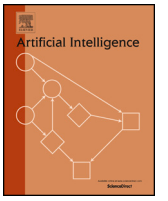
Artificial Intelligence 330 (2024) 104097

Contents lists available at ScienceDirect

Artificial Intelligence

journal homepage: www.elsevier.com/locate/artint



Generalized planning as heuristic search: A new planning search-space that leverages pointers over objects

Javier Segovia-Aguas [a,*], Sergio Jiménez [b], Anders Jonsson [a]

[a] *Universitat Pompeu Fabra, Spain*
[b] *Universitat Politècnica de València, Spain*

ARTICLE INFO

Keywords:
Generalized planning
Classical planning
Heuristic search
Planning and learning
Domain-specific control knowledge
Program synthesis
Programming by example

ABSTRACT

Planning as heuristic search is one of the most successful approaches to *classical planning* but unfortunately, it does not trivially extend to *Generalized Planning* (GP); GP aims to compute algorithmic solutions that are valid for a set of classical planning instances from a given domain, even if these instances differ in their number of objects, the initial and goal configuration of these objects and hence, in the number (and possible values) of the state variables. *State-space search*, as it is implemented by heuristic planners, becomes then impractical for GP. In this paper we adapt the *planning as heuristic search* paradigm to the generalization requirements of GP, and present the first native heuristic search approach to GP. First, the paper introduces a new pointer-based solution space for GP that is independent of the number of classical planning instances in a GP problem and the size of those instances (i.e. the number of objects, state variables and their domain sizes). Second, the paper defines an upgraded version of our GP algorithm, called *Best-First Generalized Planning* (BFGP), that implements a *best-first search* in our pointer-based solution space for GP. Lastly, the paper defines a set of evaluation and heuristic functions for BFGP that assess the structural complexity of the candidate GP solutions, as well as their fitness to a given input set of classical planning instances. The computation of these evaluation and heuristic functions does not require grounding states or actions in advance. Therefore our *GP as heuristic search* approach can handle large sets of state variables with large numerical domains, e.g. *integers*.

1. Introduction

Automated Planning (AP) is a model-based approach to the control of autonomous systems. In more detail, AP explores model-based computations to generate sequences of actions (a.k.a. *plans*) that accomplish defined objectives across various domains. In AP, plans are typically generated by taking into account a model encompassing initial conditions, available actions, and the goals to be accomplished. There is a wide palette of different AP models that deal with *partial state observability*, *non-deterministic state transitions*, *durative actions*, or *temporally extended goals* [1,2]. *Classical planning* is the simplest AP model, which assumes that the system dynamics can be modeled as a finite, deterministic, and fully observable, *transition system*. In this kind of transition systems, classical planning studies the synthesis of sequences of actions that are able to transform an initial state into a state where a set of given goals is satisfied.

* Corresponding author.
E-mail addresses: javier.segovia@upf.edu (J. Segovia-Aguas), serjice@dsic.upv.es (S. Jiménez), anders.jonsson@upf.edu (A. Jonsson).

https://doi.org/10.1016/j.artint.2024.104097
Received 22 July 2021; Received in revised form 30 January 2024; Accepted 3 February 2024

Available online 15 February 2024

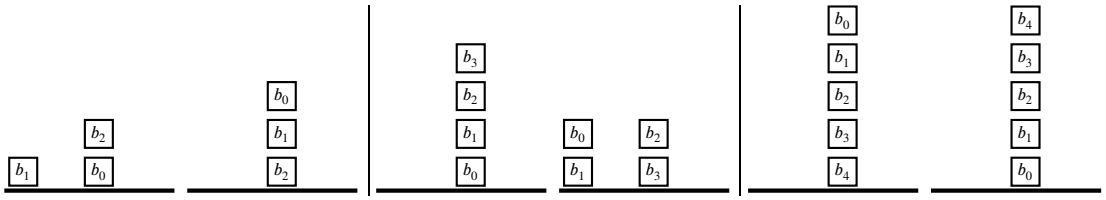


Fig. 1. Three different classical planning problems from *blocksworld*. Each problem has a different number of blocks. The figure shows the blocks configuration for their initial state (left side) and goals (right side) of each problem. The leftmost problem is the *Sussman anomaly* problem.

We illustrate the notion of a classical planning problem using the *blocksworld*, a popular classical planning domain, which consists of a set of blocks, a table, and a robot hand [3]. The robot hand can be empty or holding one block, and a block can be on top of another block or on the table. Different classical planning problems can be defined in the *blocksworld*, by changing the number of blocks, and their initial or goal configuration. A well-known *blocksworld* problem, since it is small but non-trivial, is the *Sussman anomaly* [4] (the leftmost problem shown in Fig. 1); in the *Sussman anomaly* there are three blocks that we labeled as b_0, b_1, and b_2. Initially block b_1 is on the table, b_2 on top of b_0, and b_0 on the table, and the goal is to stack the three blocks such that b_0 is on top of b_1, which in turn is on top of b_2. Fig. 1 shows three different classical planning problems from *blocksworld*. Each problem has a different number of blocks. The figure shows the blocks configuration for the initial state (left side) and goals (right side) of each problem. The leftmost classical planning problem corresponds to the *Sussman anomaly* problem.

Heuristic search is one of the most successful approaches to classical planning [5–8]. Winners of the *International Planning Competition* are often heuristic planners [9], and the *workshop on Heuristics and Search for Domain-Independent Planning* is one of the discussion forums with the longest tradition at the *International Conference on Automated Planning and Scheduling (ICAPS)*, the major conference for research on AP. Briefly, the *planning as heuristic search* approach addresses the synthesis of sequential plans as a combinatorial search in the space of the states reachable from the initial state. This combinatorial search is usually implemented as a *forward search*, guided by heuristics that are automatically extracted from the representation of the planning problem. In the last decade a wide landscape of effective search algorithms, and heuristic functions, have been developed for classical planning [10–14]. Most of these heuristic search techniques are based on the notion of *relaxed plan*, a solution to a relaxation of the classical planning problem; the cost of the relaxed plan is an informative and cheap estimate of the actual cost-to-go for many different classical planning problems.

Generalized planning (GP) addresses the representation and computation of solutions that are valid for a set of classical planning instances from a given domain [15–24]. In the worst case each classical planning instance may require a completely different solution but in practice, many classical planning domains are known to have algorithmic solutions [25,26]. In other words, one can compute a single compact general solution that exploits some common structure of the different classical planning instances in a given domain. *Generalized plans* are then algorithmic solutions that supplement sequences of planning actions with control-flow. For example, a generalized plan that solves any classical planning instance from the *blocksworld* domain [3], no matter the actual number, or identity of the blocks, and no matter the initial and goal configuration of the blocks, can be compactly specified as follows: *"put all the blocks on the table and then, in a proper order, move each block to its goal placement"*.

Unfortunately search algorithms and heuristic functions from classical planning cannot be directly applied to GP. The computation of *relaxed plans*, as it is implemented by off-the-shelf heuristic planners, requires a pre-processing step for grounding states and actions. On the other hand, GP solutions must be able to generalize to (possibly infinite) sets of classical planning instances, with different sets of objects (i.e. state variables with different domain sizes and/or different number of state variables) as well as with different initial states and goal configuration for the objects. These particular generalization requirements of GP make it impractical to ground states and actions and hence, to directly apply the state-space search or the cost-to-go estimates of heuristic planners. What is more, the knowledge represented in a given set of classical planning instances may not be enough to specify an algorithmic solution that solves them all. For example, the classical planning instances from the *blocksworld* do not include representation *features* for explicitly specifying whether *"all blocks are on the table"*, or for specifying *"the proper order for moving the blocks to their goal placements"*. A key challenge in GP is then, given a set of planning instances, to automatically discover the representation features required for computing a compact and general solution for those instances.

Because of the algorithmic kind of *generalized plans*, GP is a promising research direction to help bridge the current gap between *automated planning* and *programming* [27]. Unfortunately, most of the work on GP inherits the STRIPS representation, in which states are represented using exclusively Boolean variables (i.e. propositions that specify the properties and relations of the world objects), and state-transitions correspond to actions for object manipulations. In this work we introduce a novel pointer-based representation for GP problems and solutions, that allows us to adapt the *planning as heuristic search* paradigm to GP. Our pointer-based representation is closer to common programming languages like *Python*, *C++* or *Java*, while it also naturally applies to the STRIPS problems traditionally addressed by the AP community. Given a GP problem that comprises a finite set of classical planning instances from a given domain, our *GP as heuristic search* approach implements a combinatorial search to synthesize a program that solves the full set of input instances. With respect to previous work, our heuristic search approach to GP introduces the following contributions:

1. *A pointer-based representation for GP problems and solutions*. Our representation formalism is closer to common programming languages, while it also applies to object-centered representations (like STRIPS) traditionally used in AP.

2. *A tractable solution-space for GP*. We leverage the computational models of the *Random-Access Machine* [28] and the *Intel x86* FLAGS register [29] to define a compact and general solution search space for GP. Remarkably our new search space for GP is independent of the number of input planning instances in a GP problem, and the size of these instances (i.e. the number of objects, state variables, and their domain sizes).
3. *A heuristic search algorithm for GP with grounding-free evaluation/heuristic functions*. We present the BFGP algorithm that implements a *Best-First Search* in our GP solution-space. We also define evaluation and heuristic functions for guiding BFGP. These functions assess the structural complexity of the candidate GP solutions, as well as their fitness to an input set of classical planning instances. Evaluating these functions does not require to ground states/actions in advance, so they apply to GP problems where state variables have large domains (e.g. *integers*).
4. *A translator from the* STRIPS *fragment of PDDL to our pointer-based representation* for GP. We automate the representation change from PDDL to pointer-based, and show several solutions to planning domains from the *International Planning Competition* [30] which are validated on large random instances.

A preliminary description of our *GP as heuristic search* approach previously appeared as ICAPS and IJCAI conference papers [31, 32]. In this work we extend the seminal ideas presented in the conference papers, and provide a more exhaustive evaluation of our *GP as heuristic search* approach. Compared to the conference papers, the present paper includes the following novel material:

- We formalize the notion of *pointers* over the set of objects of a planning problem, and introduce a pointer-based formalization for states, state-constraints, and planning action schemes. We also introduce a pointer-based formalization for classical planning problems and their solutions. We show that our pointer-based formalization naturally applies to the STRIPS planning tasks traditionally addressed in AP.
- We introduce the notion of a *partially specified planning program*, that refers to the sketch of an algorithmic planning solution, and that enables a better formalization of our GP search algorithm and heuristics functions.
- We provide new theoretical results of our heuristic search algorithm for GP, that include *termination*, *soundness*, *completeness*, and *complexity* proofs. We also implemented new evaluation functions for guiding our *GP as heuristic search* approach, and extended the empirical evaluation, including more results at a wider landscape of planning domains, to characterize better the performance of our *GP as heuristic search* approach.

The paper is structured as follows: Section 2 reviews previous research work related to computing policies and generalization in planning, and it pinpoints the main differences with our approach. Section 3 provides the planning models we rely on in this work (namely the *classical planning* model and the GP model) and it also presents the formalisms we leverage for the representation of our GP solutions (i.e. *planning programs* and the *Random Access Machine*). Section 4 shows how to extend the classical planning model with a set of pointers over objects, and the corresponding primitive operations for manipulating these pointers. This extension allows us to define, in an agnostic manner, a set of features and a set of actions for computing GP solutions that can solve any instance from a given planning domain. Section 5 describes our *GP as heuristic search* approach; the section provides details on our solution space, evaluation functions, and heuristic search algorithm for GP. Section 6 presents the empirical evaluation of our *GP as heuristic search* approach and comparisons with the classical planning compilation for GP, that serves as a baseline. Finally, Section 7 wraps-up our work and discusses open issues and future work.

2. Related work

Here we first review previous work on GP according to the following three dimensions: *problem representation*, *solution representation*, and *computational approach*. Then, we connect the research work on GP with other relevant areas in AI, such as *program synthesis*, *deep learning*, and *(deep) reinforcement learning*. Last, we discuss the features that distinguish our *GP as heuristic search approach* from the reviewed related work.

Regarding *problem representation*, there are two different approaches for the specification of the set of classical planning instances that are comprised in a GP problem. The *explicit* approach, that enumerates every classical planning instance in a GP problem [33], and the *implicit* approach, that defines the constraints that hold for the set of classical planning instances of a GP problem. The implicit approach is of interest because it can compactly specify infinite sets of classical planning instances (e.g. the infinite set of the classical planning instances that belong to the *blocksworld* domain) [34,35,20]. In addition to the set of classical planning instances, extra background knowledge can be specified with the aim of reducing the space of GP solutions. For instance, *plan traces* demonstrating how to solve some of the input instances [36–38], the full state space [22], the particular subset of state *features* that can be used for computing a generalized plan [39,40], *negative examples* that specify undesired behavior for the targeted GP solutions [41,22], or *state invariants* that any state in a given domain must satisfy [42].

With respect to *solution representation*, different formalisms appeared in the AP literature to represent solutions that are valid for a set of classical planning instances; sequential plans are used in *conformant planning* [43,44], conditional *tree-like* plans are used in *contingent planning* [45,2], or policies are used in *FOND planning*, as well as in MDP/POMDP planning [46]. In all these planning settings, a set of different classical planning instances, with different initial states, can be implicitly represented as a disjunctive formulae over the state variables. Different goals can also be considered by coding them as part of the state representation, e.g. using *static* state variables [33]. Further, the notion of *feature* in GP is related to the notion of state *observation* in the POMDP formalism, where observations depend on the current state and the action just taken [47]. With this regard it can be understood that GP solvers

compute, at the same time, a generalized plan and an *observation function* that is useful for compactly representing the generalized plan. The notion of *feature* in GP is also related to *Qualitative numeric planning* [48,49,20] which leverages propositions to abstract the value of numeric state variables.

The connection between AP and programming representations is not exclusive from our GP approach; programs of different kinds are proposed as an alternative to traditional planning action languages [50–52,27]. GOLOG programs have been also used to represent planning solutions, that could branch and loop, and that could contain non-deterministic parts [53]. Furthermore the semantics compatibility between GOLOG and PDDL [50] can be exploited and a PDDL planner can be embedded [54] to address the sub-problems that are combinatorial in nature. Since the early days of AI, hierarchies, LTL formulae, and policies, are also used to specify *sketches* of general planning solutions [55]. In the AP literature these solution sketches are often called *domain-specific control knowledge*, since they are traditionally used to control the planning process, and they apply to the entire set of classical planning instances that belong to a given domain [56,57,36,37]. Last but not least, algorithmic solutions, represented either as *lifted policies*, *finite automata*, or as *programs with control-flow constructs* for branching and looping, are used to represent GP solutions [34,58,39,59–61,15,33,21,24].

Regarding the *computation of generalized plans*, there are two main approaches to GP. The *top-down/offline* approach considers the entire set of classical planning instances in a given GP problem as a single batch, and computes a solution plan that is valid for the full batch at once. A common approach for the offline computation of generalized plans is compiling the GP problem into another form of problem solving, and using an off-the-shelf solver to work out the compiled problem. For example, GP problems have been compiled into classical planning problems [61,33], conformant planning problems [39], LTL synthesis problems [62], FOND planning problems [63,20], MAXSAT problems [22], or ASP problems [64]. The compilation approach is appealing because it leverages the latest advances of other well-founded scientific communities, with robust and scalable solvers. In addition, the computational complexity of some of these tasks is theoretically characterized with respect to structural features of the input problems, which may provide insights on the difficulty of the addressed GP problem. A weak point of the compilation approach is however the size of the compiled problems to be solved; solvers are usually sensitive to the size of the input problems. On the other hand, the *bottom-up/online* approach incrementally processes the set of classical planning instances in a GP problem [15,17,65]. Given a classical planning instance, a solution to that instance is computed and then, the solution is merged with solutions computed for the previous instances. The online approach is then appealing for handling GP problems that comprise large sets of classical planning instances. The main drawback of online approaches is dealing with the over-fitting produced by the individual processing of the different classical planning instances in a GP problem.

As noted by previous work on GP, the aims of GP are connected to *program synthesis* [33,20,62,31]. Program synthesis is a task traditionally studied by the *computer-aided verification* community [66], and that aims to compute programs such that they satisfy a given correctness specification [67–69]. Program synthesis follows the *functional programming* paradigm. This means that a program is a function composition, where each function in the composition maps its input parameters into a single output, and where looping is implemented with recursion. Work on program synthesis is classified according to how the correctness specification of a program is formulated. The *programming by example* (PbE) paradigm specifies the desired program behavior with a finite and non-empty set of ground *input/output* examples. This approach is related to the explicit representation of GP problems; a ground *input/output* example can be understood as the *initial/goal* state pair that represents a classical planning instance, and the instruction set of the functional programming language can be understood as the available actions for transforming an initial state into a goal state. Program synthesis also allows the implicit representation of the input correctness specifications, e.g. using fist-order formulae specified in SMTLIB,[1] the formal language for SAT-Modulo Theories (SMT) [70]. The mainstream approach for program synthesis is to specify a formal grammar that incrementally enumerates the space of possible programs, and to leverage the satisfiability machinery of SMT solvers to validate whether a candidate program is actually a solution [71]. With this regard, work on *theorem proving* is also related to program synthesis, specially since SMT solvers allow the representation and satisfaction of first-order logic formulae [72]. Lastly, another popular trend in program synthesis is *Programming by sketches* that addresses program synthesis in the particular setting where a partially specified solution is provided as input [73].

Besides computational methods for formal verification and logic satisfaction, optimization methods (that are predominant in *Machine Learning* [74]) have also been applied to the computation of planning solutions that generalize. For instance, off-the-shelf *Deep Learning* (DL) tools, have been successfully applied to the computation of generalized policies for classical and probabilistic planning domains [75–77]. *Generalized policies* are a powerful solution representation formalism whose applicability goes beyond classical planning; generalized policies can represent planning solutions that deal with non-deterministic actions [78]. Further, *generalized policies* can represent solutions to planning problems whose aim is not the satisfaction of a given goal condition but the optimization of a given utility function [79]. The aims of GP are also related to *Reinforcement Learning* (RL) [80]; while the cited DL approaches can be viewed as off-line optimization approaches to GP, the RL paradigm can be viewed as an online optimization approach to GP. RL methods incrementally compute policies, by iteratively addressing a set of sequential decision-making episodes. In RL learning experience is however not given beforehand (learning experience is collected by the autonomous exploration of the state space), and RL assumes that there is an explicit notion of reward function (which helps to guide exploration towards the most promising portions of the state-space). Note that DL and DRL approaches learn policies, without requiring a symbolic representation of the state and the action space. This means that it is possible to compute (*deep) policies* that generalize from raw sensor data (e.g. sequences of images) [81,82]. The main disadvantage of computing solutions represented as *deep policies* is that they are black-

[1] https://smtlib.cs.uiowa.edu/.

box models that lack transparency and explanation capacity, which makes it difficult to interpret the produced solutions. This is a strong requirement in application areas that require humans in the loop, such as health, law, or defense [83].

