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Determining the Excitation Energy of Mercury Atoms

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This experiment attempted to demonstrate the presence of discrete energy levels within atoms of mercury [1] as first done by J. Franck and G. Hertz [2] in 1914., as well as determine the excitation energy of mercury atoms. This was done by firing electrons through hot mercury gas across a varied accelerating potential towards a current measuring unit and a retarding potential. The electrons first experienced elastic collisions, which conserve kinetic energy [3], before reaching an energy equal to the excitation energy of a mercury atom whereupon minima in the current were observed as the electrons could no longer overcome the retarding potential after one or multiple excitations of mercury atoms. The excitation energies were determined to be: Hg-150°: 5.0752 ± 0.2083 eV; Hg-160°: 5.0390 ± 0.1375 eV; Hg-170°: 5.0037 ± 0.1625 eV; Hg-180°: 4.9754 ± 0.09375 eV; Hg-190°: 4.9269 ± 0.08124 eV; Hg-200°: 4.8997 ± 0.2500 eV

I. INTRODUCTION

The existence of discrete energy levels within atoms is imperative to quantum physics as one of its most fundamental assumptions. In 1901 Max Planck postulated quantisation in order to marry the theory of his time with the experimental evidence [4], a postulation underpinning emerging physics in countless fields to this day - from Einstein's photoelectric effect [5] to modern-day cryptography and quantum computing [6]. The Franck-Hertz experiment aims to provide a non-optical demonstration of this fundamental quantum property through bombarding mercury atoms in the gas phase with electrons. Said electrons are introduced to the system via thermionic emission [7] and accelerated across a varying potential. Upon reaching sufficient energy and colliding with a mercury atom, the electron in the atom's valence shell will become excited into a higher unoccupied state, causing the incident electron to lose its kinetic energy. When the excited electron returns to its ground state the excess energy will be released as a quanta of electromagnetic radiation; a photon with energy given by the equation,

$$\Delta E = hf \tag{1}$$

where ΔE is the difference between the ground state and the excited energy level, h is Planck's constant, and f is the frequency of radiation. The excited level of interest for mercury in this is experiment is the 6^3P_1 level with the transition being between 6^3P_1 and 6^1S_0, as shown in Figure 1. The expected energy of this transition is $4.90eV$[2].

The mean free path λ of the electrons in this context is the average distance travelled by said electrons between successive collisions with mercury atoms. It can be calculated by the following equation,

$$\lambda = \frac{k_B T}{p\sigma} \tag{2}$$

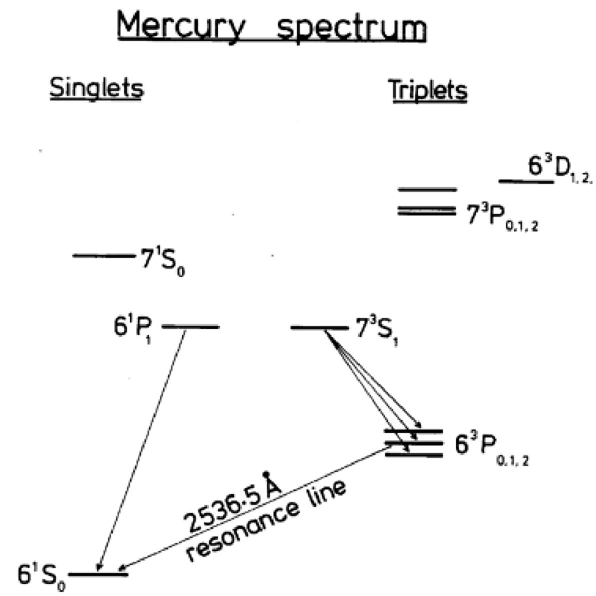

FIG. 1. Energy level diagram of the excitation transition in mercury.

where p is the mercury pressure and σ is the cross-section for inelastic collision, with a value of 2.1e$-19m^2$[8]. Over the temperature range for this experiment, the mercury pressure p is given by,

$$p = 8.7 * 10^{9-\frac{3110}{T}} \tag{3}$$

where T is the mercury gas temperature in Kelvin and p is the gas pressure in pascals. The mean free path helps to explain the increased spacing between minima at lower temperatures, as explored later on.

II. EXPERIMENTAL

A schematic of the ionisation tube used and a labelled image are shown in Figure 2.

