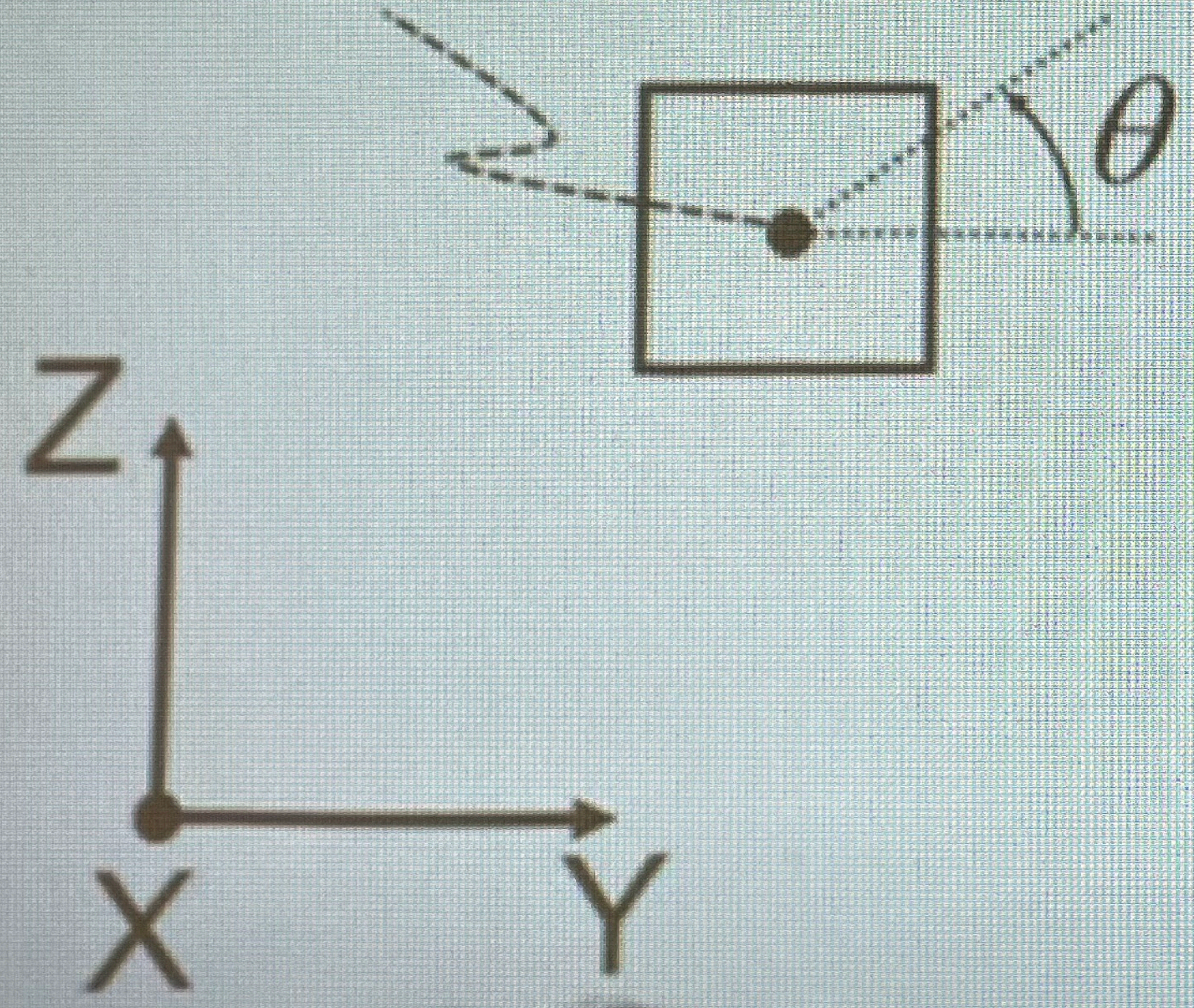

Search
Translational Velocity Vector Example
$|\mathbf{v}| = v_y = \frac{\delta d}{\delta t}$ (units are m/sec)
$\mathbf{v} = [0 \quad v_y \quad 0]^T$
Z
Y
X
View chapter
Translational velocity vectors have a magnitude and a direction only. They have no location.
4:43 / 30:00 • translational and linear velocity vectors
Compliant Mechanisms Lecture 2 Part 1

