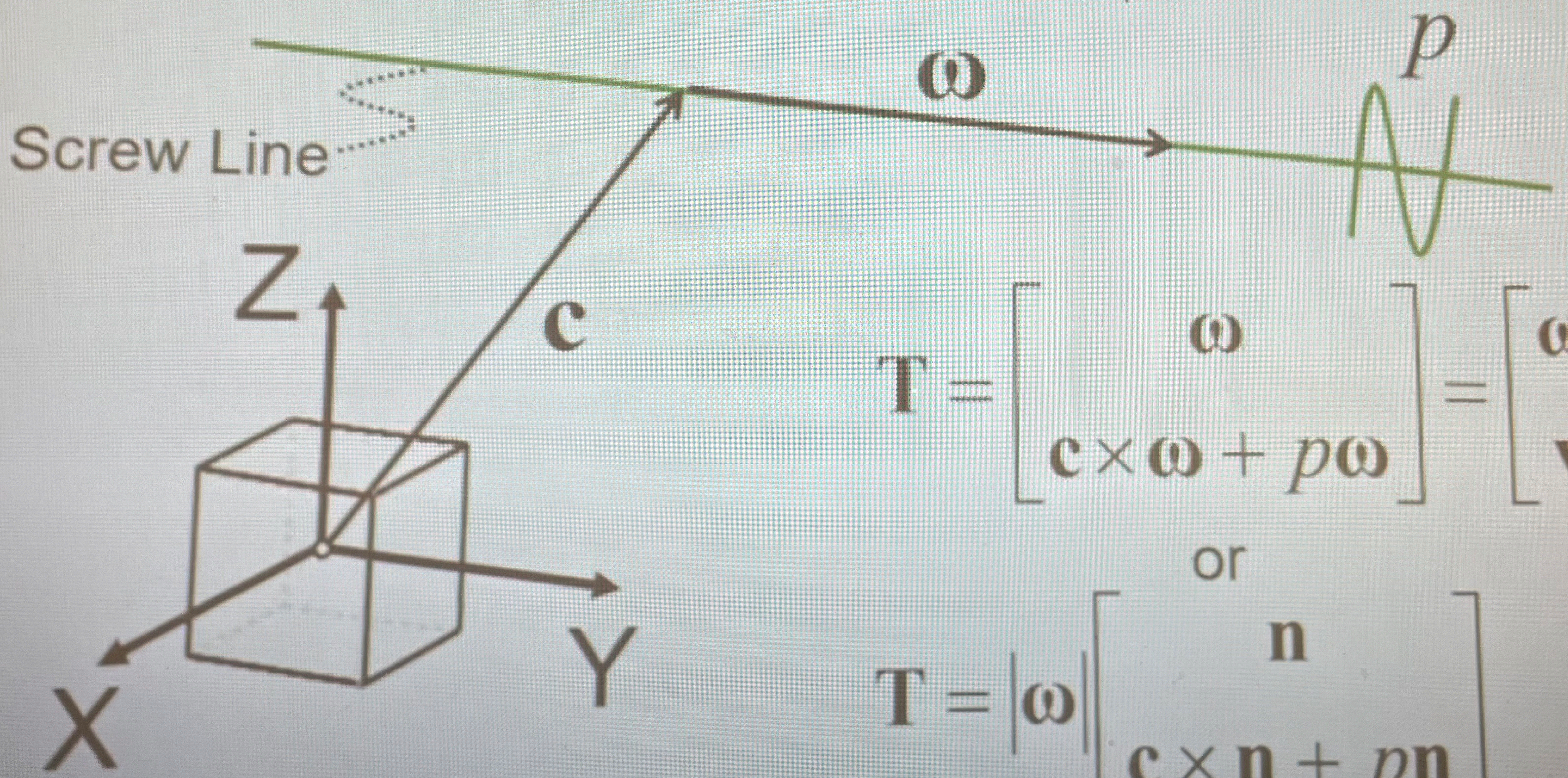
General Theory of Motion (Screw Theory)

According to Chasles' Theorem, all motions can be described as screw lines. A screw line describes a coupled translation and rotation motion where the ratio that defines this coupling is called the screw's pitch, p.