With regard to the reviewed related work, our *GP as heuristic planning* approach is framed as follows:

- *Numeric state variables*. Previous work on GP mainly followed the object-centered STRIPS representation. Addressing programming tasks with such representation is unpractical since it may require to encode all values in the domain of a state variable as objects. Other approaches, such as Qualitative Numeric Planning (QNP) [48,49], handle large numeric state variables qualitatively with propositions to denote whether a variable is equal to zero. In this work we handle GP problems with integer state variables, which allow to naturally address diverse programming tasks as if they were GP problems.
- *Explicit problem representation*. In this work, a GP problem comprises the explicit enumeration of a finite set of classical planning instances to be solved. Interestingly our experimental results show that, in several domains, solving a small set of a few randomly generated classical planning instances, is enough to obtain a solution that generalizes to the infinite set of problems that belong to a given domain.
- *No background knowledge*. Our approach does not require any additional help such as *state invariants*, *plans/traces/demonstrations*, *negative examples*, or the specification of the subset of *features* to appear in the generalized plans. With this regard, we leverage the computational models of the *Random-Access Machine* [28] and the *Intel x86* FLAGS register [29] to produce an agnostic set of state features that is shared for the different classical planning instances of a given domain (no matter their actual number of objects).
- *Generalized plans represented as structured programs*. Structured programming provides a white-box modeling paradigm that is widely popular. In this work we focus on generalized plans represented as structured programs, with control flow constructs for branching and looping the program execution flow. The application of a generalized plan on a particular classical planning instance is then a deterministic matching-free process, which makes it easier to define effective evaluation and heuristic functions. Further, the asymptotic complexity of structured programs can be assessed from their structure, which is helpful to establish preferences on the possible generalized plans.
- *Off-line satisfiability approach*. This work follows an off-line approach to GP that aims to compute, at once, a generalized plan that solves all the classical planning instances that are given as input. Because many heuristic search algorithms are easily extended to online versions, we believe that our GP as heuristic search approach is a stepping stone towards online approaches that can deal with larger sets of classical planning instances.
- *Native heuristic search for GP*. By *native* heuristic search, we mean that we defined a search space, evaluation/heuristic functions, and a search algorithm, that are specially targeted to GP. Our *GP as heuristic search* approach is related to an existing classical planning compilation for GP [33]. Our approach overcomes however the main drawback of the compilation whose search space grows exponentially with the number and domain size of the state variables; in practice, this drawback limited the applicability of the compilation to planning instances of small size since the performance of off-the-shelf classical planners is sensitive to the size of the input instances. Our experiments support this claim, and show that our BFGP algorithm significantly reduces the CPU-time required to compute and validate generalized plans, compared to the classical planning compilation approach to GP [33].

3. Background

This section introduces the necessary notation to formalize our *GP as heuristic search* approach. First, the section formalizes the *classical planning* model and the *generalized planning* model. Then the section formalizes *planning programs*, our formalism for the compact representation of planning solutions. Lastly the section formalizes the *Random Access Machine*, given that our *GP as heuristic planning* approach borrows several mechanisms from this abstract computation machine.

3.1. Classical planning

Our formalization of the classical planning model is similar to the abstract planning framework called *Finite Functional Planning*, introduced for the theoretical analysis of planning languages [84]. Let X be a set of *state variables*, where each variable $x \in X$ has a domain D_x. A *proposition* is a state variable $x \in X$ with domain $D_x = \{0, 1\}$, where $x = 0$ and $x = 1$ are interpreted as $false$ and $true$ assignments, respectively. A *state* s is a total assignment of values to the set of state variables, i.e. $s = \langle x_0 = v_0, \ldots, x_N = v_N \rangle$, such that $\forall_{0 \leq i \leq N} v_i \in D_{x_i}$ and where N is the number of state variables in X. For a subset of the state variables $X' \subseteq X$, let $D[X'] = \times_{x \in X'} D_x$ denote its joint domain. The state space is denoted as $S = D[X]$. Given a state $s \in S$, and a subset of variables $X' \subseteq X$, let $s_{|X'} = \langle x_i = v_i \rangle_{x_i \in X'}$ be the *projection* of s onto X' i.e. the partial state that is defined by the values assigned by s to the subset of state variables in X'. The *projection* of s onto X' defines the subset $\{s \mid s \in S, s_{|X'} \subseteq s\}$ of the states that are consistent with the corresponding partial state. Last, let us define a *state-constraint* as a Boolean function $C : S \to \{0, 1\}$ over the state variables, that implicitly defines the subset of states $S_C \subseteq S$ that are consistent with that constraint.

Let A be a *set of deterministic actions* such that each action $a \in A$ is characterized by two functions; an *applicability function* $\rho_a : S \to \{0, 1\}$ and a *successor function* $\theta_a : S \to S$. An action $a \in A$ is applicable in a given state $s \in S$ iff $\rho_a(s) = 1$. The execution of an applicable action $a \in A$, in a state $s \in S$ results in the *successor* state $s' = \theta_a(s)$. Please note that this definition of deterministic actions generalizes *actions with conditional effects* [85], common in GP since their state-dependent outcomes allow the adaptation of generalized plans to different classical planning instances.

A *classical planning instance* is a tuple $P = \langle X, A, I, G \rangle$, where X is a set of state variables, A is a set of actions, $I \in S$ is an initial state, and G is a constraint on the value of the state variables that induces the subset of *goal states* $S_G = \{s \mid s \vDash G, s \in S\}$. Given a classical planning instance P, a *plan* is an action sequence $\pi = \langle a_1, \ldots, a_m \rangle$ whose execution induces a *trajectory* $\tau = \langle s_0, a_1, s_1, \ldots, a_m, s_m \rangle$ such that, for each $1 \leq i \leq m$, a_i is applicable in s_{i-1} and results in the successor $s_i = \theta_{a_i}(s_{i-1})$. A plan π *solves* P if and only if the execution of π in s_0, where $s_0 = I$, finishes in a goal state, i.e. $s_m \in S_G$.

Planning languages, such as PDDL [86], can compactly represent the infinite set of classical planning instances of a given domain using a finite set of functions and action schemes. Given a finite set of objects Ω, and a finite set of functions Φ defined over that set of objects, we assume that each state variable $x \in X$ stands for a function interpretation $x \equiv \phi(\vec{o})$, where $\phi \in \Phi$ is a function with arity $ar(\phi)$, and $\vec{o} \in \Omega^{ar(\phi)}$ is a vector of objects comprised in the Cartesian product space of $\Omega^{ar(\phi)}$; objects and function signatures can by typed so the number of possible function interpretations is constrained. Functions in Φ can be Boolean e.g. to represent *PDDL predicates*, or numeric e.g. to represent *PDDL numeric fluents*. Likewise, given a set of action schemes Ξ, we assume that each action $a \in A$ is built from an action schema $\xi \in \Xi$ by substituting each variable in the action scheme with an object from Ω. An action scheme $\xi \in \Xi$ is a tuple $\xi = \langle name(\xi), par(\xi), pre(\xi), eff(\xi) \rangle$ where:

- $name(\xi)$ is the identifier of the action schema,
- $par(\xi)$ is the list of free variables, again these variables can be typed so they can only be substituted by objects of the same type,
- $pre(\xi)$ is a conjunction of Boolean formulae, where each formula is a logical evaluation, i.e. $==$, $<$, $>$, $\leq$, $\geq$, between two function symbols $\phi_1, \phi_2 \in \Phi$ defined over $par(\xi)$, or a function symbol ϕ also defined over $par(\xi)$ and a value v, that compactly represents the subset of states where the corresponding ground actions are applicable, and
- $eff(\xi)$ is a set of logical assignments, where each function symbol ϕ gets the value either from another function symbol ϕ' (both defined over $par(\xi)$) or from a constant value v, that compactly represents the updates of the state variables caused by the application of the corresponding ground actions.

3.2. Generalized planning

Generalized planning is an umbrella term that refers to more general notions of planning [21]. This work builds on top of the *inductive* formalism for GP, where a GP problem is a finite set of classical planning instances that belong to the same domain [19,62].

Definition 1 *(GP problem).* A *GP problem* is a non-empty set $\mathcal{P} = \{P_1, \ldots, P_T\}$ of T classical planning instances from a given domain $\mathcal{D}$.

Each instance $P_t \in \mathcal{P}$, $1 \leq t \leq T$, may actually differ in the set of state variables X_t, actions A_t, initial state I_t, and goals G_t, but the corresponding set of state variables X_t is induced from the common set of functions Φ. Likewise, the set of actions A_t is induced from the common set of action schemes Ξ, when grounded over the particular set of objects Ω_t of the instance.

The aim of GP is to compute algorithmic planning solutions, a.k.a. generalized plans, which work for the full input set of planning problems. There are diverse representations for GP solutions, ranging from *generalized polices* [34,58], to *finite state controllers* [39,59], formal grammars [60], hierarchies [87,61], or programs [15,33]. Each representation has its own expressiveness capacity, as well as its own validation complexity and computational complexity. In spite of this representation diversity, we can define a common condition under which a generalized plan is considered a solution to a GP problem.

Definition 2 *(GP solution).* A *generalized plan* Π solves a GP problem $\mathcal{P} = \{P_1, \ldots, P_T\}$ iff, for every classical planning instance $P_t \in \mathcal{P}$, $1 \leq t \leq T$, the execution of Π on P_t, denoted as $exec(\Pi, P_t) = \langle a_1, \ldots, a_m \rangle$, induces a classical plan that solves P_t.

Example. Fig. 2 shows the initial state and goal of two classical planning instances, $P_1 = \langle X, A, I_1, G_1 \rangle$ and $P_2 = \langle X, A, I_2, G_2 \rangle$, for sorting two six-element lists. In this particular example the two instances share the same set of state variables $X = \{x_i \equiv vector(o_i) | 0 \leq i \leq 5\}$ that is built with the one-arity function $\Phi = \{vector\}$ and the set of objects $\Omega_1 = \Omega_2 = \{o_0, \ldots, o_5\}$, and where $\forall_{x \in X} D_x = \mathbf{N}_0$. The two classical planning instances also share the set of deterministic actions A, with $\frac{6 \times 5}{2}$ actions $swap(o_i, o_j)$, that swap the content of two list positions $i < j$, and that are induced from the single action scheme $\Xi = \{swap(x, y)\}$. An example solution plan for P_1 is $\pi_1 = \langle swap(o_0, o_5), swap(o_1, o_2), swap(o_1, o_3) \rangle$ while $\pi_2 = \langle swap(o_0, o_2), swap(o_3, o_5) \rangle$ is an example of a sequential plan that solves P_2. Note that $\mathcal{P} = \{P_1, P_2\}$ is a GP problem since it comprises two classical planning instances that are built using the same set of functions Φ and action schemes Ξ. Fig. 3 shows an example of a generalized plan that solves $\mathcal{P}$, and that is represented as a *sorting network* [88]. The sorting network is illustrated using two different types of items (namely the *wires* and the *comparators*). For each state variable, there is a wire that carries the value of that variable from left to right in the network. On the other hand, comparators connect two different wires, corresponding to a pair of variables (x_i, x_j), such that $i < j$. When a pair of values traveling through a pair of wires (i, j), encounters a comparator, then the comparator applies the action $swap(o_i, o_j)$ iff $vector(o_i) \geq vector(o_j)$, which in turn is $x_i \geq x_j$. The sorting network of Fig. 3 can actually solve any instance for sorting the content of any six-element list, no matter its initial content. This solution is however not valid for sorting lists with different lengths. In this paper we will show how to represent, and compute, planning solutions that leverage *indirect memory addressing* to generalize no matter the number of objects, and corresponding state variables.

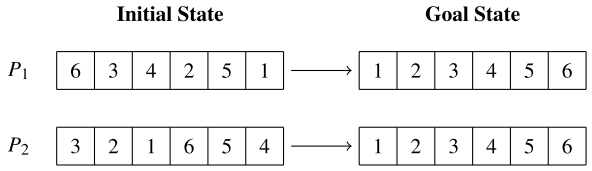


Fig. 2. Example of two classical planning instances for sorting the content of two six-element lists by swapping the list elements.

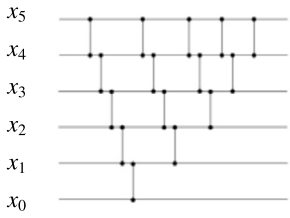


Fig. 3. Example of a generalized plan, represented as a *sorting network* that solves any classical planning instance for sorting the content of a six-element list, no matter its initial content.

3.3. Planning programs

In this work we represent planning solutions as *planning programs* [33]. Unlike sequential plans, *planning programs* include a control flow construct which allows the compact representation of solutions to classical and GP problems. Formally a *planning program* Π is a sequence of n instructions, where each instruction $\Pi[i]$ is associated with a *program line* $0 \leq i < n$, and it is either:

- A *planning action* $\Pi[i] \in A$.
- A *goto instruction* $\Pi[i] = \mathsf{go}(i', !y)$, where i' is a program line $0 \leq i' < i$ or $i + 1 < i' < n$, and y is a proposition.
- A *termination instruction* $\Pi[i] = \mathsf{end}$. The last instruction of a planning program is always a termination instruction, i.e. $\Pi[n-1] = \mathsf{end}$.

The execution model for a planning program is a *program state* (s, i), i.e. a pair of a planning state $s \in S$ and program counter $0 \leq i < n$. Given a program state (s, i), the execution of a programmed instruction $\Pi[i]$ is defined as:

- If $\Pi[i] \in A$, the new program state is $(s', i+1)$, where $s' = \theta_{\Pi[i]}(s)$ is the *successor* when applying $\Pi[i]$ in s.
- If $\Pi[i] = \mathsf{go}(i', !y)$, the new program state is $(s, i+1)$ if y holds in s, and (s, i') otherwise.[2] Proposition y can be the result of an arbitrary expression on state variables, e.g. a state *feature* [89].
- If $\Pi[i] = \mathsf{end}$, program execution terminates.

To execute a planning program Π on a classical planning instance $P = \langle X, A, I, G \rangle$, the initial program state is set to $(I, 0)$, i.e. the initial state of P and the first program line of Π. A program Π *solves* P iff the execution terminates in a program state (s, i) that satisfies the goal condition, i.e. $\Pi[i] = \mathsf{end}$ and $s \in S_G$. Otherwise the execution of the program fails. If a planning program *fails* to solve the planning instance, the only possible sources of failure are:

1. *Inapplicable program*, i.e. executing action $\Pi[i] \in A$ fails in program state (s, i) since $\Pi[i]$ is not applicable in s.
2. *Incorrect program*, i.e. execution terminates in a program state (s, i) that does not satisfy the goal condition, i.e. $(\Pi[i] = \mathsf{end}) \wedge (s \notin S_G)$.
3. *Infinite program*, i.e. execution enters into an infinite loop that never reaches an end instruction.

In this work we model instructions $\Pi[i] \in A$ as if they were always applicable but that their effects only update the current state iff the preconditions of the action hold in the current planning state. Formally, when executing $\Pi[i]$ in (s, i), the new program state is $(s', i+1)$ iff $\Pi[i]$ is applicable, otherwise it is $(s, i+1)$. Therefore, in this work the execution of a program on a classical planning instance will never return an *inapplicable program*, and only *incorrect* or *infinite program* are possible sources of failure. This particular action modeling is common in *reinforcement learning* [80], and in *conformant planning* [44], because it delivers compact solutions that apply to sets of different problems (typically with different initial states).

[2] We adopt the convention of jumping to line i' whenever y is *false*, following the semantics of JMPZ instructions in the *Random-Access Machine* that jump when a register equals zero.

3.4. The random-access machine

The *Random-Access Machine* (RAM) is an abstract computation machine, in the class of the *register machines*, that is polynomially equivalent to a *Turing machine* [90]. The RAM enhances a multiple-register *counter machine* [91] with indirect memory addressing; indirect memory addressing is useful for coding RAM programs that access an unbounded number of registers, no matter how many there are. A *register* in a RAM machine is then a memory location with both an *address* i.e. a unique identifier that works as a natural number (that we denote as r), and a *content* i.e. a single natural number (that we denote as $[r]$).

A *RAM program* Π is a finite sequence of n instructions, where each program instruction $\Pi[i]$, is associated with a *program line* $0 \leq i < n$. The execution of a RAM program starts at its first program instruction $\Pi[0]$. The execution of program instruction $\Pi[i]$ updates the RAM *registers* and the *current program line*. Diverse *base instructions sets*, that are Turing complete, can be defined. We focus on the three *base sets* of RAM instructions:

- Base1. $\{\texttt{inc}(r), \texttt{dec}(r), \texttt{jmpz}(r, i), \texttt{halt}() \mid r \in R\}$. Respectively, these instructions *increment/decrement* a register by one, jump to program line $0 \leq i < n$ if the content of a register r is zero (i.e. if $[r] == 0$), or end the program execution.
- Base2. $\{\texttt{inc}(r_1), \texttt{clear}(r_1), \texttt{jmpz}(r_1, r_2, i), \texttt{halt}() \mid r_1, r_2 \in R\}$. In this set the value of a register cannot be decremented but instead, it can be set to zero with a *clear* instruction. In addition, *jump instructions* go to program line $0 \leq i < n$ if the content of two given registers is the same (i.e. if $[r_1] == [r_2]$).
- Base3. $\{\texttt{inc}(r_1), \texttt{set}(r_1, r_2), \texttt{jmpz}(r_1, r_2, i), \texttt{halt}() \mid r_1, r_2 \in R\}$. This set comprises no instruction to decrement, or clear, a register but instead, it includes an instruction to *set* a register to the value of another register.

The three *base sets* are equivalent [90]; one can build the instructions of one base set with instructions of another base set. Further, *the expansive instruction set* (that contains the instructions of *Base* 1,2 and 3) does not alter the expressiveness of the individual *Base sets*, since each of them already is Turing complete. The choice of the set of RAM instructions depends on the convenience of the programmer for the problem being addressed.

4. Planning with a random-access machine

The synthesis of effective features for a planning domain is a challenging research question investigated since the early days of AP [92]. Furthermore, the set of ground actions for the different problems of a given domain, is usually different since it depends on the number of objects in the problem; e.g. back to the sorting example illustrated in Figs. 2 and 3, classical planning problems for sorting a vector of length six induced $\frac{6\times5}{2}$ $swap(o_i, o_j)$, $i < j$ actions, while instances for sorting a vector of length seven would induce a set of $\frac{7\times6}{2}$ $swap(o_i, o_j)$ actions. This section extends the *classical planning model* with a set of *pointers*, defined over the objects of a classical planning instance, and with the *primitive instructions* for manipulating those pointers; the extension allows the agnostic definition of a set of state features, and a set of actions, that are shared by the different instances of a classical planning domain (no matter their actual number of objects).

First, the section shows how to compactly represent a *transition system* using pointers. Then the section shows that the pointer-based representation naturally applies to the STRIPS formalism. Last, the section formalizes our extension of the classical planning model with a RAM machine, that produces the aimed set of state features and actions that are shared by all the instances of a classical planning domain. Those sets of shared state features and actions are later leveraged (at Section 5) for the computation of GP solutions, that generalize no matter the number of world objects.

4.1. Representing transition systems with pointers over objects

A *transition system* can be graphically represented as a directed graph and hence formalized as a pair $(S, \rightarrow)$, where S is a set of states, and $\rightarrow$ denotes a relation of state transitions $S \times S$. Transition systems differ from *finite automata* since the sets of states and transitions are not necessarily finite. Further a *transition system* does not necessary define a *start/initial* state or a subset of *final/goal* states. Transitions between states may be labeled,[3] and the same label may appear on more than one transition. A prominent example are transition systems that correspond to a *classical planning problem* [1,2], where state transitions are labeled with *actions* (i.e. between two states $s, s' \in S$, there exists a transition $(s \xrightarrow{a} s')$ iff the execution of action a in state s produces the state s'). Given a state s and an action label a, if there exists only a single tuple (s, a, s') in $\rightarrow$ then the transition is said to be *deterministic*. In this work we restrict ourselves to *deterministic* transitions systems, i.e. transition systems such that all their transitions are deterministic.

States. WLOG we assume that the states of a transition system are factored; given a set of world objects Ω, a *state* is factored into a finite set of variables X s.t. each variable $x \in X$ either represents a *property* of a world object, or a *relation* over k world objects. Formally $x \equiv \phi(o_1, \ldots, o_k)$, where ϕ is a k-ary function in $\mathbb{N}$, and $\{o_i\}_1^k$ are objects in Ω. For instance in the example illustrated by Figs. 2 and 3, given the one-arity function *vector* and the six-objects set $\Omega = \{o_0, o_1, o_2, o_3, o_4, o_5\}$, each state variable $x_i \in X$ is defined as $x_i \equiv vector(o_i)$.