Rotational Velocity Vector Example

Vector arrow pointing out of the page



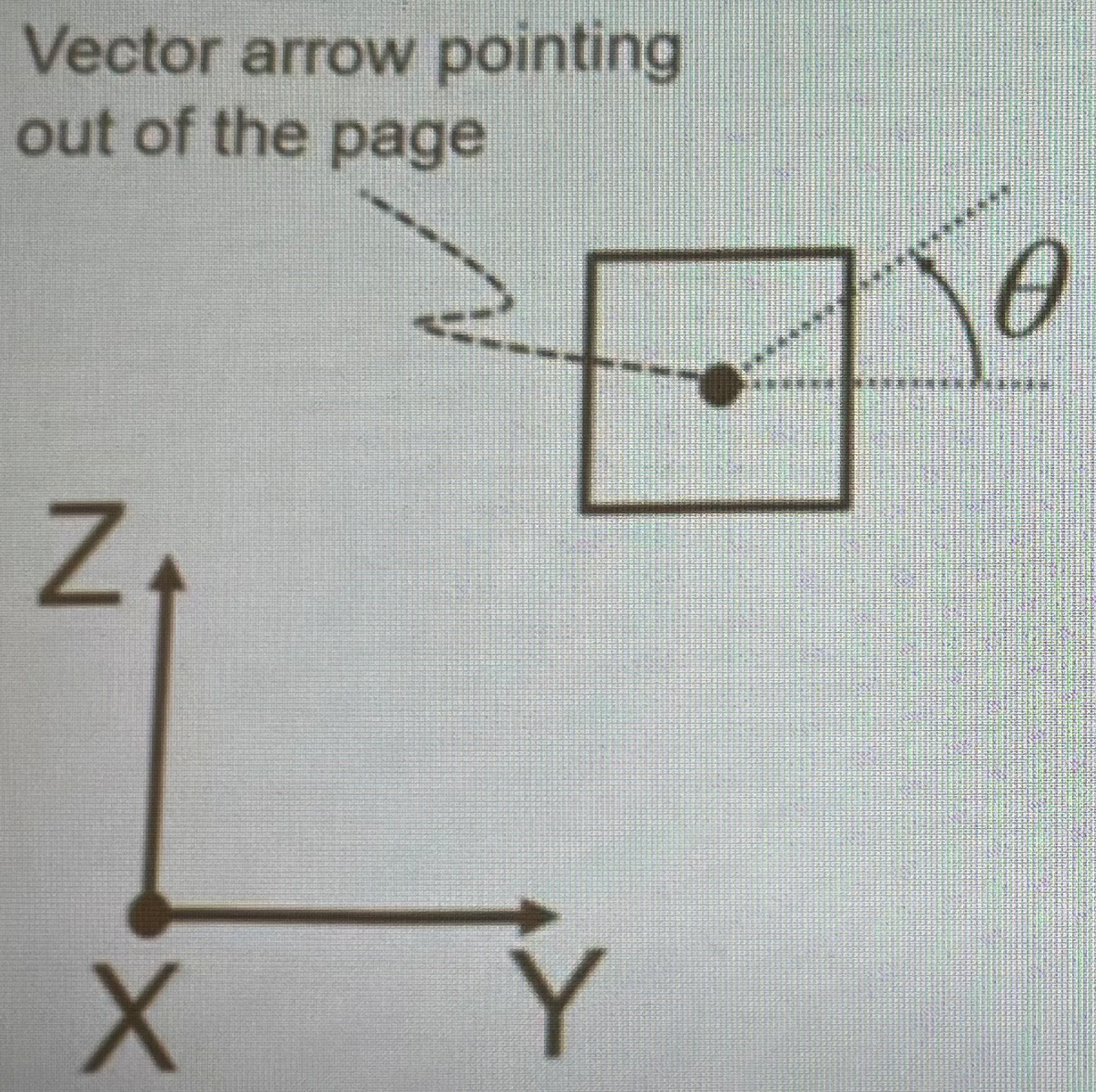
$$|\boldsymbol{\omega}| = \omega_x = \frac{\delta\theta}{\delta t} \quad \text{(units are rad/sec)}$$

$$\boldsymbol{\omega} = [\omega_x \quad 0 \quad 0]^T$$

Rotation[illegible] ctors [illegible] a magnitude
and a di[illegible] [illegible]cation.