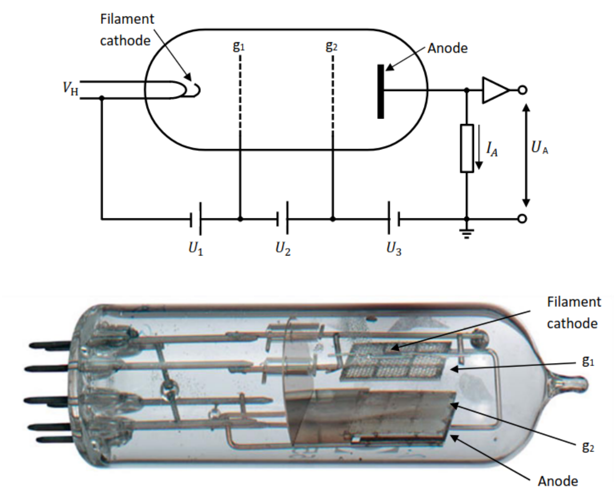

FIG. 2. Top - Schematic of the evacuated ionisation tube and the electrical circuit. Bottom - Labeled image of ionisation tube [9]

The filament cathode is indirectly heated to liberate electrons which are bought to the grid $g1$ by a potential U_1, whereupon they are accelerated by a potential U_2 to a second grid $g2$. Here a retarding potential U_3 is applied, ensuring only electrons with sufficient energy reach the anode and thus produce an excitation current I_A and a measurable potential U_A. The U_2 and resulting U_A values were recorded for a sweep of U_A from 0 to 30 Volts at temperatures of 150°C, 160°C, 170°C, 180°C, 190°C, and 200°C by a Sensor CASSY.

Approximately 5g of mercury was evaporated in the tube for these readings. Figure 3 shows more accurately how the ionisation tube was placed within the over and how the temperature was checked.

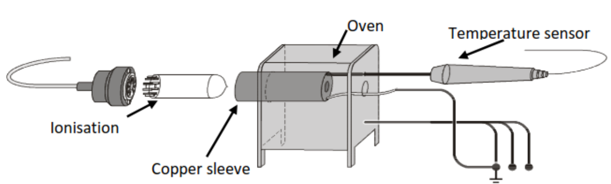

FIG. 3. How the tube and oven were fit together.

As the electrons are accelerated by a voltage U_2 across two grids $g1$ and $g2$, with mass m and charge e, they will have a velocity maximum upon reaching $g2$ given by

$$eU_2 = \frac{1}{2}m(v_{max})^2 \tag{4}$$

As U_2 is increased the energy of the electrons will of course follow until,

$$E_{ex} = \frac{1}{2}mv_{ex}^2 = eV_{ex} \tag{5}$$

where E_{ex} is the excitation energy of the mercury atoms and V_{ex} is the potential at which this energy is reached. It is at this point that minima in the readings for U_A were reached.

III. RESULTS AND SPECIFIC DISCUSSION

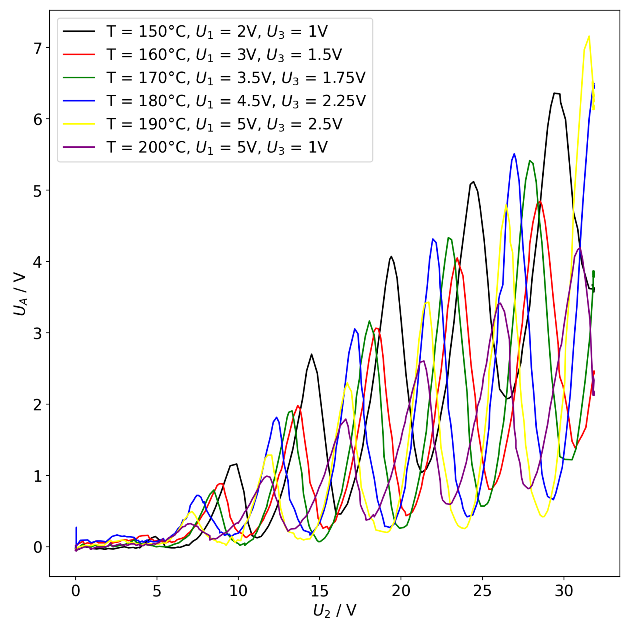

FIG. 4. The excited voltage U_A plotted against the accelerating voltage U_2 for distinct temperatures. The presence of minima for all curves is obvious and indicative of U_2 being equal to the excitation potential V_{ex}.

