Quantitative Methods

Module 8: Transportation Model

Name (LN,FN,MN): ________________________________ Program/Yr/Block: ___________

I. Introduction

In this chapter, we will explore the special type of linear programming problems: the transportation problem. This may be modeled as ***network flow problems***, with the use of nodes (points) and arcs (lines).

This chapter will explain this problem and provide network representations. The transportation problem has a special structure that will enable to be solved with very efficient algorithms.

II. Learning Objectives

After completing this module, you should be able to:

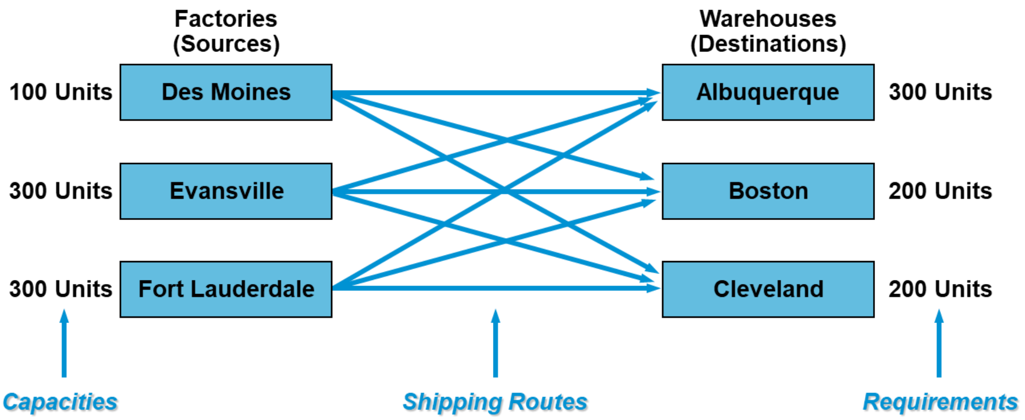
1. Structure special LP problem using the transportation model
2. Use the northwest corner and stepping-stone methods
3. Solve facility location and other application problems with transportation models

Topics and Key Concepts

A. Transportation Model

- The ***transportation problem*** deals with the distribution of goods from several points of supply (***sources***) to a number of points of demand (***destinations***)
- Usually, we are given the capacity of goods at each source and the requirements at each destination
- Typically, the objective is to minimize total transportation and production costs

Example of transportation problem in a network format



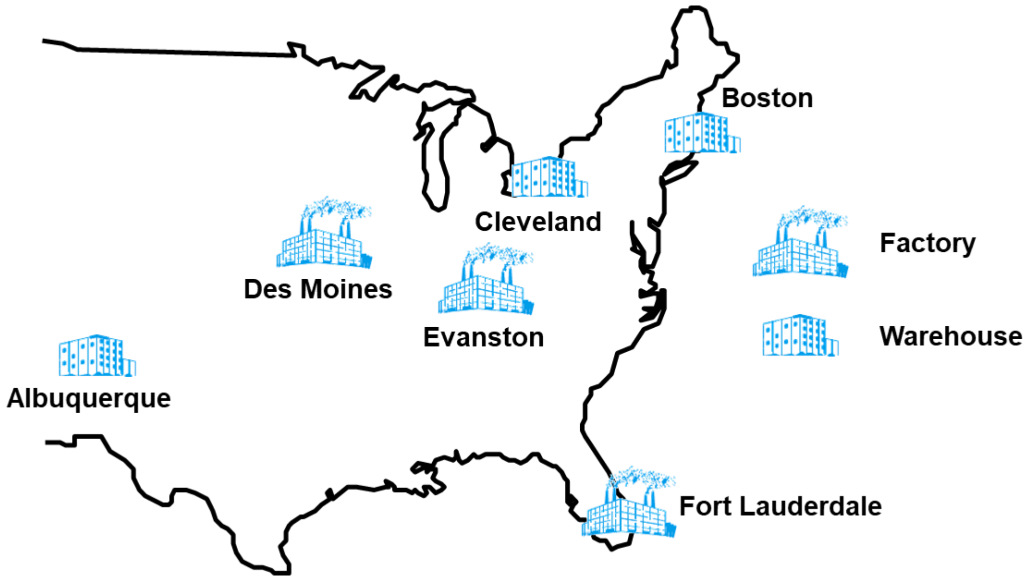
B. Setting Up a Transportation Problem

- The Executive Furniture Corporation manufactures office desks at three locations: Des Moines, Evansville, and Fort Lauderdale
- The firm distributes the desks through regional warehouses located in Boston, Albuquerque, and Cleveland
- Estimates of the monthly production capacity of each factory and the desks needed at each warehouse are shown in the figure above.
- Production costs are the same at the three factories so the only relevant costs are shipping from each ***source*** to each ***destination***
- Costs are constant no matter the quantity shipped
- The transportation problem can be described as ***how to select the shipping routes to be used and the number of desks to be shipped on each route so as to minimize total transportation cost***
- Restrictions regarding factory capacities and warehouse requirements must be observed
- The first step is setting up the transportation table
- Its purpose is to summarize all the relevant data and keep track of algorithm computations

