Propositions of solutions for *Analysis I* by Terence Tao

Frédéric Santos

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1. Introduction

No exercises in this chapter.

2. The natural numbers

Exercise 2.2.1. — *Prove that the addition is associative, i.e. that for any natural numbers* a, b, c*, we have* $(a + b) + c = a + (b + c)$.

Let's use induction on c while keeping a and b fixed.

- Base case for $c = 0$: let's prove that $(a + b) + 0 = a + (b + 0)$. The left hand side is equal to $(a + b)$ according to Lemma 2.2.3. For the right hand side, if we apply the same lemma to the $(b + 0)$ part, we get $a + (b + 0) = a + b$. Both sides are equal to $a + b$, and the base case is thus done.
- Now let's suppose inductively that $(a + b) + c = a + (b + c)$: we have to prove that $(a + b) + c{+}{+} = a + (b + c{+}{+})$. Using Lemma 2.2.3 on the right hand side leads to $a + (b + c){+}{+}$. Now consider the left hand side. Using still the same lemma, we get $(a + b) + c{+}{+} = ((a + b) + c){+}{+}$. By the inductive hypothesis, this is also equal to $(a + (b + c)){+}{+}$. And, using the lemma 2.2.3 again, this also leads to $a + b + c{+}{+}$. Therefore, both sides are equal to $a + b + c{+}{+}$, and we have closed the induction.