Math 12 Exam 2 Name: pouch

Instructions: Please answer legibly, logically, and **show all work**. Remember that explaining and words are a critical part of math – if you get stuck, try to explain what you would do if you could get past your sticking point. No credit will be given for unjustified or unclear work, including guess-and-check. Be sure to answer the question or perform the task you are asked.

1. (5 pts each) Simplify each expression completely. No negative exponents should remain.

(a) $3x^4x^{-7}y^2y^3z^5z^{-6}$ $= 3x^{4-7}y^{2+3}z^{5-6}$

$= 3x^{-3}y^5z^{-1}$

$= \dfrac{3y^5}{x^3z^1}$

(b) $\left(\dfrac{2x^{-3}yz^2}{6x^{-1}y^2z}\right)^{-1}$ $= \left(\dfrac{2x^{-3}\cancel{y}z\cdot\cancel{z}}{2\cdot 3x^{-1}\cancel{y}y\cancel{z}}\right)^{-1}$

$= \left(\dfrac{x^{-3}z}{3x^{-1}y}\right)^{-1}$

$= \left(\dfrac{yz}{3x^3x^{-1}y}\right)^{-1}$

$= \left(\dfrac{z}{3x^2y}\right)^{-1} = \left(\dfrac{3x^2y}{z}\right)$ bc of this and this product then flip fraction

2. (2 pts) Determine if the following statement is *always* true. If yes, clearly explain why. If not, show algebraically or give a simple counterexample using numbers for the variables (make sure you follow order of operations):

$(a+b)^2 = a^2 + b^2$ Never

3. (2 pts) Simplify using only positive exponents.

$(x+y)^{-1}$ $\dfrac{1}{(x+y)}$

4. (5 pts each) Perform the operations and simplify completely.

(a) $x^3 - 5x^2(x+1) - 7(x^3 - x^2)$

$x^3 - 5x^3 - 5x^2 - 7x^3 + 7x^2$

$-11x^3 + 2x^2$

(b) $(4x+7)\left(\frac{1}{2}x - 3\right)$

$= 4x\left(\frac{1}{2}x\right) + 4x(-3) + 7\left(\frac{1}{2}x\right) + 7(-3)$

$= \frac{4}{2}x^2 - 12x + \frac{7}{2}x - 21$

$= 2x^2 - \frac{2}{2}\cdot\frac{12}{1}x + \frac{7}{2}x - 21$

$= 2x^2 - \frac{24}{2}x + \frac{7}{2}x - 21$

$= 2x^2 - \frac{17}{2}x - 21$

5. (5 pts) Find the quotient and remainder.

$$\begin{array}{r} 2x - 4 \\ 2x - 1\,\overline{)\,4x^2 - 10x + 6} \\ -\,(4x^2 - 2x) \downarrow \\ \hline -8x + 6 \\ -\,(-8x + 4) \\ \hline 2 \; R \end{array}$$

$2x(2x-1) = 4x^2 - 2x$

$-4(2x-1) = -8x + 4$

6. (5 pts each) Factor completely.

(a) $3c - cd + 3d - c^2$

$$3c - c^2 - cd + 3d$$

$$\underset{c\cdot c}{}$$

$$c(3-c) + d(-c+3)$$

$$c(3-c) + d(3-c)$$

$$(3-c)(c+d)$$

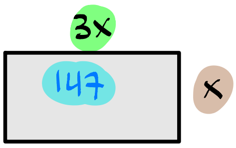
(b) $x^4 - 1$

$$= ((v^2)^2 - 1^2)$$ change v's to x's

$$= (v^2-1)(v^2+1)$$

$$= (v+1)(v-1)(v^2+1)$$

7. (6 pts) The length of a rectangle is three times as long as the width. If it's area is 147 cm^2, find the dimensions of the rectangle.



$$A = L \cdot W$$

$$147 = 3w^2$$

$$147 - 3w^2 = 0$$

$$3(49 - w^2) = 0$$

$$3(7^2 - w^2) = 0$$ difference of squares

$$\frac{3(7-w)(7+w)}{3} = \frac{0}{3}$$

$$(7-w)(7+w)$$

case 1

$$(7-w) = 0$$

$$7 = w$$

case 2

$$\cancel{7+w=0}$$

$$\cancel{w=-7}$$

cant be negative

Plug in x=7 into

$$A = L \cdot W$$

$$147 = L \cdot 7$$

$$\frac{147}{7} = L$$

$$21 = L$$

8. (5 pts) Consider the rational function: $j(x) = \dfrac{x-2}{x^2-6x+8}$

(a) Find the domain of the rational function and give it in interval notation.

