DIU-EIL bloc 5

Graphes : coloriage et transition vers la calculabilité

Laure Gonnord

http://laure.gonnord.org/pro/

Laure.Gonnord@univ-lyon1.fr

DIU EIL, Dpt Info UCBL

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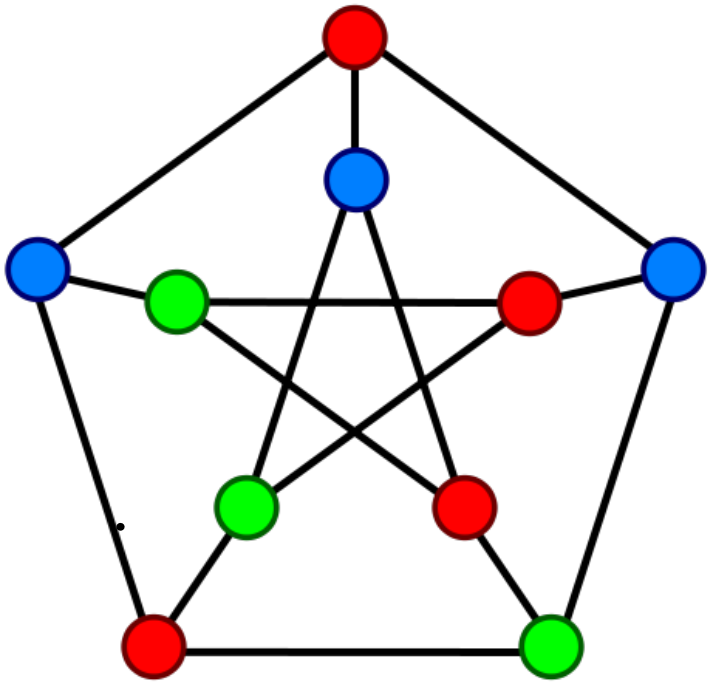
Graph Coloring Problem

