Relationship Between a General Twist and Constraint Line

$\mathbf{W} \bullet [\Delta]\mathbf{T} = 0$

$[\mathbf{f} \quad \boldsymbol{\tau}]\begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix} = [\mathbf{f} \quad (\mathbf{r}\times\mathbf{f}) + q\mathbf{f}]\begin{bmatrix} (\mathbf{c}\times\boldsymbol{\omega}) + p\boldsymbol{\omega} \\ \boldsymbol{\omega} \end{bmatrix} = 0$

In other words, twists that produce no work (or power) with the wrench of the constraint are motions that are permitted by the constraint according to the above equation.

The above equation simplifies to

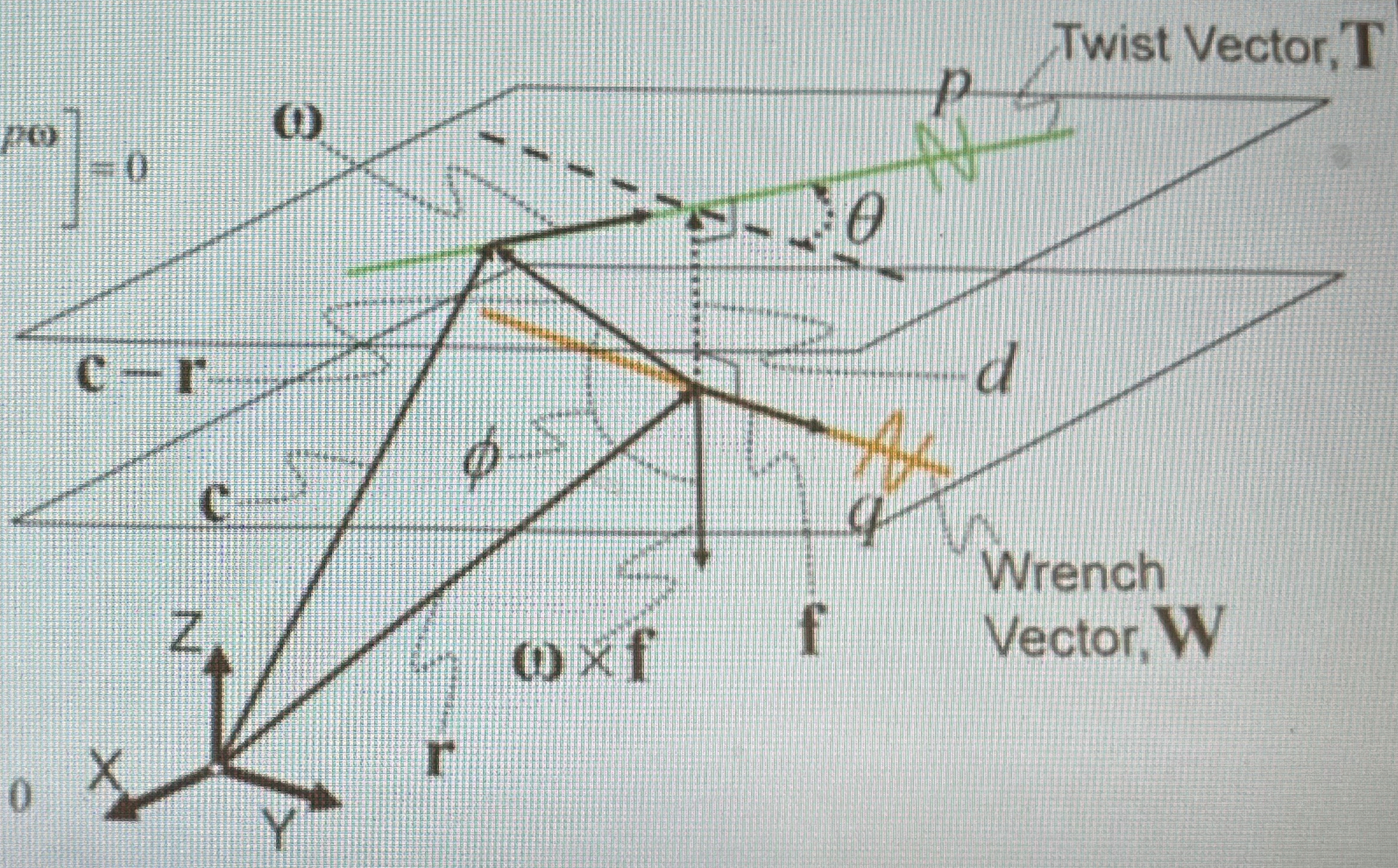
$(\mathbf{c}-\mathbf{r})\bullet(\boldsymbol{\omega}\times\mathbf{f}) + (p+q)(\mathbf{f}\bullet\boldsymbol{\omega}) = 0$

$|\mathbf{c}-\mathbf{r}||\boldsymbol{\omega}\times\mathbf{f}|\cos\phi + (p+q)|\mathbf{f}||\boldsymbol{\omega}|\cos\theta = 0$

Note that $|\mathbf{c}-\mathbf{r}|\cos\phi = -d$

Thus $-d|\boldsymbol{\omega}||\mathbf{f}|\sin\theta + (p+q)|\mathbf{f}||\boldsymbol{\omega}|\cos\theta = 0$

