BINOMIAL DISTRIBUTION

For n trials in each of which there are any two outcomes with constant probabilities q (for failure) and p (for success) respectively, the probabilities of exactly x success is given by

$$P(x) = {}^nC_x p^x q^{n-x}, \qquad x = 0, 1, 2, 3, ..., n$$

$P(x)$ is the coefficient of t^x in Probability Generating Function (PGF)

$$G(t) = \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} t^x$$

$$= \sum_{x=0}^{n} {}^nC_x q^{n-x} p^x t^x$$

$$= \sum_{x=0}^{n} {}^nC_x q^{n-x} (pt)^x$$

$$\therefore \text{G(t)} = (q + pt)^n$$

i.e. $G(t) = (q + pt)^n$ for binomial distribution.

NOTE: Putting $t = 1$ in in PGF for binomial distribution.

$$G(1) = (q + p)^n$$

Example 1

Use the PGF to show that for binomial distribution

(i) $\mu = E(x) = np$

(ii) $\text{var}(x) = npq$

Solution

(i) $G(t) = (q + pt)^n$

Differentiating $G(t)$ w.r.t $'t'$

$$G'(t) = n(q + pt)^{n-1}(p)$$

$$= np(q + pt)^{n-1}$$

Putting $t = 1$

$$G'(1) = np(q + p)^{n-1}$$

$$= np(1)^{n-1} \qquad \text{since } q + p = 1$$

$$= np$$

$$\therefore \mu = E(x) = np$$

Or $G(t) = (q + pt)^n$

Let $u = q + pt$, then $G = u^n$

$\frac{du}{dt} = p$ and $\frac{dG}{du} = nu^{n-1}$

Then by chain rule $\frac{dG}{dt} = \frac{dG}{du} \times \frac{du}{dt} = nu^{n-1} \times p = npu^{n-1} = np(q + pt)^{n-1}$

$\therefore G'(t) = np(q + pt)^{n-1}$

Putting $t = 1$ in $G'(t)$ gives

$G'(1) = np(q + p)^{n-1}$

$= np(1)^{n-1}, \quad \text{since} \quad q + p = 1$

$\therefore \mu = E(x) = np$

(ii)

$G(t) = (q + p)^n$

$G'(t) = np(q + pt)^{n-1}$

$G''(t) = np(n-1)(q + pt)^{n-1-1}(p)$

$G''(t) = np^2(n-1)(q + pt)^{n-2}$

Putting $t = 1$ in $G'(t)$ and $G''(t)$ gives

$$G'(1) = np(q + p)^{n-1} = np(1)^{n-1} = np$$

$$G''(1) = np^2(n-1)(q + p)^{n-2} = np^2(n-1)(1)^{n-2} = np^2(n-1) = n^2p^2 - np^2$$

$Var(x) = G'(1) + G'(1) - [G'(1)]^2$

$= n^2p^2 - np^2 - np^2 - (np)^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$= np - np^2$

$= np(1 - p)$

$\therefore Var(x) = \sigma^2 = npq \quad \text{since} \quad p + q = 1$

Assumptions o Binomial distribution

1. The number of trials or n is finite and fixed. The performance must be repeated for a fixed number of times.
2. There must be only two possible outcomes of the event which are mutually exclusive and exhaustive. For example, when we toss a coin once there are only two possible outcomes Head or Tail and one of them must happen.
3. The probability of the happening of the event (or success) in any trial of the event is constant. Similarly, the probability of not happening of the event (or failure) should also be constant.

4. All trials must be independent of each other. The result of any trial should not be affected by the result of a previous or subsequent trials.

Example

1. In the town of Lodwar the probability that an inhabitant catches malaria during any month is 0.2. A firm employees 15 local people.
   (i) Determine the mean and variance of the number of employees catching malaria in any month.
   (ii) Determine the probability that exactly three employees catch malaria during any month.
   (iii) Determine the probability that more six employees catch malaria during any month.
   (iv) Determine the probability that at least two employees catch malaria during any month
   (v) Determine the probability that at most three employees catch malaria during any month.