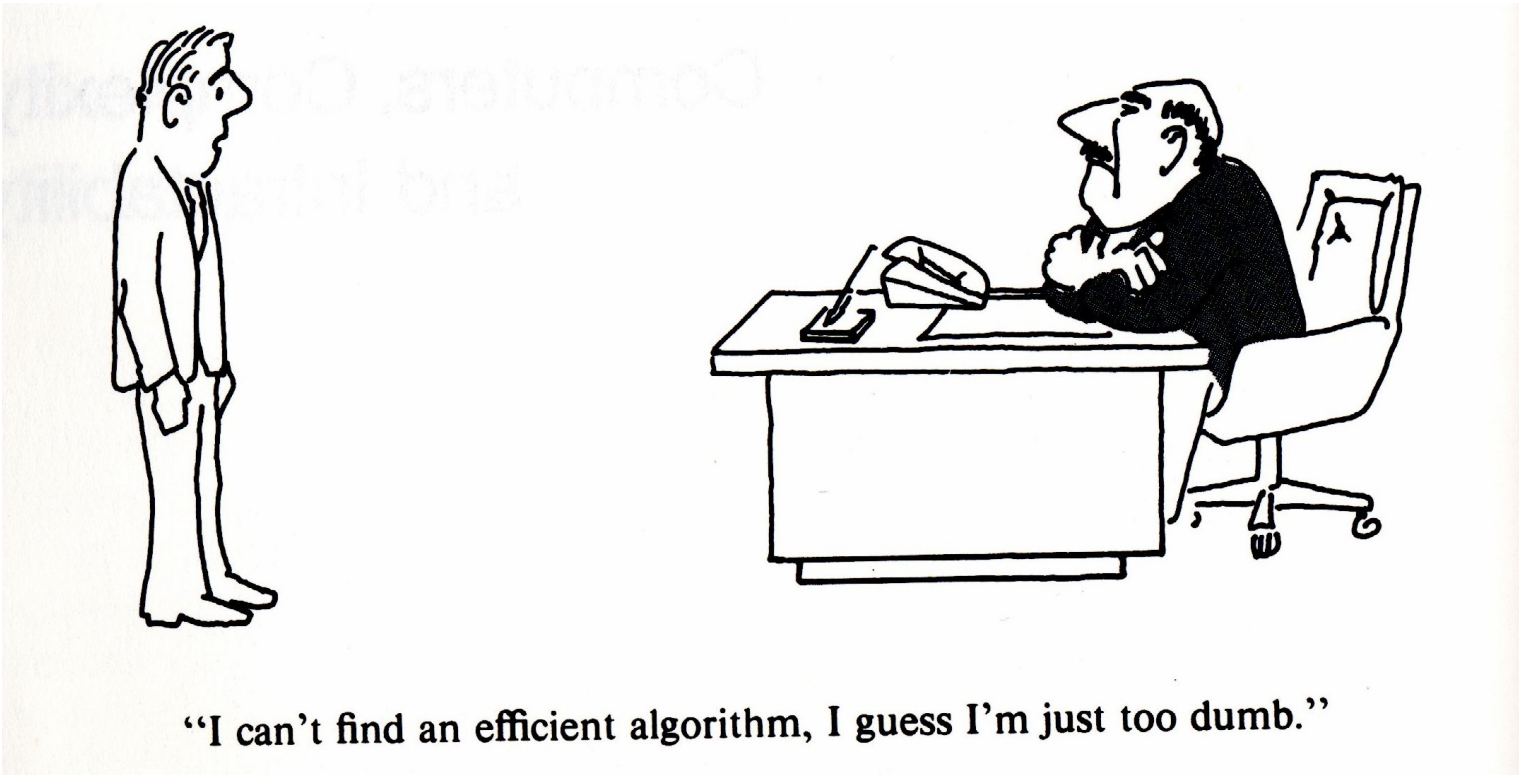


Color with the minimal number of colors !

- Application to the **register allocation** in compilers.
- The sudoku problem (9-coloring of a 81-vertices graph)

