Course:	**Latent Growth Curve Models (LGCM) & Growth Mixture Models (GMM)**	**2 Day Course**
Date(s):	**19 – 20 May, 2021**	
Venue:	**Norwegian University of Science and Technology**	

Short Summary of Course

Research questions examining within-person changes or joint within-person changes and between-person differences in the stability and change in individuals' attributes over time make longitudinal data incredibly useful. Longitudinal data offers many possibilities to describe differences in how and when people change and explain why. Methodological limitations in calculating difference score, taking residualized scores, correlation among repeated measures, and other limitations mean that appropriate growth models must be estimated.

Structural Equation Modelling (SEM) is a family of related analysis techniques – correlations, regression analyses and factor analyses, using both observed and unobserved (latent) variables to offer a flexible framework for analysing longitudinal data (and cross-sectional data too). Analysing growth models in the SEM framework provide a highly convenient and statistically rigorous framework for applied research in the social, behavioural, and educational sciences.

This course is a data analysis course, not a statistics course and will cover basic and advanced longitudinal SEM model using Mplus in a very easy and efficient implementation. Additionally, to make the course more 'theory- and practice-based' than 'equations-based', the models that will be estimated will be guided by the overarching objectives of longitudinal research described in the seminal work of Baltes and Nasselroade (1979).

Baltes, P. B., & Nesselroade, J. R. (1979). History and rationale of longitudinal research. In J. R. Nasselroade & P. B. Baltes (Eds.), *Longitudinal research in the study of behaviour and development* (pp. 1-39). New York: Academic Press.

Course Contents

DAY 1

- Objectives of longitudinal research and growth curve models?
- Latent Growth Curve Models and its extensions
    - ✓ Role of time scores/factor loadings in LGCM
    - ✓ Fitting Linear and Quadratic LGCM
    - ✓ Adding time-invariant and time-varying (dynamical) predictors, and (distal) outcomes
    - ✓ Spline/Piecewise/Multiphase LGCM
    - ✓ Multi (parallel & sequentially contingent) Process LGCM
    - ✓ Multi-group LGCM and structural invariance testing

DAY 2

- Growth Mixture Models
    - ✓ Why LGCM before growth mixture models?
    - ✓ Latent Class Growth Analysis (LCGA): A simplified GMM
    - ✓ The 'One-step' versus 'Three-step' Approach for adding predictors and (distal) outcomes.

Key Learning Outcomes

1. Be able to estimate and interpret parameters of various growth models
2. Use growth models (e.g., LGCM) and growth mixture models (e.g., LCGA and GMM)
3. Describe how and when attributes of an individual change over time
4. Explain the determinants or predictors of individual change over time
5. Describe how and when attributes of different individuals or different groups of persons change in different ways
6. Explain the factors/mechanisms for how different individuals change in different ways
7. Evaluate how changes in one variable precede, covary, and/or follow changes in another variable
8. Identify subgroups of individuals showing specific (e.g., homogeneous, or heterogeneous) growth trajectories

❖ *Understand <u>what decisions to make and why</u> when estimating growth models or growth mixture models*

Computer Software
This course includes computer workshops using Mplus.
Presented by
Frederick Anyan is postdoc researcher at the Norwegian University of Science and Technology, NTNU in Trondheim, Norway where he also completed a joint PhD in Behaviour and Health (with the Australian National University, ANU in Canberra, Australia). Frederick is currently collaborating with researchers at University of Cambridge and University of Toronto to investigate the impact of professional helping relationships on the trajectories of housing stability for 2 141 people facing severe and multiple disadvantages using data from the 'Housing First' controlled trial in five Canadian cities. He is also involved in analysing sick leave trajectories before, during and after work-focused treatment in a specialised health care centre in Oslo, Norway. Frederick manages the resilience to loneliness project (RESLON), which combines psychological network analysis and SEM to investigate concurrent and prospective relations between loneliness, worry, rumination, metacognitive beliefs and symptoms of anxiety and depression. He is also a researcher in the UPRIGHT project aimed at promoting positive mental health by enhancing resilience capacities in youths, through a holistic approach by addressing early adolescents, families, and education professionals in five EU countries.
Target Audience
The course is aimed at people from all disciplines and types of institutions that want to learn about longitudinal data analysis with a focus on applications and learning real world skills. Specifically, the applications in the course will include when to use different growth models based on research questions, what the technique is doing (in an intuitive non-mathematical way) and how to interpret results. This course is non-mathematical/equations therefore, complete beginners without any prior knowledge of either SEM or Mplus can attend. Prior knowledge of SEM or Mplus is an advantage.
Keywords
• Structural Equation Modelling • Mplus • Longitudinal analysis • Latent growth curve modelling • Growth mixture modelling
Location
Zoom/Computer lab at Building 12, Room 581. Psychology Department, NTNU
Duration
2 days 09:00 to 16:00 with lunch from 12:00 to 12:45 in both days and coffee breaks
Pre-requisites
Knowledge of regression and confirmatory factor analysis in SEM would be an advantage but not a requirement to attend.
Level of course
☒ Intermediate (some prior knowledge about SEM)
Preparatory Reading (required or <u>desirable</u>) Grimm, K. J., Ram, N., & Estabrook, R. (2016). *Growth modelling: Structural equation and multilevel modelling approaches*. Guilford Publications. Hoffman, L. (2015). *Longitudinal analysis: Modelling within-person fluctuation and change*. Routledge.

