Quiz 10 **Name:** Isabelle

Total Points possible: 21 out of 20

Math 12: Spring 2025

Instructions: Show all your work in order to receive credit. Each problem is worth 2 points. The extra credit is worth 1.

Problem 1. Simplify completely.

(a) $(81x^6)^{\frac{3}{4}}$ $= \sqrt[4]{(81x^6)^3}$

$= \sqrt[4]{81^3 x^{18}}$

3 27 3 9 3 3

$= \sqrt[4]{(3^4)^3 x^4 x^4 x^4 x^4 x^2}$

$= \sqrt[4]{(3^3)^4 x^4 x^4 x^4 x^4 x^2}$

$= 3^3 \cdot x \cdot x \cdot x \cdot x \cdot \sqrt[4]{x^2}$

$= 27x^4 \cdot \sqrt[4]{x^2}$

(b) $\sqrt[6]{(125c^6)^3}$ $= \sqrt[6]{125^3 \cdot c^{6 \cdot 3}}$

125 → 5, 25 → 5, 5

$= \sqrt[6]{(5^3)^3 \cdot c^{18}}$

$= \sqrt[6]{5^9 \cdot c^{18}}$

$= \sqrt[6]{5^6 \cdot 5^3 \cdot c^6 \cdot c^6 \cdot c^6}$

$= 5 \cdot c \cdot c \cdot c \sqrt[6]{5^3}$

$= 5c^3 \cdot \sqrt[6]{5^3}$

Problem 2. Solve for x for $f(x) = \sqrt{x^2 - 4x - 12} - 1 = 2$

$\sqrt{x^2-4x-12} - 1 = 2$

want to get this alone

$\sqrt{x^2-4x-12} = 3$ ← raise to power of 2 on both sides

$\left(\sqrt{x^2-4x-12}\right)^2 = 3^2$

$x^2 - 4x - 12 = 9$

$-9 \quad -9$ ← move everything to one side

$x^2 - 4x - 21 = 0$ ← factor

$x \quad -7$

$x \quad 3$

$(x+3)(x-7) = 0$

case 1: $x+3=0$, $x=-3$

case 2: $x-7=0$, $x=7$

Problem 3. Factory completely, using the quadratic formula. Note that, lambda, denoted as λ is just an other variable like x. However, λ is commonly seen in linear algebra.

$2\lambda^2 = \lambda - 5$

$-\lambda + 5 \quad -\lambda + 5$

$2\lambda^2 - \lambda + 5 = 0$

$2\lambda^2 - 1\cdot\lambda + 5 = 0$ (a, b, c)

using, $\lambda = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$\lambda = \dfrac{-1 \pm \sqrt{(-1)^2 - 4(2)(5)}}{2(2)}$

$\lambda = \dfrac{-1 \pm \sqrt{1 - 40}}{4}$

$\lambda = \dfrac{-1 \pm \sqrt{-39}}{4}$ note $i^2 = -1$

$\lambda = \dfrac{-1 \pm \sqrt{-1 \cdot 39}}{4}$

$\lambda = \dfrac{-1 \pm \sqrt{i^2 \cdot 39}}{4}$

$\lambda = \dfrac{-1 \pm i\sqrt{39}}{4}$

So $\lambda = \dfrac{-1 - i\sqrt{39}}{4}, \dfrac{-1 + i\sqrt{39}}{4}$

Problem 4. Simplify completely,

(a) $(3i^2 - i + 4)(-2i + i^2)$

$i^2 = -1$ $\quad i^2 = -1$

$(3(-1) - i + 4)(-2i + i^2) = (-3 - i + 4)(-2i - 1)$

$= (1 - i)(-2i - 1)$

$= -2i - 1 - 2 + i$

$= -i - 3$

	$-2i$	-1
1	$-2i$	-1
$-i$	$+2i^2 = -2$	i

$-2i^2 = -2(-1) = -2$

(b) $(-i^2 - 5i)^2$

$-i^2 = -(-1) = 1$

$= (1 - 5i)(1 - 5i) = 1 - 5i - 5i - 25$

$= 1 - 10i - 25$

$= -24 - 10i$

	1	$-5i$
1	1	$-5i$
$-5i$	$-5i$	$25i^2 = -25$

$25i^2 = 25(-1) = -25$

. **Problem 5.** Rationalize the denominator, aka divide.