Graph Coloring Problem - 2

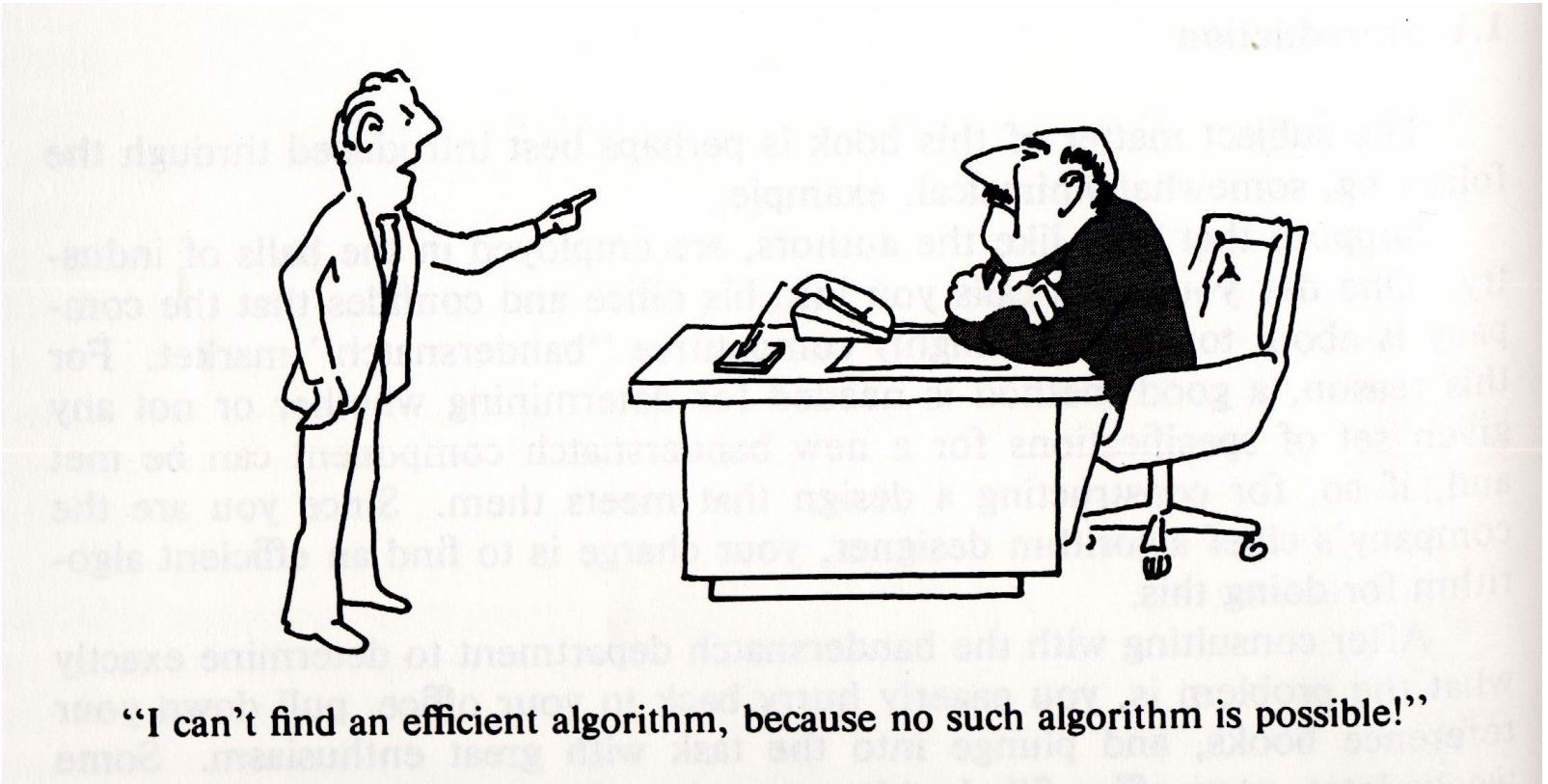
Are **you** able to design a polynomial algorithm ?



"I can't find an efficient algorithm, I guess I'm just too dumb."

Graph Coloring Problem - 3

We do not know any polynomial algorithm for this problem (see next course).

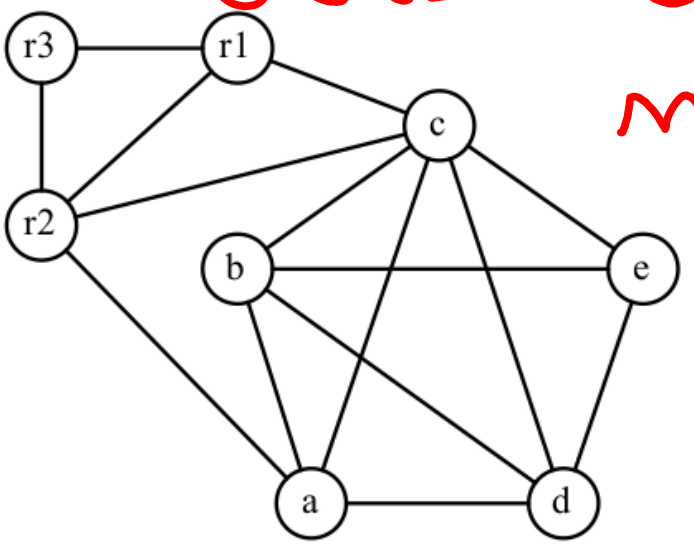


"I can't find an efficient algorithm, because no such algorithm is possible!"

Graph Coloring Problem - A Polynomial algorithm

An algorithm to color a graph (but without optimising the number) with $\leqslant K$ colors.