[3] When the set of labels is a singleton, the transition system is essentially *unlabeled*, so the simpler definition that omits labels applies.

```
Bool constraint_all_sorted () {
    For (Pointer z:=1; z<|Ω|; z++) {
        If ( vector(z-1) > vector(z) )
            Return False;
    }
    Return True;
}
```

Fig. 4. Boolean function `constraint_all_sorted` that checks whether the vector of state variables is sorted in increasing order. The constraint is implemented leveraging the single pointer z over the objects in Ω; $vector(z)$ is interpreted as $vector(o_z) \equiv x_z \in X$.

```
Bool schema_swap (Pointer z1, Pointer z2) {
     If (z1>=0 and z2>=0 and z1<|Ω| and z2<|Ω| and z1<z2){
        Variable aux;
        aux:= vector(z1);
        vector(z1):= vector(z2);
        vector(z2):= aux;
        Return True;
    }
    Return False;
}
```

Fig. 5. Pointer-based representation of the `swap` action schema. When applicable, the `swap` action schema exchanges the value of the state variables indexed by its two *parameters*, the pointers z_1 and z_2.

Definition 3 *(Pointer)*. Given a set of objects Ω, with $|\Omega|$ denoting the number of objects in the set, we define a pointer as a finite-domain variable z whose domain is $D_z = [0, \dots, |\Omega| - 1]$.

Pointers and state constraints. *Pointers* are variables for indexing the objects of a transition system. In combination with function symbols, pointers are useful to define *state constraints* that produce not only compact, but *general* representations of a possibly infinite set of states. By *general* we mean that a constraint represents a set of states that share some common structure, no matter the actual number of objects. Fig. 4 shows the Boolean function `constraint_all_sorted` that implements a *global constraint* for checking whether the content of the vector of state variables is sorted in increasing order; the `constraint_all_sorted` function is procedurally defined, leveraging a single pointer z, and it applies to any number of objects, and to any domain size of the corresponding state variables.

Action schemes. Action schemes compactly specify a (possibly infinite) set of transitions that share a common structure; action schemes generalize over any number, or identity of world objects. They do not refer to specific objects but instead, they leverage parameters to indirectly refer to the different world objects. Next we show that *pointers over objects* enable the compact and general definition of (possibly infinite) sets of state transitions via *action schemes*.

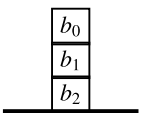
Definition 4 *(Action schema with pointers)*. Given a set of X state variables, an *action schema* with pointers is a tuple $\langle$*name, params, pre, eff*$\rangle$ where:

- *name* is the label that uniquely identifies the action schema.
- *params* is a finite set of pointers Z defined over the set Ω of objects.
- *pre* is a state constraint where state variables are indirectly addressed via the function symbols and the pointers in *params*, i.e. $x \equiv \phi(\vec{z})$ such that $\phi \in \Phi$ and $\vec{z} \in Z^{ar(\phi)}$. The *pre* state constraint implicitly represents the subset of states where the action schema is applicable.
- *eff* is a partial assignment of the state variables where a subset of the state variables is indirectly addressed via the function symbols and the pointers in *params*. The *eff* partial assignment implicitly represents the successor state that results from the execution of the action schema at a given state.

Fig. 5 illustrates our pointer-based definition of an action schema; when applicable, the `swap` action schema exchanges the value of the state variables indexed by its two parameters (the pointers z_1 and z_2). The state variables are *global*, so they can be accessed from any action schema. The `swap` action schema is succinct, because it compactly defines an infinite set of different state transitions that share a common structure. The `swap` action schema is also general, because it applies to any sorting instance, no matter the length of the vector of state variables or the domain size of those variables. What is more, the execution of the `swap` action schema is a deterministic matching-free process since the input pointers do always index a single object in Ω.

4.2. Representing STRIPS problems with pointers over objects

Since the early 1970s, the STRIPS formalism is widely used for research in *automated planning* [93]. Even today, STRIPS is an essential fragment of PDDL [86], the input language of the *International Planning Competition*, and most planners support the STRIPS representation. Here we show that our pointer-based representation naturally applies to object-centered classical planning



	State representation	
Predicates	**Strips**	**Boolean functions**
(clear ?x)	(clear b0)	clear(b0)=1
(handempty)	(handempty)	handempty()=1
(holding ?x)	-	-
(on ?x ?y)	(on b0 b1) (on b1 b2)	on(b0,b1)=1, on(b1,b2)=1
(ontable ?x)	(ontable b2)	ontable(b2)=1

Fig. 6. Example of a three-block state from the *blocksworld* (left), and its corresponding Boolean functions representation (right).

formalisms, such as STRIPS. In fact, our pointer-based representation can be understood as an instantiation of *F*-STRIPS [94], where the single level of indirection of pointers over objects is enough to represent STRIPS problems with constant memory access.

STRIPS compactly represents the set of states of a transition system using a finite set of *objects*, and a finite set of first-order logic (FOL) *predicates*, that indicate properties of the objects and their relations. Likewise, STRIPS compactly represents the space of possible state transitions using FOL *operators*, which are defined as a tuple $op = \langle name(op), args(op), pre(op), eff^-(op), eff^+(op)\rangle$ where, $name(op)$ is a unique identifier of the operator, $args(op)$ is a set of variable symbols specifying the arguments of the operator, and $pre(op)$, $eff^-(op)$, $eff^+(op)$ are sets of FOL predicates, with variables exclusively taken from $args(op)$, and that respectively specify the *preconditions*, *negative effects* and *positive effects*. The representation of a STRIPS problem is completed by specifying an *initial* state, that defines the initial situation of the objects, and the target set of *goal states*, typically specified as a partial state.

State representation. When applying our pointer-based formalism to a STRIPS problem, each state variable $x \in X$ has domain $D_x = \{0, 1\}$, and it is built as a FOL STRIPS predicate $\phi \in \Phi$ grounded by a vector of objects $\vec{o} \in \Omega^{ar(\phi)}$. Fig. 6 shows the representation of a *blocksworld* state using the STRIPS formalism, as well as using our formalism. In this state there are three blocks, $\Omega = \{b0, b1, b2\}$, that are stacked in a single tower. Predicates `clear(?x)`, `holding(?x)`, and `ontable(?x)`, are encoded as three different Boolean functions that map each vector of objects to either 0 or 1 in the current state. Omitted state variables are assumed to be zero valued. Our vector X of state variables is the result of unifying the predicates and object tuple valuations into a vector. The length of the vector of state variables is then upper bounded by $|X| \leq \sum_{k \geq 0} n_k |\Omega|^k$, where n_k is the number of first-order predicates with arity k. For instance, the X vector contains at most $|\Omega|^2 + 3|\Omega| + 1$ state variables for the *blocksworld* domain.[4]

Action representation. Given a FOL STRIPS operator $op = \langle name(op), args(op), pre(op), eff^-(op), eff^+(op)\rangle$, our pointer-based formalism produces its corresponding pointer-based action schema $\langle$*name, params, pre, eff*$\rangle$:

- The name of the action schema is $name(op)$, the name of the given FOL STRIPS operator.
- For each argument in $args(op)$, the action schema has a *pointer* that indexes an object $o \in \Omega$.
- The set $pre(op)$ is transformed into a conjunctive arithmetic-logic expression with conditions of two kinds: (i) conditions asserting that each *pointer* of the action schema is within its domain and (ii), for each precondition in $pre(op)$ a condition asserting that the state variable addressed by the pointers content equals to some specific value of its domain.
- Each negative effect in $eff^-(op)$ is transformed into an indirect variable assignment that sets the corresponding state variable to 0. Likewise, each positive effect in $eff^+(op)$ is transformed into an indirect variable assignment that sets the corresponding state variable to 1.

Next we show the grammar that formalizes our pointer-based representation of STRIPS action schemes,

$$
\begin{aligned}
\Pi ::= & \; If(CONDITIONS)\{ASSIGNMENTS\} \\
& \; Return\ False; \\
CONDITIONS ::= & \; (z >= 0 \ and \ z < |\Omega|) \ and \ CONDITIONS \mid \\
& \; (p(z_1, \ldots, z_k) == 0) \ and \ CONDITIONS \mid \\
& \; (p(z_1, \ldots, z_k) == 1) \ and \ CONDITIONS \mid \\
& \; (True) \\
ASSIGNMENTS ::= & \; p(z_1, \ldots, z_k) := 0; \ ASSIGNMENTS \mid \\
& \; p(z_1, \ldots, z_k) := 1; \ ASSIGNMENTS \mid \\
& \; Return\ True;
\end{aligned}
$$

where $CONDITIONS$ includes assertions over predicates $p(z_1, \ldots, z_k)$ instantiated with the action arguments (i.e. the pointers), and represents the operator preconditions (== denotes the equality operator, := indicates an assignment, and a semicolon denotes

[4] *State-invariants* [95] can be leveraged to save space for the memory allocation of the state variables, e.g. in the *blocksworld* one block cannot be on top of two different blocks simultaneously.

```
(:action unstack
         :parameters (?x ?y)
         :precondition (and (clear ?x) (handempty) (on ?x ?y))
         :effect (and (holding ?x) (clear ?y)
              (not (clear ?x)) (not (handempty)) (not (on ?x ?y)))))
```

Fig. 7. The `unstack` STRIPS operator from the *blocksworld* domain represented with PDDL.

```
Bool schema_unstack (Pointer z1, Pointer z2) {
    If (z1>=0 and z2>=0 and z1<|Ω| and z2<|Ω| and clear(z1)=1 and handempty()=1 and on(z1,z2)=1){
        clear(z1) := 0;
        handempty() := 0;
        on(z1,z2) := 0;
        holding(z1) := 1;
        clear(z2) := 1;
        Return True;
    }
    Return False;
}
```

Fig. 8. The `unstack` action schema from *blocksworld* defined with two pointers ($z1$ and $z2$).

```
void init() {
  clear(b0) := 1;  on(b0,b1) := 1; on(b1,b2) := 1;  ontable(b2) := 1;
}

Bool goals() {
  Return (ontable(b0)=1 and ontable(b1)=1 and ontable(b2)=1);
}
```

Fig. 9. The *init* and *goals* procedures for representing the STRIPS planning problem of unstacking the three-block tower of Fig. 6.

the end of a program instruction). $ASSIGNMENTS$ is a conjunction of assignments representing the operator positive/negative effects; in more detail $p(z_1,\dots,z_k) := 1$ denotes a *positive* effect while $p(z_1,\dots,z_k) := 0$ denotes a *negative* effect. Fig. 8 shows our pointer-based definition for the `unstack` action schema from the *blocksworld* that implements the corresponding operator represented in the STRIPS fragment of PDDL of Fig. 7. The action schema of Fig. 8 is implemented using two *pointers* (z_1 and z_2), and it applies to any *blocksworld* instance, no matter the number of blocks or their identity.

Problem representation. We complete our pointer-based representation of a STRIPS problem with the *init* and *goals* procedures: the *init procedure* is a write-only procedure, that implements a total variable assignment of the state variables for specifying the initial state of the STRIPS problem. The *goals procedure* is a read-only Boolean procedure, that encodes the state-constraint that specifies the subset of goal states. Fig. 9 shows the *init* and *goals* procedures for the planning problem of unstacking the 3-block tower of Fig. 6. The content of the *init* and *goals* procedures is inductively formalized as follows:

$$
\begin{aligned}
INIT &:= (p(o_1,\dots,o_k) := 1);\ INIT\ |;\\
GOALS &:= Return(CONDITIONS);\\
CONDITIONS &:= (p(o_1,\dots,o_k) == 0)\ and\ CONDITIONS\ |\\
&\quad\ (p(o_1,\dots,o_k) == 1)\ and\ CONDITIONS\ |\\
&\quad\ (True)
\end{aligned}
$$

Solution representation. Our pointer-based representation of a sequential plan π comprises instructions for: (i), invoking the *Boolean function* that encodes an action scheme and (ii), incrementing/decrementing the value of a pointer. Formally:

$$
\begin{aligned}
\pi &:= STATEMENTS;\\
STATEMENTS &:= a(z_1,\dots,z_k);\ STATEMENTS\ |\\
&\quad\ z{+}{+};\ STATEMENTS\ |\\
&\quad\ z{-}{-};\ STATEMENTS\ |\\
&\quad\ ;
\end{aligned}
$$

where $a(z_1,\dots,z_k)$ is an action scheme instantiated with a set of pointers $\{z_1,\dots,z_k\} \subseteq Z$, and $\{z{+}{+}, z{-} \mid z \in Z\}$ are the instructions to *increment/decrement* a pointer $z \in Z$. Pointers are always initialized to zero.

```
void ONTABLE-SEQUENTIAL-PLAN (){
    int z1=0, z2=0;
    z2++;
    act_unstack(z1,z2);
    act_putdown(z1);
    z1++;
    z2++;
    act_unstack(z1,z2);
    act_putdown(z1);
}
```

Fig. 10. Pointer-based representation of the *sequential plan* $\pi = \langle$`unstack(b0,b1), putdown(b0), unstack(b1,b2), putdown(b1)`$\rangle$ for unstacking the three-block tower of Fig. 6.

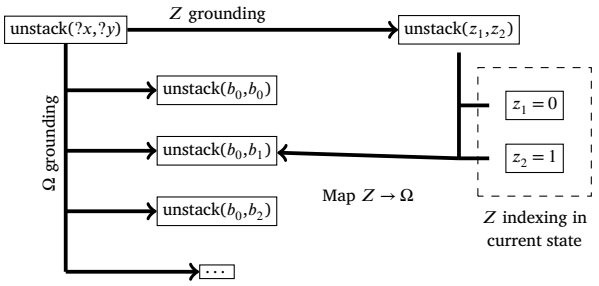


Fig. 11. Relation between the action scheme $unstack(?x, ?y)$ (i), the action $unstack(z_1, z_2)$ instantiated with pointers (z_1, z_2), and (ii), the ground actions instantiated with the set of three blocks $\Omega = \{b_0, b_1, b_2\}$. Pointers z_1 and z_2 are bound variables in $[0, \ldots, |\Omega|)$, that currently are indexing blocks b_0 and b_1, respectively.

Fig. 10 illustrates our pointer-based representation of the four-action sequential plan $\pi = \langle$`unstack(b0,b1), putdown(b0), unstack(b1,b2), putdown(b1)`$\rangle$ for unstacking the three-block tower of Fig. 6. In more detail, the `ONTABLE-SEQUENTIAL-PLAN()` program leverages two pointers, $Z = \{z_1, z_2\}$, that are initialized to zero so they initially point to the first object (block b_0 in this case). After executing the first z_2++ instruction, z_2 points to the second block, b_1, while z_1 still points to block b_0. This means that the first `act_unstack`(z_1, z_2) instruction of the program in Fig. 10 is actually executing the ground action `unstack`(b_0, b_1), which corresponds to the first step of plan π. Likewise, the first `act_putdown`(z_1) program instruction executes the ground action `putdown`(b_0), i.e. the second step of the sequential plan π. The second `act_unstack`(z_1, z_2) program instruction is executing the ground action `unstack`(b_1, b_2), since both z_1 and z_2 are increased just before that instruction is executed. Finally, the second `act_putdown(z1)` executes the ground action `putdown`(b_1), which is the fourth and last step of the sequential plan π.

Fig. 11 illustrates the particular relation between an action scheme (i), its corresponding action instantiated over pointers and (ii), the ground actions instantiated over the set of world objects Ω. In Fig. 11, the pointers z_1 and z_2 are bound variables in $[0, \ldots, |\Omega|)$, that currently are indexing blocks b_0 and b_1, respectively.

Beyond STRIPS. Our pointer-based representation supports *object typing* by simply specializing pointers to the subset of objects of a particular type [86]. Furthermore, our pointer-based representation naturally supports *classical planning with numeric state variables*, as defined in PDDL2.1 [96]. To support the representation of numeric state variables, the vector of state variables store *integers* instead of Boolean values. Goals and action preconditions can then include assertions over numeric state variables, and action effects can include assignments of the numeric state variables. For example, `distance(b0,b1)=7` can be used to indicate that the physical distance between blocks b_0 and b_1 is of seven units. Likewise `distance(z1,z2)>distance(z2,z3)` can be used to indicate that the distance between the blocks indexed by pointers $z1$ and $z2$ is larger than the distance between the blocks pointed by $z2$ and $z3$.

4.3. Extending the classical planning model with a RAM

Now we are ready to leverage our pointer-based representation, with the notion of *Random-Access Machine* (RAM), to extend the *classical planning* model. The extension produces an agnostic set of state features, and a set of actions, that are shared for the different classical planning instances of a given domain (no matter their actual number of objects).

Given a classical planning instance $P = \langle X, A, I, G\rangle$, s.t. the state variables and actions are generated with the set of functions Φ and action schemes Ξ of a given domain, grounded with a set of objects Ω. Then, the *classical planning instance* extended with a RAM machine of $|Z| + 2$ registers (i.e. $|Z|$ *pointers* that reference the planning objects, plus two dedicated FLAGS registers namely the *zero* and *carry* flags), is defined as $P'_Z = \langle X'_Z, A'_Z, I'_Z, G\rangle$ where:

- The new set of **state variables** X'_Z comprises:
  - The state variables X of the original planning instance, such that each state variable $x_i \in X$ is $x_i \equiv \phi(\vec{o})$ with $\phi \in \Phi$ and $\vec{o} \in \Omega^{ar(\phi)}$, as defined above.
  - Two *Boolean variables* $Y = \{y_z, y_c\}$, that play the role of the *zero* and *carry* FLAGS registers, respectively.
  - The *pointers* Z, a set of extra state variables s.t. each $z \in Z$ has finite domain $D_z = [0, \ldots, |\Omega| - 1]$.

  - A set of *derived state variables* $X_Z = \{ \phi(\vec{z}) \mid \phi \in \Phi, \vec{z} \in Z^{ar(\phi)} \}$ whose value is given by the interpretations of the functions of the domain with the corresponding pointers.
- The new **set of actions** A'_Z will represent the set of actions that is shared for the different classical planning instances in a given domain, and it includes:
  - The **planning actions** A' that result from reformulating each action scheme $\xi \in \Xi$ into its corresponding pointer-based version. The reformulation is a two-step procedure that requires Z to contain, at least, as many pointers as the largest arity of an scheme in Ξ: (i), each parameter in $par(\xi)$ is replaced with a pointer in Z and (ii), preconditions and effects are rewritten to refer to these pointers.
  - The **RAM actions** that implement the following sets of RAM instructions $\{\texttt{inc}(z_1), \texttt{dec}(z_1), \texttt{cmp}(z_1, z_2), \texttt{set}(z_1, z_2) \mid z_1, z_2 \in Z\}$ over the pointers in Z, and $\{\texttt{test}(\phi(\vec{z_1})), \texttt{cmp}(\phi(\vec{z_1}), \phi(\vec{z_2})) \mid \vec{z_1}, \vec{z_2} \in Z^{ar(\phi)}\}$ over the lists of pointers in $Z^{ar(\phi)}$ for each function symbol $\phi \in \Phi$. Respectively, these RAM instructions *increment/decrement* a pointer by one while keeping the values within the pointer domain, *compare* two pointers, *set* the value of a pointer z_2 to another pointer z_1, *test* whether $\phi(\vec{z_1})$ is greater than zero, and *compare* the value of $\phi(\vec{z_1})$ and $\phi(\vec{z_2})$. The $\texttt{cmp}(\phi(\vec{z_1}), \phi(\vec{z_2}))$ instructions are only required for numeric functions[5] to compare whether $\phi(\vec{z_1})$ is *greater*, *equal* or *smaller than* $\phi(\vec{z_2})$. The outcome of executing a RAM action is captured in a value, here denoted as *ret*:

$$inc(z_1) \Longrightarrow ret := \begin{cases} z_1 + 1 & \text{if } z_1 + 1 < |\Omega| \\ 0 & \text{Otherwise} \end{cases}$$

$$dec(z_1) \Longrightarrow ret := z_1 - 1,$$

$$cmp(z_1, z_2) \Longrightarrow ret := z_1 - z_2,$$

$$set(z_1, z_2) \Longrightarrow ret := z_2,$$

$$test(\phi(\vec{z_1})) \Longrightarrow ret := \phi(\vec{z_1}),$$

$$cmp(\phi(\vec{z_1}), \phi(\vec{z_2})) \Longrightarrow ret := \phi(\vec{z_1}) - \phi(\vec{z_2}).$$

    Once a RAM action is executed, its returned value is used to update the Boolean FLAGS $Y = \{y_z, y_c\}$ as defined, i.e. $y_z := (ret == 0)$ and $y_c := (ret > 0)$. FLAGS are dedicated to capture the outcome of RAM instructions. The combination of both FLAGS registers can then represent any outcome of a *three-way comparison* [97].
- The new **initial state** I'_Z is the initial state of the original planning instance, but extended with all *pointers* set to zero and the two *FLAGS* set to `False`. The **goals** are the same as those of the original planning instance.