Compliant Mechanisms Lecture 2 Part 1

Rotational Velocity Vector Example

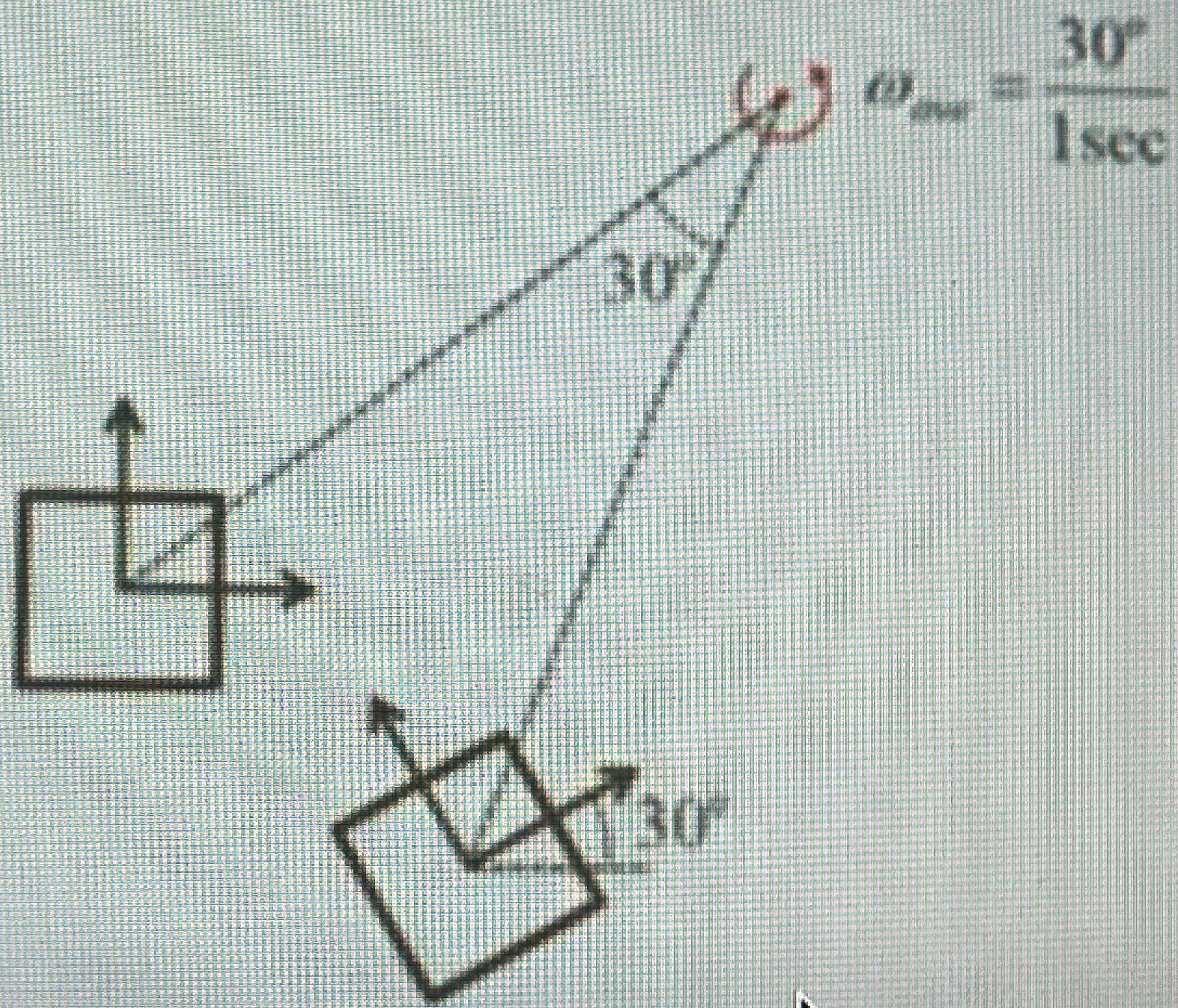


$$|\boldsymbol{\omega}| = \omega_x = \frac{\delta\theta}{\delta t}$$ (units are rad/sec)

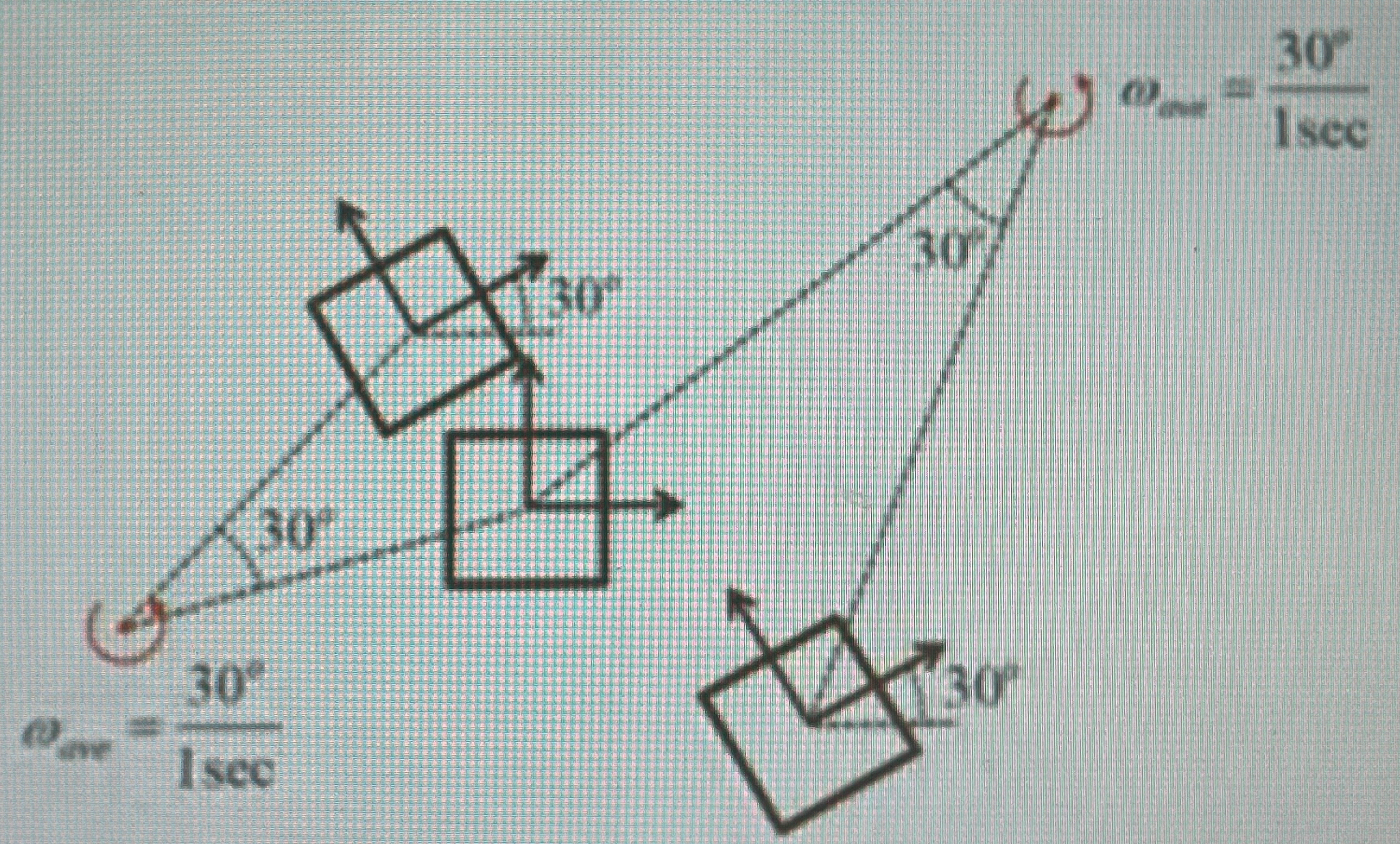
$$\boldsymbol{\omega} = [\omega_x \quad 0 \quad 0]^T$$