Therefore the main equation simplifies to $\boxed{(p+q) = d\tan\theta}$



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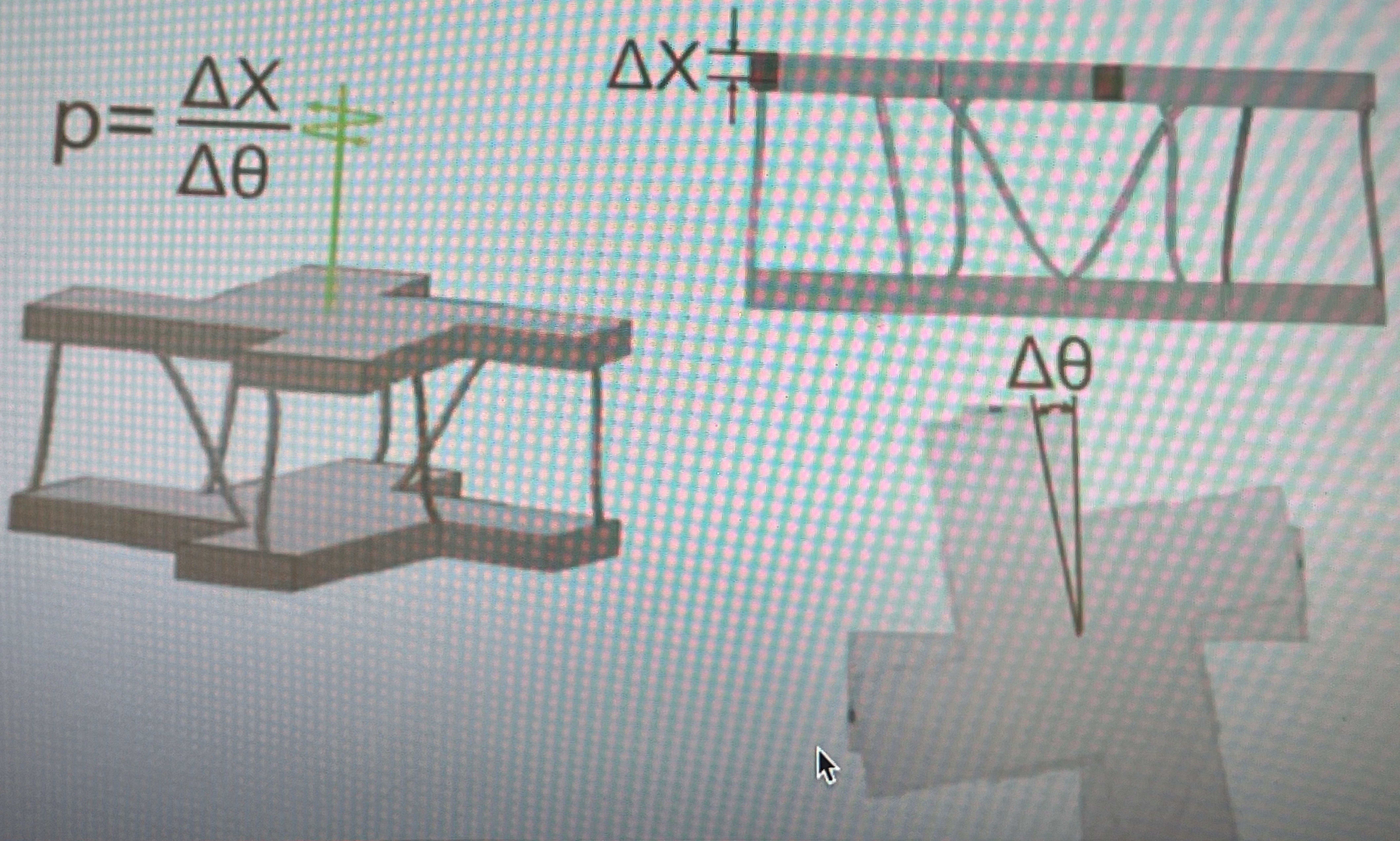
Screw Flexure

