pliant Mechanisms Lecture 2 Part 3

Finding Rotational and Translational Velocities at Any Point

$$\mathbf{T} = \omega_{x'}\begin{bmatrix}\mathbf{n}_1 \\ \mathbf{L}\times\mathbf{n}_1\end{bmatrix} + \omega_{y'}\begin{bmatrix}\mathbf{n}_2 \\ \mathbf{L}\times\mathbf{n}_2\end{bmatrix} + \omega_{z'}\begin{bmatrix}\mathbf{n}_3 \\ \mathbf{L}\times\mathbf{n}_3\end{bmatrix} + v_{x'}\begin{bmatrix}0 \\ \mathbf{n}_1\end{bmatrix} + v_{y'}\begin{bmatrix}0 \\ \mathbf{n}_2\end{bmatrix} + v_{z'}\begin{bmatrix}0 \\ \mathbf{n}_3\end{bmatrix} = \begin{bmatrix}\boldsymbol{\omega} \\ \mathbf{c}\times\boldsymbol{\omega} + p\boldsymbol{\omega}\end{bmatrix}$$

$$\mathbf{T}' = \begin{bmatrix}\omega_{x'} & \omega_{y'} & \omega_{z'} & v_{x'} & v_{y'} & v_{z'}\end{bmatrix}^T = \begin{bmatrix}\boldsymbol{\omega}' \\ \mathbf{c}'\times\boldsymbol{\omega}' + p\boldsymbol{\omega}'\end{bmatrix}$$

Thus if

$$[N] = \begin{bmatrix}\mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 & 0 & 0 & 0 \\ \mathbf{L}\times\mathbf{n}_1 & \mathbf{L}\times\mathbf{n}_2 & \mathbf{L}\times\mathbf{n}_3 & \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3\end{bmatrix}$$

then

$$[N]\mathbf{T}' = \mathbf{T}$$

or

$$\mathbf{T}' = [N]^{-1}\mathbf{T}$$

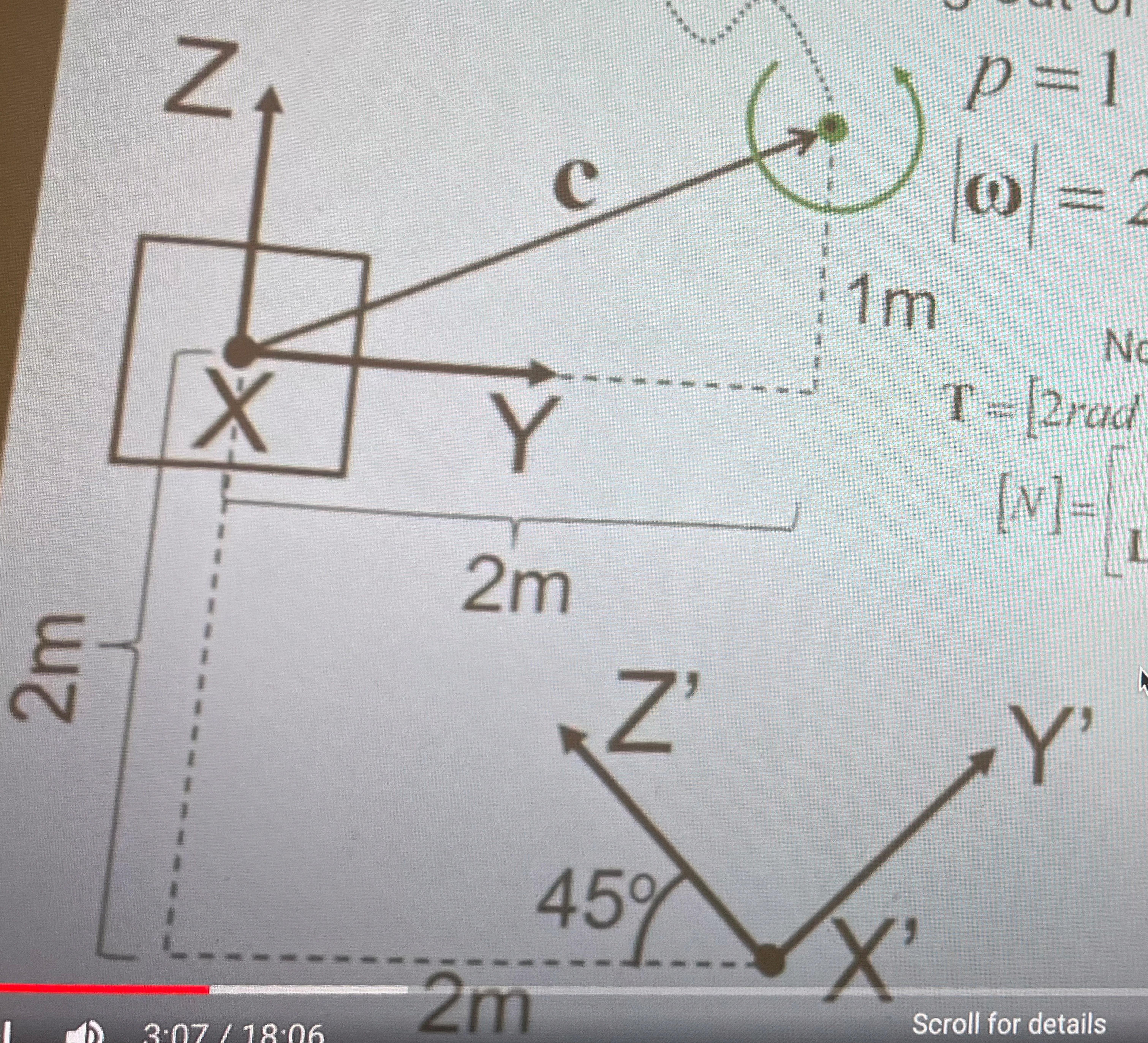
Scroll for details

Given the twist with respect to X, Y, and Z, find the same twist with respect to X' Y' and Z'.