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###          Latent Growth Curve Models (LGCM) & Growth Mixture Models (GMM)          ###
###                              Frederick Anyan                                      ###
###                          frederick.anyan@ntnu.no                                  ###
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###            DAY 1 : The Latent Growth Curve Model and its Extensions               ###
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001_No_change_No_growth_Intercept_only_model
002_Linear_growht_curve_model
003_Quadratic_growht_curve_model
004_Bilinear_spline_growht_curve_model

005_Linear_growht_curve_model_with_time_varying_covariates
006_Linear_growht_curve_model_with_time_invariant_covariates
007_Linear_growht_curve_model_with_time-invariant and time-varying covariates

008_Linear_growht_curve_model_with_growth_factors_predicting_distal_outcome

009_Multigroup_LGCM_M1_Invariance_model
009_Multigroup_LGCM_M2_Growth_factor_means
009_Multigroup_LGCM_M3_Growth_factor_means_and covariances
009_Multigroup_LGCM_M4_Growth_factor_means_and covariances_with_residual_variances

010_Parallel_growth_curve_model_anxiety_and_depressive symptoms
010_Parallel_growth_curve_model_anxiety_and_depressive symptoms_autocorrelations
010_Parallel_growth_curve_model_anxiety_and_depressive symptoms_cross_domain_correlations

011_Sequentially_contingent_growth_curve_model_anxiety_T1_T5_predicting_depression_T6_T10

###
DAY 2: The Latent Class Growth Analysis and Growth Mixture Models
###

#The Latent Class Growth Analysis - LCGA

1_CLASS_LCGA
2_CLASS_LCGA
3_CLASS_LCGA
4_CLASS_LCGA
5_CLASS_LCGA

#The Growth Mixture Model - Class varying variance-covariance components- GMM-CV

1-Class GMM
2-Class GMM_WARNING_message
2-Class GMM_WARNING_fixed
3-Class GMM_WARNING_message
3-Class GMM_WARNING_fixed
4-Class GMM_WARNING_message
4-Class GMM_WARNING_still_inadmissible

3-Class GMM_WARNING_Wald_Test_for_Equality_of_Parameters

#The Growth Mixture Model - Class Invariant variance-covariance components - GMM-CI

1-Class GMM-CI
2-Class GMM-CI
3-Class GMM-CI
4-Class GMM-CI

#The Growth Mixture Model with Covariates

001_ONE_STEP_3-Class GMM_WARNING_fixed_Adding_predictors_for_between_class_variation
002_ONE_STEP_3-Class GMM_WARNING_fixed_Adding_predictors_for_between_and_within_class_variation
003_ONE_STEP_3-Class GMM_WARNING_fixed_Adding_distal_continous_outcome

004_THREE_STEP_AUXILIARY_OPTION_FOR_ADDING_PREDICTORS_3-Class GMM_WARNING_fixed_
005_THREE_STEP_AUXILIARY_OPTION_FOR_ADDING_DISTAL_OUTCOMES_3-Class GMM_WARNING_fixed_

Norwegian University of
Science and Technology

Latent Growth Curve Models (LGCM) & Growth Mixture Models (GMM)

Frederick Anyan, PhD
frederick.anyan@ntnu.no

Overview

- DAY 1
  - Objectives of longitudinal research
  - Features of longitudinal data
  - Latent growth curve model and its extensions
    - Role of time scores/factor loadings in LGCM
    - Fitting Linear and Quadratic LGCM
    - Adding time-invariant and time-varying (dynamical) predictors, and (distal) outcomes
    - Spline/Piecewise/Multiphase LGCM
    - Multi-group LGCM and invariance testing
    - Multi (Parallel & sequentially contingent) Process LGCM

- DAY 2
  - Growth mixture models
    - Why LGCM before growth mixture models?
    - Latent Class Growth Analysis (LCGA): A simplified GMM
    - The ‘One-step’ versus ‘Three-step’ Approach for adding predictors and (distal) outcomes.

Objectives of longitudinal research

1. Identification of intraindividual (within-person) changes (and stability)
   - Is there change over time on average?
     - ✓ Growth models can address various linear and nonlinear patterns of intraindividual change

2. Analysis of causes (determinants) of intraindividual change
   - What factors/mechanisms – time-varying/dynamical predictors – drive change?
     - ✓ Inclusion of time-varying (dynamical) predictors in growth models

3. Identification of interindividual differences (or similarity) in intraindividual change
   - Do different individuals change in different ways?
     - ✓ Growth models are structured to address interindividual differences in the growth factors

4. Analysis of causes (determinants) of interindividual differences in intraindividual change
   - What factors explain between-person differences change?
     - ✓ Inclusion of time-invariant covariates in growth models, multigroup LGCM, Growth mixture models

5. Interrelationships in change
   - Does change in one variable related (*precede, covary and/or follow*) to change in another variable?
   - Correlating slopes of X and Y could provide evidence that one variable is changing in the same people as another variable – correlated changes! Or common changes!
     - ✓ Multi (parallel & sequentially contingent) process LGCM