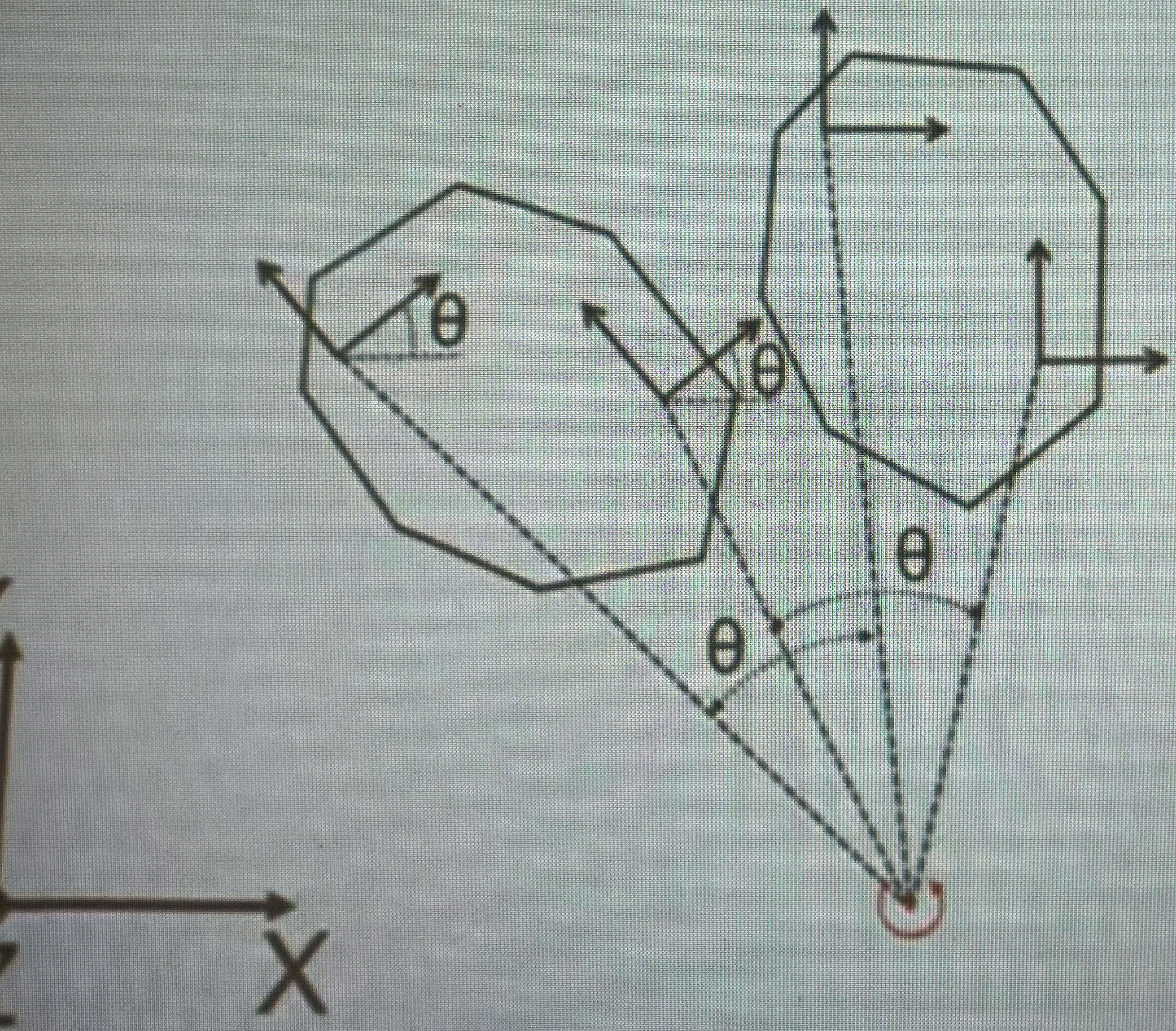
Rotational velocity vectors also only have a magnitude and a direction. They too have no location.