The number of pointers $|Z|$ is a *parameter* that indicates how many pointers are used in the extension. At least Z must contain as many pointers as the largest arity of the functions Φ and action schemes Ξ of the given domain. Therefore $\{Y \cup Z \cup X_Z\}$, becomes a subset of state variables shared by all the instances that belong to the same domain, no matter their number of objects. Likewise A'_Z becomes a set of actions that is shared by all the instances that belong to the same domain, no matter their actual number of objects.

Example. Here we extend the classical planning instance $P_1 = \langle X, A, I_1, G_1 \rangle$ (illustrated in Fig. 2) using a RAM with $Z = \{i, j\}$ two pointers. According to this extension, our pointer-based representation of the sequential plan $\pi_1 = \langle swap(o_0, o_5), swap(o_1, o_2), swap(o_1, o_3) \rangle$ is the following sequence of thirteen actions $\pi'_1 = \langle inc(j)^5, swap(i, j), inc(i), dec(j)^3, swap(i, j), inc(j), swap(i, j) \rangle$; where superscripts refer to the number of times that an instruction is sequentially repeated, and where $swap(i, j)$ refers to the pointer-based action schema defined in Fig. 5. Likewise, our pointer-based version of the sequential plan $\pi_2 = \langle swap(o_0, o_2), swap(o_3, o_5) \rangle$, that solves the classical planning problem P_2 in Fig. 2, is the ten-action sequence $\pi'_2 = \langle inc(j)^2, swap(i, j), inc(i)^3, inc(j)^3, swap(i, j) \rangle$.

4.3.1. Theoretical properties

Our extension of a classical planning problem with a RAM machine preserves the solution space of the original problem. Sequential plans in the extended planning model may however be longer (e.g. the pointer-based version of plan π_1 from the previous example required thirteen steps while the original sequential plan only had three steps). As a rule of thumb, an increment of the original plan length happens when pointers must be incremented (or decremented) multiple times to access the corresponding objects before the corresponding action is executed.

Theorem 1. *Given a classical planning instance $P = \langle X, A, I, G \rangle$, its extension P'_Z, using a RAM machine with Z pointers s.t. $|Z| \geq max_{a \in A} ar(a)$, preserves the solution space of P.*

Proof. $\Rightarrow$: Let $\pi = \langle a_1, \ldots, a_m \rangle$ be a plan that solves P, an equivalent plan π' that solves P'_Z is built as follows; for each action $a_i \in \pi$, A' contains a pointer-based action schema a'_i that replaces each parameter in $par(a_i)$ with a pointer $z \in Z$. For each such pointer z, the plan repeatedly applies RAM actions `inc(z)` or `dec(z)` until they reference the associated vector of objects $\vec{o}$, and then it

[5] *Compare instructions* are syntactic sugar for Boolean functions since they can be implemented composing test instructions.

applies a'_i. The resulting plan π' has exactly the same effect as π on the original planning state variables in X, and since the goal condition of P'_Z is the same as that of P, it follows that π' solves P'_Z.

$\Leftarrow$: Let $\pi' = \langle a'_1, \ldots, a'_m \rangle$ be a plan that solves P'_Z. Identify each action in A' among those of π', and execute π' to identify the assignment of objects to pointers when applying each action in A'. Construct a plan π corresponding to the subsequence of actions in A' from π', replacing each action schema $a'_i \in A'$ by an original action $a_i \in A$ and choosing as parameters of a_i the objects referenced by the pointers of a'_i at the moment of execution. Hence a_i has the same effect as a'_i on the state variables in X, implying that π has the same effect as π' on X. Since the goal condition of P is the same as that of P'_Z, it follows that π solves P. □

The execution of a plan corresponding to a *classical planning problem extended with a RAM machine* is a deterministic matching-free process that does not require explicit action grounding; the plan execution itself determines the values of the pointers that feed the action schemes.

Theorem 2. *The new set of actions A'_Z is independent of the number of objects, state variables, and their domain size.*

Proof. The number of actions of a classical planning instance, extended with a RAM of $|Z|$ pointers, is

$$|A'_Z| = 2|Z|^2 + \sum_{\phi \in \Phi} |Z|^{2ar(\phi)} + |A'|. \tag{1}$$

This number exclusively depends on the number of pointers in Z, on the arity of the functions in Φ, and on the arity of the action schemes in Ξ. First, the *increment/decrement* instructions induce $2|Z|$ actions, the *set* instructions over pointers induce $|Z|^2 - |Z|$ actions, and *comparison* instructions of pointers induce $|Z|^2 - |Z|$ actions; comparison instructions can compare two pointers but for symmetry breaking, we only consider the single parameter ordering (z_i, z_j) where $i < j$, i.e. we consider $\texttt{cmp}(z_1, z_2)$ but not $\texttt{cmp}(z_2, z_1)$. Second, *test* instructions are defined over each function symbol and list of pointers with the same size as its arity, inducing $\sum_\phi |Z|^{ar(\phi)}$ actions, and *comparison* of predicates with pointers induce $\sum_\phi (|Z|^{2ar(\phi)} - |Z|^{ar(\phi)})$ actions. Therefore, the total number of RAM instructions are $2|Z| + 2(|Z|^2 - |Z|) + \sum_\phi (|Z|^{ar(\phi)} + |Z|^{2ar(\phi)} - |Z|^{ar(\phi)}) = 2|Z|^2 + \sum_\phi |Z|^{2ar(\phi)}$ which only depends on the number of pointers in Z and the arity of each function symbol ϕ. Last, as defined by our abstraction procedure, the number of actions in A' is given by the number of parameters of the actions schemes Ξ and the number of pointers in Z to replace those parameters. This means that the size of A' is upper bounded by $|A'| \leq \sum_{\xi \in \Xi} |Z|^{|par(\xi)|}$. Last, it follows that A'_Z, whose size is given by $|A'_Z| = 2|Z|^2 + \sum_\phi |Z|^{2ar(\phi)} + |A'|$, it is also independent of the number of objects Ω, state variables in X and their domain size. □

5. Generalized planning as heuristic search

This section presents our *GP as heuristic search* approach; first the section details the search space of our *GP as heuristic search* approach and then, the section explains the particular details of our heuristic search algorithm for GP, called BFGP.

5.1. The search space

Here we characterize the branching factor of the space of *planning programs* introduced in Section 3.3, and we show that its size depends on the domain of the planning state variables (which unfortunately may be unbound). Then we define a tractable GP search space by conditioning the branching and looping of *planning programs* with the FLAGS registers of the RAM model. We show that our new search space for GP is independent of the number of input planning instances in a GP problem, and the size of these instances (i.e. the number of objects, state variables, and their domain sizes). This enables the definition of a heuristic search approach for GP, capable of managing large sets of state variables with large numerical domains, such as integers.

5.1.1. Planning programs conditioned over valued state variables

The branching and looping of the *planning programs* introduced in Section 3.3 are conditioned over valued state variables [33], i.e. *go to* some line i if state variable x has value v. The space of this kind of solutions can be characterized by a binary encoding; given a set of state variables X, a set of actions A, a maximum number of program lines n such that the last instruction is $\Pi_{n-1} = end$, and defining the propositions of goto instructions as $(x = v)$ atoms where $x \in X$ and $v \in D_x$, the space of possible *planning programs* is encoded with three bit-vectors:

1. The *action vector* of length $(n-1) \times |A|$, indicating whether an action $a \in A$ is programmed on line $0 \leq i < n-1$.
2. The *transition vector* of length $(n-1) \times (n-2)$, indicating whether a $\texttt{go}(i', *)$ instruction is programmed on line $0 \leq i < n-1$.
3. The *proposition vector* of length $(n-1) \times \sum_{x \in X} |D_x|$, indicating whether a $\texttt{go}(*, !\langle x = v \rangle)$ instruction is programmed on line $0 \leq i < n-1$.

Definition 5 *(Partially specified planning program).* A *partially specified planning program* Π is a planning program s.t. the content of some of its program lines may be undefined, i.e. $\exists i \in [0, n)$ s.t. $\Pi[i] =< \texttt{empty} >$.

A *partially specified planning program* is encoded as the concatenation of the previous three bit-vectors and the length of the resulting bit-vector is:

$$(n-1)\left(|A|+(n-2)+\sum_{x\in X}|D_x|\right). \tag{2}$$

The previous binary encoding allows us to quantify the similarity of two *partially specified planning programs* (e.g. the *Hamming distance* of their corresponding bit-vector representation) and more importantly, to systematically enumerate the space of all possible planning programs with a maximum of n lines. Let us define the *empty program* as the particular partially specified planning program whose instructions are all undefined (i.e. all bits of its bit-vector representation are set to `False`). Starting from the *empty program*, we can enumerate the entire set of possible planning programs with two search operators:

- `program(i,a)`, that programs an action $a \in A$ at line i of a program
- `program(i,i',x,v)`, that programs a goto$(i', !\langle x = v\rangle)$ instructions at line i of a program.

These two search operators are only applicable when $\Pi[i] =<$ `empty` $>$ (meaning that i is an undefined program line i.e. in the bit-vector representation the bits corresponding to the encoding of the program line i are set to `False`). Given the bit-vector representation of a *partially specified planning program*, the application of the `program(i,a)` or `program(i,i',x,v)` search operators set the corresponding bits to `True`. With this regard, the partially specified planning program of a given search node is at *Hamming distance 1* from its parent, when programming a planning action with `program(i,a)`, or at *Hamming distance 2*, when programming a goto instruction with `program(i,i',x,v)`. In fact, this is the search space leveraged by the classical planning compilation approach for computing planning programs with an off-the-shelf classical planner [33]. Equation (2) reveals that the number of planning programs with n lines depends on the number of grounded actions $|A|$, and the number of state variables $x \in X$ and their domain D_x. This dependence causes an scalability issue, limiting the applicability of the cited compilation to planning instances of contained size. In the worst case, the domain of state variable might be infinite, e.g. a numeric state variable, hence the search space might be intractable.

5.1.2. Planning programs conditioned over a feature language

We overcome the intractability of the previous solution space by conditioning the branching and looping of *planning programs* with a finite *feature language*. In more detail, we leverage our extension of the classical planning model with a RAM, since it produces a minimalist but general set of features for the classical planning instances of a given domain.

Definition 6 *(The feature language).* We define the feature language as the four possible joint values of the two Boolean variables $Y = \{y_z, y_c\}$, and we denote this language as $\mathcal{L} = \{(\neg y_z \wedge \neg y_c), (y_z \wedge \neg y_c), (\neg y_z \wedge y_c), (y_z \wedge y_c)\}$.

Our *feature language* $\mathcal{L}$ is minimalist, it only contains four elements. We say that $\mathcal{L}$ is general because it is independent of the number of objects (and hence, of the number of state variables and their domain). Features in $\mathcal{L}$ are a function of (i) the state variables and (ii) the last executed instruction. The value of FLAGS $Y = \{y_z, y_c\}$ depend on the last executed instruction, and considering that only RAM instructions update the variables in Y, we have a space of $2^{|Y|} \times (2|Z|^2 + \sum_{\phi} |Z|^{2ar(\phi)})$ state observations implemented with only $|Y|$ Boolean variables. The four joint values of $\{y_z, y_c\}$ can model then a large space of observations, e.g. $= 0$, $\neq 0$, $< 0, > 0, \leq 0, \geq 0$ as well as relations $=, \neq, <, >, \leq, \geq$ on pairs of state variables.

Definition 7 *(The GP with a RAM search space).* Given a *GP problem* $\mathcal{P}$, that is built extending a set $\{P_1, \ldots, P_T\}$ of classical planning instances from a given domain with a RAM of $|Z|$ pointers. Our GP search space is the set of partially specified planning programs that can be built with n program lines, the common set of planning actions A'_Z, and goto instructions that are exclusively conditioned on a feature in $\mathcal{L}$.

According to Definition 7, we represent GP solutions as *planning programs* where `goto` instructions can exclusively be conditioned on a feature in $\mathcal{L}$. Limiting the conditions of `goto` instructions to any of the four features in $\mathcal{L}$ bounds the number of planning programs. Although the $Y = \{y_z, y_c\}$ flags have four possible value combinations, the fourth case $(y_z \wedge y_c) \in \mathcal{L}$ can never happen as a result of a comparison; this fourth case is however useful for representing *unconditional* `goto`. The *proposition vector* required to encode a planning program becomes now a vector of only $(n-1) \times 4$ bits (one bit for each of the four features in $\mathcal{L}$). Equation (2) simplifies then to:

$$(n-1)\left(|A'_Z| + (n-2) + 4\right). \tag{3}$$

Equation (3) shows that the size of our new solution space for GP is independent of the number of objects and hence of the number of original state variables and their domain size; Theorem 2 already showed that A'_Z no longer grows with the number of objects. This novel GP solution space can now scale to planning problems where state variables have large domains (e.g. integers) and that have a large number of state variables.

```
0. set(j,tail)
1. swap(i,j)
2. dec(j)
3. inc(i)
4. cmp(j,i)
5. goto(1, ¬(¬y_z ∧ ¬y_c))
6. end
```

```
0. set(min,i)
1. cmp(vector(j),vector(min))
2. goto(5, ¬(¬y_z ∧ ¬y_c))
3. set(min, j)
4. swap(i,min)
5. inc(j)
6. cmp(length,j)
7. goto(1, ¬(y_z ∧ ¬y_c))
8. inc(i)
9. set(j,i)
10. cmp(length,i)
11. goto(0, ¬(y_z ∧ ¬y_c))
12. end
```

Fig. 12. Two examples of generalized plans: (left) for *reversing* a list; (left) for *sorting* a list with the *selection-sort* algorithm.

Theorem 3. *The space of planning programs that exclusively branch and loop over the features in $\mathcal{L}$ preserves the solution space of planning programs that branch and loop over valued state variables.*

Proof. Given a GP problem $\mathcal{P}$ and a planning program Π conditioned over valued state variables that solves $\mathcal{P}$. An equivalent planning program, that exclusively branches over any of the features in $\mathcal{L}$, is built replacing each `goto(i,!(x=v))` instruction in Π, where $x \equiv \phi(\vec{o})$ s.t. $\phi \in \Phi$ and $\vec{o} \in \Omega^{ar(\phi)}$, by a finite block of instructions that: (i) increments/decrements a vector of auxiliary pointers $\overrightarrow{z_{aux}}$, with size $ar(\phi)$, until they index objects $\vec{o}$, (ii) given auxiliary static state variables for each possible value, i.e. $\forall_{v \in D_x} x_v$, and a dedicated object for each new state variable o_v such that $x_v \equiv \phi(o_v)$, increments/decrements another auxiliary pointer z_{static} in a function $\phi_{static}(z_{static})$ until it reaches object o_v such that $x_v \equiv \phi_{static}(o_v)$ which equals v, (iii) compares the content of these two state variables with a $\texttt{cmp}(\phi(\overrightarrow{z_{aux}}), \phi_{static}(z_{static}))$ instruction and (iv), jumps to target line i when the state variables differ in their content with a `goto(i,!(`$y_z \wedge \neg y_c$`))` instruction. □

Example. Fig. 12 shows two examples of *planning programs* that were synthesized by our BFGP algorithm searching in our tractable GP solution-space: (left) a generalized plan for reversing a list, and (right) a generalized plan for sorting a list. Note that `goto` instructions are exclusively conditioned on a feature in $\mathcal{L}$, and that both planning programs are solutions for an infinite set of classical planning problems; they generalize with a `swap` action schema of arity 2 and a `vector` function symbol of arity 1, no matter the number of objects Ω and no matter the state variables content, i.e. $x_i \equiv \texttt{vector}(o_i)$ such that $o_i \in \Omega$, $x_i \in X$ and $D_{x_i} \subseteq \mathbb{N}_0$. In the planning program for reversing a list (left), line 0 sets pointer j to the last element of the list. Line 1 swaps in the `vector` the element pointed by i (initially set to zero) and the element pointed by j, then pointer j is decremented, pointer i is incremented, and this sequence of instructions is repeated until the condition on line 5 becomes false, i.e. when $j > i$, which means that reversing the list is finished. The planning program for *sorting* a list (right) is actually an implementation of the *selection-sort* algorithm. In this program, pointers j and i are used for inner (lines 5-7) and outer (lines 8-11) loops respectively, and *min* to point to the minimum value in the inner loop (lines 3-4); $\neg y_z \wedge \neg y_c$ on line 2 represents whether the content of $\texttt{vector}(j)$ is less than the content of $\texttt{vector}(min)$, while $y_z \wedge \neg y_c$ on line 7 represents whether $j == length$ (resp. $i == length$ on line 11). Note that object ordering affects to the actual sequential plan that is eventually produced by the execution of the generalized plan but object ordering does not affect the correctness/completeness of the generalized plan. This is a common feature of structured programs e.g. a *SelectionSort* program is sound and complete but the actual number of executed `swap` instructions depends on the input list to be sorted.

5.2. The search algorithm

Given a GP problem, our heuristic search algorithm (called BFGP) implements a *Best-First Search* (BFS) in our solution space of planning programs with n program lines, and a RAM machine with $|Z|$ pointers. Algorithm 1 shows the BFGP pseudo-code; BFGP is a *frontier search* algorithm meaning that, to reduce memory requirements, BFGP stores only the *open list* of generated nodes but not the *closed list* of expanded nodes [98]. The BFGP algorithm runs as follows:

1. **Initialization.** The *empty program* is the root node of the search-tree developed by BFGP. This means that initially, the *empty program* Π_{empty} is evaluated by the evaluation functions $f \in \mathcal{F}$ and then inserted into an $Open_{\mathcal{F}}$ list, which is implemented as a priority queue.
2. **Selection.** While $Open_{\mathcal{F}}$ list is not empty, `extractBestProgram` gets the best partial program Π from $Open_{\mathcal{F}}$. A program Π is better than another program Π' iff exists a prefix of f values for Π that is smaller than the same prefix for Π', e.g. given $\mathcal{F} = \langle f_5, f_1 \rangle$, Π is better than Π' if $f_5(\Pi, \mathcal{P}) < f_5(\Pi', \mathcal{P})$, or if they tie for f_5 but $f_1(\Pi) < f_1(\Pi')$. In case both programs tie for all $f \in \mathcal{F}$, older programs (those inserted earlier in $Open_{\mathcal{F}}$) will be considered better, hence extracted first from $Open_{\mathcal{F}}$.
3. **Expansion.** Once the best partial program Π is extracted from $Open_{\mathcal{F}}$, the `expandProgram` procedure computes all children programs of Π that are syntactically valid for a given set of pointers Z and bounded number of program lines n. In more detail, BFGP expands Π by generating one successor node Π' for each program that result from programming the maximum undefined program line that is reached after executing Π on all the instances in $\mathcal{P}$. Given a partially specified program Π, only its

$max_{P_t \in (P)} f_4(\Pi, P_t)$ line is programmable. BFGP implements this particular node expansion procedure because it guarantees that duplicate successors are not generated in the BFGP search-tree. In addition, this node expansion procedure induces a tractable branching factor of $(|A'_Z| + (n-2) \times 4)$; at a given program line BFGP can only program a planning action in A'_Z or a goto instruction that can jump to $n-2$ different destination program lines, and that is conditioned by any of the four different features in $\mathcal{L}$. The depth of the search tree developed by the BFGP algorithm is the number of program lines n, since only an undefined line can be programmed.

