 is first transmitted and then reflected. The description of the beam in path 2 is similar, but the beam is first reflected and then transmitted. This results in the same factor and the amplitude of the partial wave of the second interferometer arm is given on the screen/detector by

$$|\boldsymbol{E_2}| = \sqrt{R \cdot T} \cdot E_0 \cdot \cos(\omega t + \varphi_2) \tag{3}$$

where φ_2 is the corresponding phase for the second path. The intensity on the screen/detector is then determined by

$$I = c\,\varepsilon_0\,|\boldsymbol{E_1} + \boldsymbol{E_2}|^2 = c\,\varepsilon_0\,R\,T\,E_0^2\,[\cos(\omega t + \varphi_1) + \cos(\omega t + \varphi_2)]^2. \tag{4}$$

Naturally, we only perceive the temporal averaging of the light field oscillation on the screen, so that only the averaging

$$\frac{1}{2\pi}\int_0^{2\pi} [\cos(\omega t + \varphi_1) + \cos(\omega t + \varphi_2)]^2\, d(\omega t) = 1 + \cos(\varphi_1 - \varphi_2) \tag{5}$$

is detectable in the observed intensity. Furthermore, we are going to assume that both the transmission and the reflection have the value of 0.5, which is a good approximation for the beam splitter being used. As the average intensity, we therefore find $\bar{I}$

$$\bar{I} = \tfrac{1}{4} c\,\varepsilon_0\,E_0^2\,(1 + \cos\Delta\varphi), \tag{6}$$

where the phase difference of the two partial waves translates directly into the path length difference Δs between them:

$$\Delta\varphi = \frac{2\pi}{\lambda}\,\Delta s \tag{7}$$

Therefore, the intensity dependence on the path length difference between the two interferometer-arms is described by a cosine function as shown in Fig. 2.

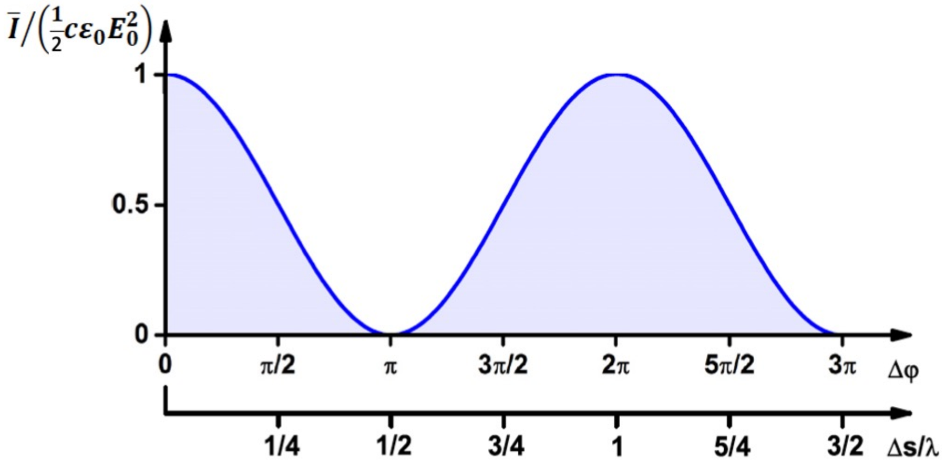

Figure 2. Normalized intensity distribution on the screen depending on the path length difference

It should be noted that the effective path difference Δs depends on the difference in the physical paths as well as on the index of refraction.

Components and equipment:

1. Breadboard with optical elements, HeNe-laser and photo-detector

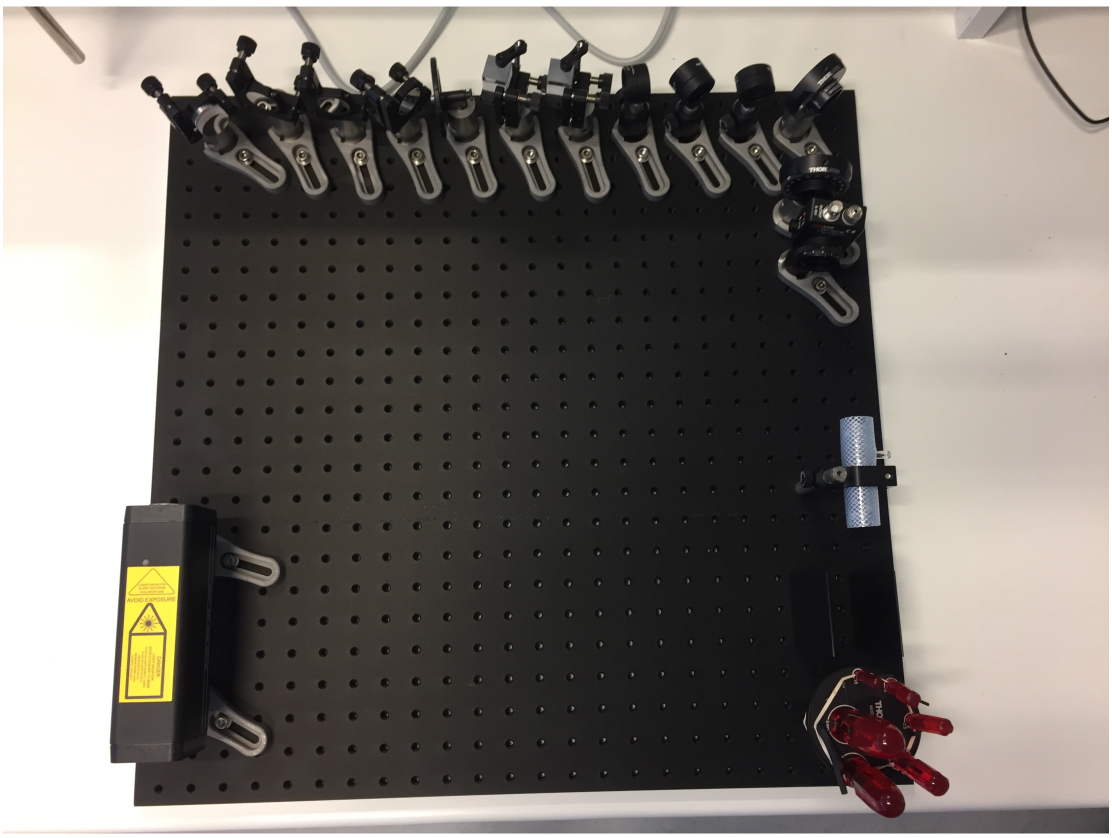

Figure 3: Optical components for the exercise

The collection of optical elements consist of 4 mirrors, where one is mounted on a piezo element, three lenses (f=50 mm, -50 mm and 150 mm), two beam splitter cubes, one $\lambda/4$ plate, one linear polarizer, a neutral density filter wheel, a glass cell. In addition there are two black "beam dumping screens" to shield potential stray laser beams. Power supplies for the laser and the photo-detector, as well as a set of screwdrivers for moving and fixing optics are to be found on the breadboard.

