Fredah Kioko-103428   CAT 1 PROBABILITY AND STATISTICS 2A

Kioko Fredah Ndila 10342

CAT One

Question One

i) $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\int_0^1 c(1-x)^9\,dx$ let $u = (1-x)$ and $\frac{du}{dx} = -1$ or $dx = -du$

$$\int_0^1 c(1-x)\,du = \int_0^1 -c(u)^9\,du = \left[\frac{-cu^{10}}{10}\right]_0^1 = \left[\frac{c(1-x)^{10}}{10}\right]_0^1$$

$$= \left[\frac{c(1-1)^{10}}{10}\right] - \left[-\frac{c(1-0)^{10}}{10}\right]$$

$$\frac{c}{10} = 1 \qquad c = 10$$

$\frac{}{10} = 1 \qquad c = 10$

ii)

Mean.

$$M = \int_0^1 10x(1-x)^9\,dx = 10\int_0^1 (x-x^2)^9$$

$\left[\cancel{\frac{10(x)}{10} - 10(x^2)} \right.$ ~~let u be x - x²~~ and

$$10\int_0^1 x - 9x^2 + 36x^3 - 84x^4 + 126x^5 - 126x^6 + 84x^7 - 36x^8 + 9x^9 - x^{10}\,dx$$

$$10\left[\frac{x^2}{2} - \frac{9x^3}{3} + \frac{36x^4}{4} - \frac{84x^5}{5} + \frac{126x^6}{6} - \frac{126x^7}{7} + \frac{84x^8}{8} - \frac{36x^9}{9} + \frac{9x^{10}}{10} - \frac{x^{11}}{11}\right]_0^1$$

$$10\left(\frac{1}{2} - 3 + 9 - \frac{84}{5} + \frac{21}{2} - 4 + \frac{9}{10} - \frac{1}{11}\right) = \frac{1}{11} = 0.1$$

iii) $P(0 \leq x \leq 0.02)$

$$= 10\int_0^{0.02} \cancel{10}(1-x)^9 \, dx$$

let u be $(1-x)$ $\qquad dx = \underline{-du}$

$\cancel{10}\int$

$$10\int_0^{0.02} \cancel{10}(u)^9 - \cancel{du}\, du$$

$$10\int_0^{0.02} -(u)^9 \, du$$

$$-10\left[\frac{u^{10}}{10}\right] = -10\left[\frac{(1-x)^{10}}{10}\right] = 10 \times 0.018292$$

$$= 0.18292.$$

Question Two

$$f(t) = \begin{cases} \frac{1}{4}e^{-0.55t} & t \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

Question Two

$$f(t) = \begin{cases} \frac{1}{4} e^{-0.55t} & t \geq 0 \quad \cdots\cdots \quad 0 \leq t \leq \infty \\ 0 & \text{elsewhere} \end{cases}$$

(i) Never out of Service

$$P(0) = \int_0^{t} \frac{1}{4} e^{-0.55t}\, dt = \frac{1}{4}\left[\frac{-e^{-0.55t}}{0.55}\right]_0^{0}$$

$$\frac{1}{4}\left[\frac{e^{-0.55t}}{-0.55}\right]_0^{0}$$

$$\cancel{-e^{-0.55(t)} + e^{-0.55(0)} = 1 - e^{-0.55t}}$$

$$\cancel{F(0) = \left(\frac{1}{0.05} - e^{-0.05(0)}\right) \quad 0.0125}$$

$$\cancel{1 - 1 = 0}$$

$$\cancel{0.0125 - 0.0125 e^{-0.05(0)}}$$

$$= \cancel{0.0125}$$

$$= \frac{1}{4}\left[\frac{e^{0}}{-0.55}\right]$$

$$= \frac{1}{4}\left[-\frac{1}{0.55}\right]$$

$$= 0.4545$$

ii) not out of service for more than 6 days

$$P(t \geq 6) = 1 - P(t \geq 6)$$

$$1 - \frac{1}{4}\int_0^6 e^{-0.55t}$$

$$1 - \frac{1}{4}\left[\frac{e^{-0.55t}}{-0.55t}\right] = 1 - \frac{1}{4}\left[\frac{e^{-0.55(6)}}{-0.55}\right] - \frac{e^{-0.55(0)}}{-0.55}$$

$$0.06706 + \frac{1}{0.55}$$

$$= 1.8852 \times \frac{1}{4}$$

$$= 0.4713$$

$$1 - 0.4713$$

$$= 0.5287.$$

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iii) b) out of service $P(t > 8)$

$$\frac{1}{4}\int_0^8 e^{-0.55t} = \frac{1}{4}\left[\frac{e^{-0.55t}}{-0.55}\right]_0^8 = \frac{1}{4} \times \left\{\frac{e^{-0.55(8)}}{-0.55} - \frac{e^{-0.55^0}}{0.55}\right\}$$

∞

$$\frac{1}{4} \times \left[-0.02232 + \frac{1}{0.55}\right]$$

$$\frac{1}{4} \times 1.79586$$

$$= 0.44896$$

$= 0.5287.$

iii) P(X> out of service $P(t > 8)$

$$\frac{1}{4}\int_0^8 e^{-0.55t} = \frac{1}{4}\left[\frac{e^{-0.55t}}{-0.55}\right]_0^8 = \frac{1}{4}\left\{\frac{e^{-0.55(8)}}{-0.55} - \frac{e^{-0.55^0}}{0.55}\right\}$$

$$\frac{1}{4} \times \left[-0.02232 + \frac{1}{0.55}\right]$$

$$\frac{1}{4} \times 1.79586$$

$$= 0.44896$$

iv) Mean

$$= \frac{1}{4}\int te^{-0.55t}\,dt = \left(\frac{1}{2}\right)$$

$$H = E(x) = \int_0^\infty \lambda x e^{-\lambda x}\,dx = H = \frac{1}{\lambda}$$

$$= \frac{1}{4} e^{-0.55(t)}$$

$\lambda = 0.55$