The curves of U_A against U_2 remained similar across all temperatures at which recordings were taken, each with distinct peaks and troughs as shown in Figure 4. Note that the values for U_1 and U_2 differ for each plot - this is done in order to yield maximum resolution for each curve. A range of values with poor resolution was observed where the difference between U_1 and U_3 was too small and a range out of scope of the Sensor CASSY was observed for a difference too large - that is, such that U_3 was relatively small and so a larger number of electrons contributed to U_A and vice-versa.

A. Excitation Energy of Mercury Atoms

The value of U_2 at each minima of each curve in Figure 5 was found by creating a quadratic fit to each trough and then finding the precise minimum of that fit using the result

$$x_{min} = \frac{-b}{2a} \tag{6}$$

for a quadratic equation of the form $ax^2 + bx + c$. This process is portrayed in Figures 6 and 7.

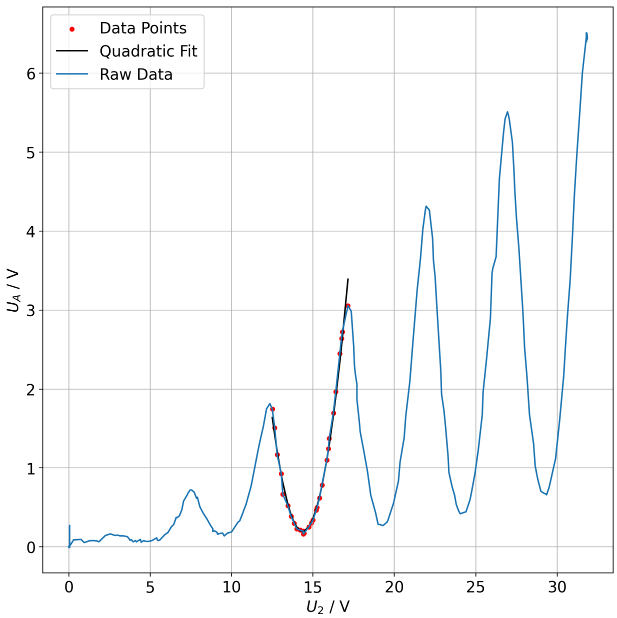

FIG. 5. The plot of data for T = 180°C with a quadratic fit applied onto the second resolvable minimum. The data points in red are those used to produce the quadratic fit.

TABLE I. Table displaying the U_2 values of minima for each U_A against U_2 curve of 150°C, 160°C, and 170°C.

Minimum	U_2 Value at Minimum Point / V		
Order (n)	150°C	160°C	170°C
1	11.4542 ± 0.25	10.6361 ± 0.25	10.2454 ± 0.25
2	16.5613 ± 0.375	15.6859 ± 0.125	15.2296 ± 0.375
3	21.6269 ± 0.375	20.7048 ± 0.25	20.229 ± 0.25
4	26.6831 ± 0.375	25.7677 ± 0.375	25.2612 ± 0.25
5	N/A	30.7902 ± 0.30	30.2481 ± 0.40

TABLE II. Table displaying the U_2 values of minima for each U_A against U_2 curve of 180°C, 190°C, and 200°C.

Minimum	U_2 Value at Minimum Point / V		
Order (n)	180°C	190°C	200°C
1	9.3625 ± 0.25	8.9982 ± 0.20	8.4653 ± 0.5
2	14.2695 ± 0.125	13.9003 ± 0.20	13.5071 ± 0.675
3	19.3245 ± 0.30	18.8515 ± 0.375	18.3895 ± 0.675
4	24.2839 ± 0.125	23.7855 ± 0.25	23.2334 ± 0.5
5	29.2323 ± 0.125	28.6900 ± 0.125	28.1006 ± 0.5

Finally the acceleration voltage U_2 was plotted against the order of each minimum, revealing a linear relationship. From equations (4) and (5) it is seen that the U_2 values at these minimum points is equal to the excitation potential V_{ex}, and so the gradient of this linear graph (example shown in Figure 7) is equal to V_{ex}. The uncertainties from Tables 1 & 2 were included on the data points which allowed for the plotting of two lines of worst fit could be plotted and their gradients calculated in order to give an accurate uncertainty value for V_{ex}, the values for which can be found in Table 3. These values are all of the same order, though they show a trend of decreasing in magnitude as the temperature increases. This likely arises from the relationship of the mean free path and the temperature - a relationship described in equation (2).