4. **Insertion.** Before a new search node is inserted into the open list, the corresponding program Π' is executed on the classical planning instances in $\mathcal{P}$. This execution can result in the three following different outcomes:
   (a) Π' *is a solution for* $\mathcal{P}$. If the execution of Π' solves all the instances $P_t \in \mathcal{P}$, then search ends, and Π' will be returned as a valid solution for the GP problem $\mathcal{P}$.
   (b) Π' *fails to solve* $\mathcal{P}$. If the execution of Π' on a given instance $P_t \in \mathcal{P}$ fails, this means that the search node corresponding to the partially planning program Π' is a dead-end. The search node will be discarded, so Π' is not inserted into the open list. The source of failure could be either because a terminal instruction is executed but the goal condition does not hold, or because an infinite loop is detected.
   (c) Π' *may still be a solution for* $\mathcal{P}$. This means that the execution of Π' on some of the classical planning instances in $\mathcal{P}$ reached an undefined program line (Π' might solve some of the instances in $\mathcal{P}$). As a consequence Π' is inserted into $Open_{\mathcal{F}}$ by calling `insertProgram` at its corresponding position according to its evaluation over $\mathcal{F}$ functions.

Algorithm 1: Best-First Generalized Planning (BFGP).

```
Data: The GP problem P, pointers Z, program lines n, a list of evaluation functions F
Result: A generalized plan Π that solves P
Open_F ← {Π_empty} ;
while Open_F ≠ ∅ do
    Π ← extractBestProgram(Open_F) ;
    ChildrenPrograms ← expandProgram(Π, Z, n) ;
    for Π' ∈ ChildrenPrograms do
        if isDeadEnd(Π', P) then
            continue
        if isGoal(Π', P) then
            return (Π')
        Open_F ← insertProgram(Open_F, Π');
    end
end
return () // no solution found
```

Example. Let us recover from the previous example (Fig. 2) the GP problem $\mathcal{P} = \{P_1, P_2\}$, and the partially specified program $\Pi =$ `0.swap(i,j) 1.inc(i) 2.dec(j) 3.goto(2,!(`$y_z \wedge \neg y_c$`)) 4.<empty> 5.end`, where lines $[0,3]$ are programmed and only line 4 is unspecified. Imagine now that BFGP extracts this program from the open list because it has the best evaluation value. In this case, the execution of Π on the classical planning instances P_1 and P_2, implemented by the node evaluation procedure, ended in both instances at the undefined program line 4. This means that the only programmable line is 4. Assuming that two pointers are available (i.e. $Z = \{i, j\}$) we can program any of following twelve actions in line 4. $\{inc(i)$, $inc(j)$, $dec(i)$, $dec(j)$, $cmp(i,j)$, $set(i,j)$, $set(j,i)$, $test(vector(i))$, $test(vector(j))$, $cmp(vector(i), vector(j))$, $swap(i,j)$, $swap(j,i)$ $\}$. A goto can only be programmed after a RAM action, which is not the case of line 4, since line 3 has another goto instruction.[6] In other words the search node corresponding to the partially specified program from the previous example would have twelve children that could be added to the open list.

5.2.1. The evaluation functions

Here we present the evaluation and heuristic functions that guide the BFGP algorithm. The functions that range from f_1 to f_6 were introduced in prior work [31], while f_7, f_8 and f_9 are introduced for first time in this article. Here we define two different families of evaluation functions, that exploit two different sources of information, to guide a combinatorial search in our GP solution space of partially specified planning programs:

- *Program structure*. Given a partially specified planning program Π, we define a set of evaluation functions $f(\Pi)$, that establish *preferences/priors* on the structure of the aimed generalized plans. For instance, following the *Occam's razor principle* a structural function can prefer generalized plans of simpler complexity or it can prefer generalized plans with more programmed lines so execution failures can be detected earlier in the search.
  - $f_1(\Pi)$, the number of goto instructions in Π.

[6] In the hypothetical case that previous line 3. would contain a RAM action, a goto instruction for jumping to lines $[0,3]$ conditioned by the corresponding four features in $\mathcal{L}$ could also be programmed at line 4.

- $f_2(\Pi)$, the number of *undefined* program lines in Π.
- $f_3(\Pi)$, the maximum number of occurrences that any instruction $w \in A'_Z$ is programmed in Π, i.e.,

$$f_3(\Pi) = max_{w \in A'_Z} \sum_{w_i \in \Pi} \mathbb{1}(w == w_i).$$

- $f_7(\Pi)$, the max number of *nested* goto instructions in Π. A goto instruction jumps from an origin program line to a destination program line. We say that a goto instruction is *nested* when it appears within the origin and destination lines of another goto instruction.

• *Fitness to the input instances*. Given a partially specified planning program Π and a GP problem $\mathcal{P} = \{P_1, \ldots, P_T\}$, we define a set of evaluation functions $f(\Pi, \mathcal{P})$ that assess the performance of Π on $\mathcal{P}$, executing Π on each of the classical planning instances $P_t \in \mathcal{P}$, $1 \leq t \leq T$. Section 3 defined the execution of a planning program on a classical planning instance as a deterministic procedure that terminates either succeeding to solve that instance or failing it. Likewise the execution of a partially specified planning program is a deterministic procedure that introduces a new termination case, *reaching an unspecified program line*. When the program execution terminates because an unspecified program line is reached, $f(\Pi, \mathcal{P})$ functions can be used to assess the cost of that program execution, as well as to estimate how far is the program from solving the given GP problem.

- $f_4(\Pi, \mathcal{P}) = n - max_{P_t \in \mathcal{P}} f_4(\Pi, P_t)$, where $f_4(\Pi, P_t)$ returns the number of the undefined program line eventually reached after executing Π on the classical planning instance $P_t \in \mathcal{P}$.
- $f_5(\Pi, \mathcal{P}) = \sum_{P_t \in \mathcal{P}} f_5(\Pi, P_t)$, where

$$f_5(\Pi, P_t) = \sum_{x \in X_t} (v_x - G_t(x))^2.$$

  Here, $v_x \in D_x$ is the value eventually reached, for the state variable $x \in X_t$, after executing Π on the classical planning instance $P_t \in \mathcal{P}$, and $G_t(x)$ is the value for this same variable as specified in the goals of P_t. Note that for Boolean variables the squared difference becomes a simple difference. This means that for STRIPS planning problems, where all the state variables are Boolean, $f_5(\Pi, P_t)$ is actually a counter of how many atomic goals in G_t are still not true.
- $f_6(\Pi, \mathcal{P}) = \sum_{P_t \in \mathcal{P}} |exec(\Pi, P_t)|$, where $exec(\Pi, P_t)$ is the sequence of actions induced from executing the planning program Π on the planning instance P_t.
- $f_8(\Pi, \mathcal{P}) = f_5(\Pi, \mathcal{P}) + f_6(\Pi, \mathcal{P})$ is the sum of an estimation to the goal and the total accumulated cost, akin to an evaluation function for A^* searching algorithm.
- $f_9(\Pi, \mathcal{P}) = W \cdot f_5(\Pi, \mathcal{P}) + f_6(\Pi, \mathcal{P})$ is similar to f_8 but the estimation to the goal is multiplied by a factor W, which is set to 5 by default, akin to an evaluation function for WA^* searching algorithm.

All these functions are *evaluation functions* (i.e. smaller values are preferred). The structural functions $f_1(\Pi)$, $f_2(\Pi)$, $f_3(\Pi)$ and $f_7(\Pi)$, are all computed in linear time by traversing the bit-vector representation of Π. On the other hand, the computational complexity of the three empirical functions $f_4(\Pi, \mathcal{P})$, $f_5(\Pi, \mathcal{P})$, $f_6(\Pi, \mathcal{P})$, $f_8(\Pi, \mathcal{P})$ and $f_9(\Pi, \mathcal{P})$ is given by the complexity of the partially specified program Π. *Performance-based* functions accumulate a set of T costs (one for each classical planning instance in the GP problem) that could actually be combined with different aggregation functions, e.g. *sum*, *max*, average, weighted average, etc. Functions $f_4(\Pi, \mathcal{P})$ and $f_5(\Pi, \mathcal{P})$ are the only *cost-to-go heuristic* functions; they provide an estimate on how far is a partially specified planning program from solving a GP problem. With this regard, $f_5(\Pi, \mathcal{P})$ requires that the goal condition of the classical planning instances in a GP problem is specified as a partial state. On the other hand $f_4(\Pi, \mathcal{P})$ does not impose requirements on the structure of the goal condition, so they can even be a *black-box* Boolean procedure over the state variables.

Example. We illustrate how our evaluation functions work on the following partially specified program Π = `0.swap(i,j) 1.inc(i) 2.dec(j) 3.goto(2,!(`$y_z \wedge \neg y_c$`)) 4.<empty> 5.end`, where only line 4 is not programmed yet. The value of the evaluation functions for this partially specified program is $f_1(\Pi) = 1$, $f_2(\Pi) = 5 - 4 = 1$, $f_3(\Pi) = 0$, $f_7(\Pi) = 1$. Given the GP problem $\mathcal{P} = \{P_1, P_2\}$ that comprises the two classical planning instances illustrated in Fig. 2, and pointers i and j starting at the first and last object indexes, respectively, we can compute f_4 and f_5 to evaluate how far Π is from solving the GP problem of sorting lists, the accumulated cost f_6, and evaluation functions f_8 and f_9 that combine heuristic-like functions with accumulated cost. In this case $f_4(\Pi, \mathcal{P}) = 5 - 4 = 1$, $f_5(\Pi, \mathcal{P}) = 32$, $f_6(\Pi, \mathcal{P}) = 14 + 14 = 28$, $f_8(\Pi, \mathcal{P}) = 32 + 28 = 60$ and $f_9(\Pi, \mathcal{P}) = 32 + 5 \cdot 28 = 172$.

5.2.2. Theoretical properties

Theorem 4 *(Termination). Given a generalized planning problem $\mathcal{P}$, a finite set of pointers Z, and a finite number of program lines n, the execution of the* BFGP *algorithm always terminates.*

Proof. By definition of the expansion procedure of the BFGP algorithm (i), only unspecified lines can be programmed and (ii), children programs always have one more line programmed than their parent. This means that BFGP increases the number of programmed lines, until all lines are programmed. When all lines are programmed BFGP necessarily terminates, either by succeeding to solve $\mathcal{P}$, or by failing to solve some of the classical planning instances in $\mathcal{P}$. The only possible cause for the non-termination of the BFGP algorithm would be that BFGP could generate duplicate search nodes, allowing the infinite re-opening of an already discarded

node. By definition of the expansion procedure of the BFGP algorithm, the re-opening of an already discarded node is impossible; BFGP only allows programming the maximum undefined program line that is reached after the execution of that program on all the instances in $\mathcal{P}$. □

Theorem 5 *(Completeness). Given a GP problem $\mathcal{P}$, a maximum number of pointers $|Z|$, and maximum number of program lines n, if there is a planning program Π within these bounds that solves $\mathcal{P}$, then the BFGP algorithm can compute it.*

Proof. The BFGP algorithm implements a complete enumeration of the entire space of planning programs with a maximum number of pointers $|Z|$ and maximum number of program lines n except: (i), a search node was identified as a dead-end or (ii), the ancestor of a search node was identified as a dead-end. BFGP is safe because it only discards a search node when its corresponding partially specified planning program failed to solve an input planning instance (which is actually the definition for not being a GP solution). Furthermore, if a partially specified planning program failed to solve an input planning instance, any planning program that can be built programming the remaining undefined program lines will also fail to solve that problem. □

Theorem 6 *(Soundness). If the execution of the* BFGP *algorithm on a GP problem $\mathcal{P}$ outputs a generalized plan Π, this means that Π is a solution for $\mathcal{P}$.*

Proof. The BFGP algorithm runs until: (i) the open list is empty, which means that search space is exhausted without finding a solution and no generalized plan is output or (ii), BFGP extracted from the open list a planning program whose execution, in all the classical planning instances $P_t \in \mathcal{P}$, resulted successful. This is actually the definition of a solution for a GP problem. □

Theorem 7 *(Time and Memory). The time and memory consumption of the BFGP algorithm are characterized by the big-Oh expression $O((|A'_Z| + (n-2)\times 4)^n)$.*

Proof. The BFGP algorithm is an implementation of a BFS, whose memory and time complexity are characterized as $O(b^d)$, where b denotes the branching factor and d denotes the depth of the corresponding search tree. The branching factor of the search tree induced by the BFGP algorithm is the number of different instructions that can be programmed at an undefined program line, which is $b \leq |A'_Z| + (n-2)\times 4$; gotos can only be programmed after RAM operations. The depth of the search tree induced by the BFGP is given by the maximum number of program lines n. □

BFGP may be incomplete in the sense that either the bound n on the maximum number of program lines, or the maximum number of pointers available $|Z|$, may be too small to accommodate a solution to a given GP problem. With respect to solution quality BFGP does not guarantee that the computed planning programs are optimal. BFGP can however be run in *anytime mode* and use $f_6(\Pi, \mathcal{P})$ to rank GP solutions according to their execution cost in the classical planning instances that are comprised in the given GP problem (e.g. to prefer a sorting planning program with smaller computational complexity).

6. Evaluation

This section evaluates the empirical performance of our *GP as heuristic search* approach. All experiments are performed in an Ubuntu 20.04 LTS, with AMD® Ryzen 7 3700x 8-core processor × 16 and 32 GB of RAM.[7]

6.1. Benchmarks

We report results in nine different domains; three *propositional* domains and six *numeric* domains. In the *propositional* domains the functions Φ that induce the state variables are Boolean. In the *numeric* domains these functions are positive numeric functions. Next, we provide more details on each of the nine domains:

- *Corridor*, an agent moves from an arbitrary initial location to a destination location in a corridor.
- *Gripper*, a robot must pick all balls from room A and drop them in room B.
- *Visitall*, starting from the bottom-left corner of a squared grid, an agent must visit all cells.
- *Fibonacci*, compute the n^{th} term of the Fibonacci sequence.
- *Find*, counts the number of occurrences of a specific value in a list.
- *Reverse*, for reversing the content of a list.
- *Select*, find the minimum value of a list.
- *Sorting*, for sorting the values of a vector.
- *Triangular Sum*, compute the n^{th} triangular number.

[7] The source code, evaluation scripts, and used benchmarks can be found at: https://github.com/jsego/bfgp-pp.

Corridor, *gripper* and *visitall* are propositional, the remaining six domains are numeric. For each domain, we build a GP problem that comprises ten randomly generated classical planning instances[8]; in the case of the *corridor* domain, instances go from corridors of length 3 to 12; in *gripper*, instances have from 2 to 11 balls in room A to be dropped in room B; in *visitall* instances are squared grids ranging from 2×2 to 11×11 cells; *Fibonacci* and *triangular sum* instances ranging from the 2^{nd} to the 11^{th} number in the sequence; and the remaining domains, *find*, *reverse*, *select* and *sorting* have instances with vectors from size 2 to 11 that are initialized with random content. The result of arithmetical operations in these domains is bounded to 10^2 in the synthesis of GP solutions, and to 10^9 in the validation of GP solutions.

All domains include the base set of RAM instructions $\{\texttt{inc}(z_1), \texttt{dec}(z_1), \texttt{cmp}(z_1, z_2), \texttt{set}(z_1, z_2) \mid z_1, z_2 \in Z\}$, such that z_1 and z_2 are pointers of the same *type*, and the RAM instructions $\{\texttt{test}(\phi(\overrightarrow{z_1})), \texttt{cmp}(\phi(\overrightarrow{z_1}), \phi(\overrightarrow{z_2})) \mid \overrightarrow{z_1}, \overrightarrow{z_2} \in Z^{ar(\phi)}\}$, for each function $\phi \in \Phi$ and where function parameters and pointers also are of the same *type*. We remind the reader that *compare* instructions are only defined for numeric functions. In addition, each domain contains the regular planning action schemes that do not affect the FLAGS. We also recall that planning actions are modeled as they are always executable, but that their effects only update the current state if their preconditions hold in the current state. Otherwise the execution of an action has no effect.

- **Propositional domains**. The *corridor* domain needs two planning action schemes, $\texttt{move\_left}(z_1, z_2)$ and $\texttt{move\_right}(z_1, z_2)$, to move from location at z_1 to location at z_2, which must be exactly one location to the left or the right of z_1, respectively. The *gripper* domain includes the following three action schemes; $\texttt{move}(z_1, z_2)$ to denote the robot is moving from the room pointed by z_1 to the one pointed by z_2, $\texttt{pick}(z_1, z_2, z_3)$ to pick the ball pointed by z_1, at room pointed by z_2, and with the gripper pointed by z_3, and $\texttt{drop}(z_1, z_2, z_3)$, to drop ball z_1 at room z_2 with gripper z_3. *Visitall* needs four action schema to move in the four cardinal directions of grid, i.e. $\texttt{move\_left}(z_1, z_2, z_3)$ to move one to the left from column z_1 to z_2, at row z_3 (resp. for `move_right`), and $\texttt{move\_up}(z_1, z_2, z_3)$ to move once up from row z_1 to z_2, at column z_3 (resp. for `move_down`); every move visits the destination cell.
- **Numeric domains**. The *triangular sum* and *Fibonacci* domains include the action schemes $\texttt{vector\_inc}(z_1)$ and $\texttt{vector\_dec}(z_1)$, to increase and decrease by one the content of the vector at z_1, and the action scheme $\texttt{vector\_add}(z_1, z_2)$ for adding the content of the vector at z_2 to the content at z_1. *Select* only requires one action schema to mark a specific vector index at z_1 as selected, i.e. $\texttt{select}(z_1)$. *Find* includes the $\texttt{accumulate}(z_1)$ action schema for counting the number of occurrences of the target element. *Reverse* and *Sorting* include the $\texttt{swap}(z_1, z_2)$ action scheme to swap the vector values addressed by z_1 and z_2.

6.2. Synthesis and validation of GP solutions

Here we present the first experiments to evaluate the performance of the BFGP algorithm. First, we assess every evaluation/heuristic function f_i by running BFGP(f_i). Then we show the solutions generated by the best configuration, $BFGP(f_5)$. Last, the synthesized solutions are validated w.r.t. test sets of larger instances (i.e. larger number of objects).