$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$$

or

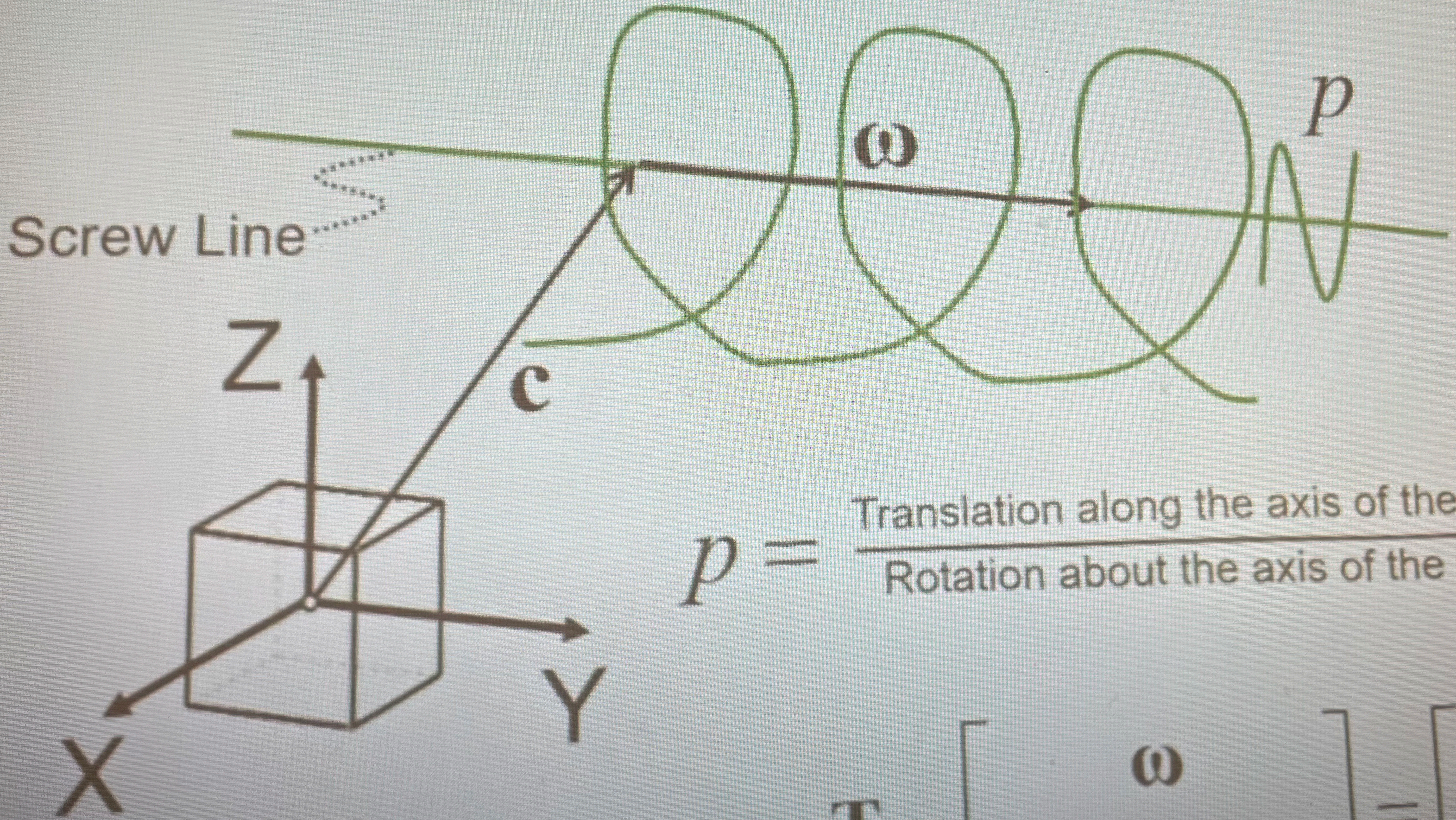
$$\mathbf{T} = |\boldsymbol{\omega}| \begin{bmatrix} \mathbf{n} \\ \mathbf{c} \times \mathbf{n} + p\mathbf{n} \end{bmatrix}$$

Here $\mathbf{v}$ is the velocity vector of the origin

Where $\mathbf{n}$ is a unit vector that poin[ts] in the direction of $\boldsymbol{\omega}$

npliant Mechanisms Lecture 2 Part 2

General Theory of Motion (Screw Theory)