Compliant Mechanisms Lecture 2 Part 1


30°
1sec
30°
30°

Axis of Rotation is Independent of Angular Velocity

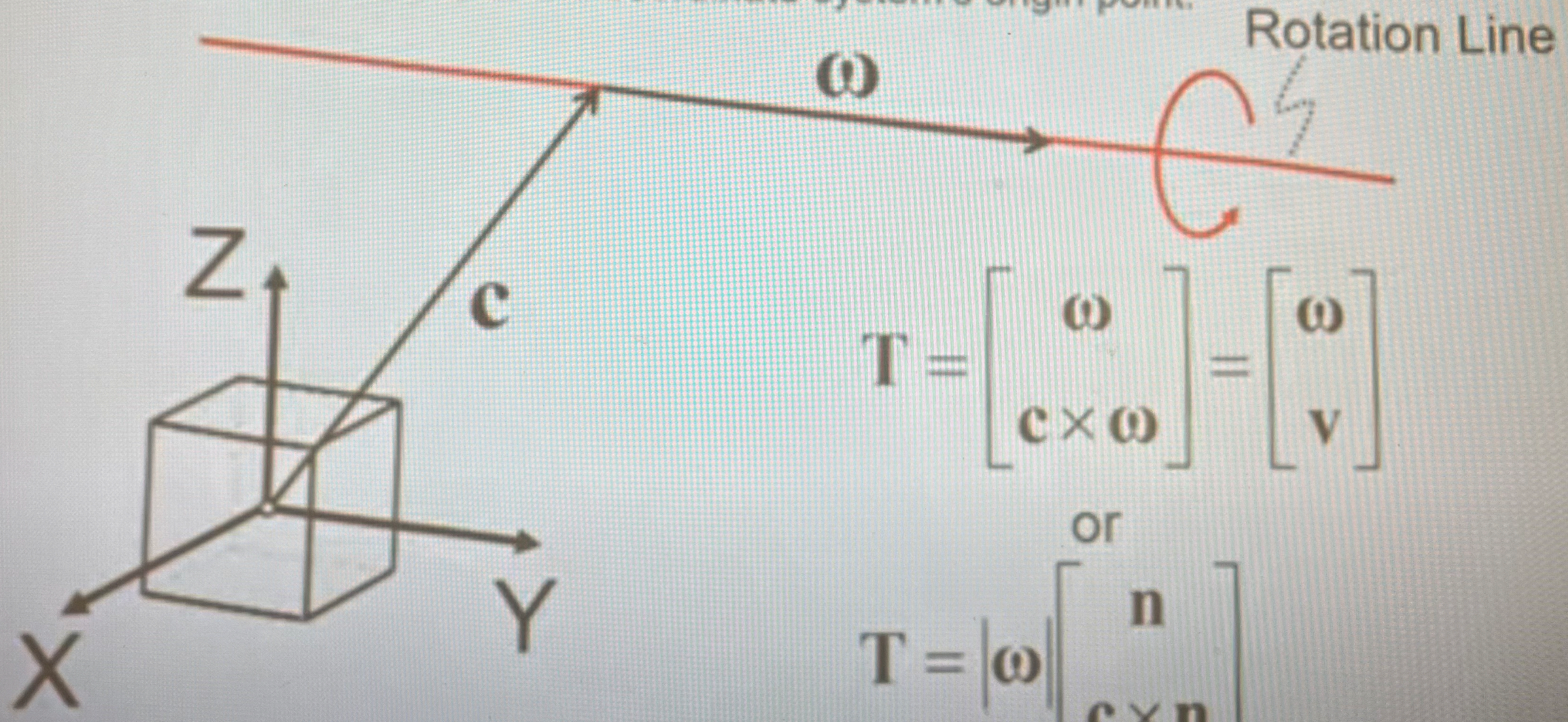



θ
θ
θ
θ
X

Compliant Mechanisms Lecture 2 Part 1

Rotation Lines as Twist Vectors

If we use another type of vector with six components, called a twist vector, we can describe the block's axis of rotation (i.e., a rotation line). A rotation line captures both rotations and translations of the coordinate system's origin point.



$$T = \begin{bmatrix} \omega \\ c \times \omega \end{bmatrix} = \begin{bmatrix} \omega \\ v \end{bmatrix}$$