6.2.1. Performance of BFGP(f_i)

Table 1 details the results of the first synthesis experiment where the BFGP algorithm uses each of our nine different evaluation/heuristic functions (the computation bounds are $3,600$ seconds of CPU time and $32GB$ of memory and best results are shown in bold). Regarding *structure*-based functions f_2 dominates in all domains and metrics (except in the *reverse* domain where f_3 is faster and f_1 expands fewer nodes) and it also has the highest coverage solving 7 out of 9 domains (f_1, f_3 and f_7 have lower coverage failing in the same four domains, namely *corridor*, *gripper*, *sorting* and *visitall*). Regarding *performance*-based functions, f_5 is the clear winner with the best scores in more than half of domains and with full coverage over the benchmarks, followed by f_4 and f_9 that cover 6 domains but improving the metrics of f_5 in several domains, e.g., f_4 has the lowest memory consumption in all domains and expands and/or evaluates fewer nodes in *Fibonacci*, *reverse* and *triangular sum*.

Table 2 summarizes the results from Table 1, grouping results by domains and averaging the metrics by the total number of functions that solved each domain. There are 5 domains which are solved by all the nine evaluation/heuristic functions. In the rest of domains, there are at least 5 or more functions that do not solve them, i.e. *gripper* is only solved by f_2, f_4, f_5 and f_8; *corridor* and *visitall* are the least solved domains (only f_5 solved them); and *sorting* which is solved by f_2 and f_5 but it is the hardest in terms of each metric average.

6.2.2. The synthesized solutions

Fig. 13 shows the programs computed by $BFGP(f_5)$. In *Corridor* there are two pointers, $l1$ and $l2$, that start pointing to the first location; the solution moves the agent to the rightmost cell, then one by one to the left while not at goal cell. In *Fibonacci*, pointers i and j are used to compute the n-th Fibonacci number, where i addresses the F_i number to which F_{i-1} and F_{i-2} are added using j as an auxiliary pointer; and finishes when i reaches the n-th element (the last one). In *Find*, there is a pointer i to iterate over a vector, accumulating for each vector location the occurrences of a target value.

The *Gripper* solution uses one pointer for balls (b_1), two for rooms (r_1 and r_2) and one for grippers g_1; for each ball b_1, the agent will pick it up from room r_1 (always room A) using gripper g_1 (always left gripper), sets r_2 to room B, moves from A to B, drop ball b_1 at room B, goes back to room A, and continues with the next ball. The *Reverse* domain uses two pointers i and j and finds a

[8] For reproducibility reasons we fix the random seed to generate the classical planning instances in the GP problems.

Table 1
We report the number of program lines n, and pointers $|Z|$ per domain, and for each evaluation/heuristic function, CPU time (secs), memory peak (MBs), and the numbers of expanded and evaluated nodes. TE stands for Time-Exceeded (>1h of CPU time). Best results in bold.

Domain	$n, \|Z\|$	f_1				f_2				f_3			
		Time	Mem.	Exp.	Eval.	Time	Mem.	Exp.	Eval.	Time	Mem.	Exp.	Eval.
Corridor	10, 2	TE	-	-	-	TE	-	-	-	TE	-	-	-
Fibonacci	7, 2	167	164	42.6K	373.8K	**72**	**5**	**38.0K**	**38.1K**	1,254	259	627.2K	783.9K
Find	4, 1	**0**	**4**	9	31	**0**	**4**	**4**	**14**	**0**	**4**	8	31
Gripper	8, 4	TE	-	-	-	1,988	**5**	1.1M	1.1M	TE	-	-	-
Reverse	7, 2	83	45	**82.6K**	148.8K	109	**5**	135.8K	**135.9K**	**80**	68	88.8K	164.0K
Select	7, 2	550	82	354.3K	361.3K	458	**5**	340.4K	340.4K	336	115	228.2K	281.2K
Sorting	9, 2	TE	-	-	-	3,555	**5**	3.5M	**3.5M**	TE	-	-	-
T. Sum	5, 2	1	6	373	2.7K	**0**	**5**	**207**	**238**	1	6	408	3.0K
Visitall	13, 4	TE	-	-	-	TE	-	-	-	TE	-	-	-
Average		160.2	60.2	96.0K	177.3K	883.1	4.9	730.6K	730.6K	334.2	90.4	188.9K	246.4K
		f_4				f_5				f_6			
Corridor	10, 2	TE	-	-	-	**86**	**41**	**16.8K**	**78.1K**	TE	-	-	-
Fibonacci	7, 2	77	**5**	38.1K	38.2K	187	189	68.5K	457.7K	287	333	104.8K	793.4K
Find	4, 1	**0**	**4**	**4**	**14**	**0**	**4**	**4**	**14**	**0**	**4**	9	31
Gripper	8, 4	2,015	6	1.1M	1.1M	**20**	39	**3.6K**	74.9K	TE	-	-	-
Reverse	7, 2	98	**5**	136.4K	137.3K	186	81	220.1K	221.0K	188	87	216.9K	220.1K
Select	7, 2	434	6	348.8K	349.6K	**78**	81	**29.0K**	**196.3K**	524	142	339.2K	346.9K
Sorting	9, 2	TE	-	-	-	**2,054**	2,483	**988.9K**	6.3M	TE	-	-	-
T. Sum	5, 2	**0**	**5**	**207**	**238**	1	6	343	2.3K	1	6	448	3.2K
Visitall	13, 4	TE	-	-	-	**998**	**67**	**116.6K**	**122.7K**	TE	-	-	-
Average		437.3	5.2	270.6K	270.9K	401.1	332.3	160.4K	828.1K	200.0	114.4	132.3K	272.7K
		f_7				f_8				f_9			
Corridor	10, 2	TE	-	-	-	TE	-	-	-	TE	-	-	-
Fibonacci	7, 2	239	264	81.3K	627.4K	273	285	86.3K	687.6K	159	182	62.5K	435.1K
Find	4, 1	**0**	**4**	9	31	**0**	**4**	**4**	**14**	**0**	**4**	**4**	**14**
Gripper	8, 4	TE	-	-	-	TE	-	-	-	22	38	**3.6K**	**74.1K**
Reverse	7, 2	146	62	169.5K	205.1K	204	78	219.8K	220.9K	203	78	219.8K	220.9K
Select	7, 2	456	95	292.7K	332.8K	559	140	339.2K	346.8K	536	82	332.6K	346.7K
Sorting	9, 2	TE	-	-	-	TE	-	-	-	TE	-	-	-
T. Sum	5, 2	1	6	428	3.0K	1	6	388	3.0K	1	6	336	2.3K
Visitall	13, 4	TE	-	-	-	TE	-	-	-	TE	-	-	-
Average		168.4	86.2	108.8K	233.7K	207.4	102.6	129.1K	251.7K	153.5	65.0	103.1K	179.9K

Table 2
We report for each domain, the time (secs), memory peak (MBs), and expanded and evaluated nodes averaged by the number of functions that solved the domain in Table 1.

Domain	Time	Mem.	Exp.	Eval.	# f_i Solved
Corridor	86.0	41.0	16.8K	78.1K	1
Fibonacci	442.6	227.9	0.2M	0.6M	9
Find	0.0	4.0	6.1	21.6	9
Gripper	1,011.3	22.0	1.1M	1.2M	4
Reverse	144.1	56.6	165.5K	186.0K	9
Select	436.8	83.1	289.4K	322.4K	9
Sorting	2,805.5	1,244	2.2M	4.9M	2
T. Sum	0.8	5.8	348.7	2.2K	9
Visitall	998.0	67.0	116.6K	122.7K	1

solution with $O(n^2)$ complexity of a vector of size n; it moves all values from j to $n-1$ indexes once to the right and places the last element in the j-th location, using i as an auxiliary pointer; then increases j by one until it reaches the end of the vector. The *Select* domain has two pointers i and j; it iterates over the vector with pointer i, and assigns i to j every time the value pointed by i is smaller than the one pointed by j, at the end j will point to the smallest value which will be selected.

The *Sorting* solution is succinct but easy to interpret; while visiting each vector index in increasing order with i and j pointers s.t. $j = i - 1$, if i^{th} value is smaller than j^{th} it moves the value backwards in the vector until it is relatively sorted, then proceeds searching for the next pair of adjacent and unsorted values, until all values are sorted. In *Triangular Sum*, given a vector initialized as $v_{idx} = idx$, each index is visited in increasing order with pointers i and j such that $j = i - 1$, executing for each one $v_i \leftarrow v_i + v_j$. The last domain, *Visitall*, has two pointers $r1$ and $r2$ for rows, and two more $c1$ and $c2$ to iterate over columns. Since the agent starts

```
CORRIDOR
0. move_right(l1,l2)
1. set(l1,l2)
2. inc(l2)
3. goto(0,¬(y_z ∧ ¬y_c))
4. move_left(l1,l2)
5. set(l1,l2)
6. dec(l2)
7. test(goal_at(l1))
8. goto(4,¬(¬y_z ∧ y_c))
9. end
```

```
FIBONACCI
0. vector_add(i,j)
1. dec(j)
2. vector_add(i,j)
3. set(j,i)
4. inc(i)
5. goto(0,¬(y_z ∧ ¬y_c))
6. end
```

```
FIND
0. accumulate(i)
1. inc(i)
2. goto(0,¬(y_z ∧ ¬y_c))
3. end
```

```
GRIPPER
0. pick(b1,r1,g1)
1. inc(r2)
2. move(r1,r2)
3. drop(b1,r2,g1)
4. move(r2,r1)
5. inc(b1)
6. goto(0,¬(y_z ∧ ¬y_c))
7. end
```

```
REVERSE
0. set(i,j)
1. swap(i,j)
2. inc(i)
3. goto(1,¬(y_z ∧ ¬y_c))
4. inc(j)
5. goto(0,¬(y_z ∧ ¬y_c))
6. end
```

```
SELECT
0. cmp(vector(i),vector(j))
1. goto(3,¬(¬y_z ∧ ¬y_c))
2. set(j,i)
3. inc(i)
4. goto(0,¬(y_z ∧ ¬y_c))
5. select(j)
6. end
```

```
SORTING
0. cmp(vector(i),vector(j))
1. goto(5,¬(¬y_z ∧ ¬y_c))
2. swap(i,j)
3. dec(i)
4. dec(i)
5. set(j,i)
6. inc(i)
7. goto(0,¬(y_z ∧ ¬y_c))
8. end
```

```
TRIANGULAR SUM
0. vector_add(i,j)
1. set(j,i)
2. inc(i)
3. goto(0,¬(y_z ∧ ¬y_c))
4. end
```

```
VISITALL
0. inc(c1)
1. move_right(c2,c1,r1)
2. inc(c2)
3. goto(0,¬(y_z ∧ ¬y_c))
4. inc(r1)
5. move_up(r2,r1,c1)
6. dec(c1)
7. move_left(c2,c1,r1)
8. dec(c2)
9. goto(5,¬(y_z ∧ ¬y_c))
10. inc(r2)
11. goto(0,¬(y_z ∧ ¬y_c))
12. end
```

Fig. 13. Solutions computed by BFGP(f_5).

```
GRIPPER
0. test(goal_at(b1,r2))
1. goto(3,¬(¬y_z ∧ y_c))
2. inc(r1)
3. test(goal_at(b1,r2))
4. goto(6,¬(y_z ∧ ¬y_c))
5. inc(r2)
6. pick(b1,r1,g1)
7. move(r1,r2)
8. drop(b1,r2,g1)
9. move(r2,r1)
10. inc(b1)
11. goto(6,¬(y_z ∧ ¬y_c))
12. end
```

Fig. 14. Alternative solution to the Gripper domain. This program is interpreted as moving $r1$ to room A if both pointers are initially in room B, otherwise move $r2$ to room B, then for each ball run a pick action, move to room B, drop it, move back to room A, until all balls have been moved.

in the bottom-left corner, the strategy consists of visiting the grid row by row, going first to the right, once up, then all to the left, until all rows are visited.

BFGP implements an *inductive* approach to GP, this means that the computed generalized plans are synthesized with the only requirement of satisfying the formal specification given by the finite set of input examples. Therefore, in domains where input examples follow a clear distribution, it may then be exploited by BFGP (e.g. the locations of a corridor or grid, or the indexes of a vector, which are naturally expressed in a specific order). However, the *object ordering* does not affect the correctness/completeness of the approach, e.g. the *Sorting* program is able to sort any input list no matter its size or content, or the *Blocks Ontable* program (Fig. 15) puts all blocks onto the table no matter the block names, their ordering, or the initial towers settings, with the cost of an extra iteration. In the particular case of *Gripper*, rooms are constants in the domain, for that reason they always appear in the same order. In case we shuffle the rooms order for each instance, we would either need more lines to solve *Gripper* (i.e. solve the problem for each possible permutation), or add the goal information in the initial state, as it is done in *Corridor* and *Grid*, to keep the solution short. In the latter, the *planning program* of Fig. 14 solves all *Gripper* instances with no particular order in rooms.

6.2.3. Validation of the synthesized solutions

Here we validate the BFGP(f_5) solutions of Fig. 13 with a larger and harder set of instances. Table 3 reports the CPU time, and peak memory, yield when running the solutions synthesized by $BFGP(f_5)$ on a validation set. All the solutions synthesized by $BFGP(f_5)$ were successfully validated. The largest CPU times and memory peaks correspond to the configuration that implements

Table 3
Validation set, CPU time (secs) and memory peak for program validation, with/out *infinite program* detection. Best results in bold.

Domain	Instances	Time_∞	Mem_∞	Time	Mem
Corridor	100	0.42	11.4MB	**0.13**	**10.8MB**
Fibonacci	33	0.03	5.8MB	**0.02**	**5.6MB**
Find	100	9.84	70.1MB	**0.69**	**48.5MB**
Gripper	1,000	153.53	1,007.6MB	**16.88**	**968.2MB**
Reverse	100	6.67	30.2MB	**1.40**	**12.5MB**
Select	100	18.39	163.8MB	**1.38**	**113.2MB**
Sorting	100	4.98	31.9MB	**1.49**	**12.5MB**
Triangular Sum	1,000	72.69	706.2MB	**9.59**	**685.7MB**
Visitall	50	44.48	403.3MB	**3.83**	**79.7MB**

Table 4
For each domain we report, CPU time (secs), memory peak (MBs), num. of expanded and evaluated nodes of BFGP(f_3, f_5), BFGP(f_5, f_3) and BFGP(f_5). Best results in bold.

Domain	BFGP(f_3, f_5)				BFGP(f_5, f_3)				BFGP(f_5)			
	Time	Mem.	Exp.	Eval.	Time	Mem.	Exp.	Eval.	Time	Mem.	Exp.	Eval.
Corridor	**38**	**25**	**7.8K**	**44.2K**	**38**	**25**	**7.8K**	**44.2K**	86	41	16.8K	78.1K
Fibonacci	1,227	**139**	579.7K	718.2K	457	**139**	293.8K	**431.2K**	**187**	189	**68.5K**	457.7K
Find	**0**	**4**	**3**	**14**	**0**	**4**	4	**14**	**0**	**4**	4	**14**
Gripper	19	**37**	**3.2K**	**69.5K**	20	**37**	3.4K	69.6K	20	39	3.6K	74.9K
Reverse	**125**	**61**	**134.6K**	**163.8K**	201	**61**	219.9K	220.9K	186	81	220.1K	221.0K
Select	**65**	**64**	**23.6K**	**152.9K**	**65**	**64**	**23.6K**	**152.9K**	78	81	29.0K	196.3K
Sorting	**1,137**	1,604	**516.4K**	**3.9M**	1,215	**1,596**	555.2K	4.0M	2,054	2,483	988.9K	6.3M
T. Sum	**1**	**6**	**304**	**2.2K**	**1**	**6**	**304**	**2.2K**	**1**	**6**	343	2.3K
Visitall	**276**	**30**	**32.3K**	**42.8K**	969	43	116.6K	121.6K	998	67	116.6K	122.7K
Average	**320.9**	**218.9**	144.2K	566.0K	329.6	219.4	**135.6K**	**560.3K**	401.1	332.3	160.4K	828.1K

the detection of *infinite programs*, which requires saving states to detect whether they are revisited during execution. Skipping this mechanism validates terminating programs with less memory and much faster [41].

In the validation set, each state variable from the planning domain is bounded by 10^9, instead of 10^2 which was the synthesis bound. *Corridor* is validated over 100 instances with corridors of length $n \in [13, 112]$ and random initial and goal locations. *Gripper* is validated over 1,000 instances of $n \in [12, 1011]$ balls that are initially in room A and need to be moved to room B. *Fibonacci* has a validation set of 33 instances, ranging from the 12^{th} Fibonacci term to the 44^{th}, i.e. the integer 701,408,733 (the 45^{th} number would overflow the validation bound). The solutions for *Select*, and *Find* domains, are validated on 100 instances each, with vector sizes ranging from 100 to 1,090, and random integer elements bounded by 10^9. Similarly, *Reverse* and *Sorting* have 100 validation instances with vectors of random integers, but their sizes range from 12 to 111. The solution for *Triangular Sum* is validated over 1,000 instances, the last one corresponding to the $1{,}011^{th}$ term in the sequence, i.e. the integer 511,566. In *Visitall*, there are 50 validation instances with squared grids ranging from 12×12 to 61×61.

6.3. Performance of BFGP with function combinations

The base performance of BFGP with a single evaluation/heuristic function is improved combining both *structural* and *cost-to-go* information; we can guide the search of BFGP with a *cost-to-go* heuristic function and break ties with a *structural* evaluation function, and vice versa. Thus, we run all configurations of BFGP(f_i, f_j) and BFGP(f_j, f_i) such that $f_i \in \{f_1, f_2, f_3, f_7\}$ and $f_j \in \{f_4, f_5, f_6, f_8, f_9\}$, and select the configuration that solves all domains with the best average time. There are 40 BFGP(f_i, f_j)/BFGP(f_j, f_i) configurations, but only BFGP(f_3, f_5) and BFGP(f_5, f_3) are able to solve all domains. The performance of these two configurations is then compared against BFGP(f_5), since it is the only single evaluation/heuristic function that solve all domains in the previous experiment. Table 4 summarizes that comparison, showing that BFGP(f_5) is improved in every domain either by BFGP(f_3, f_5) or BFGP(f_5, f_3). Furthermore, BFGP(f_3, f_5) has the best average performance in time and memory, shortly followed by BFGP(f_5, f_3) that has the best average in node expansions and evaluations, empirically proving that goal-oriented functions might be benefited when combined with *structural*-based functions.

6.4. Comparative synthesis performance with related GP solvers

The comparative analysis of GP solvers performance is complex mainly because of their different assumptions, problem specification, feature language, hyper-parameters, and different solution representations (non-/deterministic, boolean/numeric, …) [21]. In this regard, we have chosen the two closest GP solvers in terms of solution representation, i.e. *Planning Programs* (PP) [33] and *Hierarchical Finite State Controllers* (HFSC) [61]. Both approaches are compilation-based, where the input is a classical planning domain with some instances, and an upper bound in the number of program lines (n) for PP, or in the number of controller states ($|Q|$) for

Table 5
Computing CPU time (secs) for solving domains in the GP compilation approaches, i.e. PP [33] and HFSC [61], and $BFGP(f_3, f_5)$. Accumulated size of the classical plans, $\sum_i |\pi_i|$, and number of instructions, $\sum_i |\Pi_i|$.