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Features of longitudinal data

1. Longitudinal data distinguishes the levels of analysis
   i. Between-person relationships
      - Differences between people (interindividual differences)
        - How individual differences on one outcome are related to individual differences on another outcome
          - » Chronically stressed people have elevated mood problems than less chronically stressed people

   ii. Within-person relationships
      - Changes within a person relative to her/his own baseline
        - How differences in intraindividual changes on one outcome are related to intraindividual changes on another outcome
          - » Negative mood is more elevated than usual when someone is under more stress than usual

2. Longitudinal data distinguishes the data continuum
   i. Within-person fluctuation
      - Variations over repeated assessments in within-person moment-to-moment fluctuations among short term processes
        - People may fluctuate in stress and moods across days, weeks months or years

   ii. Within-person change
      - Systematic change as a function of time
        - Systematic increases or decreases as a function of time in long-term change
          - » Development of reading ability in children as a function of years in school
          - » Patient anxiety disorders improve as a function of time in metacognitive treatment

Features of longitudinal data

1. Longitudinal data distinguishes the levels of analysis
   i. Between-person relationships
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        – How individual differences on one outcome are related to individual differences on another outcome
          » Chronically stressed people have elevated mood problems than less chronically stressed people
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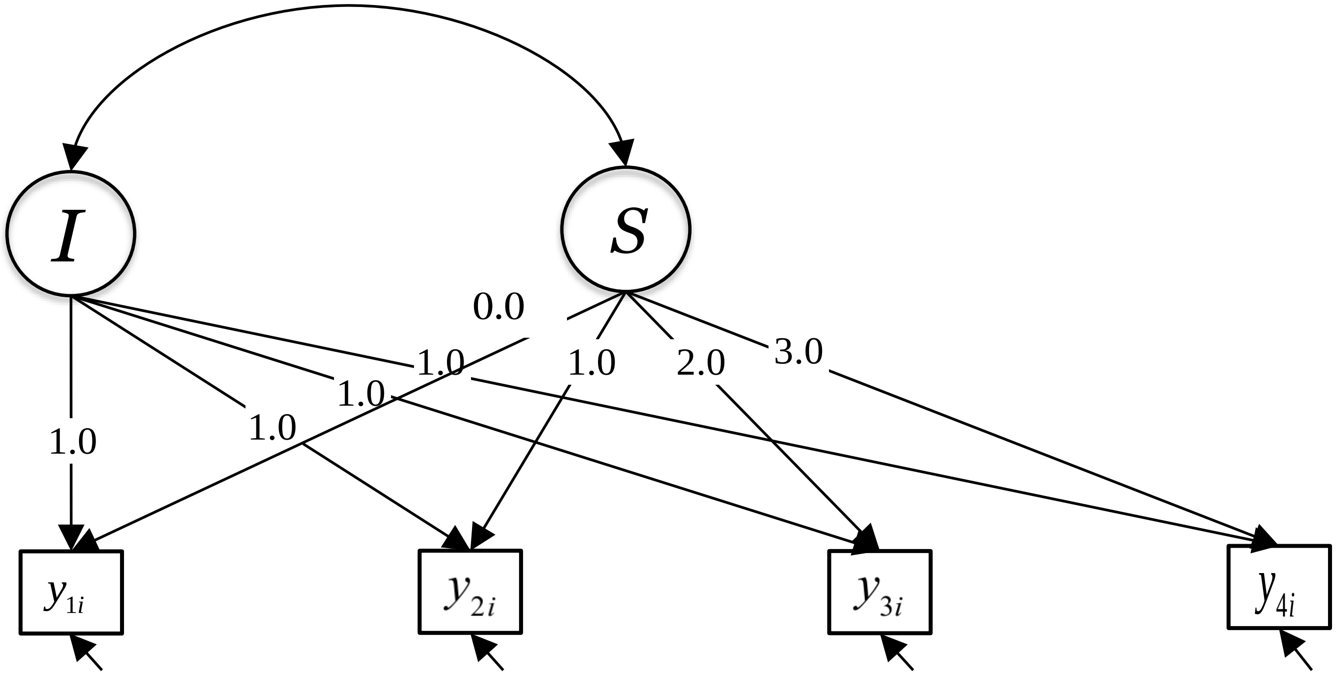
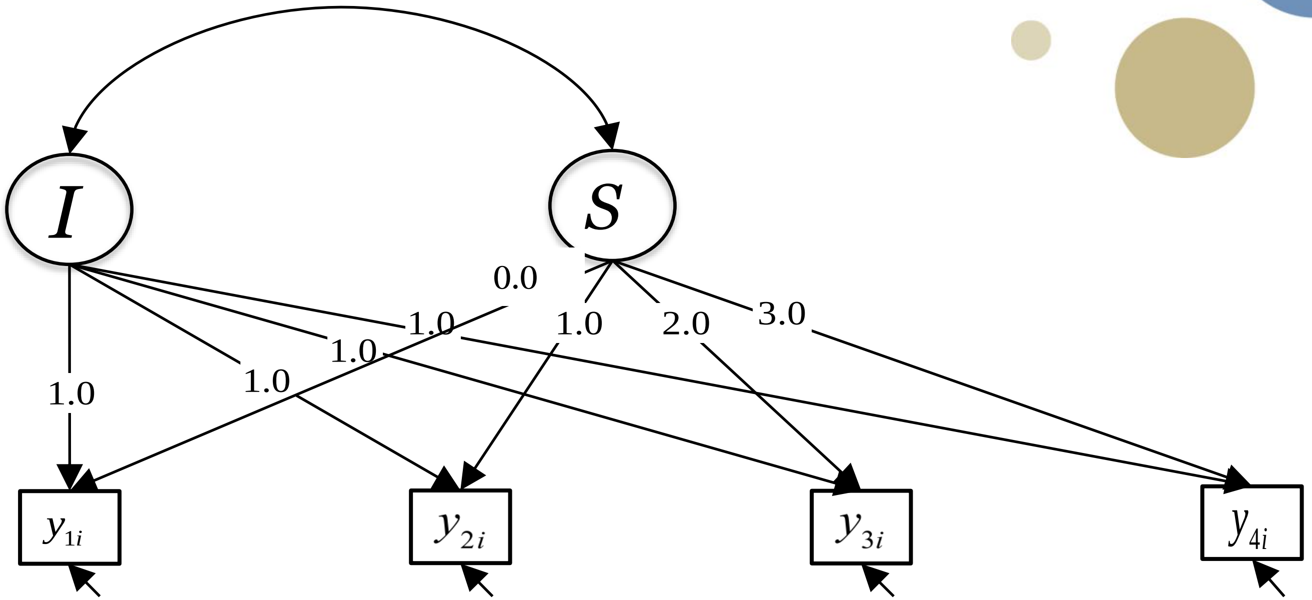
Modelling change over time