	$-i$	2
$-i$	$i^2=-1$	$-2i$
2	$-2i$	4

	$-i$	2
$-i$	$i^2=-1$	$-2i$
-2	$2i$	-4

$$\frac{-i+2}{-i-2}\cdot\frac{(-i+2)}{(-i+2)} = \frac{(-i+2)(-i+2)}{(-i-2)(-i+2)}$$

multiply by top and bottom but change sign

$$= \frac{-1-2i-2i+4}{-1-2i+2i-4}$$

$$= \frac{-4i+3}{-1-4}$$

$$= \frac{-4i+3}{-5}$$

Problem 6. Simplify completely.

$$\sqrt[3]{\frac{81x^7c^9}{3(z^0+2x)^3}} = \sqrt[3]{\frac{27\cdot 3\cdot x^3x^3x^1\cdot c^9}{3(1+2x)^3}}$$

$$= \sqrt[3]{\frac{27\cdot x^3\cdot x^3\cdot x^1\cdot c^3c^3\cdot c^3}{(1+2x)^3}}$$

$27 = 3\cdot 9 = 3\cdot 3\cdot 3$

$$= \sqrt[3]{\frac{3^3\cdot x^3\cdot x^3\cdot x^1\cdot c^3c^3\cdot c^3}{(1+2x)^3}}$$

$$= \frac{\sqrt[3]{3^3\cdot x^3\cdot x^3\cdot x^1\cdot c^3c^3\cdot c^3}}{\sqrt[3]{(1+2x)^3}}$$

$$= \frac{3\cdot x\cdot x\cdot c\cdot c\cdot c\ \sqrt[3]{x}}{(1+2x)}$$

$$= \frac{3x^2c^3\ \sqrt[3]{x}}{(1+2x)}$$

Problem 7. This will not be on the exam, expand then solve for $\frac{1}{4}$.

	f	g
f	f^2	fg
g	gf	g^2

note $gf = fg$ i.e $5 \cdot 4 = 4 \cdot 5 = 20$

	f	$-g$
f	f^2	$-fg$
$-g$	$-gf$	$(-g)(-g) = g$

note $-5(4) = 4(-5) = -20$

$$(f+g)^2-(f-g)^2 = (f+g)(f+g) - (f-g)(f-g)$$
$$= f^2+fg+fg+g^2-(f^2-fg-fg+g^2)$$
$$= f^2+2fg+g^2-(f^2-2fg+g^2)$$
$$= f^2+2fg+g^2-f^2+2fg-g^2$$

$$\frac{1}{4}\cdot (f+g)^2-(f-g)^2 = 4fg \cdot \frac{1}{4}$$

$$\frac{1}{4}\cdot\left((f+g)^2-(f-g)^2\right) = fg$$

$$\frac{1}{4} = \frac{fg}{(f+g)^2-(f-g)^2}$$

Problem 8. This will not be on the exam. Let x be any positive number bigger than 0. Which fraction is bigger and why?

$$\frac{1}{x+1} \quad \text{or} \quad \frac{1}{x}$$

$\frac{1}{x}$ is bigger, let $x = 2$ then $\frac{1}{2+1} = \frac{1}{3} \approx 0.333$

but $\frac{1}{2} = 0.5$

Indeed, $\frac{1}{2} = 0.5 > \frac{1}{3} \approx 0.333$

Problem 9. (1 points) **Extra credit**. What is your favorite K.K. Slider song? All of them