Suppose ω is pointing out of the page along the X axis

$p = 1$ m/rad

$|\omega| = 2$ rad/s



Now use the new approach. Recall that

$$\mathbf{T} = [2rad/s \quad 0 \quad 0 \quad 2m/s \quad 2m/s \quad -4m/s]^T$$

$$[N] = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 & 0 & 0 & 0 \\ \mathbf{L}\times\mathbf{n}_1 & \mathbf{L}\times\mathbf{n}_2 & \mathbf{L}\times\mathbf{n}_3 & \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 \end{bmatrix}$$

$$\mathbf{n}_1 = [1 \quad 0 \quad 0]^T$$

$$\mathbf{n}_2 = \left[0 \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right]^T$$

$$\mathbf{n}_3 = \left[0 \quad -\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right]^T$$

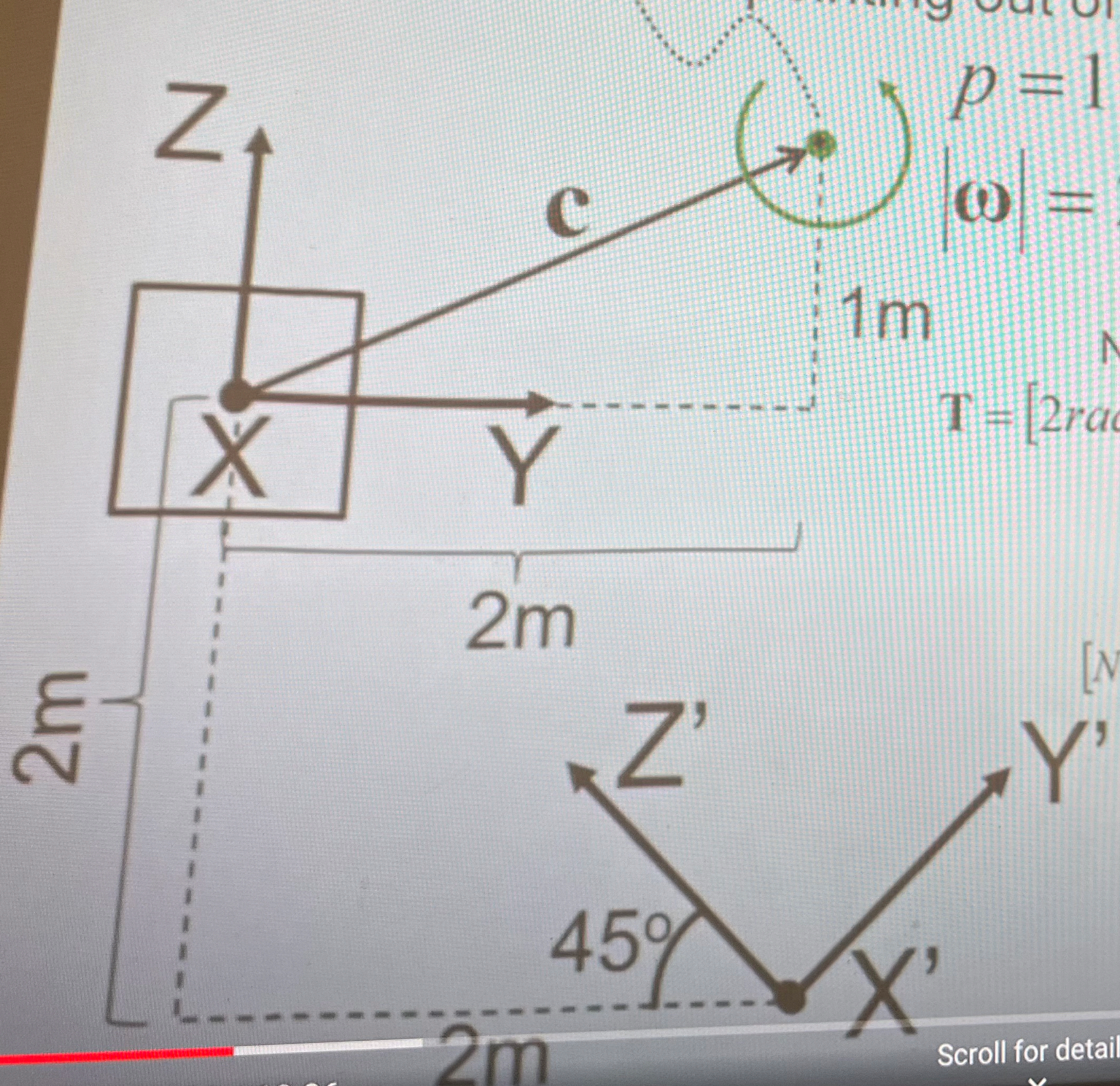
$$\mathbf{L} = [0 \quad 2m \quad -2m]^T$$

Compliant Mechanisms Lecture 2 Part 4

Example

Given the twist with respect to X, Y, and Z, find the same twist with respect to X' Y' and Z'.

Suppose ω is pointing out of the page along the X axis



$p = 1$ m/rad

$|\omega| = 2$ rad/s

Now use the new approach. Recall that

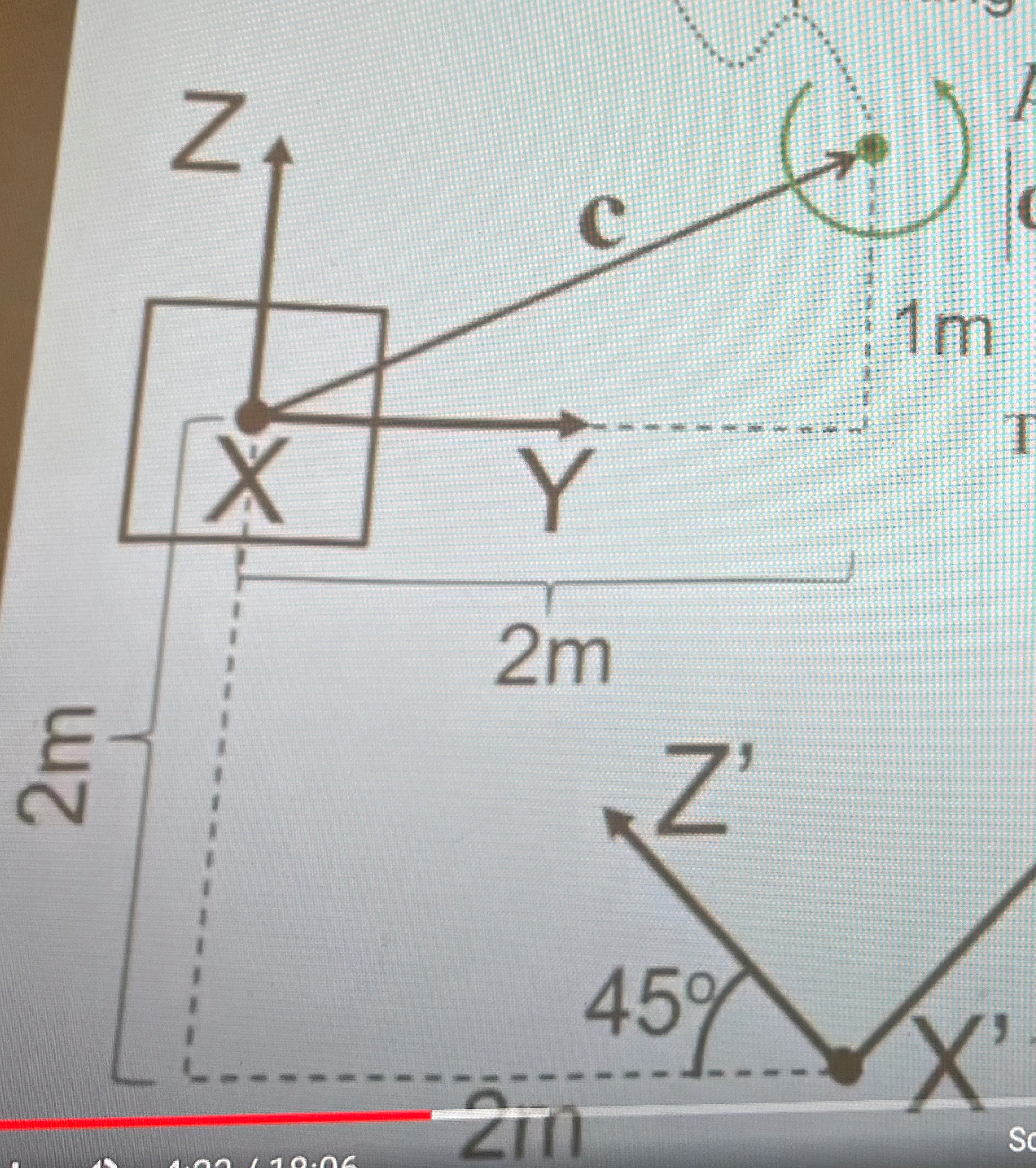
$$\mathbf{T} = [2rad/s \quad 0 \quad 0 \quad 2m/s \quad 2m/s \quad -4m/s]^T$$