$$p = \frac{\Delta X}{\Delta \theta}$$



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p
θ2
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d2
d2
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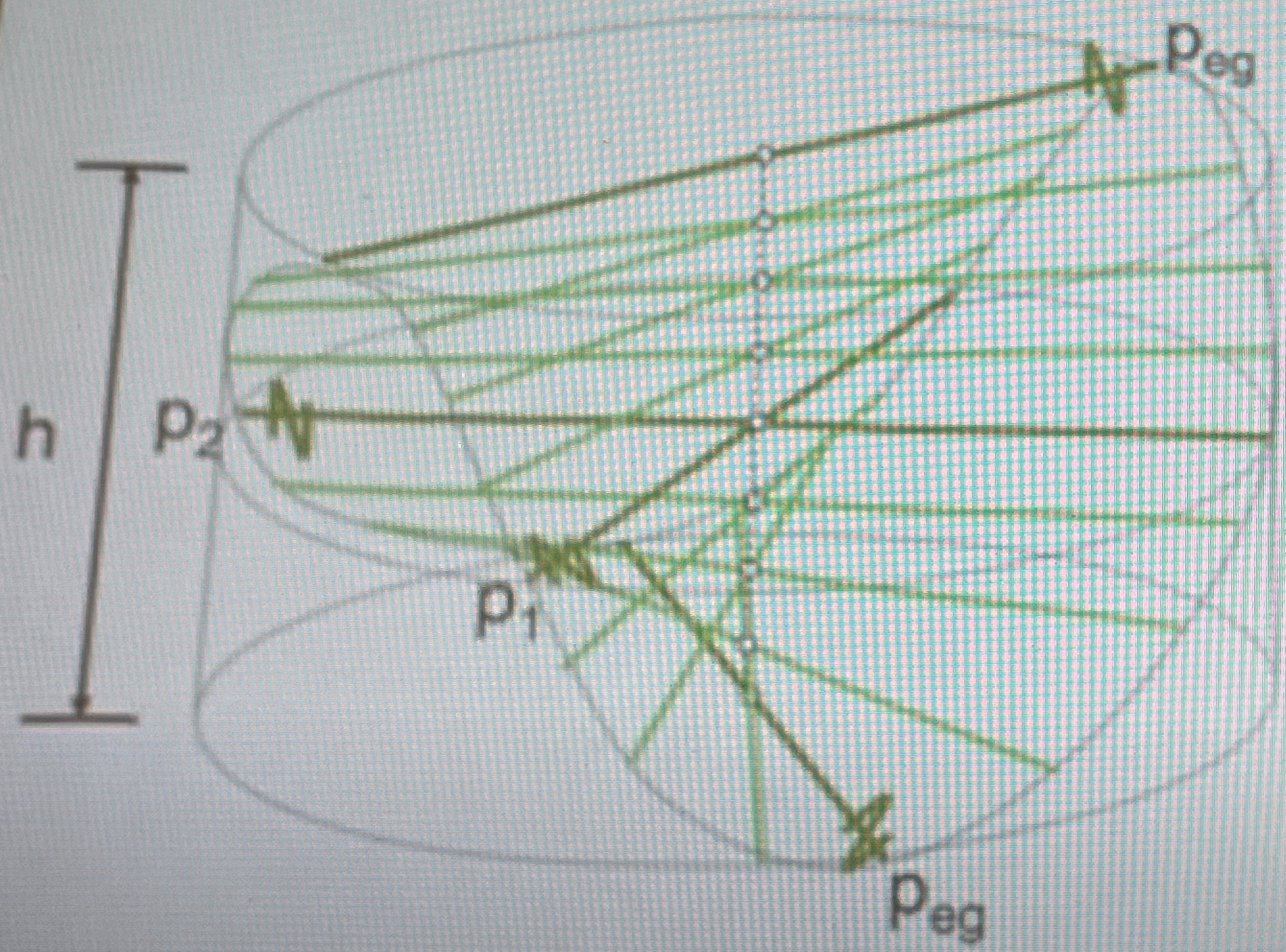
Another Look at Cylindroids



