# The famous bridge routing problem

Let us assume a country with two Cities A and B.   
Their positions are (Ax, Ay) and (Bx, By) respectively.

A road has to be build connecting these two cities. Between the two cities flows a river, strictly in west-east direction. The y-position of the lower Bank of the river is denoted as yLB. The y-position of the upper Bank of the river is yUB.

Over the river there will be a bridge with x-Position of the Lower end of the Bridge xLB and x position of the upper end of the bridge xUB.



Here is the matlab code to generate the plot above (without displaying the variable names)

yub=60 %y-position of upper bank

ylb=40 %y-position of lower bank

xl=0 %x-position of left boarder

xr=100 %x-position of right boarder

yl=0 %y-position of lower boarder

yu=100 %y-position of upper boarder

Ax=30 %x-position of City A

Ay=20 %y-position of City A

Bx=80 %x-position of City B

By=70 %y-position of City B

plot(Ax,Ay,'o','MarkerFaceColor',[1 0 0],'MarkerSize',15)

hold on

plot(Bx,By,'o','MarkerFaceColor',[0 1 0],'MarkerSize',15)

fill([xl;xl;xr;xr],[ylb;yub;yub;ylb],'b')

% replace these two lines by a brute force method

xLB=50;

xUB=60;

%################################################

plot([Ax,xLB,xUB,Bx],[Ay,ylb,yub,By],'k','LineWidth',4);

xlabel('West-East-Position X[km]');

ylabel('South-North-Position Y[km]');

legend('City A','City B','River','Street')

axis(gca,[0,100,0,100])

Let us assume that a bridge is more expensive per km than is a conventional street per km. The Ratio of costs is called R.

For a given R (ranging from 1 to infinity) the cheapest path from A to B is looked for.

## Task 1:

Write a matlab expression, that calculates a normalized length nL. The normalized Length nL is the Length of a street purely conventional invoking the same costs as the planed path from A to B. (Basically you could also calculate the cost of the street from A to B. But your are missing the information on the cost per km street over land.)  
The expression will be dependent on the following variables:

Ax, Ay, Bx, By, xLB, xUB, yLB, yUB, R

## Task 2:

Try all possible roads from A to B with Steps in x-Direction of 10 m.

Some assumptions:

The cheapest path from A to the river is a straight line. The cheapest path from the river to B is a straight line as well. The cheapest path through the river is –make an educated guess- : a straight line.

You can assume the following relations:

Ax<Bx

Ay<By

Ax<=xLB

xLB<=xUB

xUB<=Bx

Find the cheapest road and show it in the plot!

Hints:

The final code takes some CPU-time. Start with a greater step in x. Test your code and then decrease the step size.  
Carefully think about how small the ranges of your loops can be.

## Task 3:

Perform some plausibility tests on the results obtained!

For which values of R can you predict the course of the cheapest street?