$$[N] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 4/\sqrt{2}m & 0 & 1 & 0 & 0 \\ -2m & 0 & 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -2m & 0 & 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Example

Given the twist with respect to X, Y, and Z, find the same twist with respect to X' Y' and Z'.

Suppose ω is pointing out of the page along the X axis



$p = 1$ m/rad

$|\omega| = 2$ rad/s

Now use the new approach. Recall that

$$T = \begin{bmatrix} 2rad/s & 0 & 0 & 2m/s & 2m/s & -4m/s \end{bmatrix}^T$$

$$[N] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 4/\sqrt{2}m & 0 & 1 & 0 & 0 \\ -2m & 0 & 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -2m & 0 & 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$T' = \begin{bmatrix} 2rad/s & 0 & 0 & 2m/s & \frac{6}{\sqrt{2}}m/s & -\frac{6}{\sqrt{2}}m/s \end{bmatrix}^T = [N]^{-1}$$

Displacement Twists

Velocity Twist:

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$$

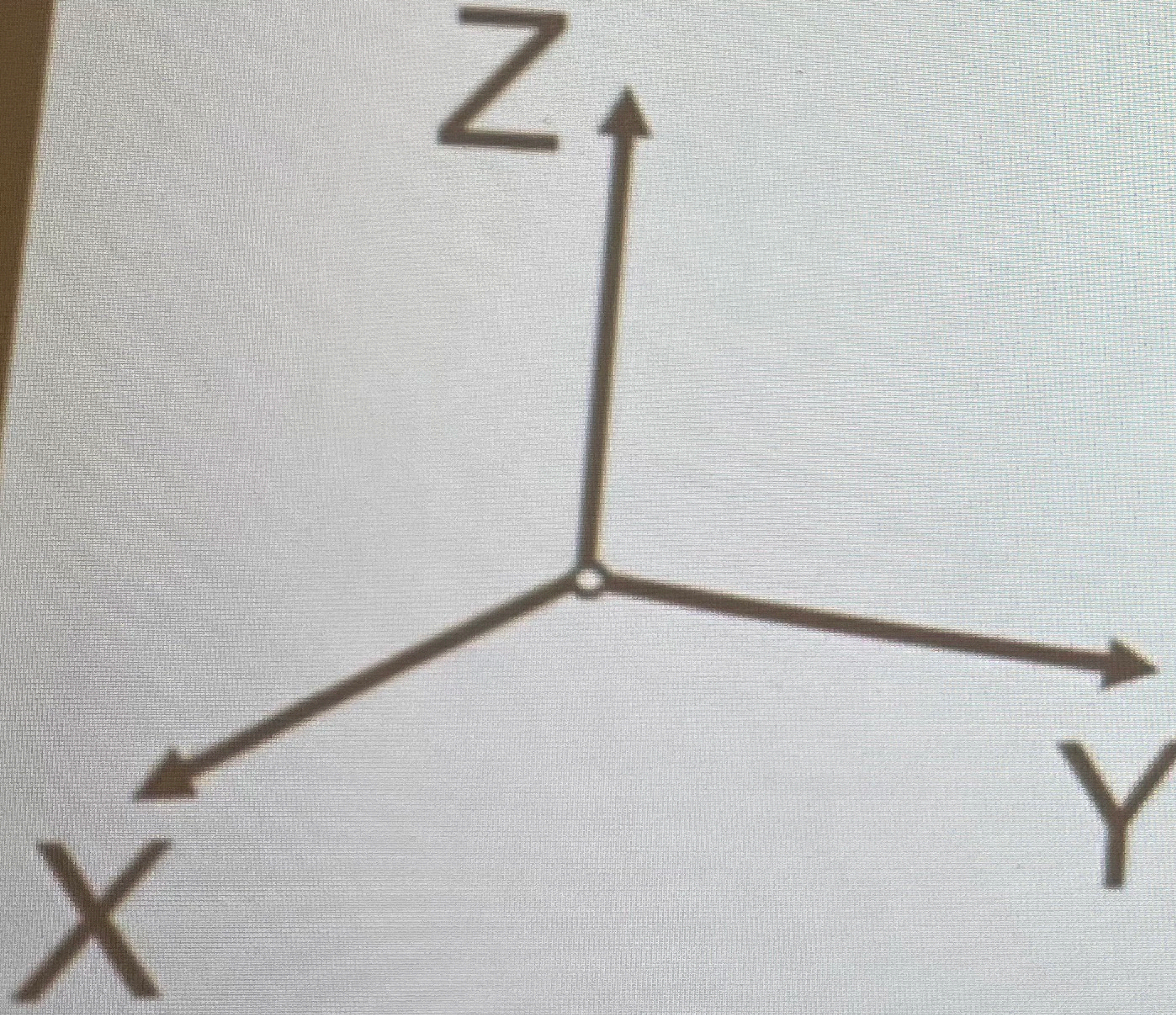
Displacement Twist:

$$\mathbf{T}\delta t = \begin{bmatrix} \boldsymbol{\omega}\delta t \\ \mathbf{c} \times \boldsymbol{\omega}\delta t + p\boldsymbol{\omega}\delta t \end{bmatrix} = |\boldsymbol{\omega}|\delta t \begin{bmatrix} \mathbf{n} \\ \mathbf{c} \times \mathbf{n} + p\mathbf{n} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega}\delta t \\ \mathbf{v}\delta t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta\theta} \\ \boldsymbol{\delta}\mathbf{d} \end{bmatrix} = \begin{bmatrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \\ \delta d_x \\ \delta d_y \\ \delta d_z \end{bmatrix}$$

