the spacing on this quiz is terrible.

Quiz 11 :(**Name:** Kumatora
Total Points possible: 10 out of 10
Math 12: Spring 2025

Instructions: Show all your work in order to receive credit.
Problem 1. (2.25 points) Consider the functions $f(x) = x^3 + 1$ and $g(x) = 2x - 1$. Find $g(f(x))$ and $f(g(x))$ and determine if these two functions are ~~commutative~~ commutative?

$f(g(x)) = f(2x+1)$ Plug in $(2x+1)$ into every x in $f(x) = x^3+1$

	$2x$	1
$2x$	$4x^2$	$2x$
1	$2x$	1

$= (2x+1)^3+1$
$= (2x+1)(2x+1)(2x+1)+1$
$= (4x^2+2x+2x+1)(2x+1)+1$
$= (4x^2+4x+1)(2x+1)+1$
$= 8x^3+4x^2+8x^2+4x+2x+1+1$
$= 8x^3+12x^2+6x+2$

	$4x^2$	$4x$	1
$2x$	$8x^3$	$8x^2$	$2x$
1	$4x^2$	$4x$	1

Now $g(f(x)) = g(x^3+1)$ plug in x^3+1 into every x in $2x+1 = g(x)$
$= 2(x^3+1)+1$
$= 2x^3+2+1$
$= 2x^3+3$

$f(g(x)) = 8x^3+12x^2+6x+2 \neq g(f(x)) = 2x^3+3$ so not commutative

Problem 2. (2.25 points) Are $h(x) = x+2$ and $j(x) = x+1002$ commutative? Show your claim.

$h(j(x)) = h(x+1002)$ Plug in $x+1002$ into every x in $h(x) = x+2$
$= x+1002+2$
$= x+1004$

$j(h(x)) = j(x+2)$ Plug in $x+2$ into every x in $j(x) = x+1004$
$= x+2+1004$
$= x+1004$

Indeed, $h(j(x)) = j(h(x)) = x+1004$ so the functions are commutative.

1) $\log_b(MN) = \log_b(M) + \log_b(N)$

2) $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

3) $\log_b(M^P) = P\log_b(M)$

Problem 3. (2.25 points) Simplify the following expression into a single logarithm.

$$\frac{1}{2}\log_3(x^2+1) + \log_3\left(\frac{x+1}{x-1}\right) - \log_3\left(\sqrt{x^2-4}\right)$$

by ③ $= \log_3(x^2+1)^{\frac{1}{2}} + \log_3\left(\frac{x+1}{x-1}\right) - \log_3\left(\sqrt{x^2-4}\right)$ note $(x^2+1)^{\frac{1}{2}} = \sqrt[2]{(x^2+1)^1}$

$= \log_3\left(\sqrt{x^2+1}\right) + \log_3\left(\frac{x+1}{x-1}\right) - \log_3\left(\sqrt{x^2-4}\right)$

by ① $= \log_3\left(\sqrt{x^2+1} \cdot \frac{x+1}{x-1}\right) - \log_3\left(\sqrt{x^2-4}\right)$

$= \log_3\left(\frac{(\sqrt{x^2+1})(x+1)}{(x-1)}\right) - \log_3\left(\sqrt{x^2-4}\right)$

by $\boxed{2}$ $= \log_3\left(\frac{(\sqrt{x^2+1})(x+1)}{(x-1)} \cdot \frac{1}{\sqrt{x^2-4}}\right)$

$= \log_3\left(\frac{(\sqrt{x^2+1})(x+1)}{(x-1)(\sqrt{x^2-4})}\right)$

Problem 4. (2.25 points) Expand the following expression completely using logarithmic properties.

$\sqrt{y} = y^{\frac{1}{2}}$

$$\log_3\left(\frac{(2x^3\sqrt{y})}{(x+2)^2}\right)$$

by $\boxed{2}$ $= \log_3(2\cdot x^3\cdot y^{\frac{1}{2}}) - \log_3(x+2)^2$

by ① $= \log_3(2) + \log_3(x^3) + \log_3(y^{\frac{1}{2}}) - \log_3(x+2)^2$

by ③ bring down exponents,

$= \log_3(2) + 3\log_3(x) + \frac{1}{2}\log_3(y) - 2\log_3(x+2)$

Problem 5. (1 point) Don't look this up. I want your opinion. If I remove an arm from a cactus and plant it, is it considered a new cactus or still part of the original?

My answer won't fit here ''