Solution

(i) $p = 0.2$ **,** $q = 0.8$**,** $n = 15$

$$mean = E(x) = np = 15 \times 0.2 = 3$$

$$Var(x) = npq = 15 \times 0.2 \times 0.8 = 2.4$$

(ii) $$P(x) = {}^{n}C_x \, p^x q^{n-x}, \qquad x = 0, 1, 2, 3, \ldots, n$$

$$P(exactly 3) = P(x = 3) = {}^{15}C_3 \times 0.2^3 \times 0.8^{12} = 0.250$$

(iii)

$$P(more\ than\ 6) = P(x > 6) = P(6) + P(7) + \ldots.. + P(15)$$

Or $P(x > 6) = 1 - P(x \le 6) = 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)]$

$$P(0) = {}^{15}C_0 \times 0.2^0 \times 0.8^{15} = 0.0351844$$

$$P(1) = {}^{15}C_1 \times 0.2^1 \times 0.8^{14} = 0.131941$$

$$P(2) = {}^{15}C_2 \times 0.2^2 \times 0.8^{13} = 0.230897$$

$$P(3) = {}^{15}C_3 \times 0.2^3 \times 0.8^{12} = 0.250139$$

$$P(4) = {}^{15}C_4 \times 0.2^4 \times 0.8^{11} = 0.187604$$

$$P(5) = {}^{15}C_5 \times 0.2^5 \times 0.8^{10} = 0.103182$$

$$P(6) = {}^{15}C_6 \times 0.2^6 \times 0.8^9 = 0.042993$$

$$Total \quad probability = 0.981940$$

$$\therefore Required probability = 1 - 0.981940 = 0.01806$$

(iv)

$$P(at \ least\, 2) = P(x \geq 2) = 1 - P(x < 2) = 1 - [P(0) + P(1)]$$
$$= 1 - [0.0351844 + 0.131941]$$
$$= 0.8328746$$

(v)

$$P(at\, most\, 2) = P(x \leq 2) = P(0) + P(1) + P(2) = 0.351844 + 0.131941 + 0.23897$$
$$= 0.3980224$$

2. The probability that an evening college student will graduate is 0.4. Determine the probability that out of 5 students.
   (i) None will graduate
   (ii) One will graduate
   (iii) At least one will graduate.

Solution

(i) $n = 5$, $p = 0.4$, $q = 0.6$

$$P(x) = {}^nC_x p^x q^{n-x}, \qquad x = 0, 1, 2, 3, ..., n$$
$$\mathrm{P(none)} = P(0) = {}^5C_0 \times 0.4^0 \times 0.6^5$$
$$= 0.046$$

(ii)

$$P(x) = {}^nC_x p^x q^{n-x}, \qquad x = 0, 1, 2, 3, ..., n$$

$$P(1) = {}^5C_1 \times 0.4^1 \times 0.6^4$$
$$= 0.2592$$

(iii)

$$P(at \ least \ 1) = P(x \geq 1) = 1 - P(x < 1) = 1 - P(0) = 1 - 0.046 = 0.954$$

3. The probability of a man hitting a target is $1/4$. How many times must he fire so that the probability of his hitting the target at least once is greater than $2/3$.

Solution

$\mathrm{P} = 1/4$, $\mathrm{P}(\mathrm{x} \geq 1) > 2/3$

To find $\mathrm{n} = ?$

$\mathrm{P}(\mathrm{x} \geq 1) = 1 - P(0) > 2/3$

i.e. $1-{}^{n}C_{0}\times\left(\frac{1}{4}\right)^{0}\times\left(\frac{3}{4}\right)^{n}>\frac{2}{3}$ [Hitting the target is success]

$$1-(0.75)^{n}>{}^{2}\!/_{3}$$

$$-(0.75)^{n}>-{}^{1}\!/_{3}$$

Or

$$(0.75)^{n}<\ 0.3333$$

$$\log(0.75)^{n}<\log 0.3333$$

$$n\log 0.75<\log 0.3333$$

$$n(-0.1249)<-0.47716$$

$$n>\frac{0.47716}{0.1249}$$

$$n>3.82$$

$$\therefore n=4$$

4. The expected number of non-defective bolts in a box is 8, and the variance is 1.6. Find the probability that there is only one non-defective bolt in the box.