Transportation costs per desk for Executive Furniture

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND
DES MOINES	$5	$4	$3
EVANSVILLE	$8	$4	$3
FORT LAUDERDALE	$9	$7	$5

- Geographical locations of Executive Furniture's factories and warehouses



- Transportation table for Executive Furniture

Des Moines capacity constraint

FROM \ TO	WAREHOUSE AT ALBUQUERQUE	WAREHOUSE AT BOSTON	WAREHOUSE AT CLEVELAND	FACTORY CAPACITY
DES MOINES FACTORY	$5	$4	$3	100
EVANSVILLE FACTORY	$8	$4	$3	300
FORT LAUDERDALE FACTORY	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total supply and demand

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

- In this table, total factory supply exactly equals total warehouse demand
- When equal demand and supply occur, a ***balanced problem*** is said to exist
- This is uncommon in the real world and we have techniques to deal with unbalanced problems

C. Developing an Initial Solution: Northwest Corner Rule

- Once we have arranged the data in a table, we must establish an initial feasible solution
- One systematic approach is known as the ***northwest corner rule***
- Start in the upper left-hand cell and allocate units to shipping routes as follows
  1. Exhaust the supply (factory capacity) of each row before moving down to the next row
  2. Exhaust the demand (warehouse) requirements of each column before moving to the right to the next column
  3. Check that all supply and demand requirements are met.
- In this problem it takes five steps to make the initial shipping assignments

  - Beginning in the upper left hand corner, we assign 100 units from Des Moines to Albuquerque. This exhaust the supply from Des Moines but leaves Albuquerque 200 desks short. We move to the second row in the same column.

FROM \ TO	ALBUQUERQUE (A)	BOSTON (B)	CLEVELAND (C)	FACTORY CAPACITY
DES MOINES (D)	100 $5	$4	$3	100
EVANSVILLE (E)	$8	$4	$3	300
FORT LAUDERDALE (F)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- Assign 200 units from Evansville to Albuquerque. This meets Albuquerque's demand. Evansville has 100 units remaining so we move to the right to the next column of the second row.

TO / FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 \$5	\$4	\$3	100
EVANSVILLE (*E*)	200 \$8	\$4	\$3	300
FORT LAUDERDALE (*F*)	\$9	\$7	\$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- Assign 100 units from Evansville to Boston. The Evansville supply has now been exhausted but Boston is still 100 units short. We move down vertically to the next row in the Boston column.

TO / FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 \$5	\$4	\$3	100
EVANSVILLE (*E*)	200 \$8	100 \$4	\$3	300
FORT LAUDERDALE (*F*)	\$9	\$7	\$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- Assign 100 units from Evansville to Boston. The Evansville supply has now been exhausted but Boston is still 100 units short. We move down vertically to the next row in the Boston column.

TO / FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 \$5	\$4	\$3	100
EVANSVILLE (*E*)	200 \$8	100 \$4	\$3	300
FORT LAUDERDALE (*F*)	\$9	100 \$7	\$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- Assign 200 units from Fort Lauderdale to Cleveland. This exhausts Fort Lauderdale's supply and Cleveland's demand. The initial shipment schedule is now complete.