Running example :



On fait un compromis
colorier le graphe
mais pas
forcément
de façon
optimale

Kempe's simplification algorithm 1/2

A "simple remark" :

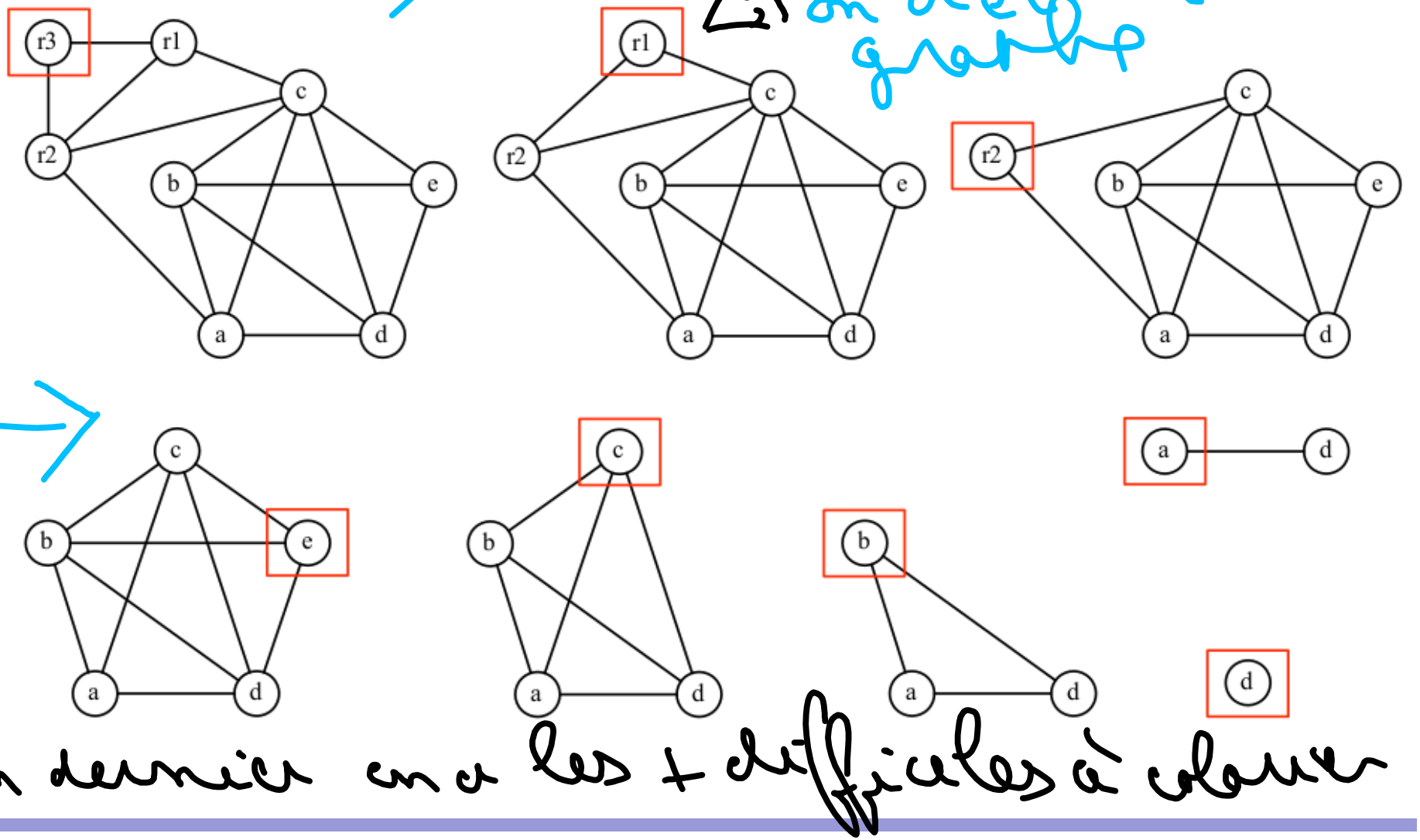
Proposition (Kempe 1879)

Suppose the graph contains a node m with fewer than K neighbours. Then if $G' = G \setminus \{m\}$ can be colored, then G can be colored as well.

- Pick a low degree node, and remove it, and continue until remove all (the graph is K-colorable) or ...