Solution

$$mean=\mu=E(x)=np$$

$$np=8 \quad \textbf{……..} \text{ (i)}$$

$$\mathrm{Var}(\mathrm{x})=npq$$

$$\mathrm{npq}=1.6 \quad \text{……. (ii)}$$

Substituting $\mathrm{np}=8$ into equation (ii)

$$8\mathrm{q}=1.6$$

$$\mathrm{q}=0.2,\quad \text{then } \mathrm{p}=0.8$$

But $\mathrm{np}=8$ or $0.8\mathrm{n}=8$ or $\mathrm{n}=10$

$$P(x)={}^{n}C_{x}p^{x}q^{n-x},\qquad x=0,1,2,3,\ldots,n$$

$$P(1)={}^{10}C_{1}\times 0.8^{1}\times 0.2^{9},\qquad x=0,1,2,3,\ldots,10$$

$$=0.00004096$$

5. Eight coins are thrown simultaneously. Show that the probability of obtaining at least 6 heads is ${}^{37}\!/_{256}$.

Solution

$$\mathrm{n}=8,\quad \mathrm{P}={}^{1}\!/_{2},\quad \mathrm{q}={}^{1}\!/_{2}$$

$$\text{P}(\text{at least } 6) = P(x \geq 6) = P(6) + P(7) + P(8)$$

$$= {}^8C_6 \times \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2 + {}^8C_7 \times \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_8 \times \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$$

$$= \frac{28}{256} + \frac{8}{256} + \frac{1}{256}$$

$$= \frac{37}{256}$$

6. Assuming the probability of a male birth as $1/2$, find the probability that a family of 3 children will have
   (i) At least one girl
   (ii) Two boys and one girl
   (iii) At most two girls

Solution

(i) $\text{n} = 3$, $\text{P} = \frac{1}{2}$, $\text{q} = \frac{1}{2}$

$$\text{P}(\text{x} \geq 1) = 1 - P(x < 1) = 1 - P(0)$$

$$= 1 - {}^3C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^3$$

$$= 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

(ii) $$\text{P}(2 \text{ boys and } 2 \text{ girls}) = {}^3C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^1 = \frac{3}{8}$$

(iii)

$$\text{P}(\text{at most } 2 \text{ girls}) = P(x \leq 2) = P(0) + P(1) + P(2)$$

$$= {}^3C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^3 + {}^3C_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^2 + {}^3C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^1$$

$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$$

$$= \frac{7}{8}$$

7. A man takes a step forward with a probability 0.4 and backwards with probability 0.6. Find the probability that at the end of 11 steps he is one step a way from the starting point.

Solution

The man will take either 6 steps forward and 5 backwards.

Required probability

$$={}^{11}C_6 \times 0.4^6 \times 0.6^5 + {}^{11}C_5 \times 0.4^5 \times 0.6^6$$
$$= 0.367873228$$
$$= 0.3679$$

Exercises

1. Four coins are tossed simultaneously, what is the probability of getting
   (i) 2 heads and 2 tails
   (ii) At least 2 heads
   (iii) At least 1 head

   Answer [$3/8$, $11/16$, $15/16$]

2. Three percent of a given lot of manufactured parts are defective. What is the probability that in a sample of four items none will be defective?

   Answer [0.885]

3. The incidence of occupational disease in an industry is such that the workmen have a 20% chance of suffering from it. What is the probability that out of six workmen 4 or more will contract the disease?

   Answer [0.016]

4. A box contains 100 transistors, 20 of which are defective. 10 are selected for inspection. Indicate what is the probability that
   (i) All 10 are defective
   (ii) All 10 are good
   (iii) At least 1 is defective
   (iv) At most 3 are defective

   Answer $\left(\frac{1}{5}\right)^{10}$, $\left[1-\left(\frac{1}{5}\right)^{10}\right]$, $\left[1-\left(\frac{4}{5}\right)^{10}\right]$, 0.859

5. Assuming that half of the population is vegetarian so that choice of an individual being a vegetarian is $1/2$. Assuming that 100 investigators take a sample of 10 individuals each to see whether they are vegetarian. How many investigators would you expect to report that three people or less were vegetarian?