TO / FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	100 $4	$3	300
FORT LAUDERDALE (*F*)	$9	100 $7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- We can easily compute the cost of this shipping assignment

ROUTE FROM	ROUTE TO	UNITS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*A*	200		8		1,600
E	*B*	100		4		400
F	*B*	100		7		700
F	*C*	200		5		1,000
						4,200

This solution is feasible but we need to check to see if it is optimal

D. Stepping-Stone Method: Finding a Least Cost Solution

- The ***stepping-stone method*** is an iterative technique for moving from an initial feasible solution to an optimal feasible solution
- There are two distinct parts to the process
  - Testing the current solution to determine if improvement is possible
  - Making changes to the current solution to obtain an improved solution
- This process continues until the optimal solution is reached
- There is one very important rule

The number of occupied routes (or squares) must always be equal to one less than the sum of the number of rows plus the number of columns

- In the Executive Furniture problem this means the initial solution must have 3 + 3 – 1 = 5 squares used

$$\text{Occupied shipping routes (squares)} = \text{Number of rows} + \text{Number of columns} - 1$$

- When the number of occupied rows is less than this, the solution is called ***degenerate***

E. Testing the Solution for Possible Improvement

- The stepping-stone method works by testing each unused square in the transportation table to see what would happen to total shipping costs if one unit of the product were tentatively shipped on an unused route

Five Steps to Test Unused Squares with the Stepping-Stone Method

1. Select an unused square to evaluate
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used with only horizontal or vertical moves allowed
3. Beginning with a plus (+) sign at the unused square, place alternate minus (–) signs and plus signs on each corner square of the closed path just traced
4. Calculate an ***improvement index*** by adding together the unit cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign
5. Repeat steps 1 to 4 until an improvement index has been calculated for all unused squares. If all indices computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease total shipping costs.

For the Executive Furniture Corporation data example

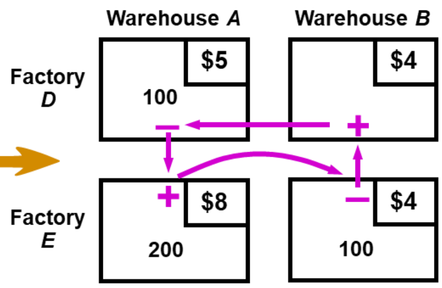
Steps 1 and 2. Beginning with Des Moines–Boston route we trace a closed path using only currently occupied squares, alternately placing plus and minus signs in the corners of the path

- In a ***closed path***, only squares currently used for shipping can be used in turning corners
- ***Only one*** closed route is possible for each square we wish to test

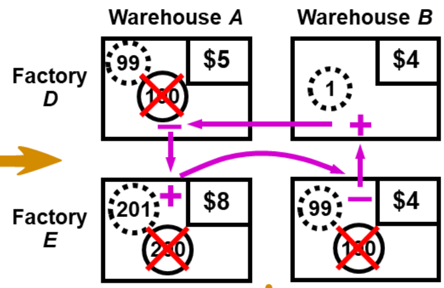
Step 3. We want to test the cost-effectiveness of the Des Moines–Boston shipping route so we pretend we are shipping one desk from Des Moines to Boston and put a plus in that box

- But if we ship one ***more*** unit out of Des Moines we will be sending out 101 units
- Since the Des Moines factory capacity is only 100, we must ship ***fewer*** desks from Des Moines to Albuquerque so we place a minus sign in that box
- But that leaves Albuquerque one unit short so we must increase the shipment from Evansville to Albuquerque by one unit and so on until we complete the entire closed path

Evaluate the unused Des Moines-Boston shipping route



TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5)	($4)	($3)	100
EVANSVILLE	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700



TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	
DES MOINES	100 ($5)	($4)	($3)	
EVANSVILLE	200 ($8)	100 ($4)	($3)	
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Result of Proposed Shift in Allocation

= 1 x $4
– 1 x $5
+ 1 x $8
– 1 x $4 = +$3

Step 4. We can now compute an ***improvement index*** (**I_{ij}**) for the Des Moines–Boston route

- We add the costs in the squares with plus signs and subtract the costs in the squares with minus signs

Des Moines–Boston index $= I_{DB} = +\$4 - \$5 + \$5 - \$4 = +\$3$

- This means for every desk shipped via the Des Moines–Boston route, total transportation cost will ***increase*** by $3 over their current level

Step 5. We can now examine the Des Moines–Cleveland unused route which is slightly more difficult to draw

- Again, we can only turn corners at squares that represent existing routes
- We must pass through the Evansville–Cleveland square but we cannot turn there or put a + or – sign
- The closed path we will use is

+ DC – DA + EA – EB + FB – FC

Evaluating the Des Moines–Cleveland shipping route

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5)	($4)	Start ($3)	100
EVANSVILLE	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Des Moines–Cleveland improvement index $= I_{DC} = +\$3 - \$5 + \$8 - \$4 + \$7 - \$5 = +\$4$

- Opening the Des Moines–Cleveland route will not lower our total shipping costs
- Evaluating the other two routes we find