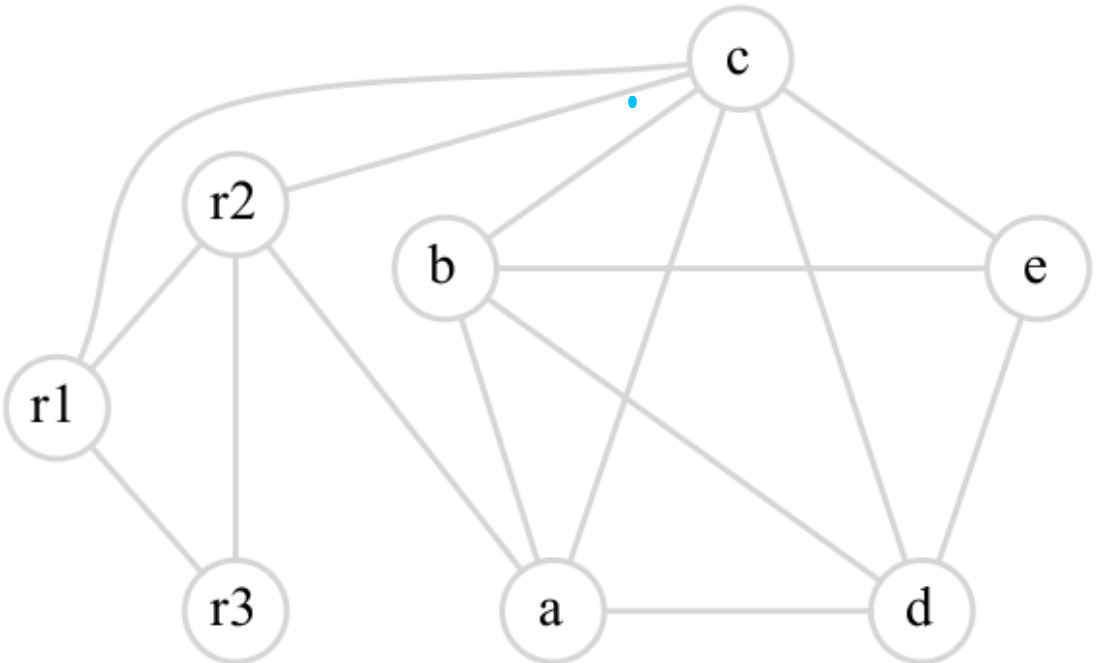
trouve le graphe est colorable en K couleurs mais notre heuristique polynomiale ne permet pas de le colorier

Kempe's simplification algorithm 2/2



Let's color !

