Math 12 **Exam 3** Name:________________

Instructions: Please answer legibly, logically, and **show all work**. Your goal is to convince me you understand the material, so no credit will be given for unjustified or unclear work, including guess-and-check. Remember that explaining and words are a critical part of math – if you get stuck, try to explain what you would do if you could get past your sticking point. Be sure to answer the question or perform the task you are asked.

1. (4 pts) Simplify into a real number. If there are no real number solutions, then state "No real solution."

(a) $\sqrt{100}$ $= \sqrt[\cancel{2}]{10^{\cancel{2}}} = 10$

(c) $-\sqrt{100}$ $-\cancel{\sqrt{}}10^{\cancel{2}} = -10$

(b) $\sqrt{-100}$ Not real

(d) $(\cancel{\sqrt{}}100)^{\cancel{2}}$ $= 100$

2. (6 pts) Simplify completely. Your answer should contain no radicals.

new method

(a) $\sqrt[123]{x^{123}y^{246}}$ $= \sqrt[\cancel{123}]{x^{\cancel{123}}y^{2\cancel{(123)}}}$

$= x \cdot y^2$

(b) $\sqrt[3]{125x^9y^0}$ $= \sqrt[3]{5^3x^9\cdot 1}$ $= \sqrt[\cancel{3}]{5^{\cancel{3}}\cdot (x^3)^{\cancel{3}}}$ $= 5x^3$

125: 5, 25
25: 5, 5

3. (5 pts) Find the domain of each function below. Express your answer in interval notation.

(a) $r(x) = \sqrt[12]{2 - \frac{1}{7}x}$ ≥ 0

$\left(\sqrt[12]{2-\frac{1}{7}x}\right)^{12} \geq 0^{12}$

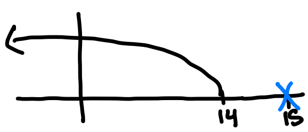
$2 - \frac{1}{7}x \geq 0$

$(7)\ 2 \geq \frac{1}{\cancel{7}}x\ \cancel{(7)}$

$14 \geq x$

$14 \geq 15$ not true

Domain: $(-\infty, 14]$



(b) $g(x) = \sqrt[101]{x}$

odd

so Domain: $(-\infty, \infty)$

4. (5 pts each) Simplify the expression so that no negative exponents remain. Assume all variables are positive.

(a) $(-64a^3)^{2/3}$

$x^{\frac{1}{2}} = \sqrt[2]{x^1}$

$$\sqrt[3]{(-64a^3)^2}$$
$$= \sqrt[3]{(-1\cdot 64\cdot a^3)^2}$$
$$= \sqrt[3]{(-1)^2}\cdot\sqrt[3]{(64)^2}\cdot\sqrt[3]{(a^3)^2}$$
$$= \sqrt[3]{1}\cdot\sqrt[3]{(2^6)^2}\cdot\cancel{\sqrt[3]{(a^2)^3}}$$
$$= \cancel{\sqrt[3]{1^3}}\cdot\sqrt[3]{(2^2)^6}\cdot a^2$$

$1^3 = 1\cdot1\cdot1 = 1$

64 → 2, 32 → 2, 16 → 2, 8 → 2, 4 → 2, 2

$$1\cdot\sqrt[3]{4^6}\cdot a^2$$
$$= \cancel{\sqrt[3]{4^3\cdot 4^3}}\cdot a^2$$
$$= 4\cdot 4\cdot a^2$$
$$= 16a^2$$

(b) $\sqrt[3]{(-27)^4}$

$$= \cancel{\sqrt[3]{(-27)^3}}\cdot(-27)^1$$
$$= -27\cdot\sqrt[3]{(-27)}$$
$$= -27\,\cancel{\sqrt[3]{(-3)^3}}$$
$$= -27(-3)$$
$$= 81$$

$-27 = (-3)(-3)(-3)$
$= (-3)^3$

or ...

$$\sqrt[3]{(-27)^4} = \sqrt[3]{((-3)^3)^4}$$
$$= \cancel{\sqrt[3]{((-3)^4)^3}}$$
$$= (-3)^4$$
$$= 81$$

(c) $\sqrt[3]{\dfrac{54x^4y^6}{2xy}}$

$\frac{54}{2} = \frac{27\cdot\cancel{2}}{\cancel{2}} = \frac{27}{1}$

$$= \sqrt[3]{\frac{27x^4y^6}{x^1y^1}} = \sqrt[3]{3^3x^3y^5} = \cancel{\sqrt[3]{3^3x^3y^3}}\cdot y^2$$
$$= 3xy\sqrt[3]{y^2}$$

27 → 3, 9 → 3, 3
$9 = 3^3$

5. (5 pts) Perform the operation(s) and simplify completely.

$$\sqrt[3]{16} - \sqrt[3]{54} + \sqrt{8} - \sqrt{50}$$

16 → 8, 2 → 4, 2 → 2, 2: 2^4
54 → 27, 2 → 9, 3 → 3, 3: $3^3\cdot 2$
8 → 4, 2 → 2, 2: 2^3
50 → 25, 2 → 5, 5: $5^2\cdot 2$

$$= \sqrt[3]{2^4} - \sqrt[3]{3^4} + \sqrt{2^3} - \sqrt{5^2\cdot 2^1}$$
$$= \sqrt[3]{2^3\cdot 2^1} - \sqrt[3]{3^3\cdot 2} + \sqrt{2^2\cdot 2^1} - 5\cdot\sqrt{2^1}$$
$$= 2\sqrt[3]{2} - 3\sqrt[3]{2} + 2\sqrt{2} - 5\sqrt{2}$$
$$= -\sqrt[3]{2} \qquad -3\sqrt{2}$$