Evansville-Cleveland index $= I_{EC} = +\$3 - \$4 + \$7 - \$5 = +\$1$

- The closed path is

+ EC – EB + FB – FC

Fort Lauderdale–Albuquerque index $= I_{FA} = +\$9 - \$7 + \$4 - \$8 = -\$2$

- The closed path is

+ FA – FB + EB – EA

Opening the Fort Lauderdale-Albuquerque route will lower our total transportation costs

F. Obtaining an Improved Solution

- In the Executive Furniture problem there is only one unused route with a negative index (Fort Lauderdale-Albuquerque)
  - If there was more than one route with a negative index, we would choose the one with the largest improvement
  - We now want to ship the maximum allowable number of units on the new route
  - The quantity to ship is found by referring to the closed path of plus and minus signs for the new route and selecting the ***smallest number*** found in those squares containing minus signs

- To obtain a ***new solution***, that number is added to all squares on the closed path with plus signs and subtracted from all squares the closed path with minus signs
  - All other squares are unchanged
  - In this case, the maximum number that can be shipped is 100 desks as this is the smallest value in a box with a negative sign (*FB* route)
  - We add 100 units to the *FA* and *EB* routes and subtract 100 from *FB* and *EA* routes
  - This leaves balanced rows and columns and an improved solution

Stepping-stone path used to evaluate route FA

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 ($5)	($4)	($3)	100
E	200 ($8) −	100 ($4) +	($3)	300
F	($9) +	100 ($7) −	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Total shipping costs have been reduced by
(100 units) x ($2 saved per unit) and now equals $4,000

- This second solution may or may not be optimal

- To determine whether further improvement is possible, we return to the first five steps to test each square that is ***now*** unused
  - The four new improvement indices are

D to B = I_{DB} = + \$4 – \$5 + \$8 – \$4 = + \$3
(closed path: + DB – DA + EA – EB)
D to C = I_{DC} = + \$3 – \$5 + \$9 – \$5 = + \$2
(closed path: + DC – DA + FA – FC)
E to C = I_{EC} = + \$3 – \$8 + \$9 – \$5 = – \$1
(closed path: + EC – EA + FA – FC)
F to B = I_{FB} = + \$7 – \$4 + \$8 – \$9 = + \$2
(closed path: + FB – EB + EA – FA)

Path to evaluate for the EC route

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 (\$5)	(\$4)	(\$3)	100
E	100 (\$8) –	200 (\$4)	Start (\$3) +	300
F	100 (\$9) +	(\$7)	200 (\$5) –	300
WAREHOUSE REQUIREMENTS	300	200	200	700

An improvement can be made by shipping the maximum allowable number of units from E to C

Total cost of third solution

ROUTE FROM	ROUTE TO	DESKS SHIPPED	x	PER UNIT COST (\$)	=	TOTAL COST (\$)
D	A	100		5		500
E	B	200		4		800
E	C	100		3		300
F	A	200		9		1,800
F	C	100		5		500
						3,900

Third and optimal solution

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 ($5)	($4)	($3)	100
E	($8)	200 ($4)	100 ($3)	300
F	200 ($9)	($7)	100 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- This solution is optimal as the improvement indices that can be computed are all greater than or equal to zero

D to *B* = I_{DB} = + $4 – $5 + $9 – $5 + $3 – $4 = + $2
(closed path: + *DB* – *DA* + *FA* – *FC* + *EC* – *EB*)
D to *C* = I_{DC} = + $3 – $5 + $9 – $5 = + $2
(closed path: + *DC* – *DA* + *FA* – *FC*)
E to *A* = I_{EA} = + $8 – $9 + $5 – $3 = + $1
(closed path: + *EA* – *FA* + *FC* – *EC*)
F to *B* = I_{FB} = + $7 – $5 + $3 – $4 = + $1
(closed path: + *FB* – *FC* + *EC* – *EB*)

Summary of Steps in Transportation Algorithm (Minimization)

1. Set up a balanced transportation table
2. Develop initial solution using northwest corner method
3. Calculate an improvement index for each empty cell using stepping-stone method. If improvement indices are all nonnegative, stop as the optimal solution has been found. If any index is negative, continue to step 4.
4. Select the cell with the improvement index indicating the greatest decrease in cost. Fill this cell using the stepping-stone path and go to step 3.