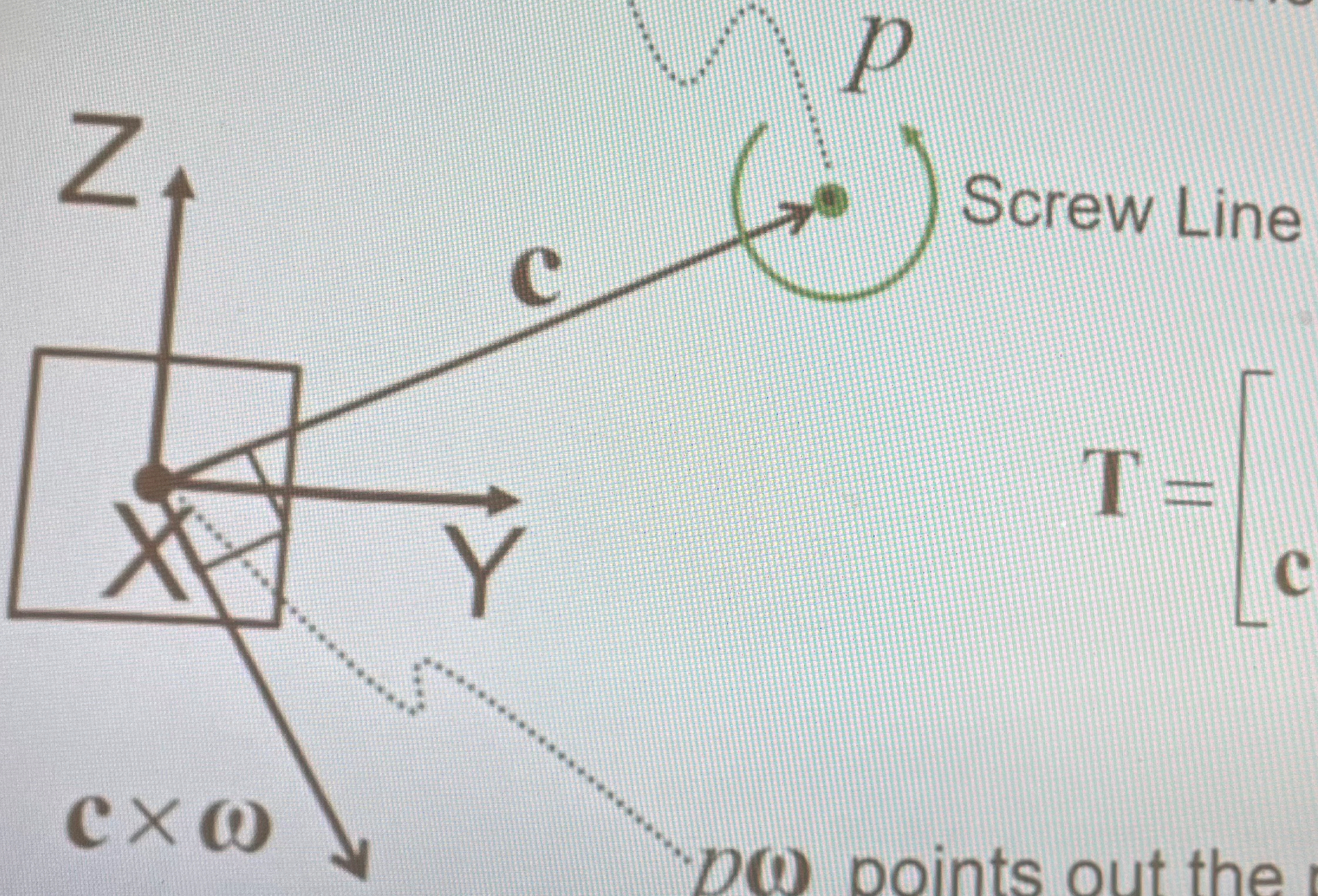


$$p = \frac{\text{Translation along the axis of the screw line}}{\text{Rotation about the axis of the screw line}}$$

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$$

A Closer Look at Screw Twists

Suppose ω is pointing out of the page along the X axis



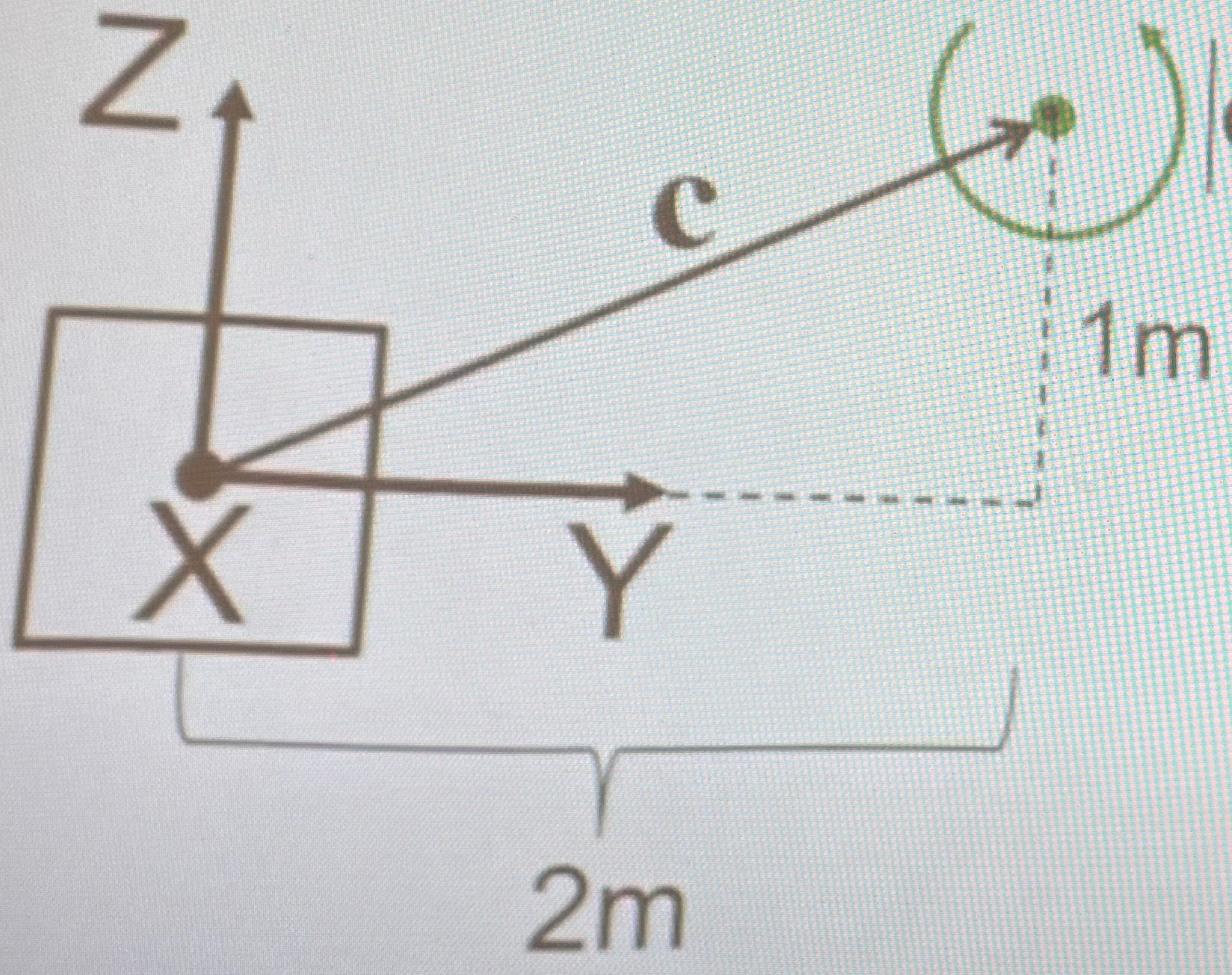
$$\mathbf{T} = \begin{bmatrix} \omega \\ \mathbf{c} \times \omega + \boxed{p\omega} \end{bmatrix} = \begin{bmatrix} \omega \\ \mathbf{v} \end{bmatrix}$$