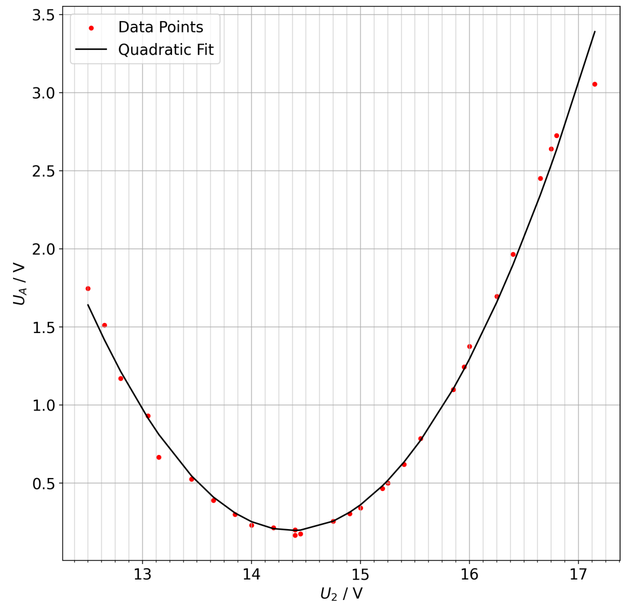

FIG. 6. A closer look at the quadratic fit from Figure 5. The divisions seen here were those used to produce the uncertainties quoted within Tables 1 & 2.

TABLE III. Table displaying the determined V_{ex} values for each temperature at which data was collected.

Temperature, T / °C	V_{ex} / V
150	5.0752 ± 0.2083
160	5.0390 ± 0.1375
170	5.0037 ± 0.1625
180	4.9754 ± 0.09375
190	4.9269 ± 0.08124
200	4.8997 ± 0.2500

B. Mean Free Path

Using equation (2) it is possible to plot a graph of the mean free path, λ against the temperature, T for all temperatures in the range of those in this experiment as

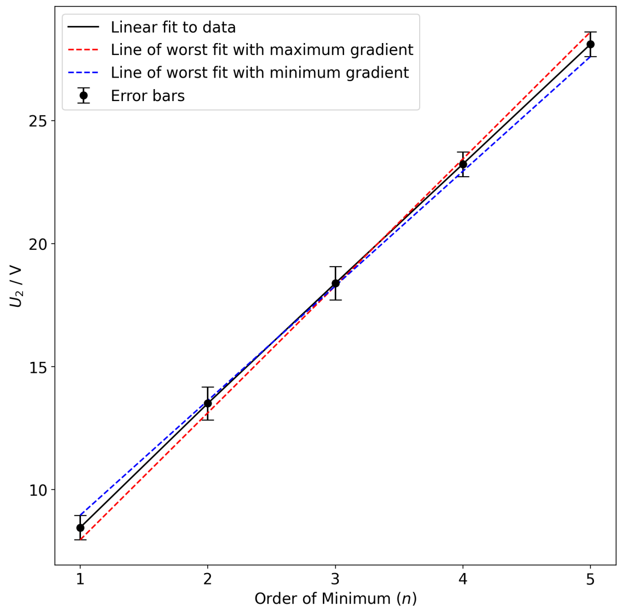

FIG. 7. A linear fit of the minimum U_2 values against Order (n) for $T = 200$°C. Two lines of worst fit are included: one of steepest gradient and one of least steep gradient, which were used to determine the uncertainty in gradient for the linear fit to the data.