Modelling change over time

- Role of time scores
  - Time of observation as factor loading
  - Time scores determine
    1. Form/shape of growth process
    2. Centering point of growth process
    3. Scaling of growth factors



Factor loading matrix

1	0	Time 1
1	1	Time 2
1	2	Time 3
1	3	Time 4

I S | y1@0 y2@1 y3@2 y4@3

Factor loading matrix

1	-3	Time 1
1	-2	Time 2
1	-1	Time 3
1	0	Time 4

I S | y1@-3 y2@-2 y3@-1 y4@0

I S | y1@-1 y2@0 y3@1 y4@2; centering at Time 2

Factor loadings should be proportionate to time intervals/spacing

Factor loading matrix

1	0	Time 1
1	1	Time 2
1	2	Time 3
1	3	Time 4

Factor loading matrix

1	0	Month 1
1	1	Month 6
1	2	Month 12
1	3	Month 18
1	4	Month 24

Factor loading matrix (unequal time interval)

1	0	Month 1
1	1	Month 2
1	2	Month 3
1	3	Month 4
1	4	Month 5
1	5	Month 6
1	6	Month 7

Factor loading matrix

1	-5	Month 1
1	-4	Month 2
1	-3	Month 3
1	-2	Month 4
1	-1	Month 5
1	0	Month 6

I S | y1@0 y2@1 y3@2 y4@3)

I S | y1@0 y2@1 y3@2 y4@3 y5@4
I S | y1@0 y2@6 y3@12 y4@18 y5@24

Either one is correct, same mode fit indices

I S | y1@0 y2@1 y3@3 y4@4 y5@6 [No observations on Months 3 & 6, time scores specified to match observed time points]

I S | y1@-5 y2@-4 y3@-3 y4@-2 y5@-1 y6@0 [End of observation defined as centering point; intercept status; predicted level of y at end of obs.]

<u>Free time score/Unspecified LGCM/Model estimated time score = empirically determines shape of outcome growth</u>