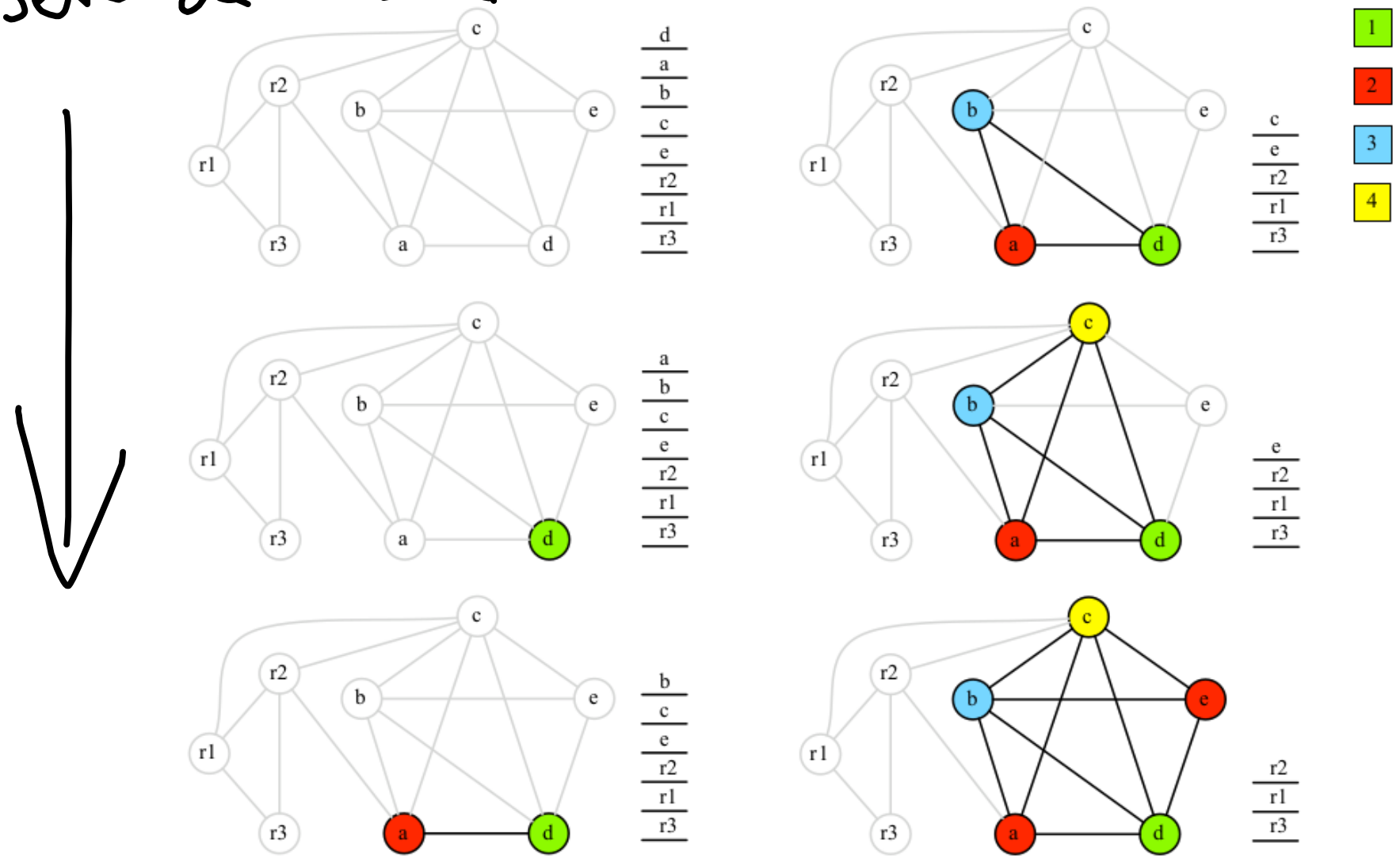
- We assign colors to the nodes greedily, in the reverse order in which nodes are removed from the graph.
- The color of the next node is the first color that is available, *i.e.* not used by any neighbour.