absolute value

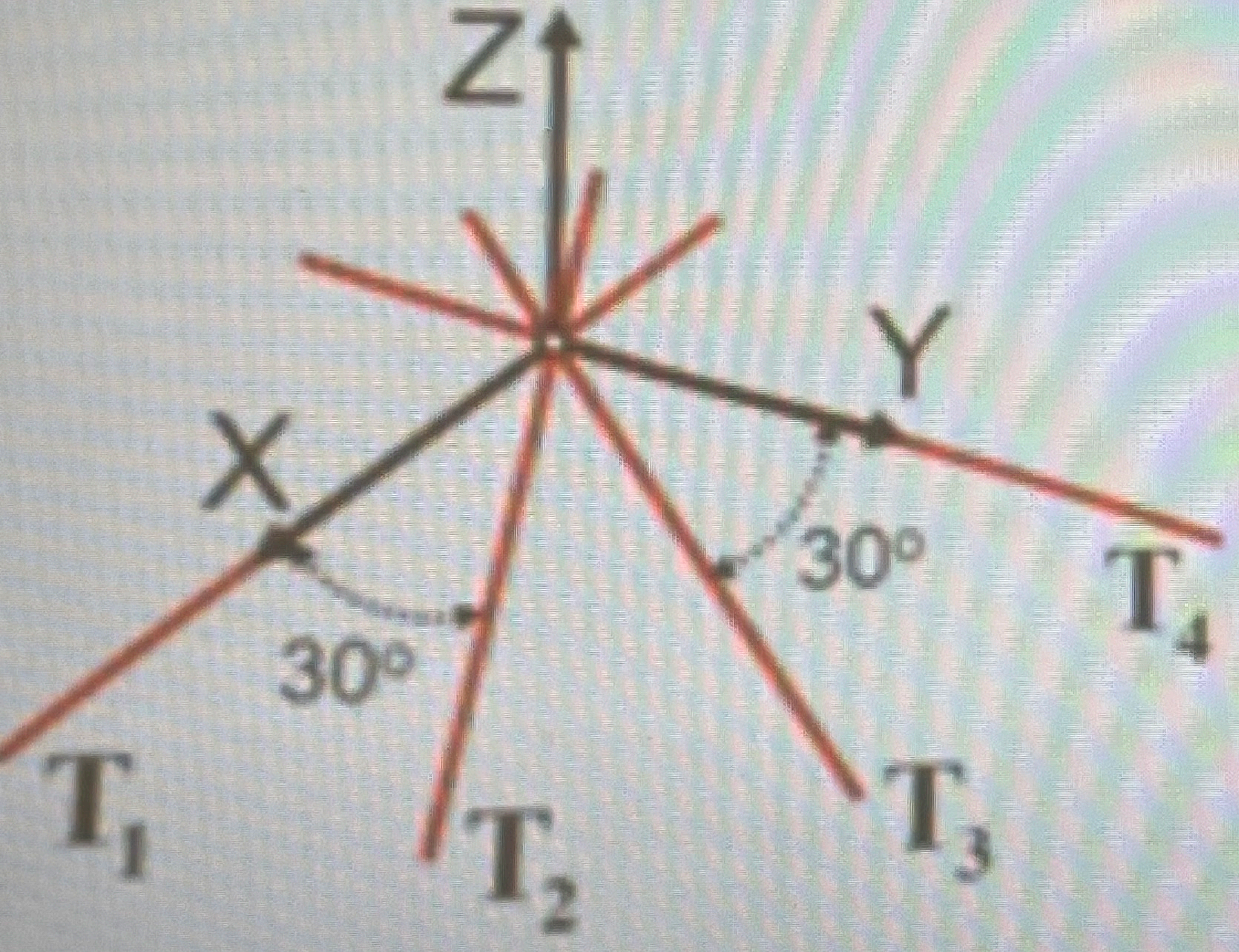
$$h = abs(p_2 - p_1)$$

$$p_{eg} = \frac{p_2 + p_1}{2}$$

Note: If $p_1 = p_2$, the cylindroid becomes a disk

Link Motions to Freedom Spaces

1) Define the motions as twists and linearly combine them. Then interpret the equations to identify the resulting freedom space.



$$T_1 = \omega_1 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$T_2 = \omega_2 \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$T_3 = \omega_3 \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$T_4 = \omega_4 \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$T_{FS} = T_1 + T_2 + T_3 + T_4$$


Compliant Mechanisms Lecture 6 Part 8
Rotation & Translation Combinations
(1)
d ≠ 0
=
(2)
=
θ ≠ 0° or 180°
(3)
θ ≠ 0° or 180°
=
d ≠ 0
(4)
=
θ ≠ 0° or 180°
(5)
=
(6)
θ = 90°
=
(7)
=
θ ≠ 0° or 90°
2:32 / 27:25
Scroll for details
AMD
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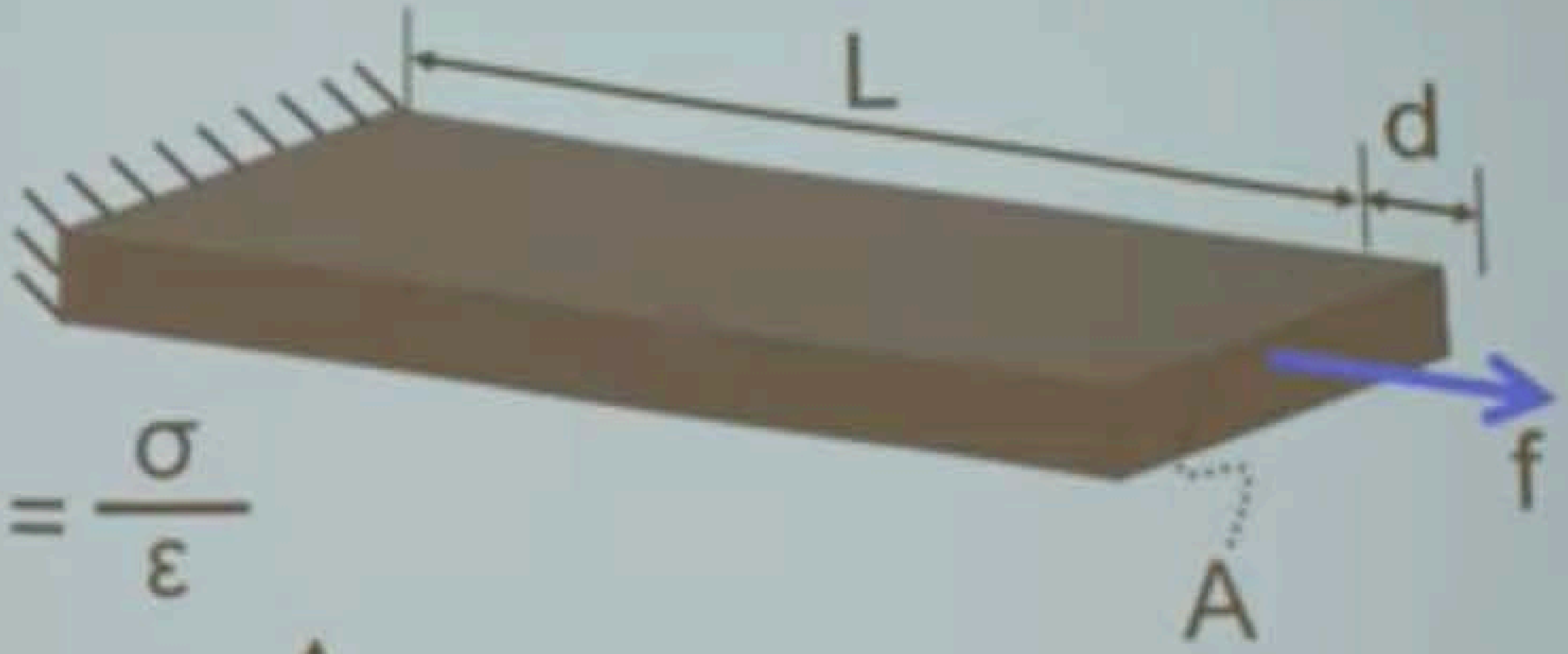
	Normalized Values			
Material	**$\sigma y/E$**	**$\alpha_{diff}/\alpha_{CTE}$**	**E/ρ**	**Cost**
Titanium V	1.00	0.14	0.92	3.77
Aluminum 7075	0.70	1.00	1.00	1.00
Stainless 316	0.09	0.13	0.94	3.50
Invar - Annealed	0.19	0.87	0.70	5.21