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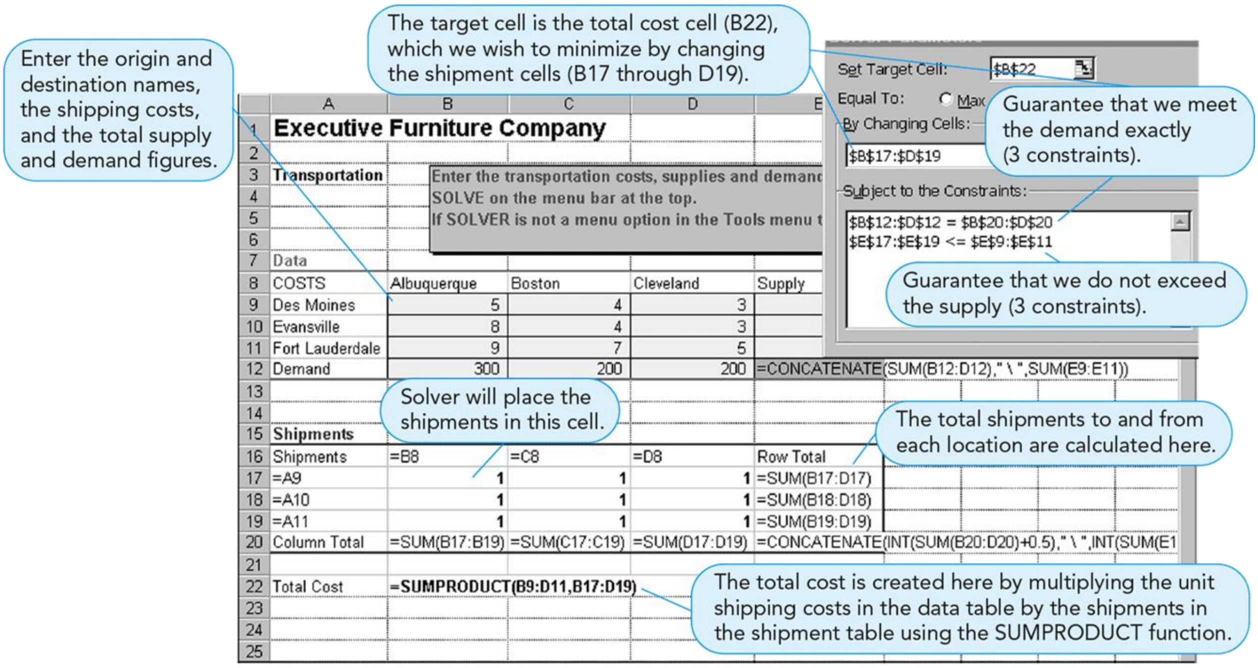
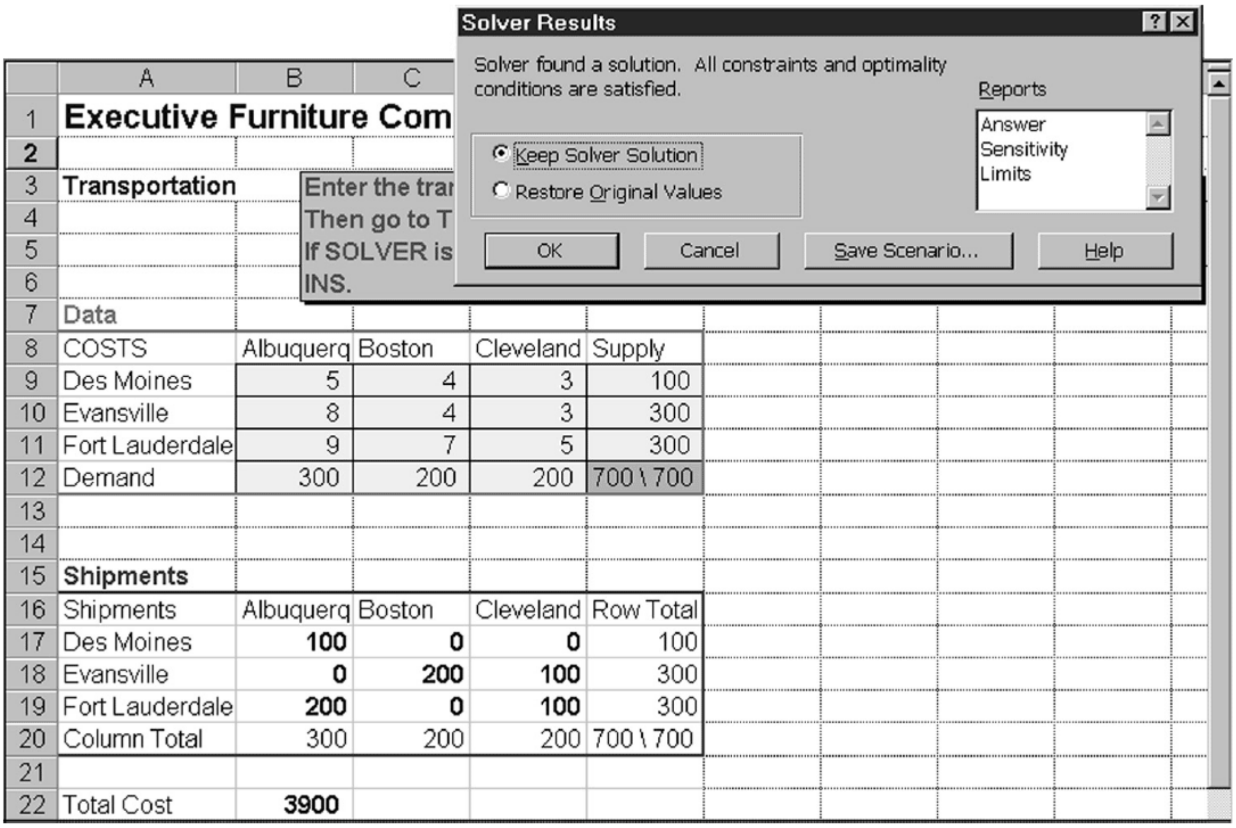
G. Using Excel QM to Solve Transportation Problems