$p\omega$ points out the page along the X axis

Example

Find the screw line's twist vector, $\mathbf{T}$

$p = 1$ m/rad



$$\omega = \begin{bmatrix} 2rad/s & 0 & 0 \end{bmatrix}^T$$

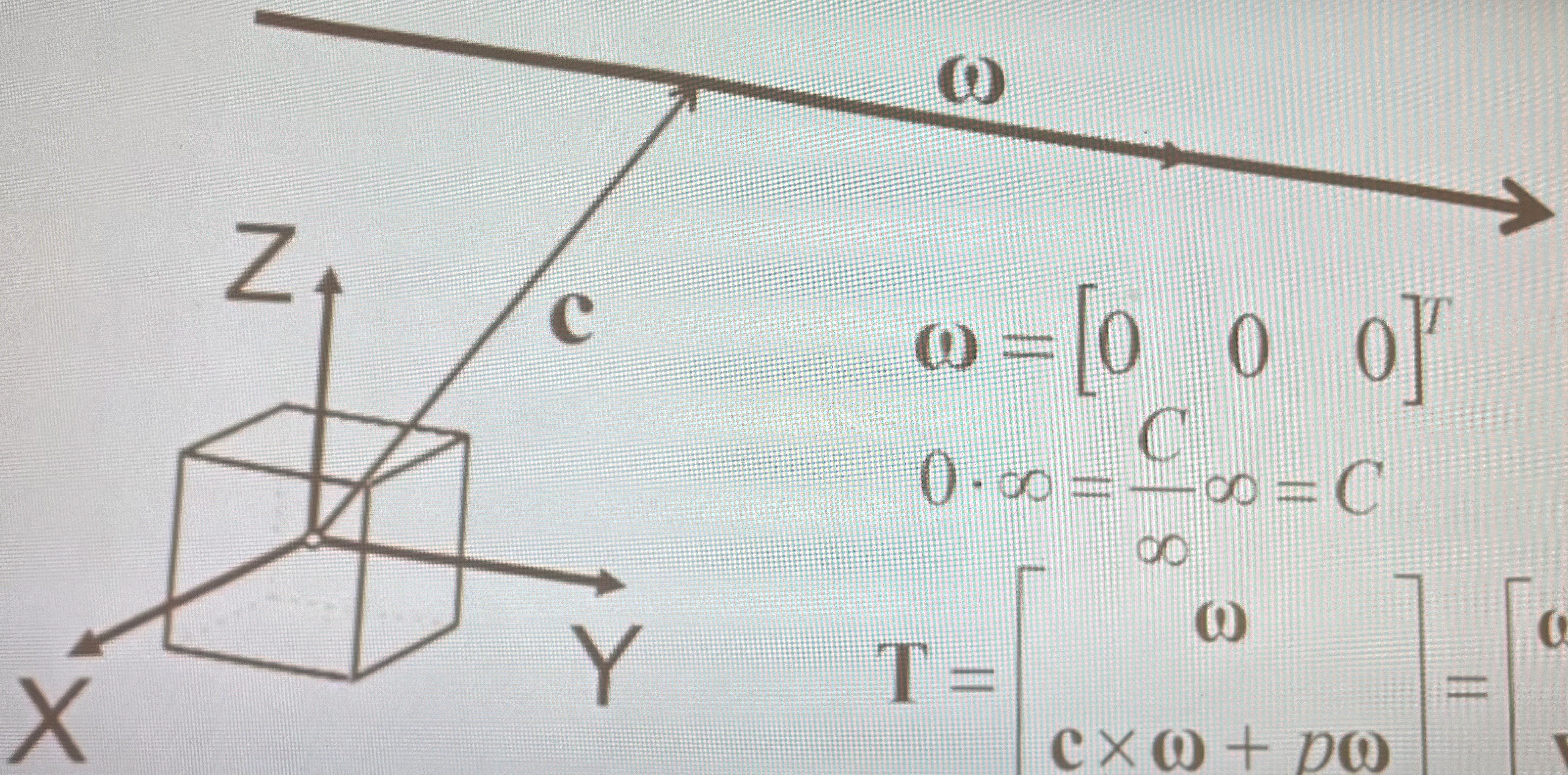
$$\mathbf{c} = \begin{bmatrix} A & 2m & 1m \end{bmatrix}^T$$

$$p = 1m/rad$$

$$\mathbf{T} = \begin{bmatrix} \omega \\ \mathbf{c} \times \omega + p\omega \end{bmatrix} = \begin{bmatrix} \omega \\ \mathbf{v} \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} 2rad/s & 0 & 0 & 2m/s & 2m/s & -4m/s \end{bmatrix}^T$$