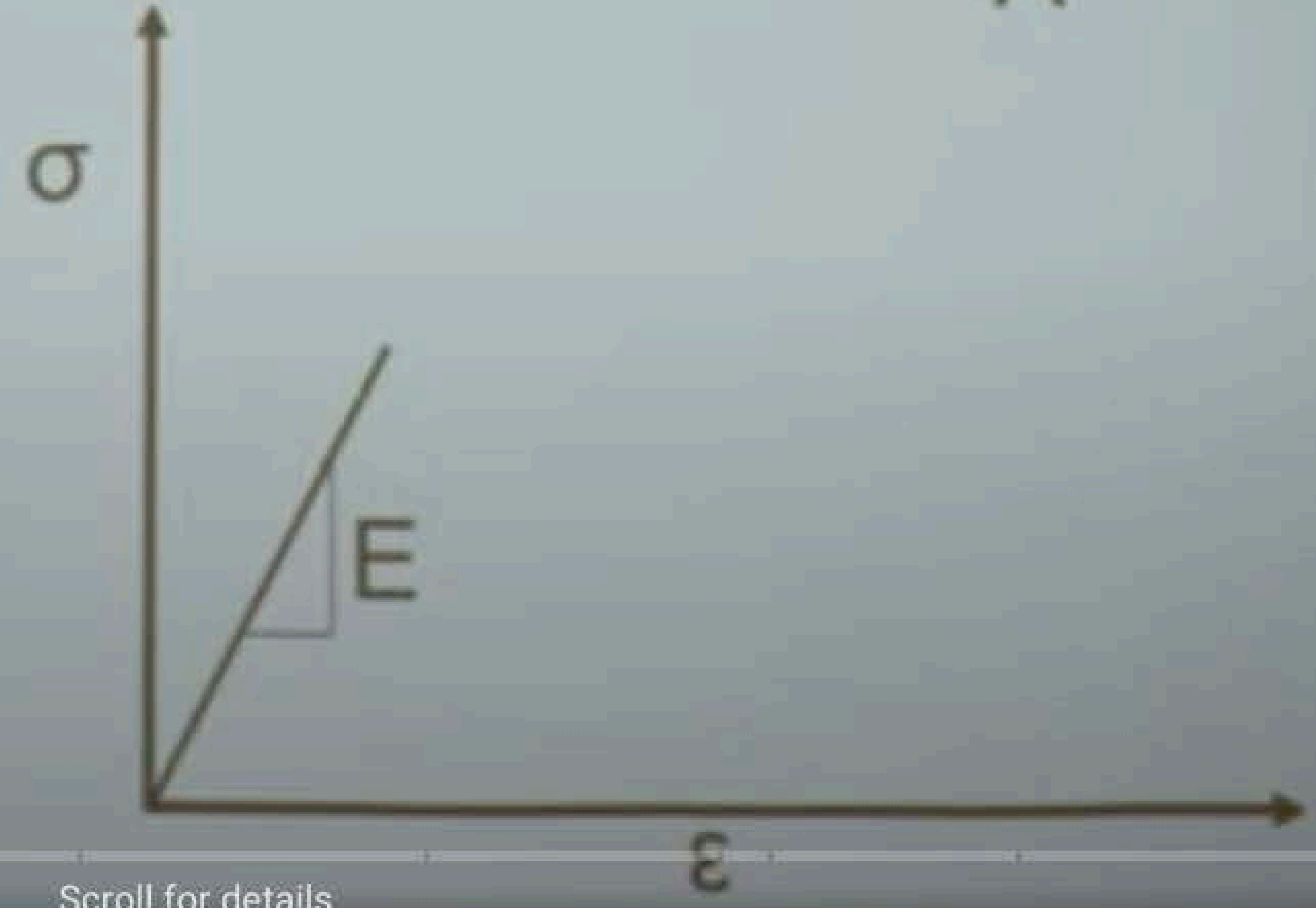
Stress and Strain

Stress: $\sigma = \dfrac{f}{A}$

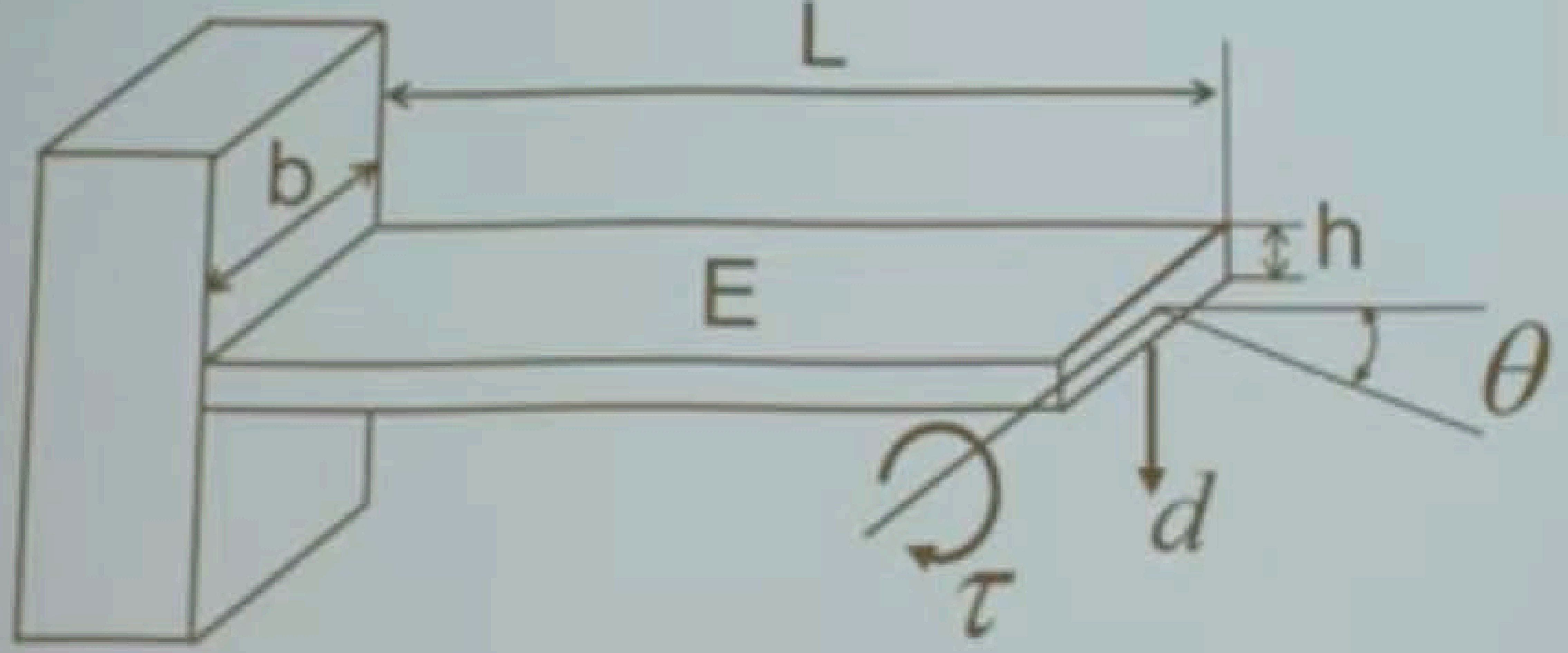
Strain: $\varepsilon = \dfrac{d}{L}$

Young's Modulus: $E = \dfrac{\sigma}{\varepsilon}$





Bending Moment Displacement



$$\tau = \boxed{\frac{2EI}{L^2}}\, d, \quad \text{where } I = \frac{bh^3}{12}$$