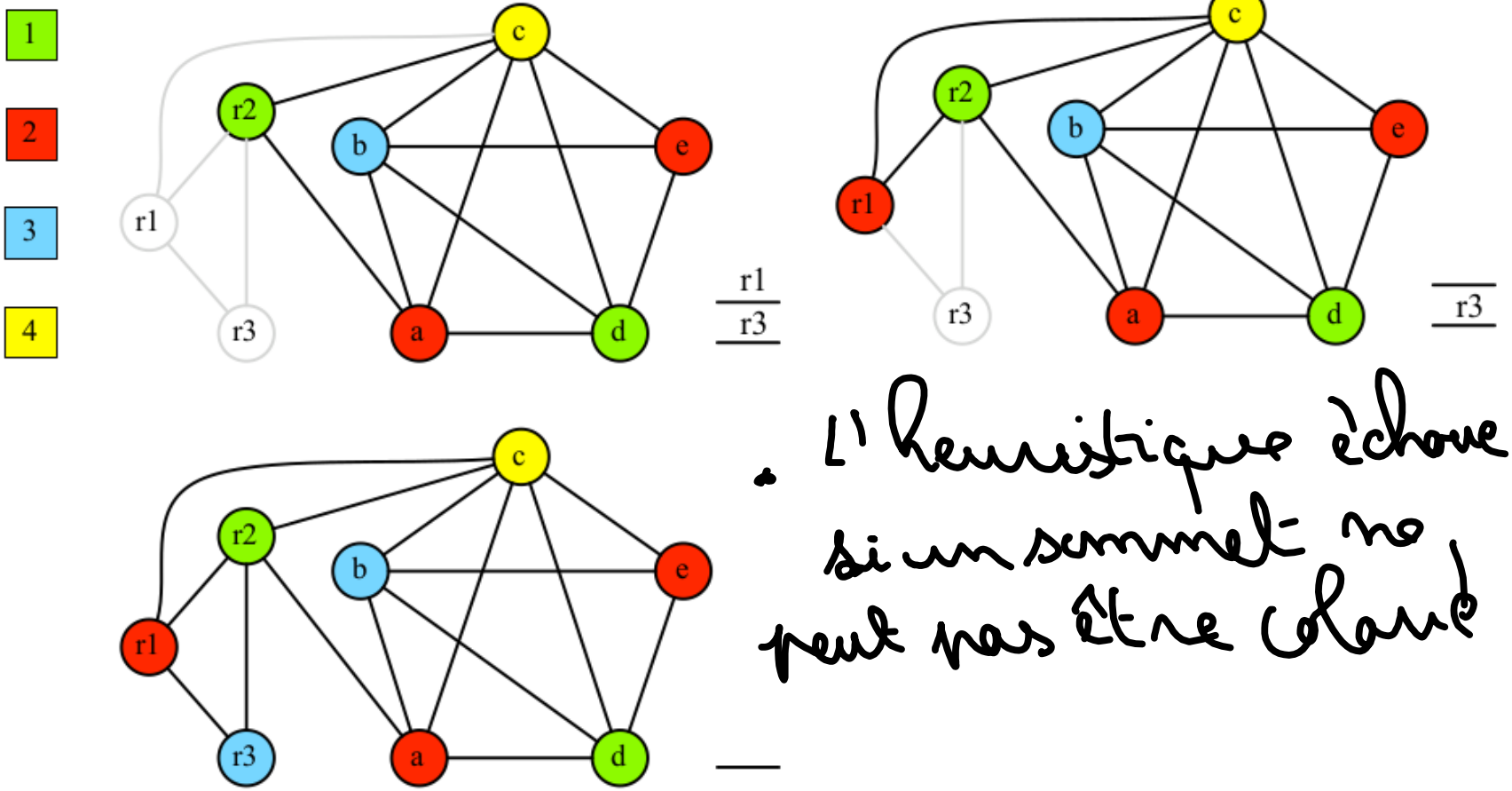
on colorie en commençant par le haut de la pile

Greedy coloring example 1/2

sens de lecture



Greedy coloring example 2/2

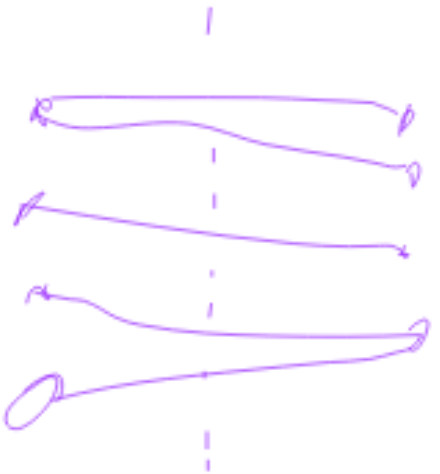


- see the Python implementation !

Cette heuristique polynomiale ne permet pas de trouver le nombre de couleur minimale mais il peu toujours à trouver une solution si on autorise à augmenter le nombre de couleurs

La coloration à 2 couleurs est liée au parcours en profondeur.

Caractérisation d'un graphe 2-colorable par une propriété de bipartisme. On peut diviser les sommets en 2 ensembles de sommets qui n'ont pas d'arêtes internes



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