or

$$T = |\omega| \begin{bmatrix} n \\ c \times n \end{bmatrix}$$

Here **v** is the velocity vector of the origin

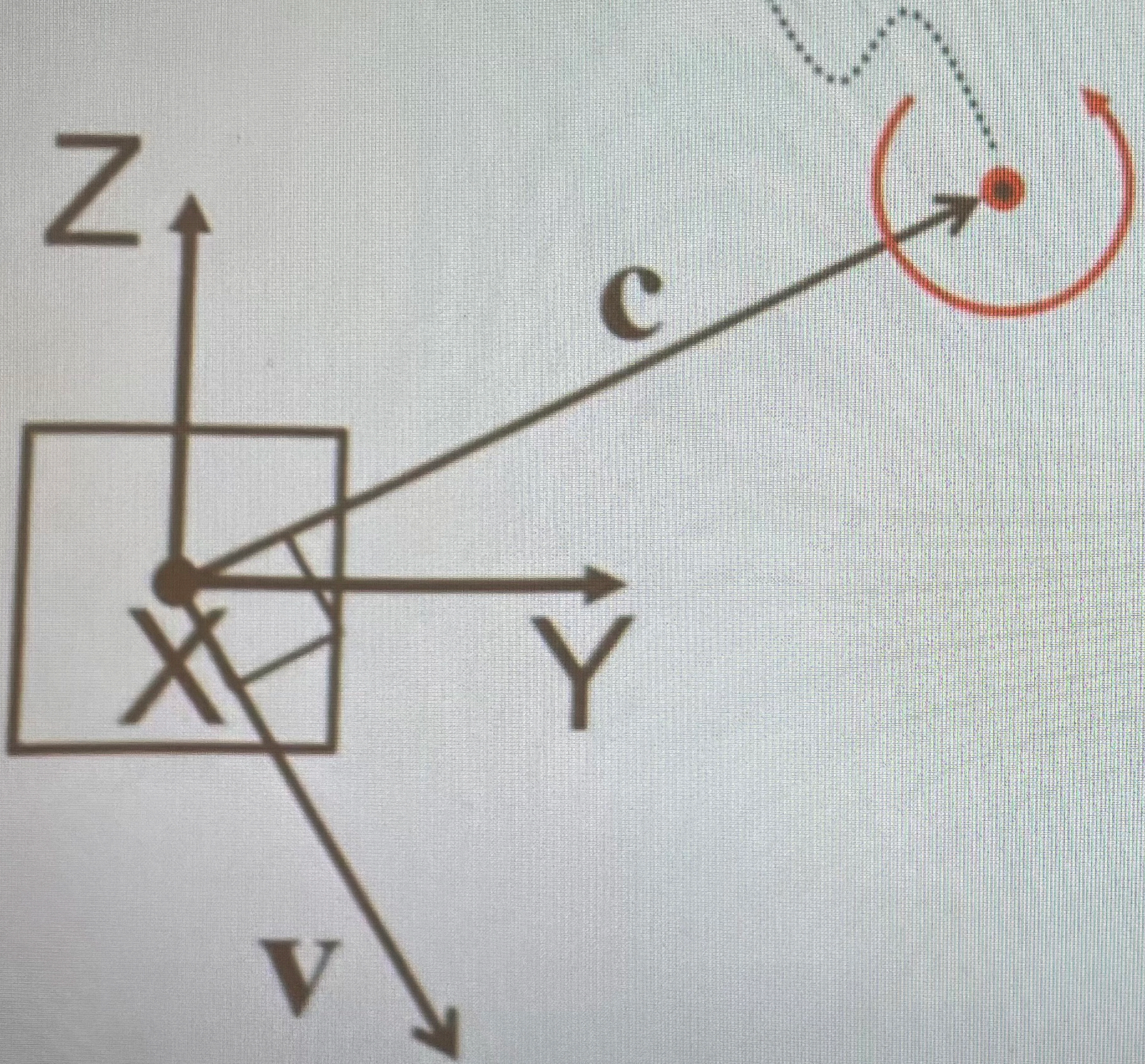
Where **n** is a unit vector that points in the direction of **ω**