   Answer [17]

6. The probability of failure in Physics practical examination is 20%. If 25 batches of 6 students each take the examination, in how many batches 4 or more students would pass?

   Answer [23]

7. A binomially distributed random variable X has expected value, $E(x) = 27.52$ and variance , $Var(x) = 15.69$. Find the value of the two parameter n and p, where n is the sample size and p is the probability of success.

Answer $[\ 64,\ 0.57\ ]$

8. Four percent of resistors produced by machine are defective. In a random sample of 10 resistors, determine the probability that
   (i) Three resistors are defective.
   (ii) At least two resistors are defective
   (iii) At most three resistors are defective

   Answer $[0.00577,\quad 0.05815,\ 0.9995574]$

9. It is generally known that 60% of match boxes from a manufacturing process in a factory have exactly 40 match sticks. Based on this, determine the probability that among 12 randomly selected match boxes from the factory
   (i) Exactly 4 boxes will have 40 match sticks.
   (ii) Between 2 and 4 boxes inclusive will have 40 match sticks.
   (iii) At least 4 boxes will have 40 match sticks.
   Answer $[0.04204,\quad 0.05699,\quad 0.98473]$

POISSON DOISTRIBUTION

Is a discrete probability distribution and is useful in such cases where the value of p is very small and the value of n is very large.

Definition

Is a limiting form of binomial distribution as n moves towards infinity and p moves towards zero but np or mean remains constant and finite.

Conditions of Poisson distribution

(i) The variable is discrete.
(ii) The number of trials i.e. n should be very large.
(iii) The probability of success i.e. p is very small.
(iv) The probability of success in each trial is constant.
(v) np or mean is constant and finite.

Form of Poisson distribution.

(i) The Poisson distribution is a discrete distribution.
(ii) It has a single parameter which is the mean of the distribution and is denoted by λ or m or μ.
(iii) The probability of exactly $0,\ 1,\ 2,\ 3,......$ success is found by

$$P(x)=\frac{\lambda^{x}e^{-\lambda}}{x!},\qquad 0,\ 1,\ 2,\ 3,.......$$

The PGF of Poisson distribution of the random variable X given by

$$G'(t) = E(t^x) = \sum_{x=0}^{\infty} p(x) t^x = \sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x t^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda t)^x}{x!}$$

$$= e^{-\lambda} \left[\frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^x}{3!} + ... \right]$$

$$= e^{-\lambda} \left[e^{\lambda t} \right]$$

$$= e^{-\lambda + \lambda t}$$

$$= e^{\lambda t - \lambda}$$

$$\therefore G(t) = e^{\lambda(t-1)}$$

Example

Find the mean and the variance of Poisson distribution using PGF.

Solution

$$G(t) = e^{\lambda(t-1)}$$

Differentiating $G(t)$ with respect to $'t'$ gives

$$G'(t) = \lambda e^{\lambda(t-1)}$$

Differentiating $G''(t) = \lambda^2 e^{\lambda(t-1)}$

Putting $t = 1$ in $G''(t) = \lambda e^{\lambda(1-1)} = \lambda e^{\lambda(0)} = \lambda e^0 = \lambda$

$\therefore E(x) = \mu = G'(1) = \lambda$

Putting $t = 1$ in $G''(t)$ gives

$$G''(1) = \lambda^2 e^{\lambda(1-1)} = \lambda^2 e^{\lambda(0)} = \lambda^2 e^0 = \lambda^2$$

$$Var(x) = G''(1) + G'(1) - [G'(1)]^2$$

$$= \lambda^2 + \lambda - [\lambda]^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$\therefore Var(x) = \lambda$$

NOTE: In a Poisson distribution mean is equal to the variance.

i.e. $mean = \lambda = np = \text{var}\, iance$

Example

1. It is known from past experience that in a certain plant there are on the average 4 industrial accidents per year. Find the probability that in a given year there will be less than 4 accidents. Assume Poisson distribution.