A Screw with an Infinite Pitch is a Translation



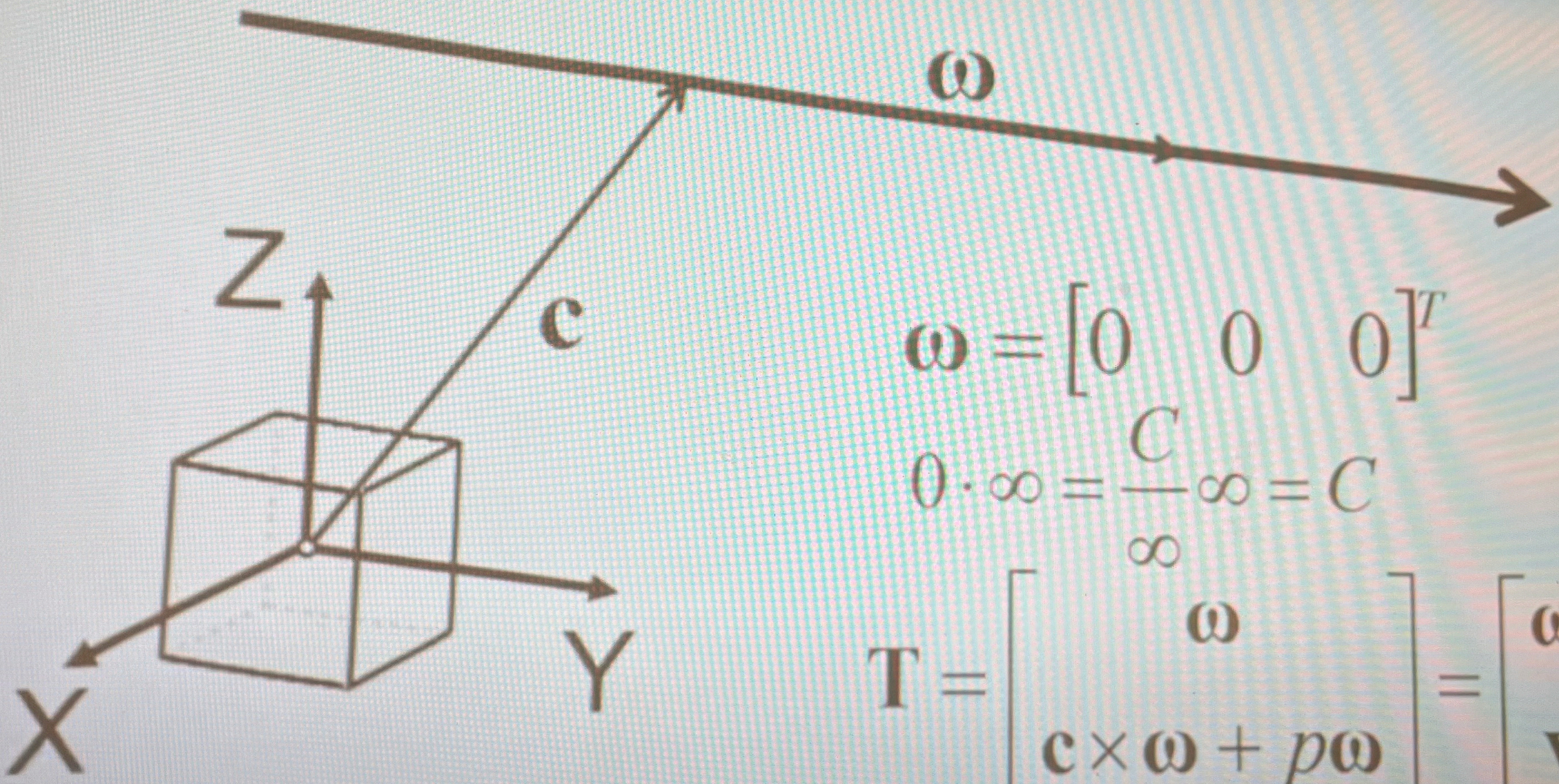
$$\boldsymbol{\omega} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$0 \cdot \infty = \frac{C}{\infty}\infty = C$$

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{v} \end{bmatrix}$$

$$\frac{C}{0} = p = \frac{\text{Translation along the axis of the screw line}}{\text{Rotation about the axis of the screw line}} = \infty$$

A Screw With an Infinite Pitch is a Translation



$$\boldsymbol{\omega} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$$

$$0 \cdot \infty = \frac{C}{\infty}\infty = C$$

$$\mathbf{T} = \begin{bmatrix}\boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega}\end{bmatrix} = \begin{bmatrix}\boldsymbol{\omega} \\ \mathbf{v}\end{bmatrix} = \begin{bmatrix}\mathbf{0} \\ \mathbf{v}\end{bmatrix}$$