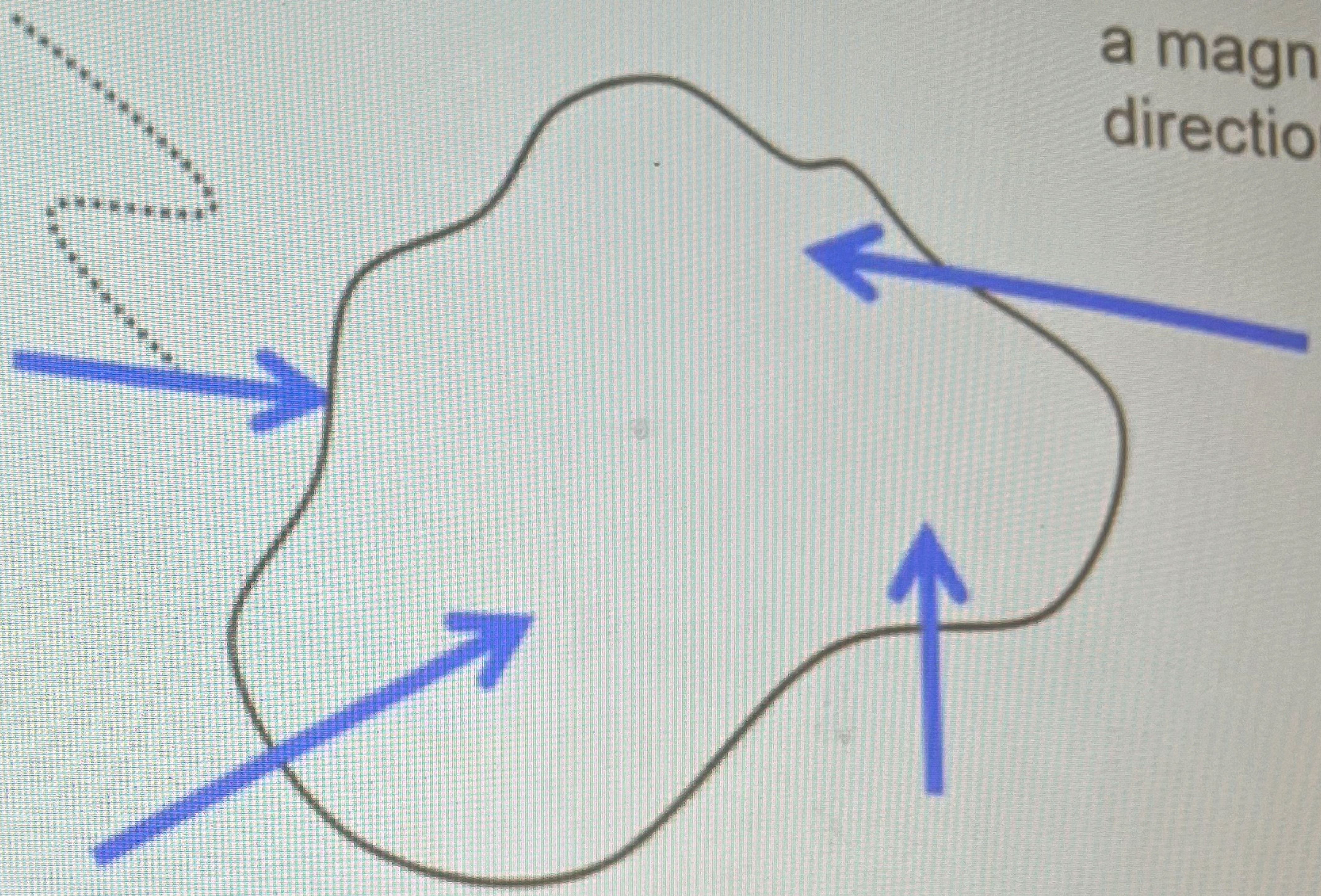
Force Actions

$$\mathbf{f} = \begin{bmatrix} f_x & f_y & f_z \end{bmatrix}^T$$

$$|\mathbf{f}| = \sqrt{f_x^2 + f_y^2 + f_z^2}$$



Force vectors have a magnitude and direction only.



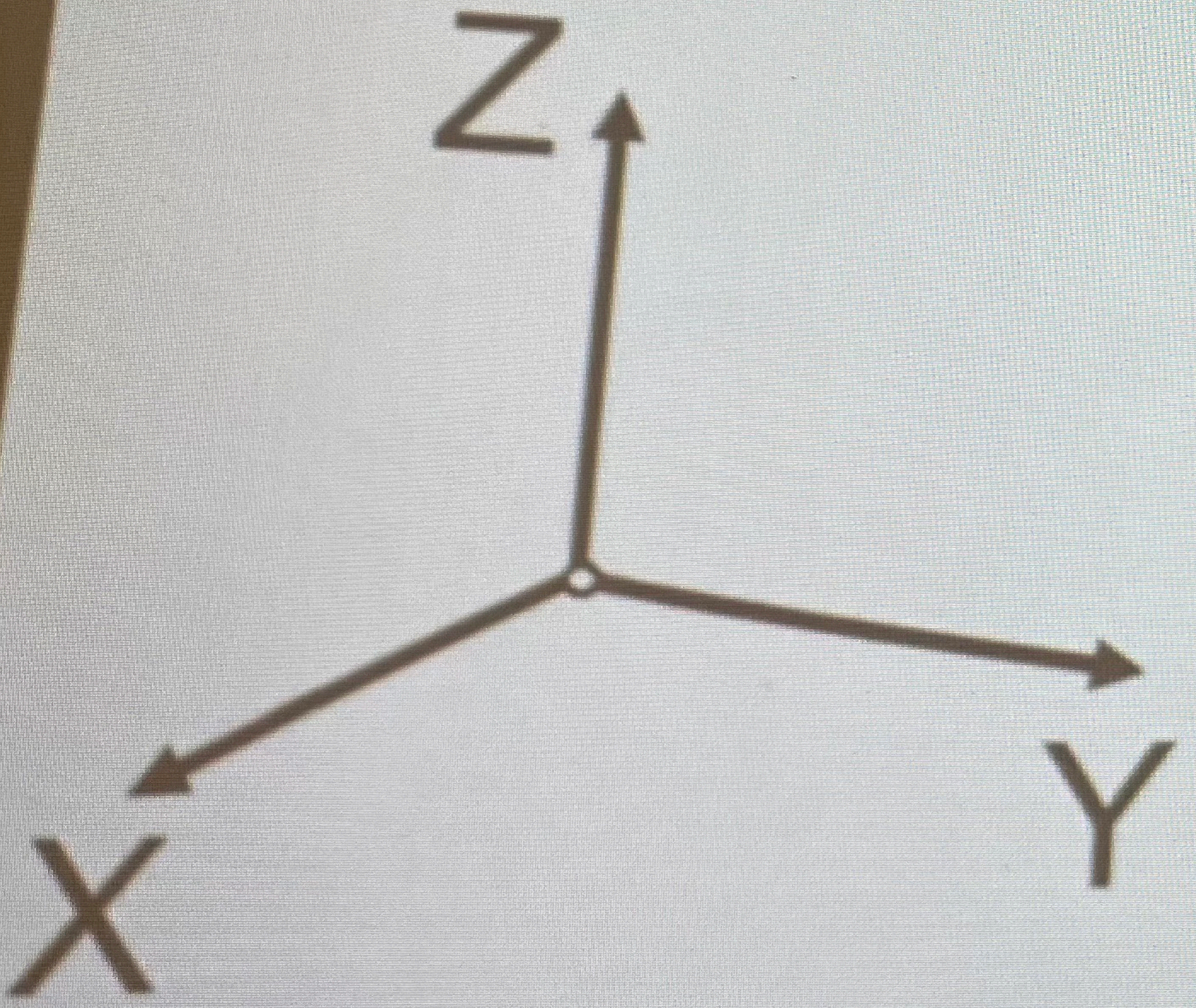
When finding the net force on a body, we don't care about the location where each force is applied. We only care about the magnitude and direction of each force.