   Solution

   $$mean = \lambda = 4$$

   $$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, 3, ...$$

   $$P(x < 4) = P(0) + P(1) + P(2) + P(3)$$

   $$= \frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!} + \frac{4^3 e^{-4}}{3!}$$

   $$= e^{-4}\left[\frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!}\right]$$

   $$= e^{-4}\left[1 + 4 + 8 + \frac{32}{3}\right]$$

   $$= 0.43347$$

2. In a town 10 accidents take place in a span of 50 days. Assuming that the number of accidents follows the Poisson distribution, find the probability that there will be 3 or more accidents in a day.

   Solution

   The average number of accidents per day $= \frac{10}{50} = 0.2$

   $$P(3\, or\ more) = P(x \geq 3) = 1 - P(x \leq 2) = 1 - [P(0) + P(1) + P(2)]$$

   $$= 1 - \left[\frac{0.2^0 e^{-0.2}}{0!} + \frac{0.2^1 e^{-0.2}}{1!} + \frac{0.2^2 e^{-0.2}}{2!}\right]$$

   $$= 1 - e^{-0.2}\left[\frac{0.2^0}{0!} + \frac{0.2^1}{1!} + \frac{0.2^2}{2!}\right]$$

   $$= 1 - e^{-0.2}[1 + 0.2 + 0.02]$$

   $$= 1 - 1.22e^{0.2}$$

   $$= 1 - 0.9988$$

   $$= 0.0012$$

3. If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly five bulbs are defective.

   Solution

   $$n = 100, \; p = 3\% = 0.03, \; mean = \lambda = np = 0.03 \times 100 = 3$$

   $$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, ...$$

   $$P(5) = \frac{3^5 e^{-3}}{5!} = 0.10082$$

4. Suppose that a manufacturer product has 2 defects per unit of product inspected. Using Poisson's distribution, calculate the probabilities of finding a product
   (i) without any defect
   (ii) with 4 defects

Solution

(i) $\lambda = 2$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

$$P(0) = \frac{2^0 e^{-2}}{0!} = 0.135$$

(ii) $$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

$$P(4) = \frac{2^4 e^{-2}}{4!} = 0.09$$

5. Between the hours of 2 and 4 p.m. the average number of phone calls per minute coming into the switch board of a company is 2.5 .Find the probability that during a particular minute there will be no phone call at all.

Solution

$$mean = \lambda = 2.5$$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

$$P(0) = \frac{2.5^0 e^{-2.5}}{0!} = 0.08208$$

6. If a random variable X follows Poisson distribution such that $P(X = 1) = P(X = 2)$,find
   (i) The mean of the distribution
   (ii) $P(X = 0)$
   (iii) Standard of the distribution

Solution

(i)

$$P(X=1)=P(X=2)$$
$$\frac{\lambda^1 e^{-\lambda}}{1!}=\frac{\lambda^2 e^{-\lambda}}{2!}$$
$$\frac{\lambda}{1}=\frac{\lambda^2}{2}$$
$$\lambda^2=2\lambda$$
$$\therefore \lambda=2$$

(ii) $P(x)=\dfrac{\lambda^x e^{-\lambda}}{x!}, \quad x=0,1,2,3,\ldots$

$$P(0)=\frac{2^0 e^{-2}}{0!}=0.1353$$

(iii) Standard deviation $=\sqrt{\lambda}=\sqrt{2}=1.4142$

7. The probability that a Poisson variate x takes a positive value is $(1-e^{-1.5})$. Find the variances and also the probability that x lies between -1.5 and 1.5.

Solution

In a Poisson distribution of x successes

$$P(x)=\frac{\lambda^x e^{-\lambda}}{x!}, \quad x=0,1,2,3,\ldots.. \qquad where \quad mean=\lambda=np$$

The Poisson variate x takes a positive value $\Rightarrow$ there is at least one success.