6. Rationalize the denominator.

$$\frac{2}{\sqrt[4]{2}^{1}}$$

want bottom to be $\sqrt[4]{2^4}$, so muliply $\sqrt[4]{2^3}$

$$\frac{2}{\sqrt[4]{2^1}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \frac{2 \cdot \sqrt[4]{2^3}}{\sqrt[4]{2^1 \cdot 2^3}} = \frac{2\sqrt[4]{2^3}}{\cancel{\sqrt[4]{2^4}}} = \frac{\cancel{2}\sqrt[4]{2^3}}{\cancel{2}} = \sqrt[4]{2^3}$$

7. (7 pts) Solve for x. State any extraneous solutions, if they exist.

$$\sqrt{-2x+12} + 2 = x$$
$$\quad -2 \quad -2$$

$$\sqrt[2]{-2x+12} = x-2$$

square both sides

$$\left(\cancel{\sqrt[2]{-2x+12}}\right)^{\cancel{2}} = (x-2)^2$$

$$-2x+12 = (x-2)(x-2)$$

$$-2x+12 = x^2-2x-2x+4$$

$$-2x+12 = x^2-4x+4$$
$$+2x \; -12 \qquad\quad +2x \; -12$$

$$0 = x^2-2x-8$$

x	-4
x	2

$$0 = (x+2)(x-4)$$

$$x=-2, \; x=4$$

8. (5 pts each) Multiply and simplify completely.

(a) $5i(3+4i)$ $= 5i(3) + 5i(4i)$

$= 15i + 20i^2$ $\qquad i^2 = -1$

$= 15i + 20(-1)$

$= 15i - 20$

(b) $(-4-4i)^2$ $= (-4-4i)(-4-4i)$

$= \cancel{16} + 16i + 16i - \cancel{16}$

$= 32i$

	-4	$-4i$
-4	16	$16i$
$-4i$	$16i$	$16i^2 = -16$

$i^2 = -1$

9. (5 pts) Divide and simplify. Your answer should be in the form of a complex number $a + bi$ or $a - bi$.

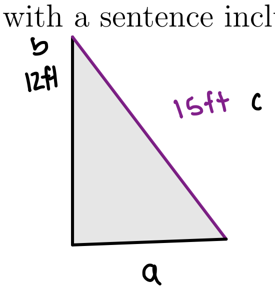
$$\frac{10}{-5-i} \cdot \frac{(-5+i)}{(-5+i)} = \frac{10(-5+i)}{(-5-i)(-5+i)} = \frac{-50+10i}{25 - \cancel{5i} + \cancel{5i} + 1}$$

	-5	$-i$
-5	25	$5i$
i	$-5i$	$-i^2 = 1$

$-i^2 = -(-1) = 1$

$$= \frac{-50+10i}{26}$$

10. (5 pts) A broken window is 12-feet up the side of a house. How far from the base of the house do you need to put a 15-foot ladder so that it reaches exactly 12-feet up to the window? Be sure to answer the question with a sentence including units.



$a^2 + b^2 = c^2$

$a^2 + 12^2 = 15^2$

$a^2 + 144 = 225$

$a^2 = 225 - 144$

$a^2 = 81$

$\sqrt{a^2} = \sqrt{81}$

$\sqrt{\cancel{a^2}} = \sqrt{\cancel{9^2}} \Rightarrow a = 9$

11. (5 pts) Solve for x **using the quadratic formula.**

$$x^2 + x = -1$$

move everything to one side

$$1 \cdot x^2 + x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{-1 \cdot 3}}{2} \qquad \sqrt{-1} = i^2$$

$$= \frac{-1 \pm \sqrt{-1} \cdot \sqrt{3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2} \rightarrow \quad x = \frac{-1 + i\sqrt{3}}{2} \quad \text{or} \quad x = \frac{-1 - i\sqrt{3}}{2}$$

12. (6 pts) A client wants a rectangular garden with an area of 96 sq. ft. in their yard. The lenght of the garden needs to be 4 feet more than the width. Find the dimensions (length and width) of the garden.

$$A = L \times W$$

$$96 = (4 + w)(w)$$

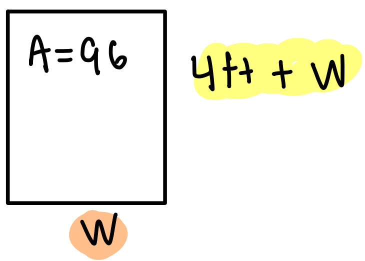
$$96 = 4w + w^2$$

$$0 = \underbrace{1 \cdot}_{a} w^2 + \underbrace{4}_{b} w \underbrace{- 96}_{c}$$

w	12
w	-8

$$0 = (w + 12)(w - 8)$$

$$w = -12, \quad w = 8$$



13. Consider the function: $\overset{y}{f(x)} = 1x^2 - 6x + 8$

(a) (2 pts) Find the vertex.

x-vertex: $\left(-\frac{b}{2a}\right) = \left(-\frac{(-6)}{2(1)}\right) = \frac{6}{2} = 3$

y-vertex: $f(3) = 3^2 - 6(3) + 8$

$= 9 - 18 + 8$

$= -1$

$(3, -1)$

(b) (2 pts) Find the $y-$intercept.

set $x=0$

$y = x^2 - 6x + 8$

$y = 0^2 - 6(0) + 8$

$y = 8$ $\quad (0, 8)$

(c) (2 pts) Find the $x-$intercept(s) (if they exist).

$y = 0$

$0 = x^2 - 6x + 8$

x	-4
x	-2

$0 = (x-2)(x-4)$

$x = 2, \quad x = 4$

$(2, 0)$ and $(4, 0)$

(d) (2 pts) Graph the function using parts (a) - (c).