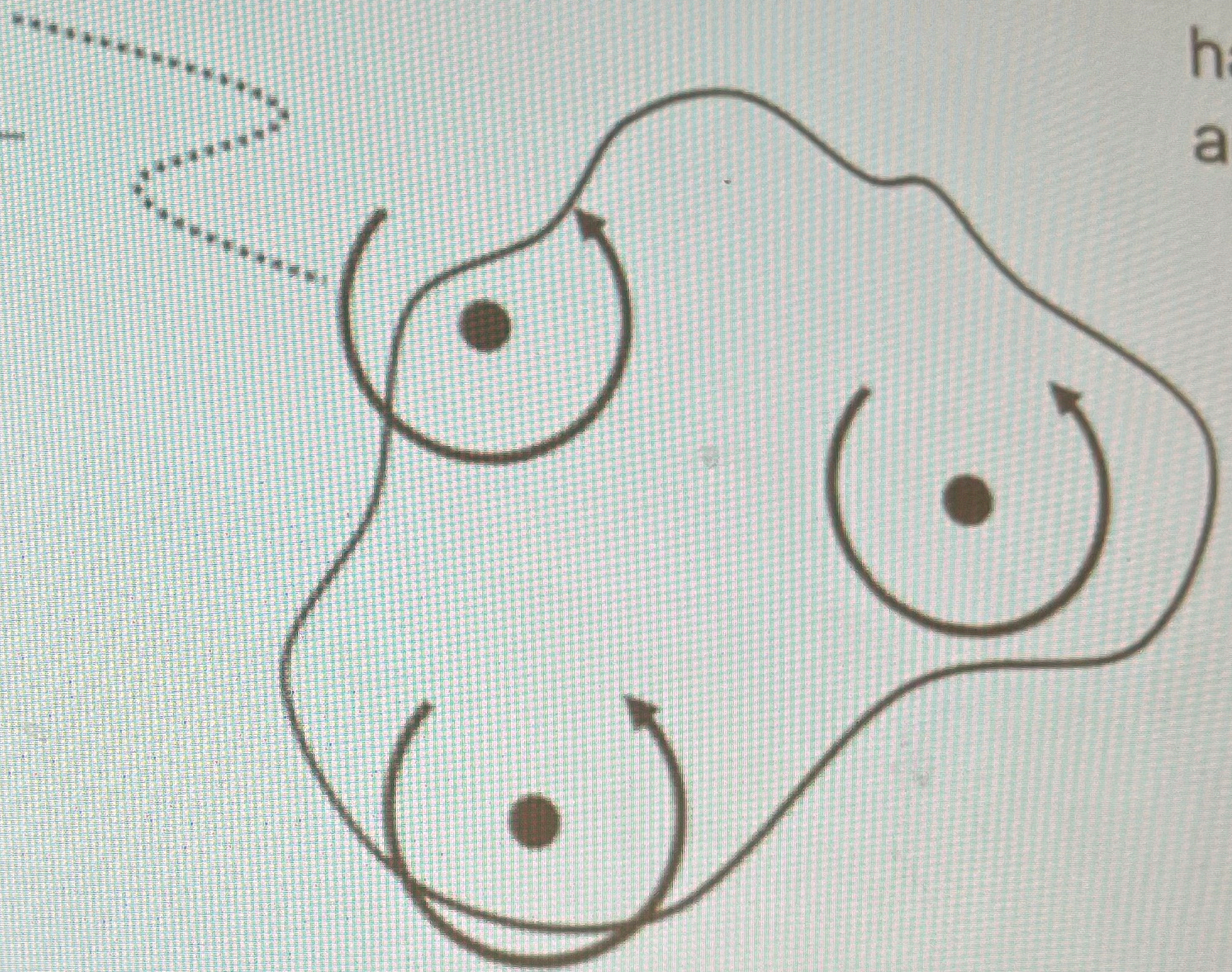
Moment Actions

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T$$

$$|\boldsymbol{\tau}| = \sqrt{\tau_x^2 + \tau_y^2 + \tau_z^2}$$

Moment vectors have a magnitude and direction only.