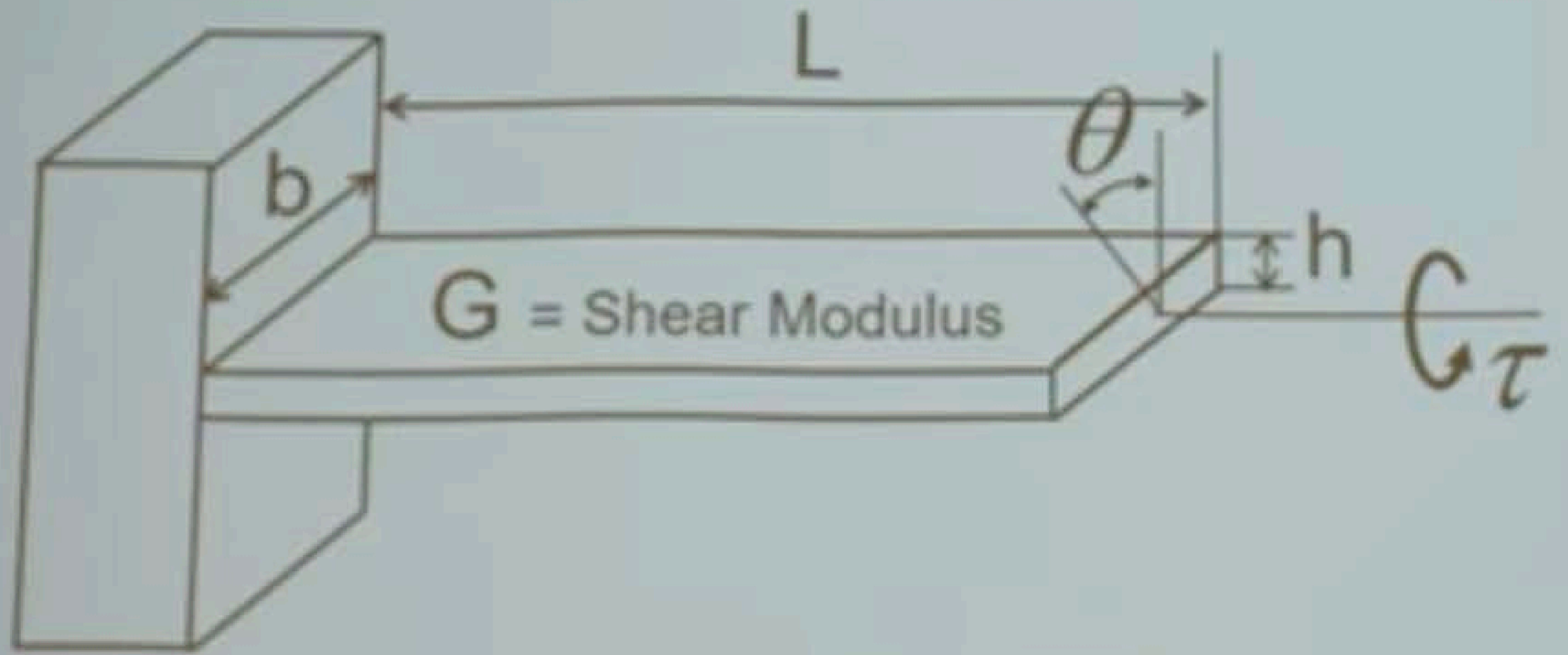
$$\tau = \boxed{\frac{EI}{L}}\, \theta$$

Bending Moment to Lateral Displacement Stiffness

Bending Moment of Ine

Bending Moment to Bending Angle Stiffness

Torsional Moment Displacement



As long as b>h:

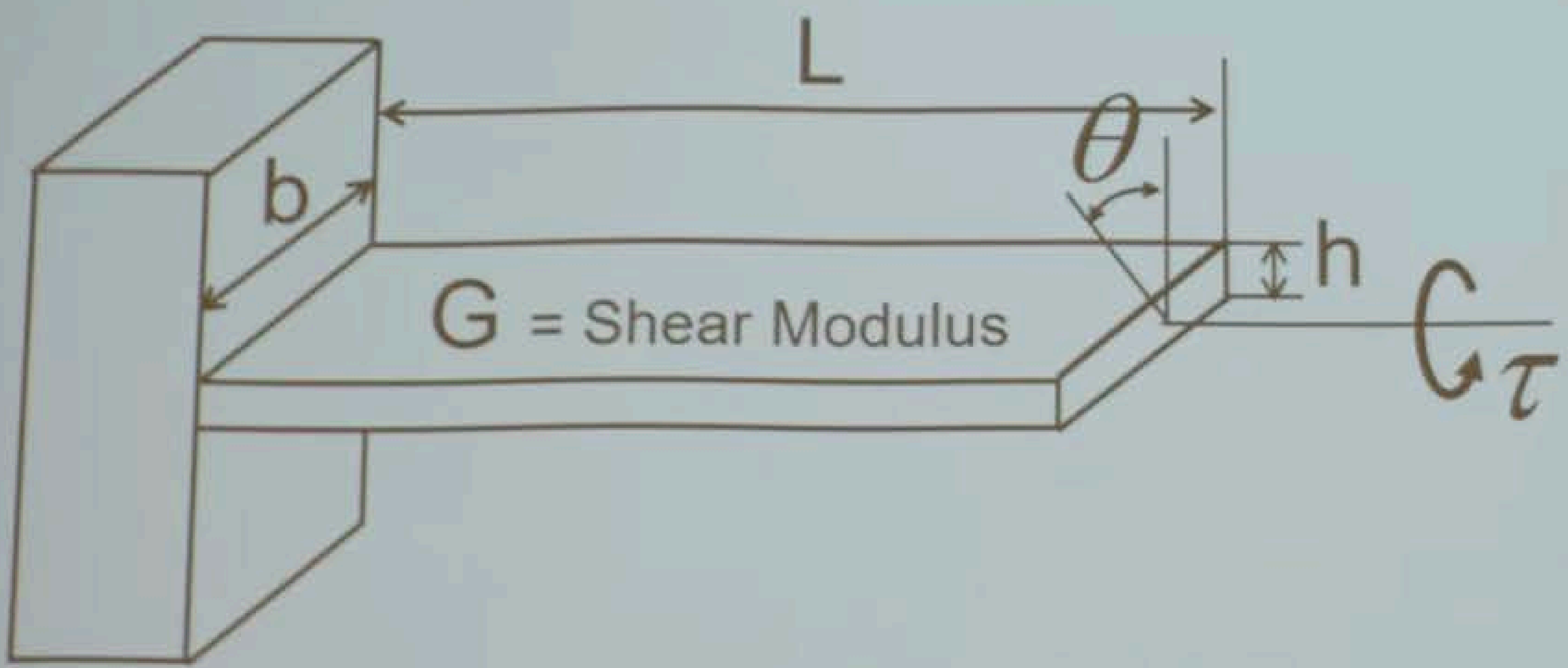
$$\tau = \boxed{\frac{GJ}{L}}\,\theta \qquad J = \frac{h^3 b}{3}\left(1 - \frac{192}{\pi^5}\left(\frac{h}{b}\right)\sum_{n=1,3,5}^{n=\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi b}{2h}\right)\right)$$

Torsional Stiffness

Polar Moment of Inertia

If b<h, swap b and h in the equation above.