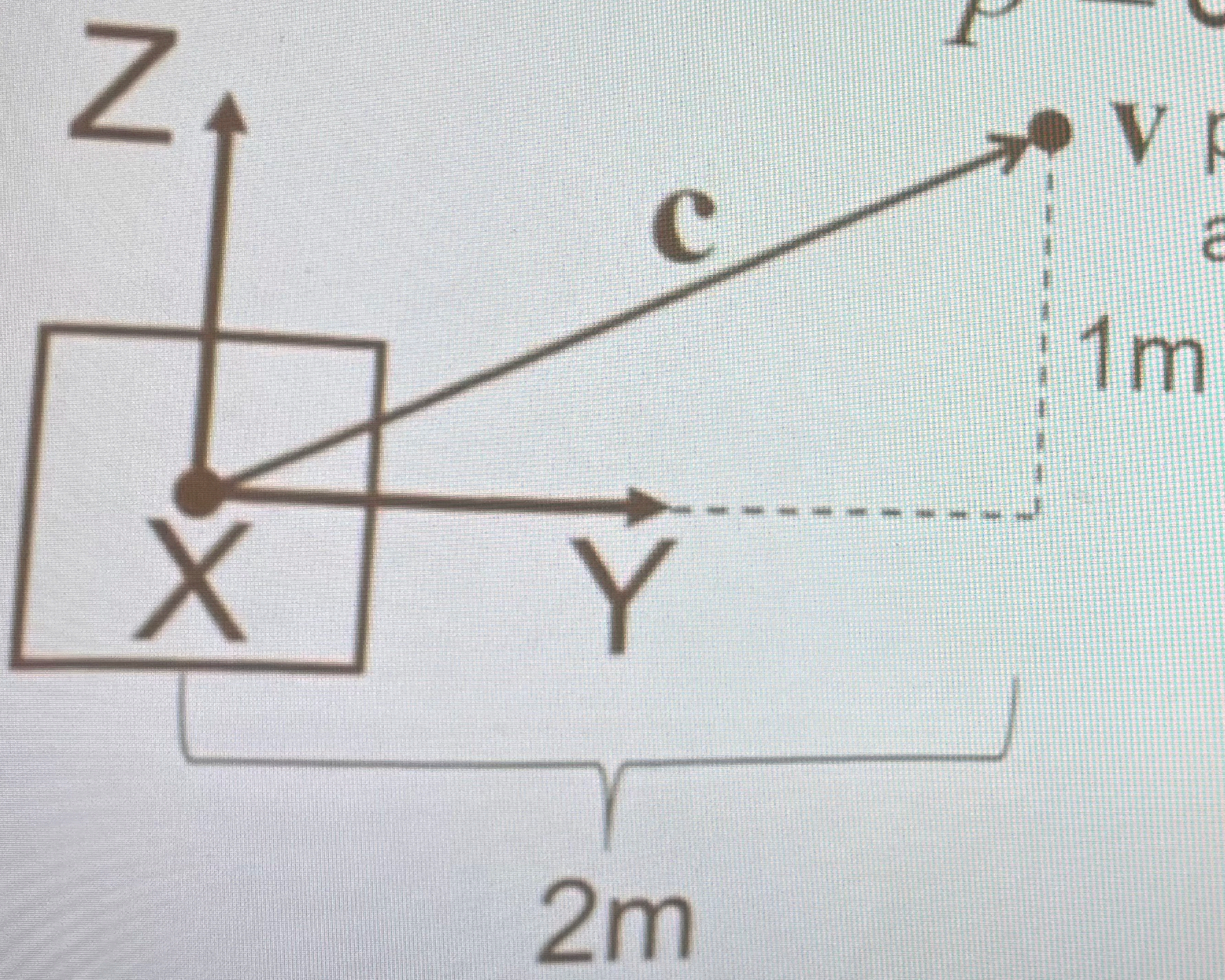
Note that when $\boldsymbol{\omega}$ is a zero vector, $\mathbf{c}$ is irrelevant.

$$\frac{C}{0} = p = \frac{\text{Translation along the axis of the screw line}}{\text{Rotation about the axis of the screw line}} = \infty$$

Example

Find the translation's twist vector, $\mathbf{T}$

$p = \infty$ m/rad



$\mathbf{v}$ points out of the page along the X axis and $|\mathbf{v}| = 2$ m/s

$$\boldsymbol{\omega} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{c} = \begin{bmatrix} A & 2m & 1m \end{bmatrix}^T$$

$$p = \infty \ m/rad$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{0} \\ \mathbf{v} \end{bmatrix}$$

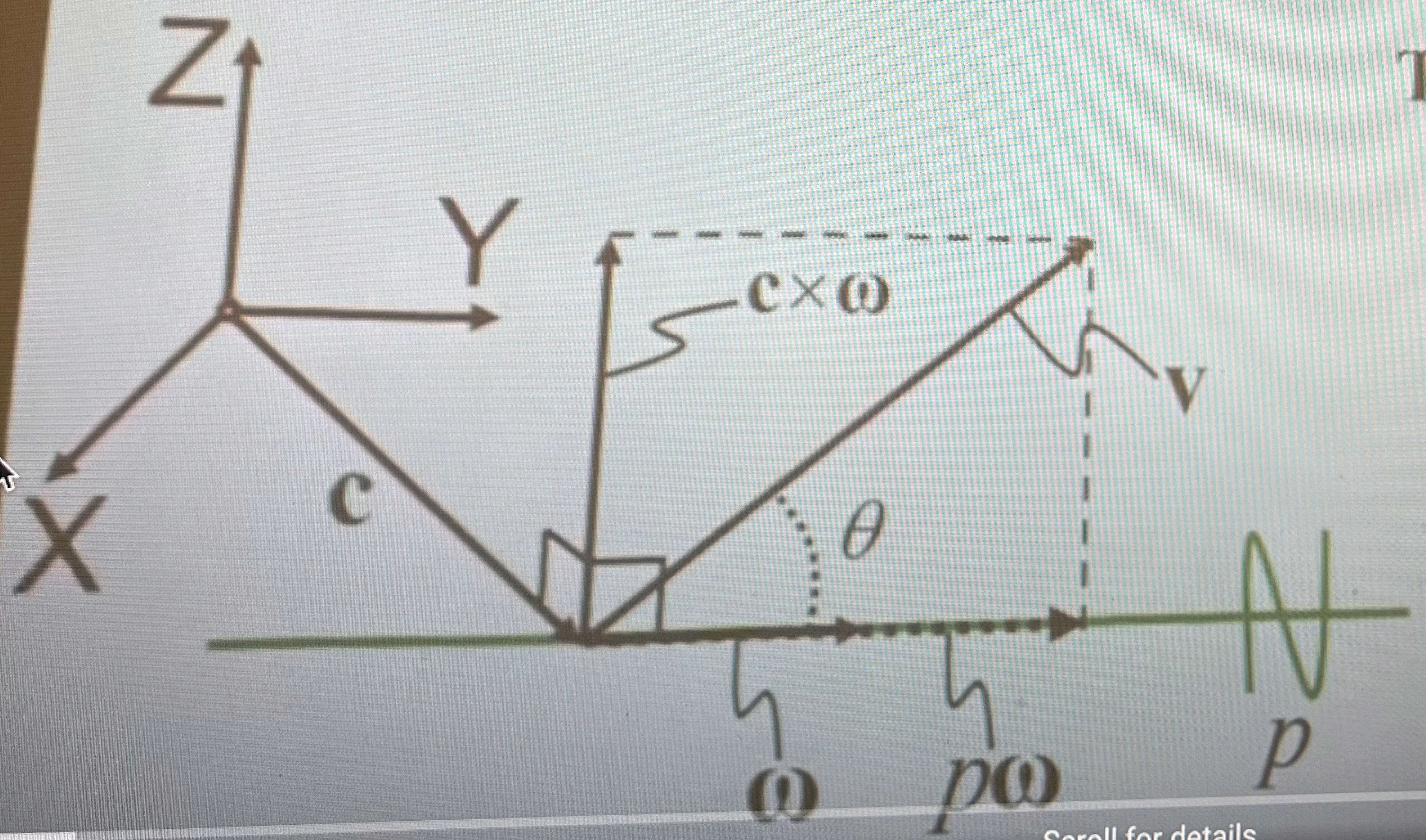
$$\mathbf{T} = \begin{bmatrix} 0 & 0 & 0 & 2m/s & 0 & 0 \end{bmatrix}^T$$