28:58 / 30:00 • Twist vectors ›

Scroll for details

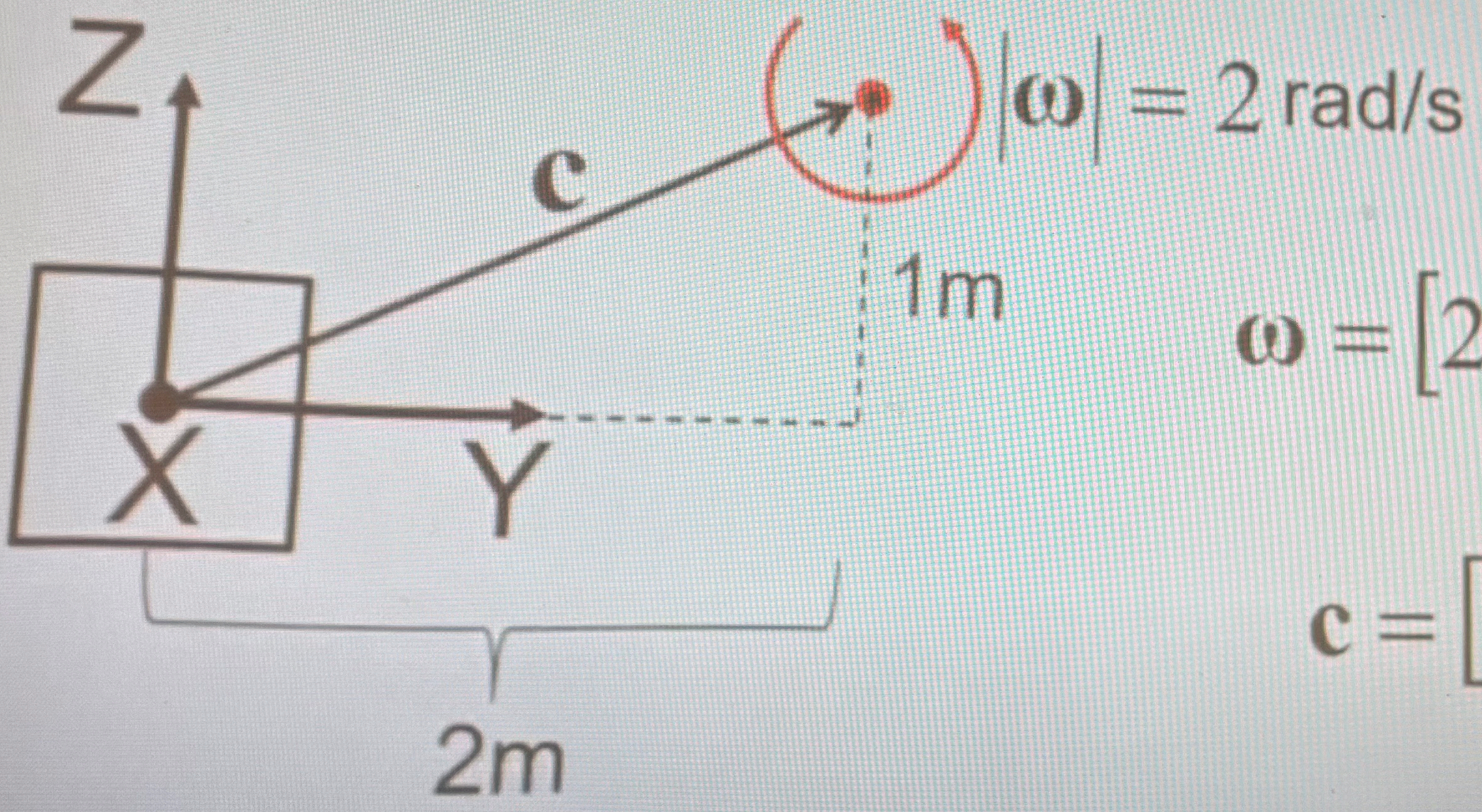
AMD FreeSync

Suppose ω is pointing out of the page along the X



$$T = \begin{bmatrix} \omega \\ c \times \omega \end{bmatrix} = \begin{bmatrix} \omega \\ v \end{bmatrix}$$

Find the rotation line's twist vector, $\mathbf{T}$



$$\omega = \begin{bmatrix} 2rad/s & 0 & 0 \end{bmatrix}^T$$

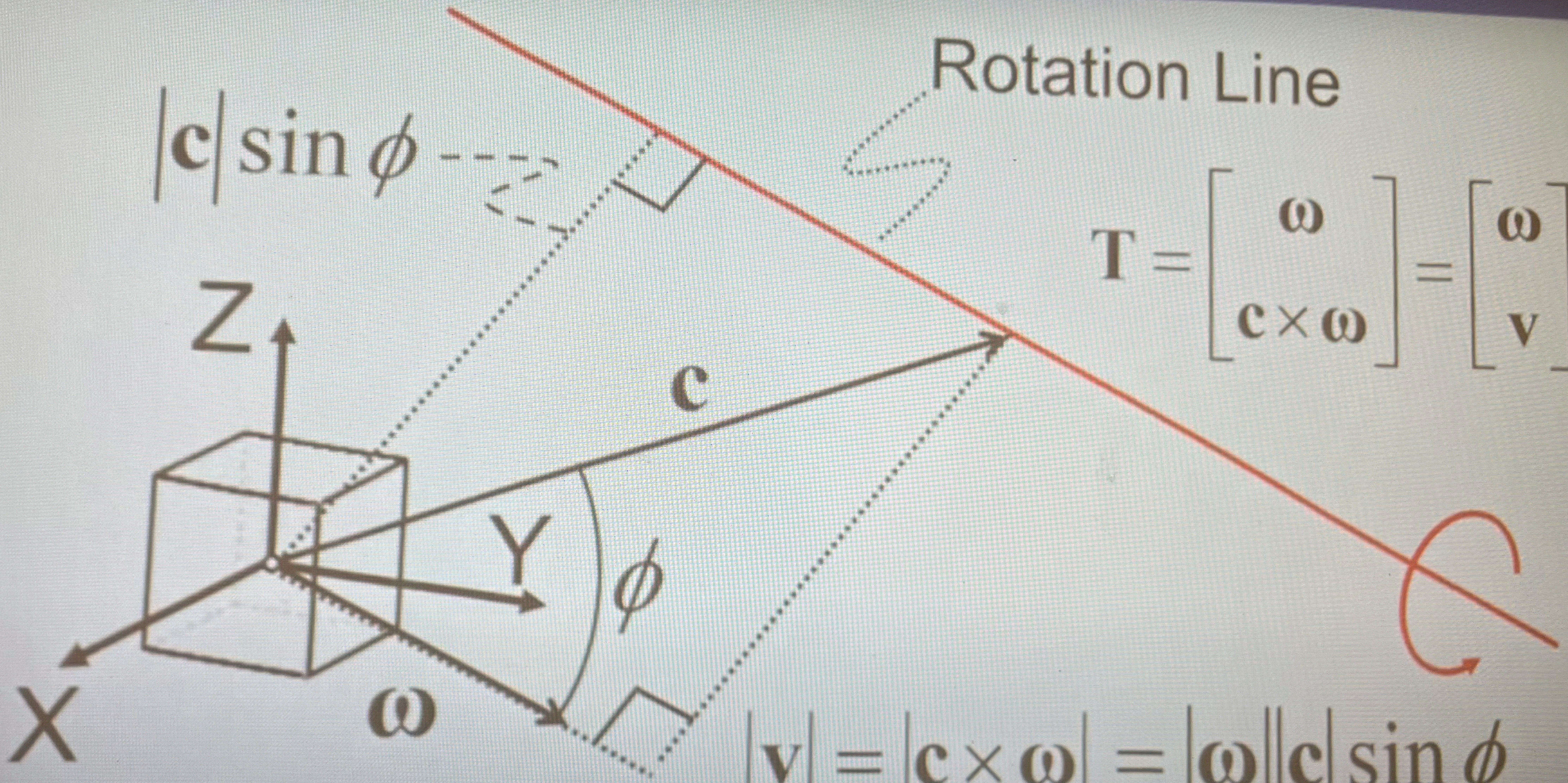
$$\mathbf{c} = \begin{bmatrix} A & 2m & 1m \end{bmatrix}^T$$

$$\mathbf{T} = \begin{bmatrix} \omega \\ \mathbf{c} \times \omega \end{bmatrix} = \begin{bmatrix} \omega \\ \mathbf{v} \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} 2rad/s & 0 & 0 & 0 & 2m/s & -4m/s \end{bmatrix}^T$$

Notice that it doesn't matter what A is.

