

Instructions: Please answer legibly, logically, and **show all work**. Remember that explaining and words are a critical part of math – if you get stuck, try to explain what you would do if you could get past your sticking point. No credit will be given for unjustified or unclear work, including guess-and-check. Be sure to answer the question or perform the task you are asked.

1. (5 pts each) Simplify each expression completely. No negative exponents should remain.

(a) $3x^4x^{-7}y^2y^3z^5z^{-6}$

(b) $\left(\frac{2x^{-3}yz^2}{6x^{-1}y^2z}\right)^{-1}$

2. (2 pts) Determine if the following statement is *always* true. If yes, clearly explain why. If not, show algebraically or give a simple counterexample using numbers for the variables (make sure you follow order of operations):

$$(a + b)^2 = a^2 + b^2$$

3. (2 pts) Simplify using only positive exponents.

$$(x + y)^{-1}$$

4. (5 pts each) Perform the operations and simplify completely.

(a) $x^3 - 5x^2(x + 1) - 7(x^3 - x^2)$

(b) $(4x + 7)\left(\frac{1}{2}x - 3\right)$

5. (5 pts) Find the quotient and remainder.

$$2x - 1 \overline{)4x^2 - 10x + 6}$$

6. (5 pts each) Factor completely.

(a) $3c - cd + 3d - c^2$

(b) $x^4 - 1$

7. (6 pts) The length of a rectangle is three times as long as the width. If it's area is 147 cm^2 , find the dimensions of the rectangle.

8. (5 pts) Consider the rational function: $j(x) = \frac{x-2}{x^2-6x+8}$

(a) Find the domain of the rational function and give it in interval notation.

(b) Simplify the function, if possible.

9. (8 pts) Divide and simplify completely.

$$\frac{y^2 - 36}{y^2 - 8y + 16} \div \frac{3y - 18}{y^2 - y - 12}$$

10. (6 pts) Perform the operations and simplify completely.

$$\frac{1}{x+1} - \frac{x}{x-2} + \frac{x^2+2}{x^2-x-2}$$

11. (5 pts) Simplify the complex fraction completely into a single fraction.

$$\frac{\frac{5}{x}}{\frac{2}{x} + \frac{3}{y}}$$

12. (6 pts) Solve for x . If any extraneous solutions exist, be sure to identify them and/or cross them out.

$$\frac{2}{x^2 - x} = \frac{1}{x - 1}$$

13. (5 pts) A new type of blood analyzer can process a batch of samples in 3 hours. An older model can process the same batch in 4 hours. How long will it take to process the batch if both machines are working together at the same time?