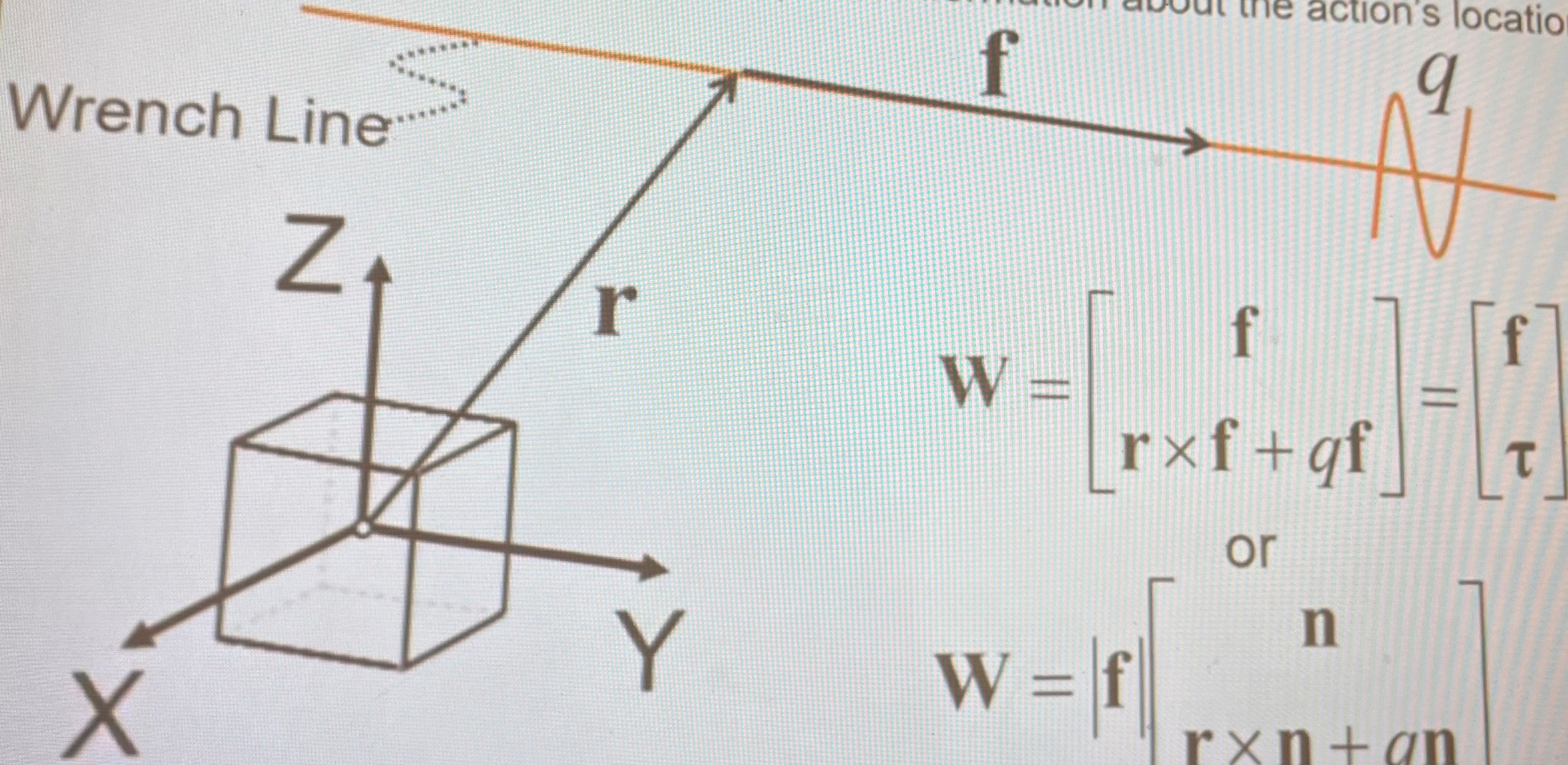




When finding the net moment on a body, we don't care about the location where each moment is applied. We only care about the magnitude and direction of each moment.

General Action Vectors

According to screw theory, all actions can be described as wrench lines. A wrench line describes a coupled moment and force action where the ratio that defines this coupling is defined by q. Note that wrench vectors consider both moment and force vectors together and thus they requires information about the action's location.



$$\mathbf{W} = \begin{bmatrix} \mathbf{f} \\ \mathbf{r} \times \mathbf{f} + q\mathbf{f} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{bmatrix}$$

or

$$\mathbf{W} = |\mathbf{f}| \begin{bmatrix} \mathbf{n} \\ \mathbf{r} \times \mathbf{n} + q\mathbf{n} \end{bmatrix}$$

Here $\boldsymbol{\tau}$ is the moment imposed on the origin

Where $\mathbf{n}$ is a unit vector that points in the direction of $\mathbf{f}$

Compliant Mechanisms Lecture 2 Part 4

General Action Vectors

Things to Note:

It doesn't matter where $\mathbf{r}$ points along the wrench line as long as it points from the origin to a point on the line. Ultimately, all that will matter in the cross product of $\mathbf{r}$ and $\mathbf{f}$ is the magnitude of the shortest distance from the origin to the wrench line (i.e., the length of the moment arm).

$$q = \frac{\text{Moment about the axis of the wrench line}}{\text{Force along the axis of the wrench line}} = \left(\frac{\mathbf{f} \bullet \boldsymbol{\tau}}{\mathbf{f} \bullet \mathbf{f}}\right)$$

When q is zero, the wrench line becomes a pure force, when q is infinite it becomes a pure moment.

$$\begin{bmatrix} q & -r_z & r_y \\ r_z & q & -r_x \\ -r_y & r_x & q \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}$$

Scroll for details

15:02 / 18:06

Compliant Mechanisms Lecture 2 Part 4

Conventions and Comparisons

Name	Pitch	Line Color
Rotation	$p = 0$	
Translation	$p = \infty$	
Screw	$p \neq 0 \neq \infty$	

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$$

Name	q-value	Line Color
Force	$q = 0$	
Moment	$q = \infty$	
Wrench	$q \neq 0 \neq \infty$	

$$\mathbf{W} = \begin{bmatrix} \mathbf{f} \\ \mathbf{r} \times \mathbf{f} + q\mathbf{f} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{bmatrix}$$