Domain	$\|\mathcal{P}\|$	PP				HFSC				BFGP(f_3, f_5)			
		n	Time (s)	$\sum_i \|\pi_i\|$	$\sum_i \|\Pi_i\|$	$\|Q\|$	Time (s)	$\sum_i \|\pi_i\|$	$\sum_i \|\Pi_i\|$	$n, \|Z\|$	Time (s)	$\sum_i \|\pi_i\|$	$\sum_i \|\Pi_i\|$
Corridor	10	6	188.72	68	**260**	3	**1.31**	66	279	10,2	39.39	**64**	500
Fibonacci	4	6	**68.84**	40	**70**	5	1,296.52	39	138	7,2	675.99	**36**	108
Find	8	5	27.99	80	165	3	119.86	88	283	4,1	**0.02**	**24**	**132**
Gripper	10	6	**2.48**	**260**	**406**	4	8.28	**260**	806	8,4	19.22	**260**	455
Reverse	4	6	TE	-	-	5	TE	-	-	7,2	**40.61**	**34**	**144**
Select	6	5	TE	-	-	3	599.28	36	161	7,2	**27.15**	**6**	**123**
Sorting	10	8	TE	-	-	7	TE	-	-	9,2	**1,158.12**	**107**	**1,641**
Triangular Sum	3	4	55.23	18	**43**	3	218.44	30	104	5,2	**0.16**	**12**	48
Visitall	3	7	TE	-	-	4	201.02	49	**193**	13,4	**60.84**	**46**	243

HFSC. Both approaches produce a single domain and instance that can be solved with an off-the-shelf classical planner. Solutions to the produced instances are computed following a top-down strategy, with the LAMA-2011 [99] planner (*first solution* setting) of the *Fast-Downward* [8] system, that produce sequences of interleaved programming and executing actions, from which the programs, controllers and classical plans can be induced.

Section 5 already discussed the relation between the search spaces of PP (worst case intractable, e.g. integer representation) and BFGP (tractable with bounded size). PP and HFSC are proven to have equivalent solutions in the other representation [61], so the theory proven in Section 5 actually applies to both. A priori, there are some strengths and weaknesses that were already previously identified for PP and HFSC [61,33]:

- *Strengths*:
  - GP problems can be solved with an off-the-shelf classical planner,
  - classical planners are biased towards shorter solution plans (which may produce succinct generalized plans),
  - good performance for small bounds and problems with small number of objects.
- *Weaknesses*:
  - Computation sensitive to the order of the input instances and their number of objects,
  - scalability dramatically drops down when the number of input instances grows,
  - require hand-coding features as *derived predicates* in several domains or extra knowledge to compute generalized plans with deterministic behavior, e.g. in *gripper* domain the balls need to be serialized to know which ball is the next one to be picked,
  - the synthesized generalized plans can over-fit because the feature language includes all fluents in the set of classical planing instances given as input.

Compared to PP and HFSC, the strengths and weakness of BFGP are:

- *Strengths*:
  - The computation of generalized plans is not affected by the order of input instances,
  - the scalability of *BFGP* decreases smoothly and continuously with the number of objects,
  - solutions do not over-fit because of the feature language (over-fitting is only due to poorly sampled input instances),
  - domains do not require extra knowledge (BFGP implements an agnostic feature generation).
- *Weaknesses*:
  - BFGP is not using off-the-shelf classical planning machinery, although most of them could be adapted [65],
  - generalized plans are longer because of pointer manipulations, which may reduce performance in simple instances,
  - BFGP may require to increase the default number of input pointers.

The experimental setting for comparing GP solvers uses the same $|\mathcal{P}| = 10$ random input instances as in BFGP synthesis experiments. Since we observed that PP and HFSC were limited by this number, we have however proceed on removing the largest instance from the pool of instances, until one of them solved the problem without over-fitting. In addition, we encoded extra knowledge in the PP and HFSC domains, when necessary, with the form of *derived predicates* to guarantee generalization, which in turn helped to compute shorter generalized plans, e.g. in *corridor* domain it requires to know whether the agent is at the goal or in the rightmost location; and in *gripper* it requires to serialize the balls and know whether no more balls are in room A; and so on. The comparison of the three GP solvers is shown in Table 5; even with the advantage given to PP and HFSC, with the hand-crafted extra knowledge in their domains and the reduction of input instances (yielding to shortest number of executed instructions in half of domains i.e. $\sum_i |\Pi_i|$), BFGP outperforms PP and HFSC in many aspects, i.e. full coverage over the benchmarks, best computation time in 6 out of 9 domains, and shortest classical plans in all domains (i.e. $\sum_i |\pi_i|$), with less assumptions.

6.5. Validation of GP solutions in more complex domains

Here we present several GP benchmarks, with known polynomial time solutions, that are too complex for our current BFGP algorithm (within the given time and memory bounds). Our aim is showing that our approach is expressive enough to represent solutions to GP problems coming from IPC planning domains, noise-free supervised classification tasks, and numeric domains. These solutions are succinctly represented as GP plans, instead of long sequences of grounded actions for large problems, and validated efficiently without being affected by the grounding methods of current off-the-shelf classical planners.

- *Blocks Ontable*, towers of blocks where all blocks must be placed on the table.
- *Grid*, an agent has to move from an arbitrary location to a destination one in a 2D grid.
- *Miconic*, is an elevator problem where passengers at origin, wait for the elevator to enter, and then served at their destination floor.
- *Michalski Trains*, is a classic of relational supervised machine learning. A binary noise-free classification task with 10 trains that either go east or west, and multiple features such as the number of wheels, wagons, or their shape for each train among others. The goal is to learn the features that classify all trains in the right direction.
- *Satellite*, consists of taking images of different targets with instruments that are boarded in satellites. In addition, instruments need to be calibrated and in specific modes for taking each image; and each satellite has only power for one instrument at a time, so it needs to switch the current instrument off, switch on the next and calibrate it, before using a new instrument for taking images.
- *Sieve of Erathostenes*, is a method to find prime numbers up to a certain bound using only additive and iterative mechanisms.
- *Spanner*, consists of tighten all loose nuts at the end of a corridor, with the picked spanners along the corridor. Spanners can only be used once, and when the agent moves to the next room it can not go back, so if there are unpicked spanners in visited rooms the task could become unsolvable.

Fig. 15 shows the hand-coded solutions for these benchmarks. In *Blocks Ontable*, given n blocks the complexity of the solution is cubic, i.e. $O(n^3)$, where it searches n times, every $o1$ block that is on top of an $o2$ block, then unstack and put $o1$ down on the table. In *Spanner*, an agent picks up all available spanners in location $l2$, walks to the next $l1$ location and repeats the process until it reaches the last location (the gate), collecting all spanners on its path; once in the gate, it tightens each loose nut with a spanner. The solution to *Michalski Trains* is summarized as, each train $t1$ will go east if it has a car which is closed and short, otherwise it will go west. In *Sieve of Eratosthenes* all numbers are initially classified as primes, and it should decide whether they are not; so it iterates over i and uses j and k as auxiliary pointers, where the first acts as a counter that ranges from 0 to i, and second adds up to the next multiple of i, i.e. $k \,\%\, i = 0$; then every k-th number will be set to no prime, i is increased by one and the process repeats until i reaches the last element. In *Grid*, the agent moves to the top-right corner, then it moves left while not at the goal column, then down while not at the goal row, visiting the resulting coordinate. In *Miconic*, the elevator moves to the upmost floor, then for every floor down, it boards and departs all available passengers, once in the bottom floor, it moves upwards serving the remaining passengers that are in the elevator until reaching the upmost floor again. The last domain, *Satellite*, is the most complex because it requires iteration over multiple variable types, i.e. satellites, instruments, modes and directions. The solution to this domain consists of switching off all instruments and turning all satellites to the first direction; then for each satellite, the $i1$ instrument is switched on, calibrated with its calibration target direction $d2$, and used to take images of every direction $d2$ in every mode $m1$; once it finishes, the satellite turns to the first direction $d1$ again, switches off the current instrument, and continues with the next one, until all satellites have used all their instruments.

We get some main take away lessons from the analysis of Fig. 15 solutions; solutions have common high-level structures, that either iterate over all combinations of variable types (i.e. *Blocks*, *Miconic*, *Satellite*, ...), or build complex logic queries (i.e. *Michalski Trains*) which in some cases require the goal information to be coded in the initial state (i.e. *Grid* with `goal_column` and `goal_row`). This suggests that planning programs may be synthesized more efficiently using predefined structures (such as `FOR` or `IF-THEN-ELSE` constructs [100]) although this is out of the scope of this paper.

Table 6 shows the validation results in complex domains, where validation without infinite detection again scales better. All domains are successfully validated (except *Blocks Ontable* and *Satellite* with infinite detection mode that get a time- and memory-exceeded, respectively). *Blocks Ontable* can be solved with 13 lines and 3 pointers, and the validation set consists of 100 instances that range from 12 to 111 blocks. *Grid* requires 21 lines of code and 4 pointers, and it is validated with 50 instances with grids between 12×12 and 111×111 size. *Miconic* needs 25 lines and 3 pointers, and 100 instances that validates from 12 floors and 18 passengers to 111 floors and 166 passengers. *Michalski Trains* uses 15 lines and 6 pointers to classify all the trains in the unique classical task with 10 trains and their features. *Satellite* is by difference the most complex in terms of required lines and pointers, which are 43 and 5, respectively. Its validation set consists of 20 instances, starting with 11 satellites, 22 instruments and modes, and 44 directions, and finishing with 30 satellites, 60 instruments and modes and 120 directions. *Sieve of Erathostenes* requires 16 lines and 3 pointers to classify either as prime or non-prime, all the numbers comprised in the first 114 natural numbers. *Spanner*, uses 14 lines and 5 pointers to solve all 100 instances of the validation set, that range from 24 spanners and nuts and a corridor with 12 locations, to 222 spanners and nuts and a corridor with 111 locations.

```
BLOCKS ONTABLE
0. dec(o2)
1. goto(0,¬(y_z ∧ ¬y_c))
2. dec(o1)
3. goto(2,¬(y_z ∧ ¬y_c))
4. unstack(o1,o2)
5. put_down(o1)
6. inc(o1)
7. goto(4,¬(y_z ∧ ¬y_c))
8. inc(o2)
9. goto(2,¬(y_z ∧ ¬y_c))
10. inc(o3)
11. goto(0,¬(y_z ∧ ¬y_c))
12. end
```

```
SPANNER
0. pickup_spanner(l2,s1,m1)
1. inc(s1)
2. goto(0,¬(y_z ∧ ¬y_c))
3. dec(s1)
4. goto(3,¬(y_z ∧ ¬y_c))
5. inc(l2)
6. walk(l1,l2,m1)
7. inc(l1)
8. goto(0,¬(y_z ∧ ¬y_c))
9. tighten_nut(l1,s1,m1,n1)
10. inc(s1)
11. inc(n1)
12. goto(9,¬(y_z ∧ ¬y_c))
13. end
```

```
MICHALSKI TRAINS
0. test(has_car(t1,c1))
1. goto(7,¬(¬y_z ∧ y_c))
2. test(closed(c1))
3. goto(7,¬(¬y_z ∧ y_c))
4. test(short(c1))
5. goto(7,¬(¬y_z ∧ y_c))
6. set_eastbound(t1)
7. inc(c1)
8. goto(0,¬(y_z ∧ ¬y_c))
9. set_westbound(t1)
10. dec(c1)
11. goto(10,¬(y_z ∧ ¬y_c))
12. inc(t1)
13. goto(0,¬(y_z ∧ ¬y_c))
14. end
```

```
SIEVE OF ERATHOSTENES
0. inc(i)
1. inc(i)
2. set(k,i)
3. dec(j)
4. goto(3,¬(y_z ∧ ¬y_c))
5. inc(k)
6. goto(13,¬(¬y_z ∧ y_c))
7. inc(j)
8. cmp(i,j)
9. goto(5,¬(y_z ∧ ¬y_c))
10. set_no_prime(k)
11. cmp(i,j)
12. goto(3,¬(¬y_z ∧ y_c))
13. inc(i)
14. goto(2,¬(y_z ∧ ¬y_c))
15. end
```

```
GRID
0. inc(c2)
1. move_right(c1,c2)
2. inc(c1)
3. goto(0,¬(y_z ∧ ¬y_c))
4. inc(r2)
5. move_up(r1,r2)
6. inc(r1)
7. goto(4,¬(y_z ∧ ¬y_c))
8. test(goal_column(c1))
9. goto(14,¬(y_z ∧ ¬y_c))
10. dec(c2)
11. move_left(c1,c2)
12. dec(c1)
13. goto(8,¬(y_z ∧ ¬y_c))
14. test(goal_row(r1))
15. goto(20,¬(y_z ∧ ¬y_c))
16. dec(r2)
17. move_down(r1,r2)
18. dec(r1)
19. goto(14,¬(y_z ∧ ¬y_c))
20. end
```

```
MICONIC
0. inc(f2)
1. up(f1,f2)
2. inc(f1)
3. goto(0,¬(y_z ∧ ¬y_c))
4. board(p1,f1)
5. depart(p1,f1)
6. inc(p1)
7. goto(4,¬(y_z ∧ ¬y_c))
8. dec(p1)
9. goto(8,¬(y_z ∧ ¬y_c))
10. dec(f2)
11. down(f1,f2)
12. dec(f1)
13. goto(4,¬(y_z ∧ ¬y_c))
14. board(p1,f1)
15. depart(p1,f1)
16. inc(p1)
17. goto(14,¬(y_z ∧ ¬y_c))
18. dec(p1)
19. goto(18,¬(y_z ∧ ¬y_c))
20. inc(f2)
21. up(f1,f2)
22. inc(f1)
23. goto(14,¬(y_z ∧ ¬y_c))
24. end
```

```
SATELLITE
0. switch_off(i1,s1)
1. inc(i1)
2. goto(0,¬(y_z ∧ ¬y_c))
3. dec(i1)
4. goto(3,¬(y_z ∧ ¬y_c))
5. turn_to(s1,d1,d2)
6. inc(d2)
7. goto(5,¬(y_z ∧ ¬y_c))
8. set(d2,d1)
9. inc(s1)
10. goto(0,¬(y_z ∧ ¬y_c))
11. dec(s1)
12. goto(11,¬(y_z ∧ ¬y_c))
13. switch_on(i1,s1)
14. test(calibration_target(i1,d2))
15. goto(19,¬(¬y_z ∧ y_c))
16. turn_to(s1,d2,d1)
17. calibrate(s1,i1,d2)
18. turn_to(s1,d1,d2)
19. inc(d2)
20. goto(14,¬(y_z ∧ ¬y_c))
21. set(d2,d1)
22. take_image(s1,d2,i1,m1)
23. inc(m1)
24. goto(22,¬(y_z ∧ ¬y_c))
25. dec(m1)
26. goto(25,¬(y_z ∧ ¬y_c))
27. inc(d2)
28. turn_to(s1,d2,d1)
29. inc(d1)
30. goto(22,¬(y_z ∧ ¬y_c))
31. dec(d1)
32. goto(31,¬(y_z ∧ ¬y_c))
33. turn_to(s1,d1,d2)
34. set(d2,d1)
35. switch_off(i1,s1)
36. inc(i1)
37. goto(13,¬(y_z ∧ ¬y_c))
38. dec(i1)
39. goto(38,¬(y_z ∧ ¬y_c))
40. inc(s1)
41. goto(13,¬(y_z ∧ ¬y_c))
42. end
```

Fig. 15. Solutions to complex domains.

7. Conclusions

The paper presented an innovative solution space for GP that enables the definition of a heuristic search approach to GP. This novel solution space for GP is independent of the number of input planning instances in a GP problem, and the size of these instances (i.e. the number of objects, state variables, and their domain sizes). Therefore our *GP as heuristic search* approach can handle large sets of state variables with large numerical domains, e.g. integers.

Table 6
Validation of complex domains, CPU time (secs) and memory peak for program validation, with/out *infinite program* detection. TE and ME stands for time (1 h) and memory exceeded, respectively. Best results in bold.

Domain	$n, \|Z\|$	$\|P\|$	Time_∞	Mem_∞	Time	Mem
Blocks Ontable	13, 3	100	TE	–	**148.59**	**17 MB**
Grid	21, 4	50	0.67	14 MB	**0.15**	**10 MB**
Miconic	25, 3	100	2,248.59	13,129 MB	**12.68**	**166 MB**
Michalski Trains	15, 6	1	0.04	7 MB	**0.00**	**4 MB**
Satellite	43, 5	20	–	ME	**89.83**	**51 MB**
Sieve of Erathostenes	16, 3	100	9.60	39 MB	**0.43**	**13 MB**
Spanner	14, 5	100	326.79	1,512 MB	**5.10**	**61 MB**

We believe that this work is a step-forward towards building stronger connections between the areas of *automated planning* and programming. The work presented a formalization of classical planning as a vector transformation task, which is a common programming task. According to this formalism, computing a sequential plan for this tasks is computing a composition of vector transformation operations. Likewise computing a generalized plan is computing an algorithmic expression of the vector transformations.[9] With this regard, the BFGP algorithm starts from an empty program, but nothing prevents us from starting search from a partially specified generalized plan [102] to develop new online approaches that scale up better. In fact, local search approaches have already shown successful for both planning [103] and program synthesis [104,68].

Our *cost-to-go heuristics* are still less informed than current state-of-the-art heuristics for classical planning; note that our heuristics only consider goals that are explicitly provided in the problem representation. A clear example is $f_5(\Pi, P_l)$, that builds on top of the *Euclidean distance*, and that for STRIPS planning problems is actually a goal counter. We believe that better estimates may be obtained by building on top of the powerful ideas of modern planning heuristics [105,8,106]. In more detail, a promising approach for the development of more informative heuristics for GP is to consider sub-goals, that are not explicitly given in the problem representation [10,65]. For instance sets of sub-goals can be discovered as a pre-processing step, without grounding, regarding the set of *relevant* atoms that are traversed by the polynomial IW(1) algorithm, when achieving individual goals [107]. Besides landmarks, heuristic planners implement complementary ideas such as *helpful actions* [7], *multiple queues* for combining different heuristics [8], or *novelty-based exploration* [108]. Incorporating those classical planning technologies into the *GP as heuristic search* approach is a promising research direction [109]. In addition to more informative heuristics, we are interested in more expressive solution representations, where not only *goal-agnostic* (e.g. *Blocks Ontable* in Fig. 6) and *goal-oriented* (e.g. *Grid* in Fig. 6) generalized plans are computed, but also solutions which include *distance functions* that measure progress towards (sub-)goals; *distance functions* are known to be required to represent compact algorithmic solutions to polynomially-approximable (**poly-APX**) domains [110].

Since we are approaching GP as a classic tree search, a wide landscape of effective techniques, coming from *combinatorial search* and *classical planning*, could help to improve the base performance of our approach. We mention some of the more promising ones. Exploration in search can be more effective when adding one open list per evaluation function [8] and more sophisticated mechanisms could be implemented for handling closed nodes. For instance, *delayed duplicate detection* could be implemented to manage large closed lists with magnetic disk memory [111]. Further, once a search node is canceled (e.g. because $f_i(\Pi, P)$ identified that the *planning program* fails on a given instance), any program equivalent to this node should also be canceled, e.g. any program that can be built with transpositions of the causally-independent instructions. Given that the depth of the search-tree is bounded, techniques coming from SAT/CSP/SMTs, such a *non-chronological backtracking*, *limited discrepancy search* [112], or *taboo search* [113], might also result effective to improve our approach. SATPLAN planners exploit multiple-thread computing to parallelize search in solution spaces with different bounds [114]. This same idea could be applied to multiple searches for GP solutions with different program sizes.

Last but not least, another interesting research direction is the extension of our *GP as heuristic search* approach for computing generalized plans starting from different input settings. For instance, the computation of generalized plans from a set of *plan traces* that demonstrates how to solve several planning problems. We are also interested on exploring the application of our *GP as heuristic search* approach to planning problems that are not goal-oriented, where the objective is to maximize a given *utility function* [115]. In this particular setting, ideas from *approximated policy iteration* [116], and *reinforcement learning* [80], could be incorporated to our framework. With this regard, we are exploring the extension of our approach to GP problems that include *real* state variables. We believe that we can address this kind of GP problems by introducing the notion of *precision* for the comparison of real variables, and redefining accordingly our mechanism for the update of the FLAGS registers.