Breaking a Twist into its Parts

We could now find the twist vector that results from any $\boldsymbol{\omega}$, $\mathbf{c}$, and p.

But how could we find the $\boldsymbol{\omega}$, p, and a $\mathbf{c}$ vector for a given twist vector?

Finding $\boldsymbol{\omega}$ is obvious but what about finding p.



$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{c} \times \boldsymbol{\omega} + p\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$$

$$|\mathbf{v}|\cos\theta = p|\boldsymbol{\omega}|$$

$$\boldsymbol{\omega} \bullet \mathbf{v} = |\boldsymbol{\omega}||\mathbf{v}|\cos\theta$$

$$\boldsymbol{\omega} \bullet \boldsymbol{\omega} = |\boldsymbol{\omega}||\boldsymbol{\omega}|$$

$$p = \left(\frac{\boldsymbol{\omega} \bullet \mathbf{v}}{\boldsymbol{\omega} \bullet \boldsymbol{\omega}}\right)$$

Example

What is the $\boldsymbol{\omega}$ vector and the pitch p of $\mathbf{T} = [1 \quad 2 \quad 0 \quad 0 \quad 1 \quad 3]^T$?

$$\boldsymbol{\omega} = [1 \quad 2 \quad 0]^T$$

$$\mathbf{v} = [0 \quad 1 \quad 3]^T$$

$$p = \left(\frac{\boldsymbol{\omega} \bullet \mathbf{v}}{\boldsymbol{\omega} \bullet \boldsymbol{\omega}}\right) = \left(\frac{2}{5}\right)$$

Finding the Location Vector, c, of a Twist

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\omega} \\ \boxed{\mathbf{c}\times\boldsymbol{\omega} + p\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \boxed{\mathbf{v}} \end{bmatrix}$$

$$\mathbf{c}\times\boldsymbol{\omega} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ c_x & c_y & c_z \\ \omega_x & \omega_y & \omega_z \end{vmatrix} = (c_y\omega_z - c_z\omega_y)\vec{i} - (c_x\omega_z - c_z\omega_x)\vec{j} + (c_x\omega_y - c_y\omega_x)\vec{k}$$

$$v_x = c_y\omega_z - c_z\omega_y + p\omega_x$$

$$v_y = c_z\omega_x - c_x\omega_z + p\omega_y$$

$$v_z = c_x\omega_y - c_y\omega_x + p\omega_z$$

$$\begin{bmatrix} p & -c_z & c_y \\ c_z & p & -c_x \\ -c_y & c_x & p \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

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What is the $\mathbf{c}$ vector of $\mathbf{T}=\begin{bmatrix}1 & 2 & 0 & 0 & 1 & 3\end{bmatrix}^T$?

$$\boldsymbol{\omega}=\begin{bmatrix}1 & 2 & 0\end{bmatrix}^T$$

$$\mathbf{v}=\begin{bmatrix}0 & 1 & 3\end{bmatrix}^T$$

$$p=\left(\frac{\boldsymbol{\omega}\bullet\mathbf{v}}{\boldsymbol{\omega}\bullet\boldsymbol{\omega}}\right)=\left(\frac{2}{5}\right)$$

$$\begin{bmatrix}p & -c_z & c_y \\ c_z & p & -c_x \\ -c_y & c_x & p\end{bmatrix}\begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix}=\begin{bmatrix}v_x \\ v_y \\ v_z\end{bmatrix}$$

$$\begin{bmatrix}2/5 & -c_z & c_y \\ c_z & 2/5 & -c_x \\ -c_y & c_x & 2/5\end{bmatrix}\begin{bmatrix}1 \\ 2 \\ 0\end{bmatrix}=\begin{bmatrix}0 \\ 1 \\ 3\end{bmatrix}$$