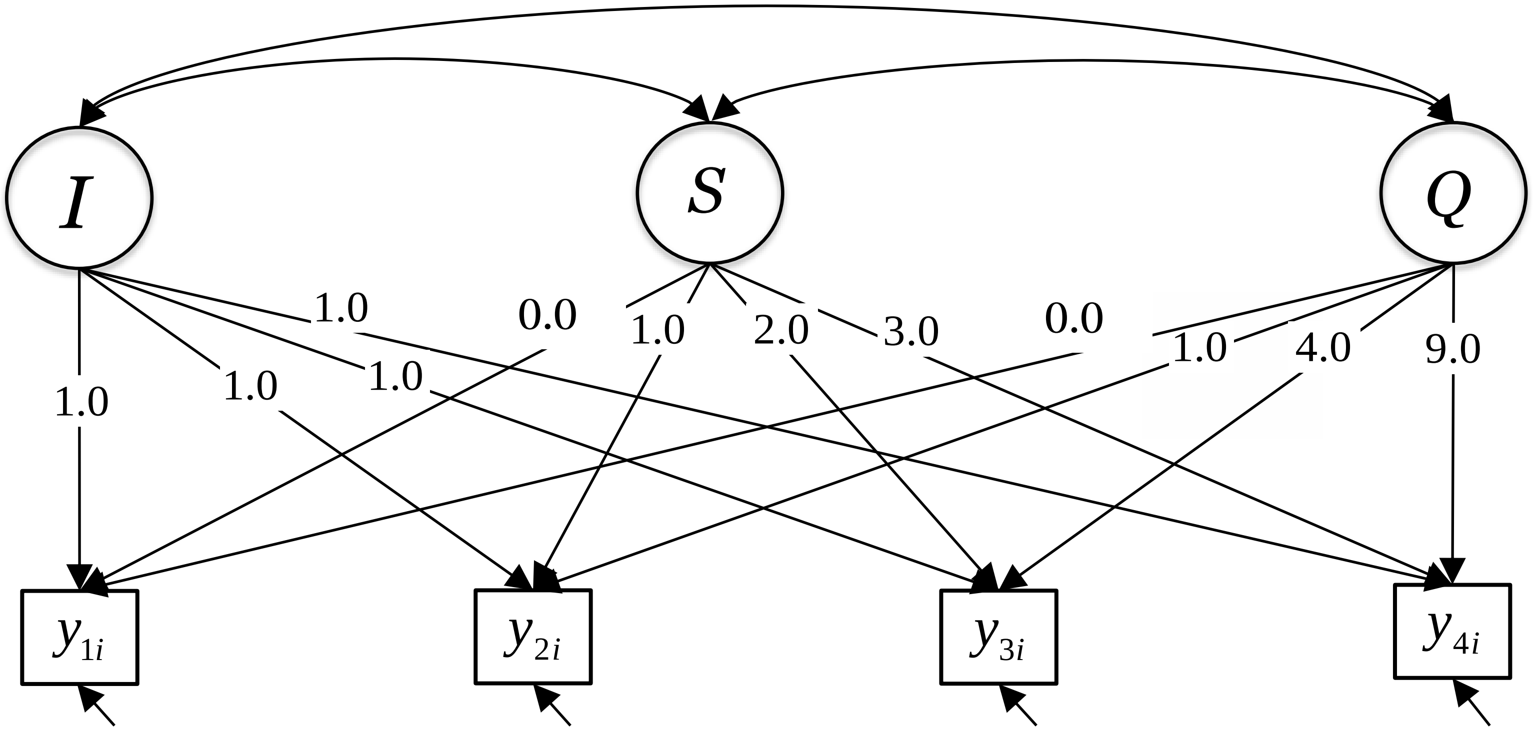
I S | y1@0 y2@1 y3* y4* y5*

<u>Latent basis model/Unstructured growth model</u>

I S | y1@0 y2* y3* y4@1

- Freely estimated parameters represent the proportion of predicted overall change that occurred up to a specific point in time eg Time 2, for y2*
- The Slope growth factor represents the total amount of change from the first to the last timepoints

Non-linear change (e.g., quadratic)



Factor loading matrix

1	0	0	Time 1
1	1	1	Time 2
1	2	4	Time 3
1	3	9	Time 4

(I S Q| y1@0 y2@1 y3@2 y4@3)

Dataset

Variable Names	Time 1 (2015)	Time 2 (2016)	Time 3 (2018)	Time 4 (2019)	Time 5 (2021)
Id	id				
Female	female				
Parental drug use	pdu1				
Adverse childhood environment	ace1				
Worry	worry1				
Loneliness	lone1	lone2	lone3	lone4	lone5
Anxiety symptoms	anx1	anx2	anx3	anx4	anx5
Depression symptoms	dep1	dep2	dep3	dep4	dep5
Hostility	host1	host2	host3	host4	host5
Peer conflict					percon5
Peer conflict					percon5b
Substance use					subs5
Substance use					subs5b
Difficulty sleeping					sleep5
Difficulty sleeping					sleep5b