The probability of at least one success

$$=1-P(0)$$
$$=1-\frac{\lambda^0 e^{-\lambda}}{0!}$$
$$=1-e^{-\lambda}$$
$$=1-e^{-1.5}, \quad given \quad \lambda=1.5$$

$\therefore Variance=\lambda=1.5$

As the success cannot be negative or a fraction, the probability that x lies between -1.5 and 1.5

$$=P(0)+P(1)$$
$$=\frac{\lambda^0 e^{-\lambda}}{0!}+\frac{\lambda^1 e^{-\lambda}}{1!}$$
$$=e^{-\lambda}+\lambda e^{-\lambda}$$
$$=e^{-\lambda}[1+\lambda]$$
$$=e^{-1.5}[1+1.5]$$
$$=2.5e^{-1.5}$$
$$=0.5578254$$

8. Records show that the probability is 0.00002 that a car will have a flat tyre while driving over a certain bridge. Use Poisson distribution to find the probability that among $20{,}000$ cars driven over the bridge , not more than one will have a flat tyre.

Solution

$mean = \lambda = np = 0.00002 \times 20{,}000 = 0.4$

$P(at\ least\ one) = P(x \leq 1) = P(0) + P(1)$

$$= \frac{0.4^0 e^{-0.4}}{0!} + \frac{0.4^1 e^{-0.4}}{1!}$$

$= 0.938$

9. If the probability of a defective bolt is 0.2, find
   (i) The mean
   (ii) The standard deviation of defective bolts in a total of 900 bolts.

Solution

(i) $mean = \lambda = np = 0.2 \times 900 = 180$

(ii) $s\tan dard \quad deviation = \sqrt{\text{var}\, iance} = \sqrt{mean} = \sqrt{180} = 13.4$

10. A manufacturer who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain:
    (i) No defectives
    (ii) At least two defectives

Solution

(i) $mean = \lambda = np = 500 \times \dfrac{0.1}{100} = 0.5$

$$P(0) = \frac{0.5^0 e^{-0.5}}{0!} = 0.6065$$

$number\, of\ boxes = 100 \times 0.6065 = 60.65 \approx 61$

(ii)

$P(at\ least\, one) = P(x \geq 2) = 1 - P(x < 2) = 1 - [P(0) + P(1)]$

$$= 1 - \left[\frac{0.5^0 e^{-05}}{0!} + \frac{0.5^1 e^{-0.5}}{1!}\right]$$

$$= 1 - e^{-0.5}\left[\frac{0.5^0}{0!} + \frac{0.5^1}{1!}\right]$$

$= 1 - e^{-0.5}[1 + 0.5]$

$= 1 - 1.5e^{-0.5}$

$= 0.090204$

$Numberofboxes = 100 \div 0.090204 = 9.0204 \approx 9$

Exercises

1. It is given that 3% of electric bulbs manufactured by a company are defective. Using the Poisson approximation find the probability that a sample of 100bulbs will contain:
   (i) No defective
   (ii) Exactly one defective

   Answer [$0.049790 \quad 0.14936$ **]**

2. Assuming that on average , 5% of the output of a factory making certain parts is defective and that 200units are in a package. What is the probability that at most 4 defective parts may be found in a package.

   Answer [0.264 **]**

3. A book contains 100misprints distributed randomly throughout its 100pages, what is the probability that a page observed at random contains at least 2 misprints.
   Answer [0.264 **]**
4. One fifth percent of the blades produced by a blade manufacturing factory turn out to be defective. The blades are supplied in packets of 10 use Poisson distribution to calculate the approximate number of packets containing:
   (i) no defective
   (ii) One defective
   (iii) Two defective blades in consignment of $100{,}000$ packets.

   Answer [$98010 \quad 1960 \quad 19.6$ **]**

5. A discrete random variable X with $E(x) = 2.5$ follows the Poisson distribution. Determine the;
   (i) $P(x > 2)$
   (ii) $P(x \leq 2)$

   Answer [$0.7127 \quad 0.54380$

6. The number of accidents per week in a certain factory follows a Poisson distribution with variance 1.2. Find the probability that:
   (i) No accident occurs in a particular week
   (ii) More than three accident occur in a particular week.

   Answer [$0.30119 \quad 0.033769$]

7.