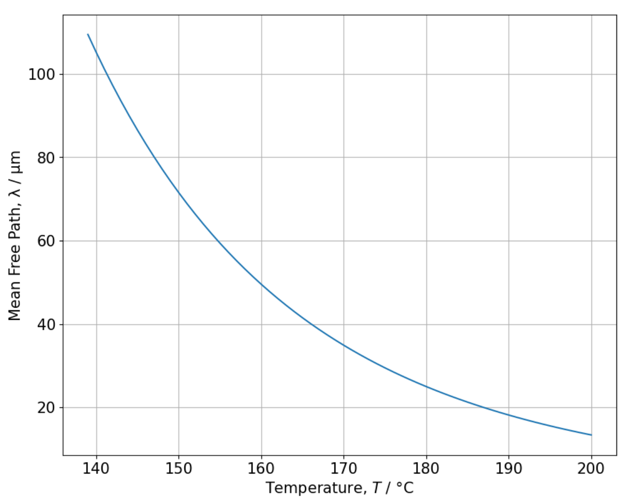

FIG. 8. A plot of the mean free path against temperature. The mean free path clearly increases as the temperature decreases, potentially shifting the location of minima shown in Figure 4 to higher values of U_2.

shown in Figure 8.

It becomes evident upon seeing this relationship that the electrons will travel further (on average) at lower temperatures and therefore are more likely to pass through grid g_2 and contribute to the excited current I_A even after exciting a mercury atom explaining the higher V_{ex} voltages produced at lower temperatures. Upon considering that the separation of g_1 and g_2 was measured to be 1.05 $\pm$ 0.05cm it becomes clear that λ quickly becomes small in comparison to the grid separation and so the effects of this phenomenon are minimised at higher temperatures.

IV. GENERAL DISCUSSION

All data, regardless of temperature, maintained a similar form upon plotting displaying successive troughs as the acceleration voltage U_2 was increased and thus each plot demonstrated the existence of quanta within atoms. As the electrons were accelerated through the mercury gas by the potential U_2 they elastically collided with the mercury atoms, conserving kinetic energy overall. Upon reaching an energy equal to E_{ex} they in-elastically collided with a mercury atom and excited a valence electron of the atom to a higher state, losing all of its kinetic energy.

Electrons with sufficient energy to overcome the retarding potential U_3 once past grid g_2 reached the anode and so contributed to an excitation current I_A, which ultimately produced a measurable potential U_A. This potential was found to have successive minima at which a large amount of electrons reached E_{ex} and so had insufficient energy to contribute to I_A. Given a high enough acceleration potential electrons could once again reach E_{ex} and excite another atom which resulted in multiple minima in the plots of U_A against U_2 which were spaced uniformly as seen in Figure 7.

The mean free path of the electrons contributed to the shifting of minima at lower temperatures at which the mean free path was higher (shown by Figure 8) allowing a greater proportion of electrons to reach the anode before being overcome by the retarding potential.

A range of values for E_{ex} was found, however taking into consideration the effect of the temperature and, ultimately, the mean free path on the V_{ex} values it stands to reason that the values produced by higher temperature experiments should be regarded as more accurate representations of the true value. Of course, this assumes that the E_{ex} value will be constant for mercury atoms across the range of temperatures at which they are studied - this is to be expected due to the nature of energy quanta[1]. The expected value of 4.9eV[10] was within close range of all values determined throughout this experiment, with the most accurate result being that found at the highest temperature, 200°C, perhaps due to this temperature cultivating the smallest mean free path. This value had a relatively high uncertainty however, due to poor alignment of quadratic fittings to the relevant data points.

Within the scope of this experiment however it is difficult to quantify exactly the effect of the relationship between the mean free path and temperature ultimately had - though steps can certainly be taken to ensure this effect is minimised. For example a greater retarding potential U_3 or a greater distance between grid g_2 and the anode would decrease the likeliness of electrons contributing to the excitation current I_A, though a more sensitive voltmeter may be required to collect a reasonably resolvable set of results due to the overall decrease of electrons contributing to I_A. Assuming appropriate temperatures for equation (2), as shown by Figure 8 the value of λ will decrease independent of any potentials applied within the system and so data collection at higher temperatures may be most appropriate for acquiring the most accurate value of E_{ex}.

V. CONCLUSION

In conclusion the initial aims of demonstrating quanta within atoms and determining the excitation energy E_{ex} of mercury were successfully realised.

The presence of minima in the plots of the excited U_A against the accelerating potential U_2 was consistent with quantum theory regarding excitation levels of mercury atoms. Said minima occurred at separations equal to values of excitation potential V_{ex}, producing E_{ex} values close to the documented value of 4.9eV. The effects of the mean free path were evidenced in the shifting of the minima to higher values of U_2 at lower temperatures, resulting in determined E_{ex} values further from 4.9eV thus promoting the use of higher temperatures in future experimentation of a similar nature.

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