set bottom equal to 0 and solve

$x^2-6x+8=0$

x	-4
x	-2

$(x-2)(x-4)=0$

case 1

$x-2=0$

$x=2$

case 2

$x-4=0$

$x=4$

$(-\infty,2)\cup(2,4)\cup(4,\infty)$

(b) Simplify the function, if possible.

$$\frac{x-2}{x^2-6x+8} = \frac{\cancel{x-2}}{\cancel{(x-2)}(x-4)} = \frac{1}{(x-4)}$$

9. (8 pts) Divide and simplify completely.

$$\frac{y^2-36}{y^2-8y+16} \div \frac{3y-18}{y^2-y-12} = \frac{y^2-36}{y^2-8y+16} \cdot \frac{y^2-y-12}{3y-18}$$

$y^2-36 = y^2-6^2$

$=(y-6)(y+6)$

$y^2-8y+16$

x	-4
x	-4

$(x-4)(x-4)$

$3y-18 = 3(y-6)$

y^2-y-12

y	-4
y	3

$(y+3)(y-4)$

$$= \frac{\cancel{(y-6)}(y+6)}{(x-4)\cancel{(x-4)}} \cdot \frac{(y+3)\cancel{(y-4)}}{3\cancel{(y-6)}}$$

$$= \frac{(y+6)(y+3)}{3(x-4)}$$

10. (6 pts) Perform the opperations and simplify completely.

$$\frac{1}{x+1} - \frac{x}{x-2} + \frac{x^2+2}{x^2-x-2}$$

$$\begin{array}{c|c} x & -2 \\ x & 1 \end{array}$$

$(x+1)(x-2)$

$$= \frac{1}{x+1} - \frac{x}{x-2} + \frac{x^2+2}{(x+1)(x-2)}$$

$$= \frac{1}{x+1}\cdot\frac{(x-2)}{(x-2)} - \frac{x}{x-2}\cdot\frac{(x+1)}{(x+1)} + \frac{x^2+2}{(x+1)(x-2)}$$

$$= \frac{x-2 - x(x+1) + x^2+2}{(x+1)(x-2)}$$

$$= \frac{\cancel{x}\cancel{-2} - \cancel{x^2} \cancel{-x} + \cancel{x^2} \cancel{+2}}{(x+1)(x-2)}$$

$$= \frac{0}{(x+1)(x-2)}$$

$$= 0$$

11. (5 pts) Simplify the complex fraction completely into a single fraction.

$$\frac{y}{y}\cdot\frac{\frac{5}{x}}{\frac{2}{x}+\frac{3}{y}}\cdot\frac{x}{x} = \frac{\frac{5}{x}}{\frac{2y}{yx}+\frac{3x}{yx}}$$

add top
keep bottom
the same

keep

$$= \frac{\frac{5}{x}}{\frac{2y+3x}{yx}}$$

change

flip fraction

same as

$$\frac{5}{x} \div \frac{2y+3x}{yx}$$

$$= \frac{5}{\cancel{x}}\cdot\frac{y\cancel{x}}{2y+3x}$$

$$= \frac{5y}{2y+3x}$$

12. (6 pts) Solve for x. If any extraneous solutions exist, be sure to identify them and/or cross them out.

$$\frac{2}{x^2 - x} = \frac{1}{x - 1}$$

cross mulitply bc there is equal sign

$$1(x^2-x) = 2(x-1)$$

$$x^2 - x = 2x - 2$$

$$-2x \quad -2x$$

$$x^2 - 3x = -2$$

$$+2 \quad +2$$

$$x^2 - 3x + 2 = 0$$

x	-2
x	-1

$$(x-1)(x-2)$$

so $(x-1)(x-2)=0$

case 1: $x - 1 = 0$

$x = 1$ ← this is extraneous since

$$\frac{2}{1^2-1} = \frac{1}{1-1}$$

$$\underbrace{\frac{2}{0} = \frac{1}{0}}_{\text{undefine}}$$

X

case 2: $x - 2 = 0$

$x = 2$

13. (5 pts) A new type of blood analyzer can process a batch of samples in 3 hours. An older model can process the same batch in 4 hours. How long will it take to process the batch if both machines are working together at the same time?

$$\frac{1}{3} + \frac{1}{4} = \frac{1}{x}$$

$$\frac{1}{3}\cdot\frac{4}{4} + \frac{1}{4}\cdot\frac{3}{3} = \frac{1}{x}$$

$$\frac{4}{12} + \frac{3}{12} = \frac{1}{x}$$

$$\frac{4+3}{12} = \frac{1}{x}$$

$$\frac{7}{12} = \frac{1}{x}$$

$$\frac{7}{12} = \frac{1}{x}$$

cross mulitply

$$12 \cdot 1 = 7x$$

$$\frac{12}{7} = \frac{\cancel{7}x}{\cancel{7}}$$

$$\frac{12}{7} = x$$