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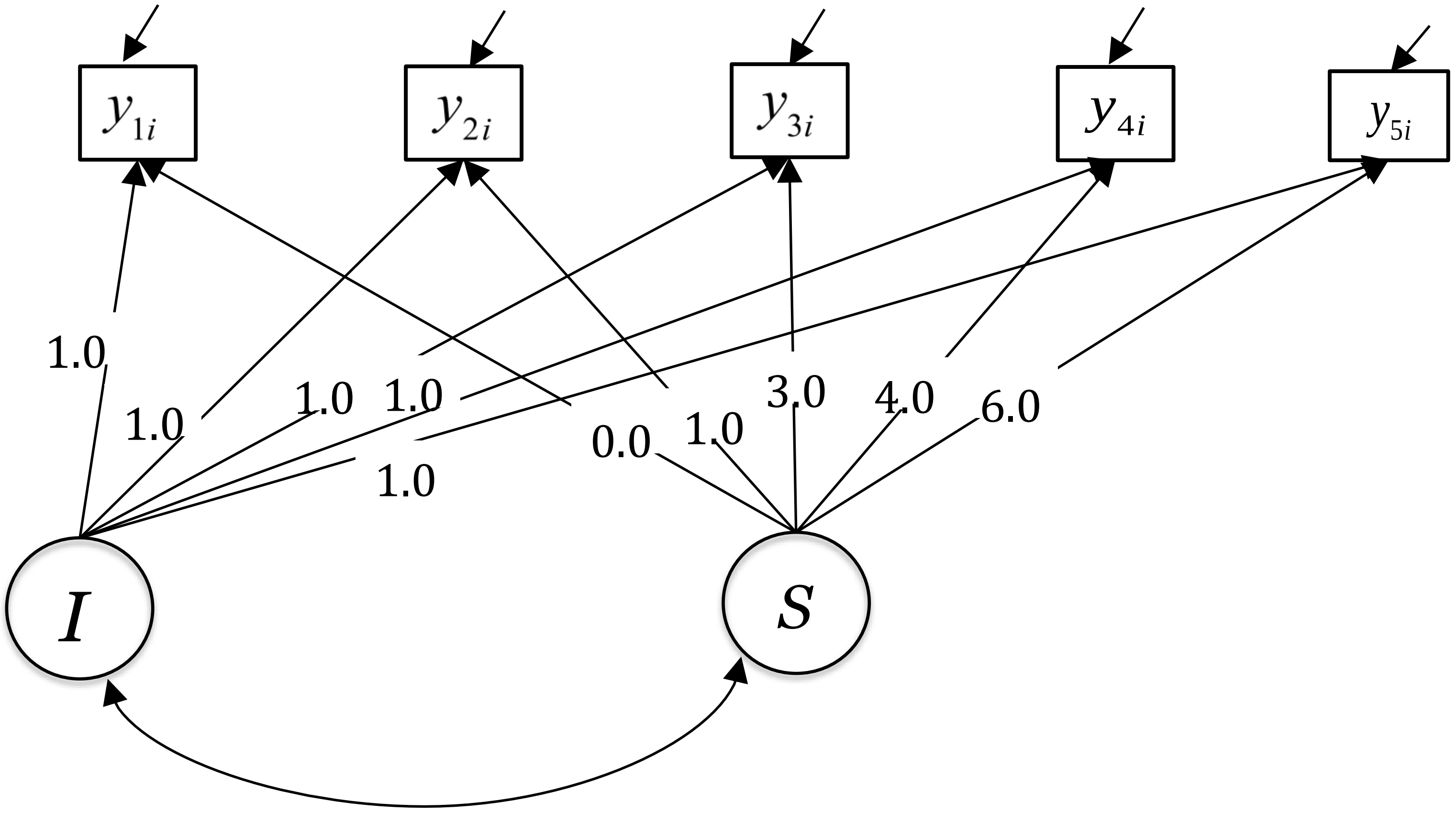
- What does the overall trajectory of loneliness look like?

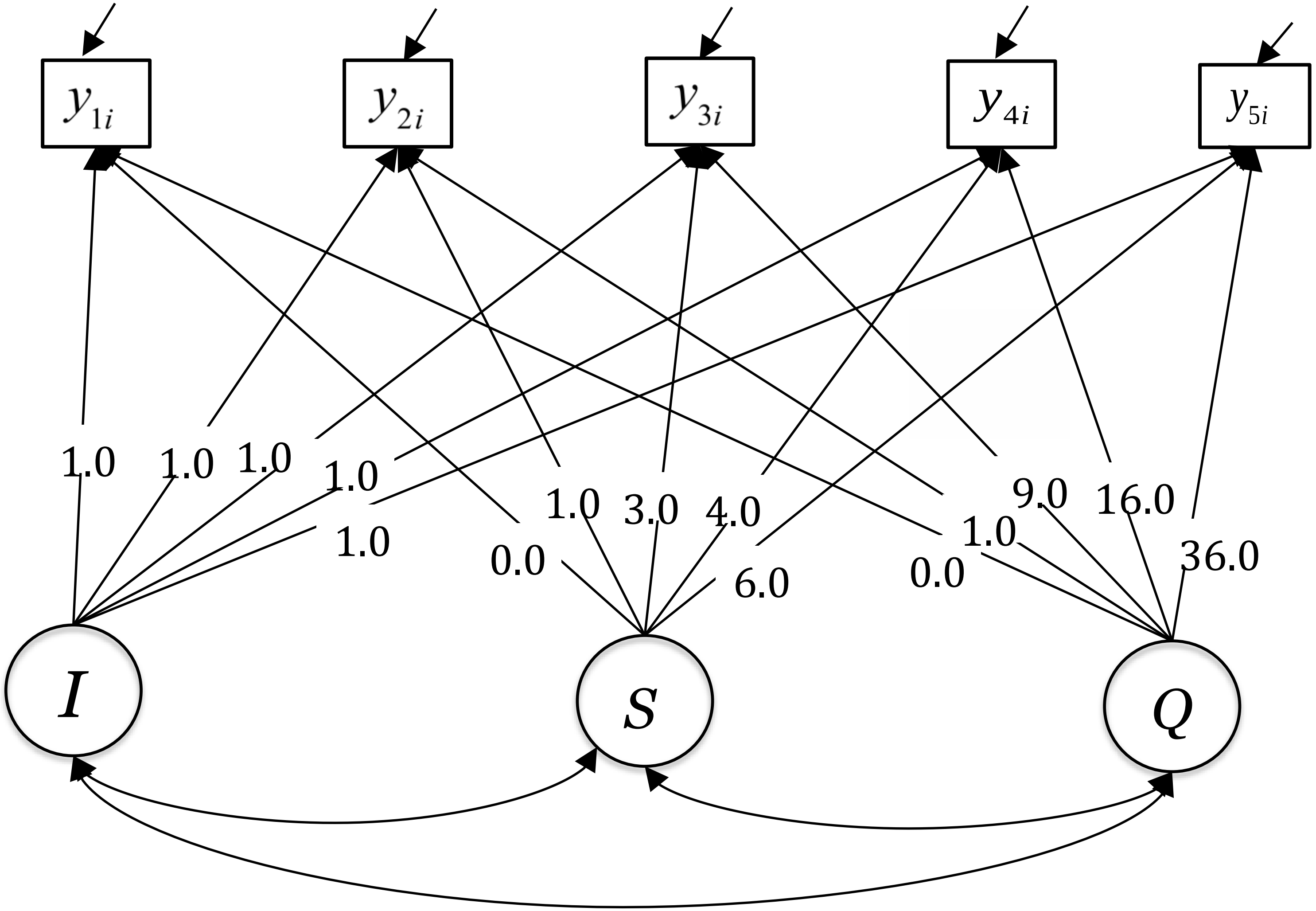
  - Is there change over time on average?
    - Do different individuals change in different ways, or everybody change the same?