A viable $\mathbf{c}$ vector is:

$$\mathbf{c}=\begin{bmatrix}0 & -3 & 1/5\end{bmatrix}^T$$

$$c_z=1/5$$

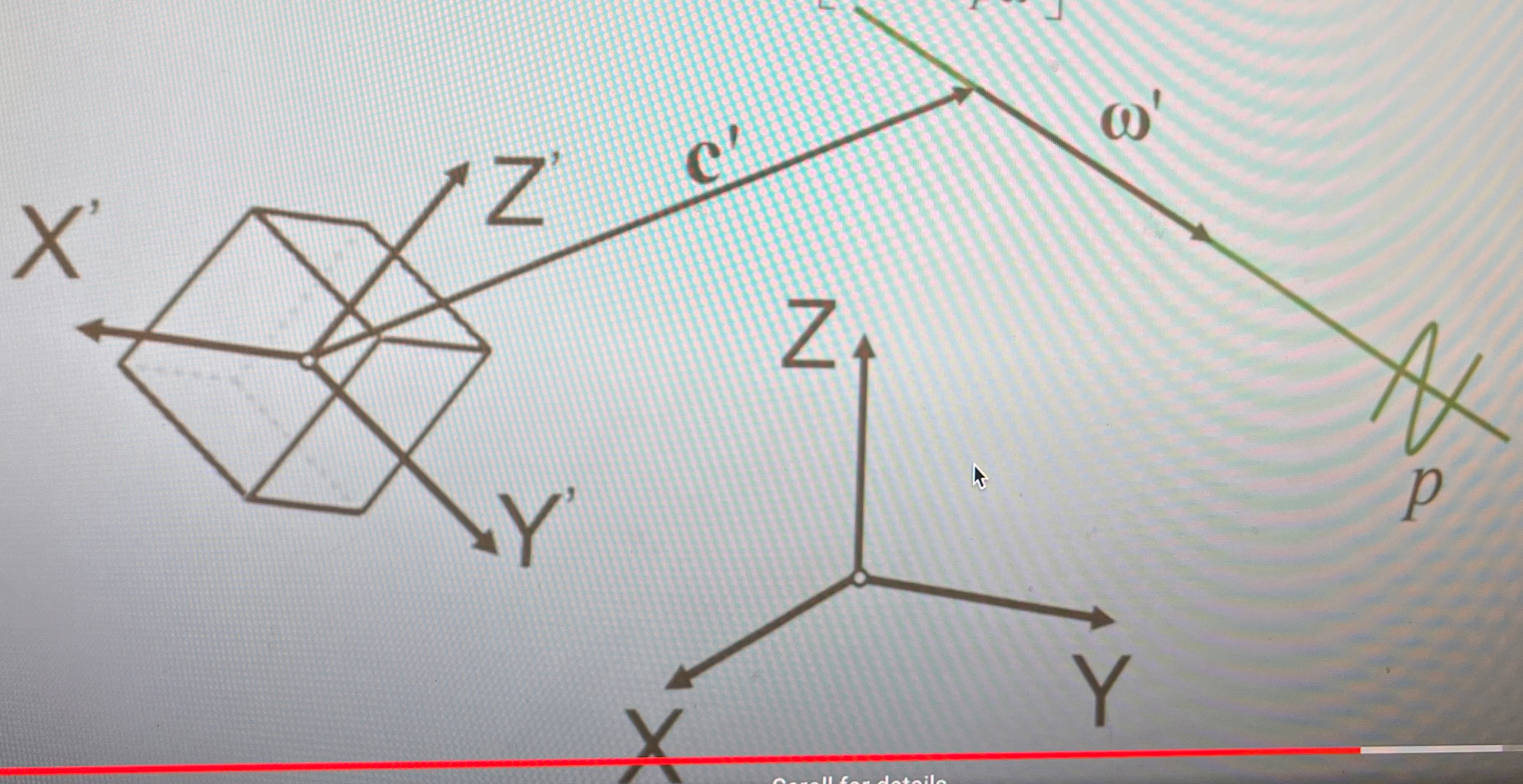
$$-c_y+2c_x=3$$

Compliant Mechanisms Lecture 2 Part 3

Finding Rotational and Translational Velocities at Any Point

$$\mathbf{T} = \omega_{x'}\begin{bmatrix}\mathbf{n}_1 \\ \mathbf{L}\times\mathbf{n}_1\end{bmatrix} + \omega_{y'}\begin{bmatrix}\mathbf{n}_2 \\ \mathbf{L}\times\mathbf{n}_2\end{bmatrix} + \omega_{z'}\begin{bmatrix}\mathbf{n}_3 \\ \mathbf{L}\times\mathbf{n}_3\end{bmatrix} + v_{x'}\begin{bmatrix}0 \\ \mathbf{n}_1\end{bmatrix} + v_{y'}\begin{bmatrix}0 \\ \mathbf{n}_2\end{bmatrix} + v_{z'}\begin{bmatrix}0 \\ \mathbf{n}_3\end{bmatrix} = \begin{bmatrix}\boldsymbol{\omega} \\ \mathbf{c}\times\boldsymbol{\omega} + p\boldsymbol{\omega}\end{bmatrix}$$

$$\mathbf{T}' = \begin{bmatrix}\omega_{x'} & \omega_{y'} & \omega_{z'} & v_{x'} & v_{y'} & v_{z'}\end{bmatrix}^T = \begin{bmatrix}\boldsymbol{\omega}' \\ \mathbf{c}'\times\boldsymbol{\omega}' + p\boldsymbol{\omega}'\end{bmatrix}$$



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