Torsional Moment Displacement



As long as b>h:

$$\tau = \boxed{\frac{GJ}{L}}\,\theta \qquad J = \frac{h^3 b}{3}\left(1 - \frac{192}{\pi^5}\left(\frac{h}{b}\right)\sum_{n=1,3,5}^{n=\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi b}{2h}\right)\right)$$

Torsional Stiffness

Polar Moment of Inertia

If b<h, swap b and h in the equation above.

Bernoulli-Euler Beam Stiffness Matrix

$$\begin{bmatrix}\delta\theta_x\\ \delta\theta_y\\ \delta\theta_z\\ \delta d_x\\ \delta d_y\\ \delta d_z\end{bmatrix}=\begin{bmatrix}\frac{l}{EI_x} & 0 & 0 & 0 & -\frac{l^2}{2EI_x} & 0\\ 0 & \frac{l}{EI_y} & 0 & \frac{l^2}{2EI_y} & 0 & 0\\ 0 & 0 & \frac{l}{GJ} & 0 & 0 & 0\\ 0 & \frac{l^2}{2EI_y} & 0 & \frac{l^3}{3EI_y} & 0 & 0\\ -\frac{l^2}{2EI_x} & 0 & 0 & 0 & \frac{l^3}{3EI_x} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{l}{EA}\end{bmatrix}\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\\ f_x\\ f_y\\ f_z\end{bmatrix}$$

where $I_x=\dfrac{bh^3}{12}$, $I_y=\dfrac{hb^3}{12}$, $A=bh$, and

$$J=\frac{h^3b}{3}\left(1-\frac{192}{\pi^5}\left(\frac{h}{b}\right)\sum_{n=1,3,5}^{n=\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi b}{2h}\right)\right) \quad \text{for} \quad b\geq h$$

Bernoulli-Euler Beam Stiffness Matrix

$[C]$ = compliance matrix

$$\begin{bmatrix}\delta\theta_x\\ \delta\theta_y\\ \delta\theta_z\\ \delta d_x\\ \delta d_y\\ \delta d_z\end{bmatrix}=\underbrace{\begin{bmatrix}\frac{l}{EI_x} & 0 & 0 & 0 & -\frac{l^2}{2EI_x} & 0\\ 0 & \frac{l}{EI_y} & 0 & \frac{l^2}{2EI_y} & 0 & 0\\ 0 & 0 & \frac{l}{GJ} & 0 & 0 & 0\\ 0 & \frac{l^2}{2EI_y} & 0 & \frac{l^3}{3EI_y} & 0 & 0\\ -\frac{l^2}{2EI_x} & 0 & 0 & 0 & \frac{l^3}{3EI_x} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{l}{EA}\end{bmatrix}}\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\\ f_x\\ f_y\\ f_z\end{bmatrix}$$

where $I_x=\frac{bh^3}{12}$, $I_y=\frac{hb^3}{12}$, $A=bh$, and

$$J=\frac{h^3b}{3}\left(1-\frac{192}{\pi^5}\left(\frac{h}{b}\right)\sum_{n=1,3,5}^{n=\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi b}{2h}\right)\right)\ \text{for}\ \ b\geq h$$

$$[S]=[C]^{-1}=\text{stiffness matrix}$$

Bernoulli-Euler Beam Stiffness Matrix

$$\begin{bmatrix}\delta\theta_x\\ \delta\theta_y\\ \delta\theta_z\\ \delta d_x\\ \delta d_y\\ \delta d_z\end{bmatrix} = \overbrace{\begin{bmatrix} \frac{l}{EI_x} & 0 & 0 & 0 & -\frac{l^2}{2EI_x} & 0\\ 0 & \frac{l}{EI_y} & 0 & \frac{l^2}{2EI_y} & 0 & 0\\ 0 & 0 & \frac{l}{GJ} & 0 & 0 & 0\\ 0 & \frac{l^2}{2EI_y} & 0 & \frac{l^3}{3EI_y} & 0 & 0\\ -\frac{l^2}{2EI_x} & 0 & 0 & 0 & \frac{l^3}{3EI_x} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{l}{EA} \end{bmatrix}}^{[C] = \text{compliance matrix}} \begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\\ f_x\\ f_y\\ f_z\end{bmatrix}$$

where $I_x = \frac{bh^3}{12}$, $I_y = \frac{hb^3}{12}$, $A = bh$, and

$$J = \frac{h^3 b}{3}\left(1 - \frac{192}{\pi^5}\left(\frac{h}{b}\right)\sum_{n=1,3,5}^{n=\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi b}{2h}\right)\right) \quad \text{for} \quad b \ge h$$

$$[S] = [C]^{-1} = \text{stiffness matrix}$$

Examples