A Twist Vector is Not Affected by Which Location Vector is Selected



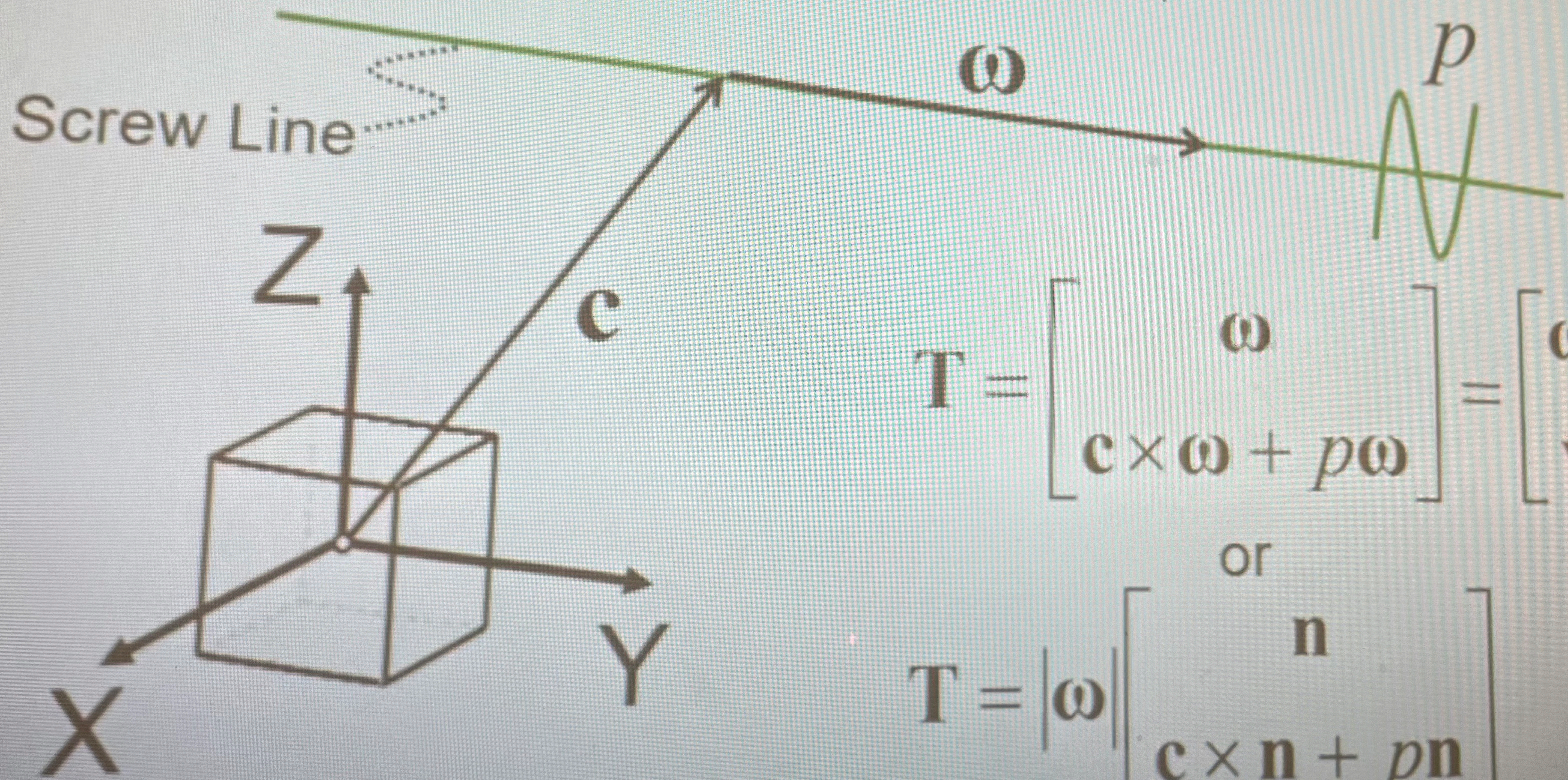
$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$$

$$|\mathbf{v}| = |\mathbf{c} \times \boldsymbol{\omega}| = |\boldsymbol{\omega}||\mathbf{c}| \sin\phi$$

Thus, no matter where $\mathbf{c}$ points along the rotation line, the magnitude of $\mathbf{v}$ will always be the magnitude of $\boldsymbol{\omega}$ multiplied by the shortest distance from the origin to the rotation line. The direction of $\mathbf{v}$ will always be perpendicular to both the rotation line and the shortest distance line that intersects the rotation line and the origin.

General Theory of Motion (Screw Theory)

According to Chasles' Theorem, all motions can be described as screw lines. A screw line describes a coupled translation and rotation motion where the ratio that defines this coupling is called the screw's pitch, p.



$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$$

or

$$\mathbf{T} = |\boldsymbol{\omega}| \begin{bmatrix} \mathbf{n} \\ \mathbf{c} \times \mathbf{n} + p\mathbf{n} \end{bmatrix}$$

Here $\mathbf{v}$ is the velocity vector of the origin

Where $\mathbf{n}$ is a unit vector that points in the direction of $\boldsymbol{\omega}$