Excel QM input screen and formulas



Output from Excel QM with optimal solution

Teaching and Learning Materials Resources

- PC Computer | Laptop | Android Phone
- GC LAMP
- Google Meet
- Facebook Messenger

Learning Tasks

A. Explore

a. Use MS Excel QM/POM with the data given below to calculate transportation using northwest corner method. Take a screenshot of the output and paste it inside the box. (15 pts.)

Don Yale, president of Hardrock Concrete Company, has plants in three locations and is currently working on three major construction projects, located at different sites. The shipping cost per truckload of concrete, plant capacities, and project requirements are provided in the accompanying table. *(minimize cost)*

TO / FROM	PROJECT *A*	PROJECT *B*	PROJECT *C*	PLANT CAPACITIES
PLANT 1	$10	$4	$11	70
PLANT 2	$12	$5	$8	50
PLANT 3	$9	$7	$6	30
PROJECT REQUIREMENTS	40	50	60	150

Output Screenshot

b. Use MS Excel QM/POM with the data given below to minimize cost using northwest corner method. Take a screenshot of the output and paste it inside the box. (15 pts.)

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	34

Output Screenshot

B. Explain

a. Is the transportation model an example of decision making under certainty or decision making under uncertainty? Why? (15 pts)

____
____
____
____
____
____
____
____

C. Engage

a. Show your solutions. (40 pts)

The management of the Executive Furniture Corporation decided to expand the production capacity at its Des Moines factory and to cut back production at its other factories. It also recognizes a shifting market for its desks and revises the requirements at its three warehouses.

1. Use the northwest corner rule to establish an initial feasible shipping schedule and calculate its cost.
2. Use the stepping-stone method to test whether an improved solution is possible

NEW WAREHOUSE REQUIREMENTS		NEW FACTORY CAPACITIES	
Albuquerque (*A*)	200 desks	Des Moines (*D*)	300 desks
Boston (*B*)	200 desks	Evansville (*E*)	150 desks
Cleveland (*C*)	300 desks	Fort Lauderdale (*F*)	250 desks

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND
DES MOINES	5	4	3
EVANSVILLE	8	4	3
FORT LAUDERDALE	9	7	5

References

1. Render, Stair, Hannah. 2012. "Quantitative Analysis for Management Global Edition 11th Edition. Retrieve from: https://wps.pearsoned.co.uk/ema_ge_render_qam_11/202/51951/13299709.cw/-/t/index.html
2. MECHANICAL ENGINEERING EXPLAINED OFFICIAL (2017, February 8). north west corner method transportation problem | transportation problem north west corner rule [Video]. YouTube. https://www.youtube.com/watch?v=TUYSD649PEY
3. SSK Edutech (2020, June 23). STEPPING STONE METHOD | TRANSPORTATION PROBLEM | OPTIMAL SOLUTION | LATEST 2020 [Video]. YouTube. https://www.youtube.com/watch?v=fYXCY1aXg7Y