Linear growth



Unconditional Linear LGCM

Quadratic growth



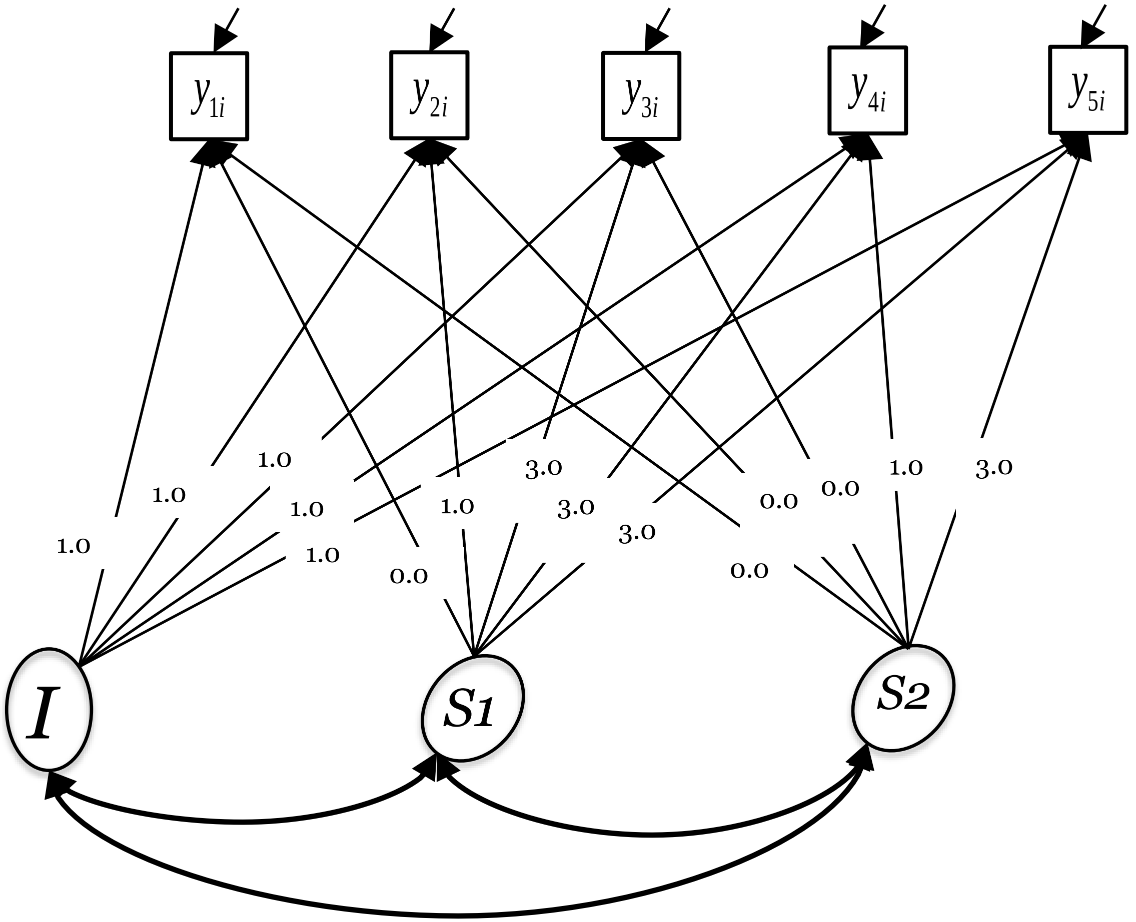
Unconditional Quadratic LGCM

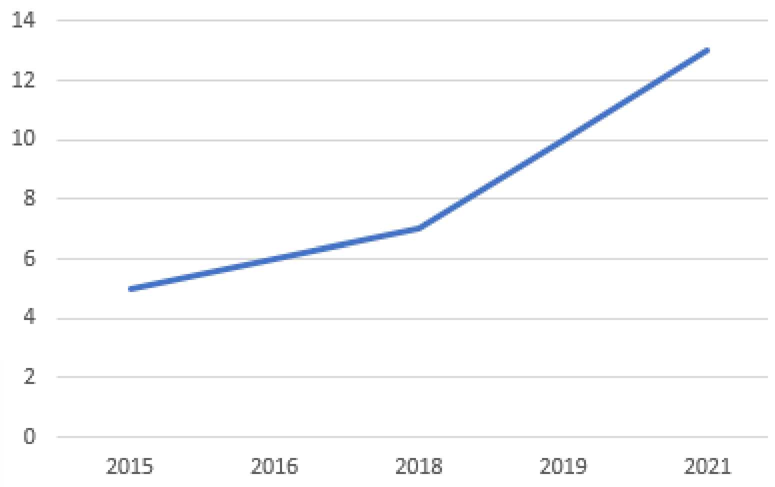
Spline/Piecewise/Multiphase LGCM

- When theoretical reasons suggest to separate time/development into discrete phases
    - Trajectory of loneliness before divorce and post-divorce
    - Trajectory of depression before and after intervention
    - Pre-crawling and post-crawling in children
    - Childhood and adolescence