Scroll for details

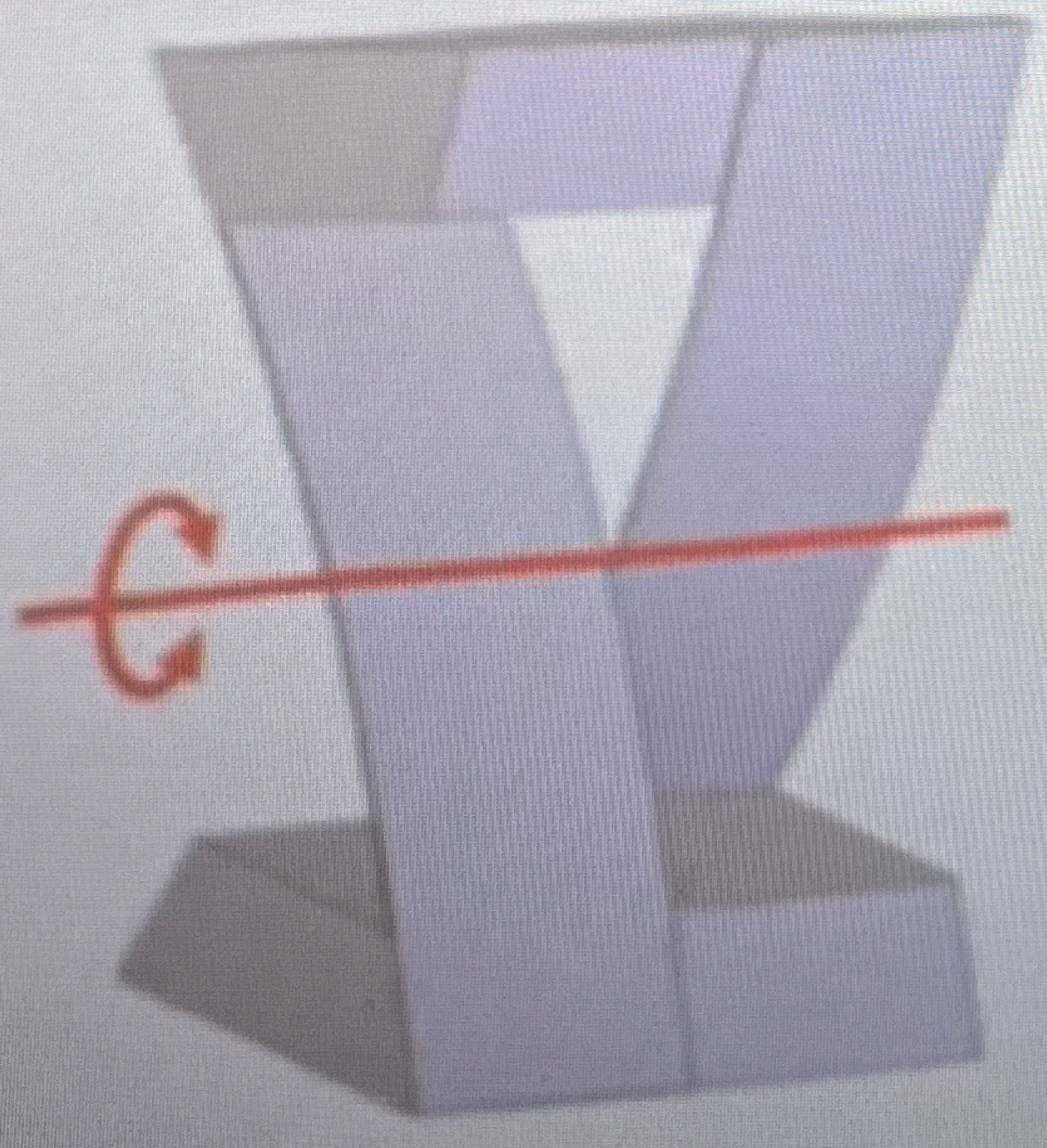
15:17 / 18:06

Chasle's Theorem

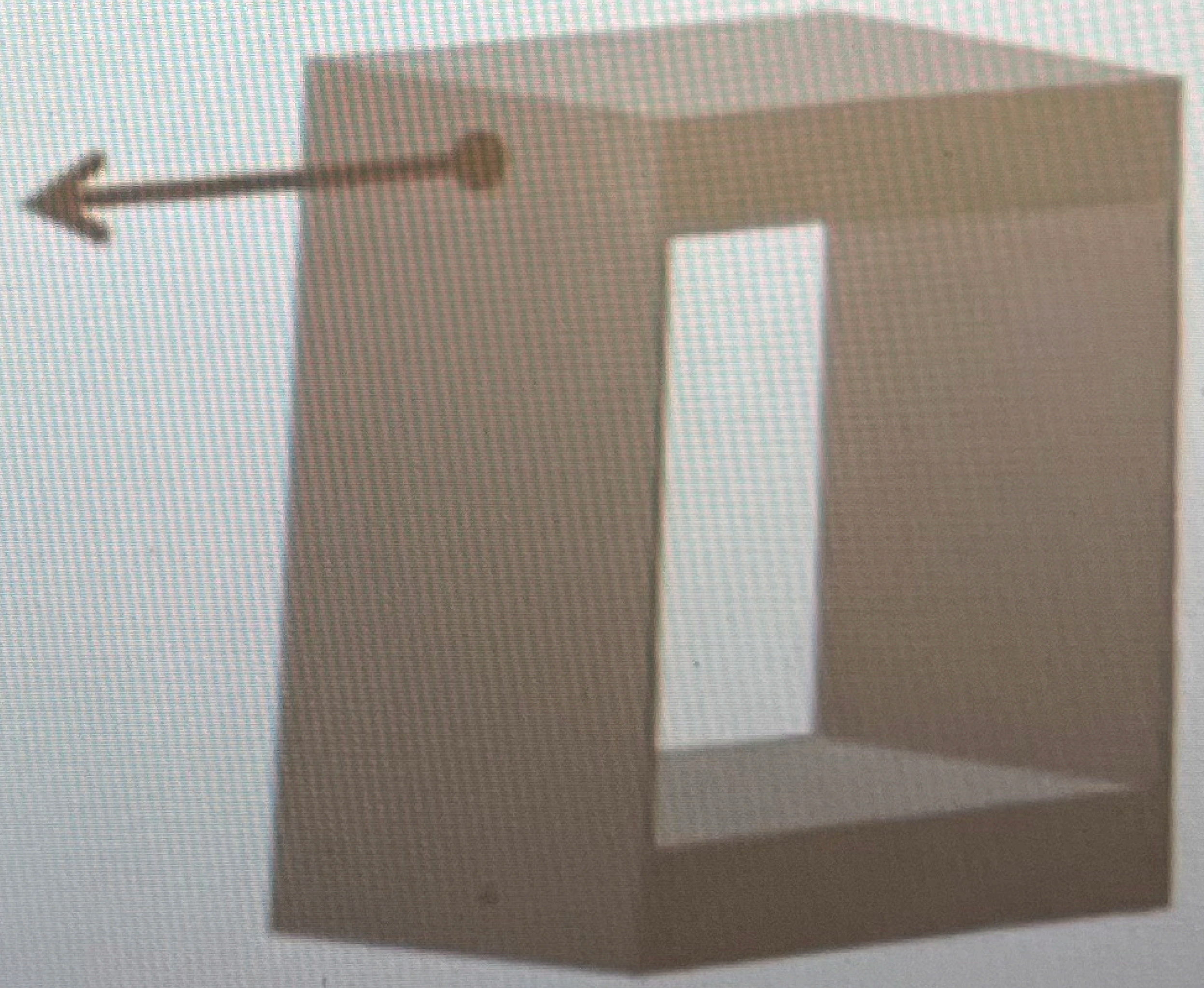
"Any motion of a rigid body in space may be described as a screw motion."

A screw is a coupled translational & rotational motion.
The pitch of a screw, p, determines this coupling.