CRediT authorship contribution statement

Javier Segovia-Aguas: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Sergio Jiménez:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Software, Supervision, Validation, Writing – original draft, Writing – re-

[9] We have already explored this general scope of our GP approach to synthesize complex action models from examples [101].

view & editing. **Anders Jonsson:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Project administration, Resources, Supervision, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

A link to the source code is included in the article.

Acknowledgements

This work has been co-funded by MCIN/AEI /10.13039/501100011033 under the Maria de Maeztu Units of Excellence Programme (CEX2021-001195-M), TAILOR (H2020 #952215) and AIPLAN4EU (H2020 #101016442) projects. Javier Segovia-Aguas is also supported by AGAUR SGR and the Spanish grant PID2019-108141 GB-I00. Sergio Jiménez is supported by the Spanish MINECO project PID2021-127647NB-C22. Anders Jonsson is partially supported by Spanish grant PID2019-108141 GB-I00.

References

[1] M. Ghallab, D. Nau, P. Traverso, Automated Planning: Theory and Practice, Elsevier, 2004.
[2] H. Geffner, B. Bonet, A Concise Introduction to Models and Methods for Automated Planning, Morgan & Claypool Publishers, 2013.
[3] J. Slaney, S. Thiébaux, Blocks world revisited, Artif. Intell. 125 (1–2) (2001) 119–153.
[4] S.J. Russell, Artificial Intelligence a Modern Approach, Pearson Education, Inc., 2010.
[5] D.V. McDermott, A heuristic estimator for means-ends analysis in planning, in: AIPS, Vol. 96, 1996, pp. 142–149.
[6] B. Bonet, H. Geffner, Planning as heuristic search, Artif. Intell. 129 (1–2) (2001 Jun) 5–33.
[7] J. Hoffmann, FF: the fast-forward planning system, AI Mag. 22 (3) (2001) 57.
[8] M. Helmert, The fast downward planning system, J. Artif. Intell. Res. 26 (2006) 191–246.
[9] M. Vallati, L. Chrpa, M. Grześ, T.L. McCluskey, M. Roberts, S. Sanner, et al., The 2014 international planning competition: progress and trends, AI Mag. 36 (3) (2015) 90–98.
[10] J. Hoffmann, J. Porteous, L. Sebastia, Ordered landmarks in planning, J. Artif. Intell. Res. 22 (2004) 215–278.
[11] M. Helmert, C. Domshlak, Landmarks, critical paths and abstractions: what's the difference anyway?, in: Proceedings of the International Conference on Automated Planning and Scheduling, Vol. 19, 2009.
[12] S. Richter, M. Westphal, The LAMA planner: guiding cost-based anytime planning with landmarks, J. Artif. Intell. Res. 39 (2010) 127–177.
[13] G. Frances, H. Geffner, Modeling and computation in planning: better heuristics from more expressive languages, in: Proceedings of the International Conference on Automated Planning and Scheduling, Vol. 25, 2015.
[14] N. Lipovetzky, H. Geffner, Best-first width search: exploration and exploitation in classical planning, in: AAAI, 2017.
[15] E. Winner, M. Veloso, DISTILL: learning domain-specific planners by example, in: ICML, 2003, pp. 800–807.
[16] Y. Hu, H.J. Levesque, A correctness result for reasoning about one-dimensional planning problems, in: IJCAI, 2011, pp. 2638–2643.
[17] S. Srivastava, N. Immerman, S. Zilberstein, A new representation and associated algorithms for generalized planning, Artif. Intell. 175 (2) (2011) 615–647.
[18] S. Siddharth, I. Neil, Z. Shlomo, Z. Tianjiao, Directed search for generalized plans using classical planners, in: ICAPS, 2011, pp. 226–233.
[19] Y. Hu, G. De Giacomo, Generalized planning: synthesizing plans that work for multiple environments, in: IJCAI, 2011.
[20] L. Illanes, S.A. McIlraith, Generalized planning via abstraction: arbitrary numbers of objects, in: AAAI, Vol. 33, 2019, pp. 7610–7618.
[21] S. Jiménez, J. Segovia-Aguas, A. Jonsson, A review of generalized planning, Knowl. Eng. Rev. 34 (2019) e5.
[22] G. Francès, B. Bonet, H. Geffner, Learning general policies from small examples without supervision, in: AAAI, 2021.
[23] B. Bonet, H. Geffner, General policies, representations, and planning width, in: Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 35, 2021, pp. 11764–11773.
[24] S. Ståhlberg, B. Bonet, H. Geffner, Learning generalized policies without supervision using gnns, in: KR, 2022.
[25] A. Fern, R. Khardon, P. Tadepalli, The first learning track of the international planning competition, Mach. Learn. 84 (1–2) (2011) 81–107.
[26] J. Seipp, F. Pommerening, G. Röger, M. Helmert, Correlation complexity of classical planning domains, in: IJCAI, 2016.
[27] R.I. Brafman, D. Tolpin, O. Wertheim, Probabilistic programs as an action description language, in: AAAI, 2023.
[28] S.S. Skiena, The Algorithm Design Manual: Text, vol. 1, Springer Science & Business Media, 1998.
[29] S.P. Dandamudi, Installing and using nasm, in: Guide to Assembly Language Programming in Linux, 2005, pp. 153–166.
[30] M. Vallati, L. Chrpa, M. Grzes, T.L. McCluskey, M. Roberts, S. Sanner, The 2014 international planning competition: progress and trends, AI Mag. 36 (3) (2015) 90–98.
[31] J. Segovia-Aguas, S. Jiménez, A. Jonsson, Generalized planning as heuristic search, in: ICAPS, 2021.
[32] J. Segovia-Aguas, S. Jiménez, A. Jonsson, Computing programs for generalized planning as heuristic search, in: IJCAI, 2022.
[33] J. Segovia-Aguas, S. Jiménez, A. Jonsson, Computing programs for generalized planning using a classical planner, Artif. Intell. 272 (2019) 52–85.
[34] R. Khardon, Learning action strategies for planning domains, Artif. Intell. 113 (1–2) (1999) 125–148.
[35] M. Martin, H. Geffner, Learning generalized policies from planning examples using concept languages, Appl. Intell. 20 (2004) 9–19.
[36] S. Yoon, A. Fern, R. Givan, Learning control knowledge for forward search planning, J. Mach. Learn. Res. 9 (4) (2008) 683–718.
[37] T. De la Rosa, S. Jiménez, R. Fuentetaja, D. Borrajo, Scaling up heuristic planning with relational decision trees, J. Artif. Intell. Res. 40 (2011) 767–813.
[38] T. Silver, K.R. Allen, A.K. Lew, L.P. Kaelbling, J. Tenenbaum, Few-shot bayesian imitation learning with logical program policies, in: Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 34, 2020, pp. 10251–10258.
[39] B. Bonet, H. Palacios, H. Geffner, Automatic derivation of finite-state machines for behavior control, in: AAAI, 2010.
[40] B. Bonet, H. Geffner, Features, projections, and representation change for generalized planning, in: International Joint Conference on Artificial Intelligence, 2018, pp. 4667–4673.
[41] J. Segovia-Aguas, S. Jiménez, A. Jonsson, Generalized planning with positive and negative examples, in: AAAI, 2020, pp. 9949–9956.

[42] D. Lotinac, A. Jonsson, Constructing hierarchical task models using invariance analysis, in: Proceedings of the Twenty-Second European Conference on Artificial Intelligence, 2016, pp. 1274–1282.
[43] D.E. Smith, D.S. Weld, Conformant graphplan, in: AAAI/IAAI, 1998, pp. 889–896.
[44] H. Palacios, H. Geffner, Compiling uncertainty away in conformant planning problems with bounded width, J. Artif. Intell. Res. 35 (2009) 623–675.
[45] L. Pryor, G. Collins, Planning for contingencies: a decision-based approach, J. Artif. Intell. Res. 4 (1996) 287–339.
[46] A. Kolobov, Planning with Markov decision processes: an AI perspective, Synth. Lect. Artif. Intell. Mach. Learn. 6 (1) (2012) 1–210.
[47] N. Roy, G. Gordon, S. Thrun, Finding approximate POMDP solutions through belief compression, J. Artif. Intell. Res. 23 (2005) 1–40.
[48] S. Srivastava, S. Zilberstein, N. Immerman, H. Geffner, Qualitative numeric planning, in: AAAI, 2011.
[49] B. Bonet, H. Geffner, Qualitative numeric planning: reductions and complexity, J. Artif. Intell. Res. 69 (2020) 923–961.
[50] G. Röger, M. Helmert, B. Nebel, On the relative expressiveness of adl and golog: the last piece in the puzzle, in: KR, 2008, pp. 544–550.
[51] M. Katz, D. Moshkovich, E. Karpas, Semi-black box: rapid development of planning based solutions, in: AAAI, 2018.
[52] J.A. Baier, S.A. McIlraith, Knowledge-based programs as building blocks for planning, Artif. Intell. 303 (2022) 103634.
[53] S. Sardina, G. De Giacomo, Y. Lespérance, H.J. Levesque, On the semantics of deliberation in indigolog—from theory to implementation, Ann. Math. Artif. Intell. 41 (2–4) (2004) 259–299.
[54] J. Claßen, V. Engelmann, G. Lakemeyer, G. Röger, Integrating golog and planning: an empirical evaluation, in: Non-Monotonic Reasoning Workshop, 2008.
[55] S. Jiménez, T. De La Rosa, S. Fernández, F. Fernández, D. Borrajo, A review of machine learning for automated planning, Knowl. Eng. Rev. 27 (4) (2012) 433–467.
[56] F. Bacchus, F. Kabanza, Using temporal logics to express search control knowledge for planning, Artif. Intell. 116 (1–2) (2000) 123–191.
[57] D. Nau, Y. Cao, A. Lotem, H. Munoz-Avila, The SHOP planning system, AI Mag. 22 (3) (2001) 91.
[58] M. Martín, H. Geffner, Learning generalized policies from planning examples using concept languages, Appl. Intell. 20 (1) (2004) 9–19.
[59] J. Segovia-Aguas, S. Jiménez, A. Jonsson, Hierarchical finite state controllers for generalized planning, in: IJCAI, 2016.
[60] M. Ramirez, H. Geffner, Heuristics for planning, plan recognition and parsing, arXiv preprint, arXiv:1605.05807.
[61] J. Segovia-Aguas, S. Jiménez, A. Jonsson, Computing hierarchical finite state controllers with classical planning, J. Artif. Intell. Res. 62 (2018) 755–797.
[62] B. Bonet, G. De Giacomo, H. Geffner, F. Patrizi, S. Rubin, High-level programming via generalized planning and LTL synthesis, in: KR, 2020.
[63] B. Bonet, G. Frances, H. Geffner, Learning features and abstract actions for computing generalized plans, in: AAAI, Vol. 33, 2019, pp. 2703–2710.
[64] D. Drexler, J. Seipp, H. Geffner, Learning sketches for decomposing planning problems into subproblems of bounded width, in: Proceedings of the International Conference on Automated Planning and Scheduling, Vol. 32, 2022, pp. 62–70.
[65] J. Segovia-Aguas, S.J. Celorrio, L. Sebastiá, A. Jonsson, Scaling-up generalized planning as heuristic search with landmarks, in: International Symposium on Combinatorial Search, 2022, pp. 171–179.
[66] R.P. Kurshan, Computer-Aided Verification of Coordinating Processes: the Automata-Theoretic Approach, vol. 302, Princeton University Press, 2014.
[67] A. Solar-Lezama, Program synthesis by sketching, Citeseer (2008).
[68] S. Gulwani, O. Polozov, R. Singh, et al., Program synthesis, Found. Trends® Program. Lang. 4 (1–2) (2017) 1–119.
[69] R. Alur, R. Singh, D. Fisman, A. Solar-Lezama, Search-based program synthesis, Commun. ACM 61 (12) (2018) 84–93.
[70] C. Barrett, A. Stump, C. Tinelli, et al., The smt-lib standard: Version 2.0, in: Proceedings of the 8th International Workshop on Satisfiability Modulo Theories (Edinburgh, England), Vol. 13, 2010, p. 14.
[71] H. Barbosa, C.W. Barrett, M. Brain, G. Kremer, H. Lachnitt, M. Mann, A. Mohamed, M. Mohamed, A. Niemetz, A. Nötzli, A. Ozdemir, M. Preiner, A. Reynolds, Y. Sheng, C. Tinelli, Y. Zohar, cvc5: a versatile and industrial-strength SMT solver, in: Tools and Algorithms for the Construction and Analysis of Systems, 2022.
[72] D.W. Loveland, Automated Theorem Proving: a Logical Basis, Elsevier, 2016.
[73] A. Solar-Lezama, Program sketching, Int. J. Softw. Tools Technol. Transf. 15 (5) (2013) 475–495.
[74] A. Burkov, The Hundred-Page Machine Learning Book, vol. 1, Andriy Burkov, Canada, 2019.
[75] S. Toyer, F. Trevizan, S. Thiébaux, L. Xie, Action schema networks: generalised policies with deep learning, in: Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 32, 2018.
[76] T.P. Bueno, L.N. de Barros, D.D. Mauá, S. Sanner, Deep reactive policies for planning in stochastic nonlinear domains, in: Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 33, 2019, pp. 7530–7537.
[77] S. Garg, A. Bajpai Mausam, Symbolic network: generalized neural policies for relational MDPs, in: International Conference on Machine Learning, PMLR, 2020, pp. 3397–3407.
[78] S. Sanner, C. Boutilier, Practical solution techniques for first-order MDPs, Artif. Intell. 173 (5–6) (2009) 748–788.
[79] A. Fern, S. Yoon, R. Givan, Approximate policy iteration with a policy language bias: solving relational Markov decision processes, J. Artif. Intell. Res. 25 (2006) 75–118.
[80] R.S. Sutton, A.G. Barto, Reinforcement Learning: An Introduction, MIT Press, 2018.
[81] E. Groshev, M. Goldstein, A. Tamar, S. Srivastava, P. Abbeel, Learning generalized reactive policies using deep neural networks, in: Proceedings of the International Conference on Automated Planning and Scheduling, Vol. 28, 2018.
[82] M. Junyent, A. Jonsson, V. Gómez, Deep policies for width-based planning in pixel domains, in: ICAPS, Vol. 29, 2019, pp. 646–654.
[83] J. Pearl, The limitations of opaque learning machines, in: Possible Minds: Twenty-Five Ways of Looking at AI, 2019, pp. 13–19.
[84] C. Bäckström, P. Jonsson, All PSPACE-complete planning problems are equal but some are more equal than others, in: SOCS, 2011.
[85] B. Nebel, On the compilability and expressive power of propositional planning formalisms, J. Artif. Intell. Res. 12 (2000) 271–315.
[86] P. Haslum, N. Lipovetzky, D. Magazzeni, C. Muise, An introduction to the planning domain definition language, Synth. Lect. Artif. Intell. Mach. Learn. 13 (2) (2019) 1–187.
[87] D.S. Nau, T.-C. Au, O. Ilghami, U. Kuter, J.W. Murdock, D. Wu, F. Yaman, SHOP2: an HTN planning system, J. Artif. Intell. Res. 20 (2003) 379–404.
[88] M. Ajtai, J. Komlós, E. Szemerédi, An 0 (n log n) sorting network, in: Proceedings of the Fifteenth Annual ACM Symposium on Theory of Computing, 1983, pp. 1–9.
[89] D. Lotinac, J. Segovia-Aguas, S. Jiménez, A. Jonsson, Automatic generation of high-level state features for generalized planning, in: IJCAI, 2016.
[90] G.S. Boolos, J.P. Burgess, R.C. Jeffrey, Computability and Logic, Cambridge University Press, 2002.
[91] M.L. Minsky, Recursive unsolvability of post's problem of "tag" and other topics in theory of Turing machines, Ann. Math. (1961) 437–455.
[92] M. Veloso, J. Carbonell, A. Perez, D. Borrajo, E. Fink, J. Blythe, Integrating planning and learning: the PRODIGY architecture, J. Exp. Theor. Artif. Intell. 7 (1) (1995) 81–120.
[93] R.E. Fikes, N.J. Nilsson, STRIPS: a new approach to the application of theorem proving to problem solving, Artif. Intell. 2 (3–4) (1971) 189–208.
[94] H. Geffner, Functional STRIPS: a more flexible language for planning and problem solving, in: Logic-Based Artificial Intelligence, Springer, 2000, pp. 187–209.
[95] M. Fox, D. Long, The automatic inference of state invariants in TIM, J. Artif. Intell. Res. 9 (1998) 367–421.
[96] M. Fox, D. Long, PDDL2.1: an extension to PDDL for expressing temporal planning domains, J. Artif. Intell. Res. 20 (2003) 61–124.
[97] J.B. Browning, B. Sutherland, Working with numbers, in: C++ 20 Recipes, Springer, 2020, pp. 115–145.
[98] R.E. Korf, W. Zhang, I. Thayer, H. Hohwald, Frontier search, J. ACM 52 (5) (2005) 715–748.
[99] S. Richter, M. Westphal, M. Helmert, Lama 2008 and 2011, in: International Planning Competition, 2011.
[100] J. Segovia-Aguas, Y. E-Martín, S. Jiménez, Representation and synthesis of c++ programs for generalized planning, in: Workshop on Generalization in Planning, IJCAI, 2022.

[101] J. Segovia-Aguas, J. Ferrer-Mestres, S. Jiménez, Synthesis of procedural models for deterministic transition systems, in: ECAI, 2023.
[102] B. Bonet, H. Geffner, General policies, representations, and planning width, in: Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 35, 2021, pp. 11764–11773.
[103] A. Gerevini, A. Saetti, I. Serina, Planning through stochastic local search and temporal action graphs in LPG, J. Artif. Intell. Res. 20 (2003) 239–290.
[104] A. Solar-Lezama, The sketching approach to program synthesis, in: Asian Symposium on Programming Languages and Systems, Springer, 2009, pp. 4–13.
[105] J. Hoffmann, The metric-FF planning system: translating "ignoring delete lists" to numeric state variables, J. Artif. Intell. Res. 20 (2003) 291–341.
[106] G. Francès, et al., Effective planning with expressive languages, Ph.D. thesis, Universitat Pompeu Fabra, 2017.
[107] G. Frances, M. Ramírez Jávega, N. Lipovetzky, H. Geffner, Purely declarative action descriptions are overrated: classical planning with simulators, in: IJCAI, International Joint Conferences on Artificial Intelligence Organization (IJCAI), 2017.
[108] N. Lipovetzky, H. Geffner, Width and serialization of classical planning problems, in: ECAI, 2012, pp. 540–545.
[109] C. Lei, N. Lipovetzky, K.A. Ehinger, Novelty and lifted helpful actions in generalized planning, in: International Symposium on Combinatorial Search, 2023.
[110] M. Helmert, R. Mattmüller, G. Röger, Approximation properties of planning benchmarks, in: ECAI 2006, IOS Press, 2006, pp. 585–589.
[111] R.E. Korf, Linear-time disk-based implicit graph search, J. ACM 55 (6) (2008) 1–40.
[112] R.E. Korf, Improved limited discrepancy search, in: AAAI/IAAI, Vol. 1, 1996, pp. 286–291.
[113] E. Nowicki, C. Smutnicki, A fast taboo search algorithm for the job shop problem, Manag. Sci. 42 (6) (1996) 797–813.
[114] J. Rintanen, Planning as satisfiability: heuristics, Artif. Intell. 193 (2012) 45–86.
[115] N. Lipovetzky, M. Ramirez, H. Geffner, Classical planning with simulators: results on the atari video games, in: IJCAI, 2015.
[116] D.P. Bertsekas, Approximate policy iteration: a survey and some new methods, J. Control Theory Appl. 9 (3) (2011) 310–335.