2. Computer with Pico-Scope, function generator, and piezo-driver

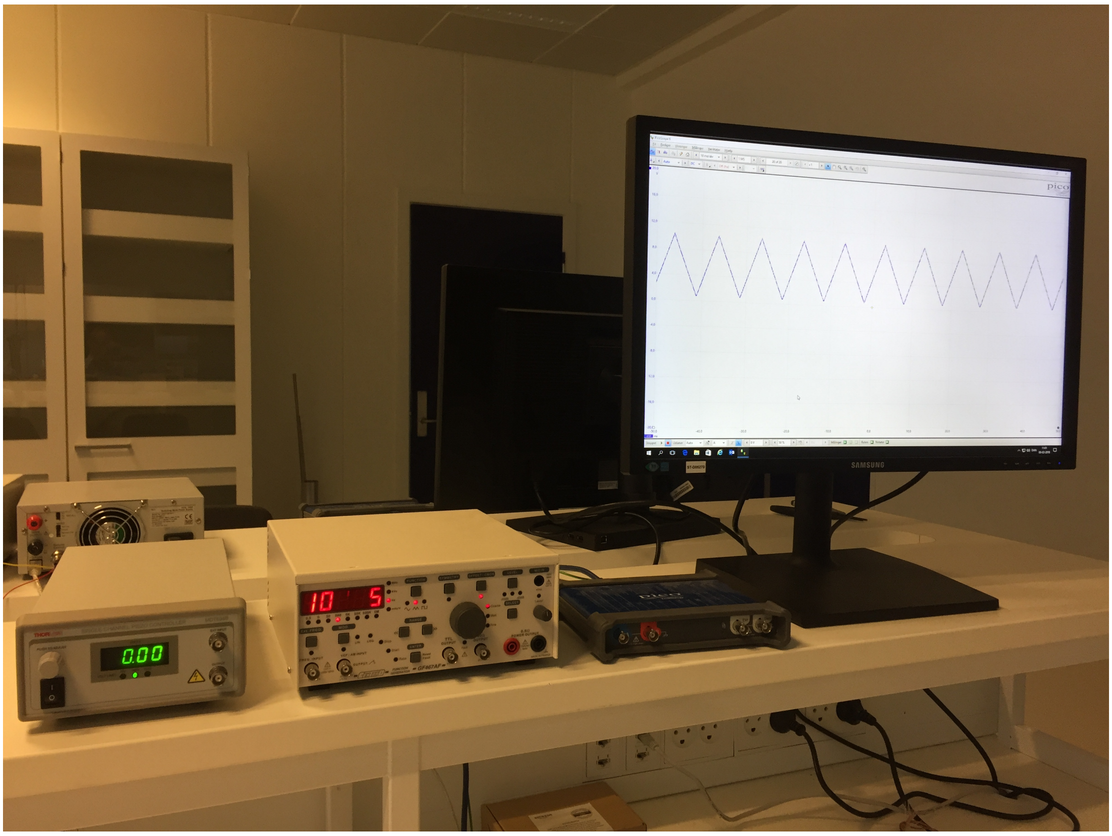

Figure 4: Electronic devices for the exercise

In addition to the devices shown in the figurer above, the setup includes four cables: 2 standard BNC-cables (+ a BNC T-connector) for hooking up the function generator to the Pico-Scope and the piezo-driver, and two flexible cables for connecting the detector to the Pico Scope and the piezo-actuated mirror to the piezo-driver, respectively.

Program for the exercise

1. Discuss with your team mate(s) and Teaching Assistant (TA) how you can setup a Michelson interferometer with the optics available, and how it can be used to measure the expansion coefficient of a piezoelectric element mounted

behind one of the mirrors.
2. Make a detailed sketch of the setup you want to construct and discuss it with your TA.
3. Through interacting with your team mate(s) and TA discuss how to setup the electronics part of the setup, and make a sketch of it.
4. Reflect on the measurement procedure you want to apply, and discuss it with the TA.
5. When cleared by the TA, start setting up the interferometer.

6. Carry out the experiments characterizing the piezo. How do the results match the expectations (see data sheet for the piezo applied and https://en.wikipedia.org/wiki/Piezoelectricity for details on the piezoelctrical effect)
7. Reflect on how small distance changes one should be able to measure.

8. Use the neutral density filter wheel and investigate the interferometer signal versus intensity imbalance of the interferometer arms.
9. Based on the theory discussed above for equal arm intensities, establish an expression for the interferometer signal you observe.

Practical/technical notes

1. The optical breadboard with all its components is very heavy, so take care when taking it out of the cabinet and back again!
2. Remember laser light can be harmful, so be careful also with parasitical beams!
3. Remember not to touch any optics on the surfaces on which light in impinging!
4. Do only apply voltages in the range 0-10V to the control input of the piezo-driver, and do not drive it at a frequency of more than 200 Hz. Hence, check and adjust the output of the function generator with the Pico Scope before connecting it to the piezo-driver. The voltage delivered to the piezo should be a factor of 10 higher than the control voltage (check backside of the piezo-driver).
5. Remember to take clear pictures of you various setups, including the electronic wiring.
6. At the end of each experimental session, remember to safely fix *all* the optical elements to the breadboard in positions similar to those on Fig. 1., and bring *everything* back in good order in the cabinets!

Historical notes on the Michelson interferometer

The Michelson Interferometer, first developed by Albert Michelson in 1881, has proved of vital importance in the development of modern physics. This versatile instrument was used to establish experimental evidence for the development for the validity of the special theory of relativity, to detect and measure hyperfine structure in line spectra and to provide a substitute standard for the meter in terms of wavelengths of light. Most recently Michelson interferometers with kilometer long arms have been used to directly detect gravitational waves for the first time ever within the collaboration LIGO.