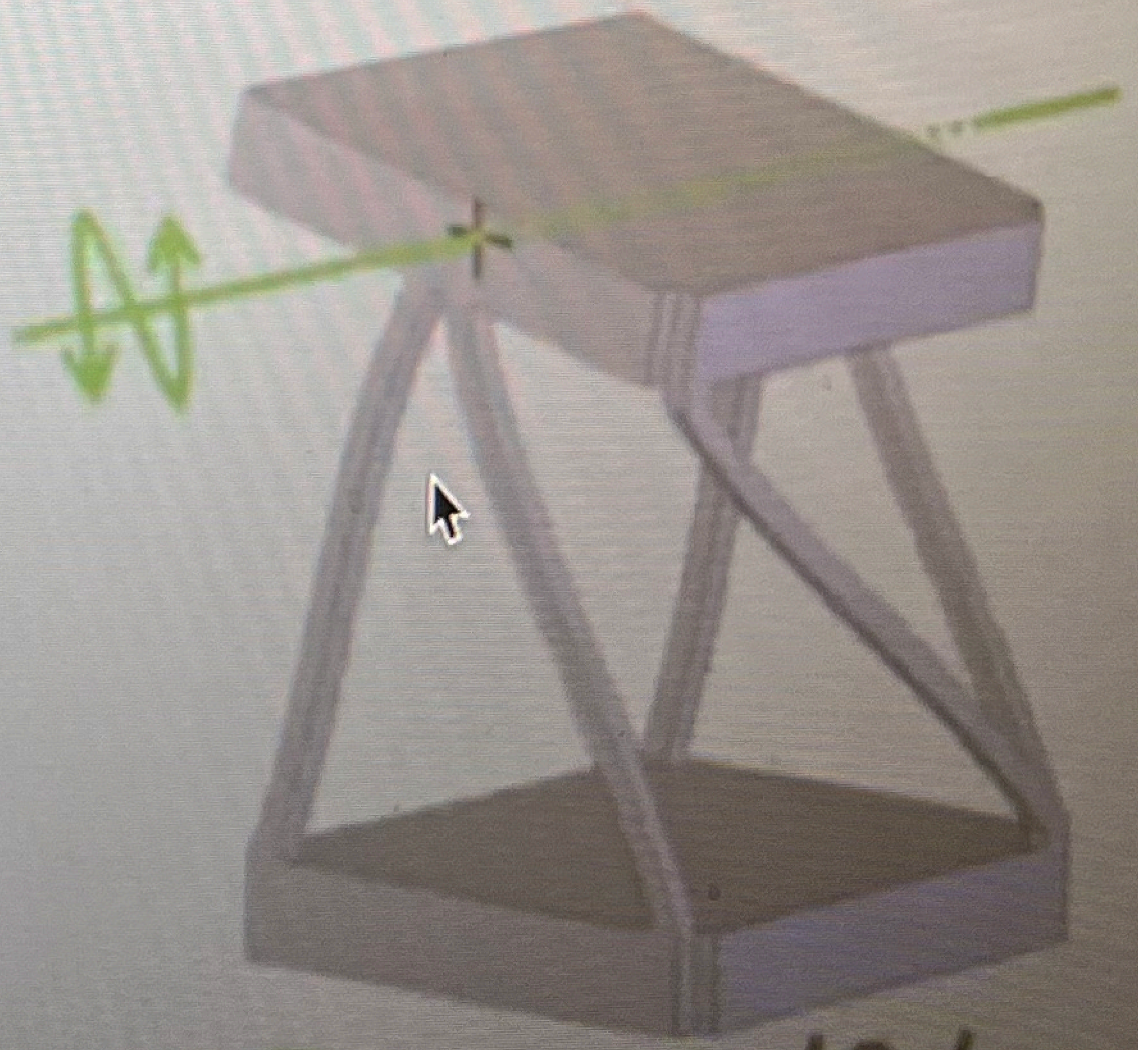
$$p = \frac{\text{How much a body translates}}{\text{How much a body rotates}}$$



Rotation p=0



Translation p=∞

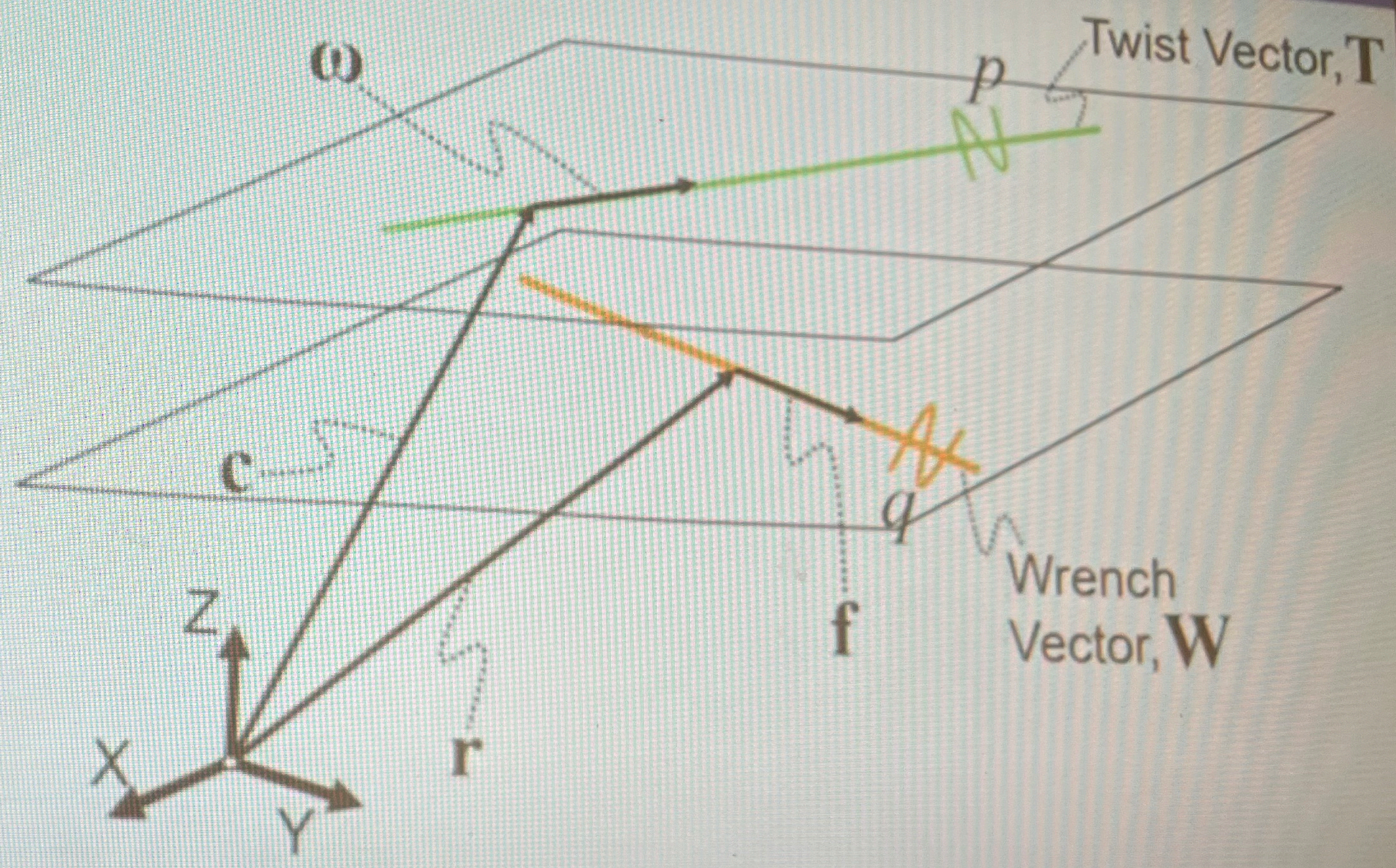


Screw p≠0≠∞

Compliant Mechanisms Lecture 6 Part 5

Relationship Between a General Twist and Constraint Line

$$\mathbf{W} \bullet [\Delta]\mathbf{T} = 0$$



Scroll for details

...ween a General Twist and Constraint Lin

$$W \bullet [\Delta]T = 0$$

$$[f \quad \tau]\begin{bmatrix} v \\ \omega \end{bmatrix} = [f \quad (r\times f)+qf]\begin{bmatrix} (c\times\omega)+p\omega \\ \omega \end{bmatrix} = 0$$