- Different phases of development are captured by more than one slope growth factor
    - Knots or transition points show where one phase is ending, and another phase is beginning

Bilinear Spline/Piecewise/Multiphase LGCM



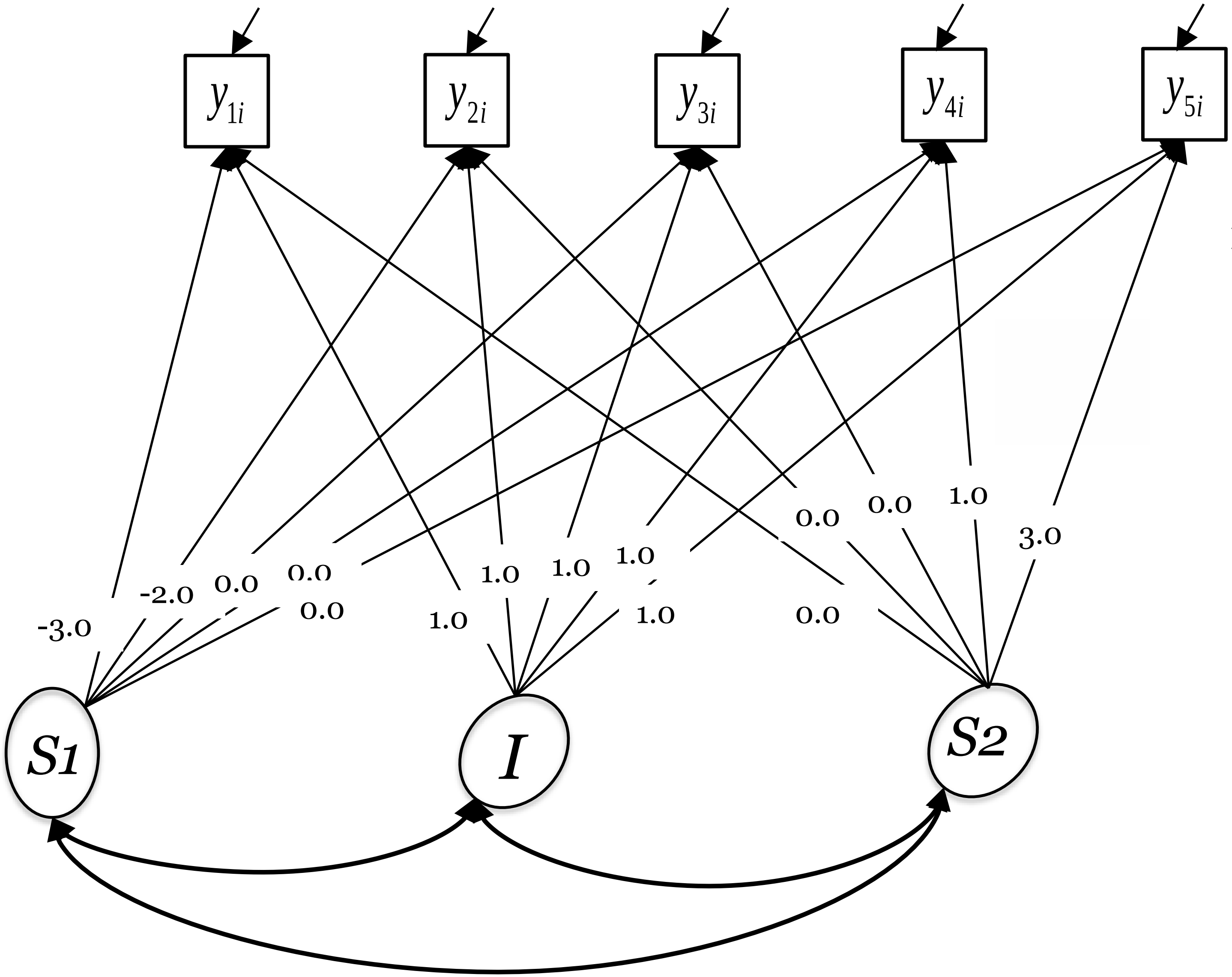


Intercept at Time 1, knot at Time 3

1	0	0	Time 1, 2015
1	1	0	Time 2, 2016
1	3	0	Time 3, 2018
1	3	1	Time 4, 2019
1	3	3	Time 5, 2021

I = Predicted average value of outcome in 2015
S1 = Average rate of change in outcome from 2015 to 2018
S2 = Average rate of change in outcome from 2018 to 2021

Spline/Piecewise/Multiphase LGCM - 2

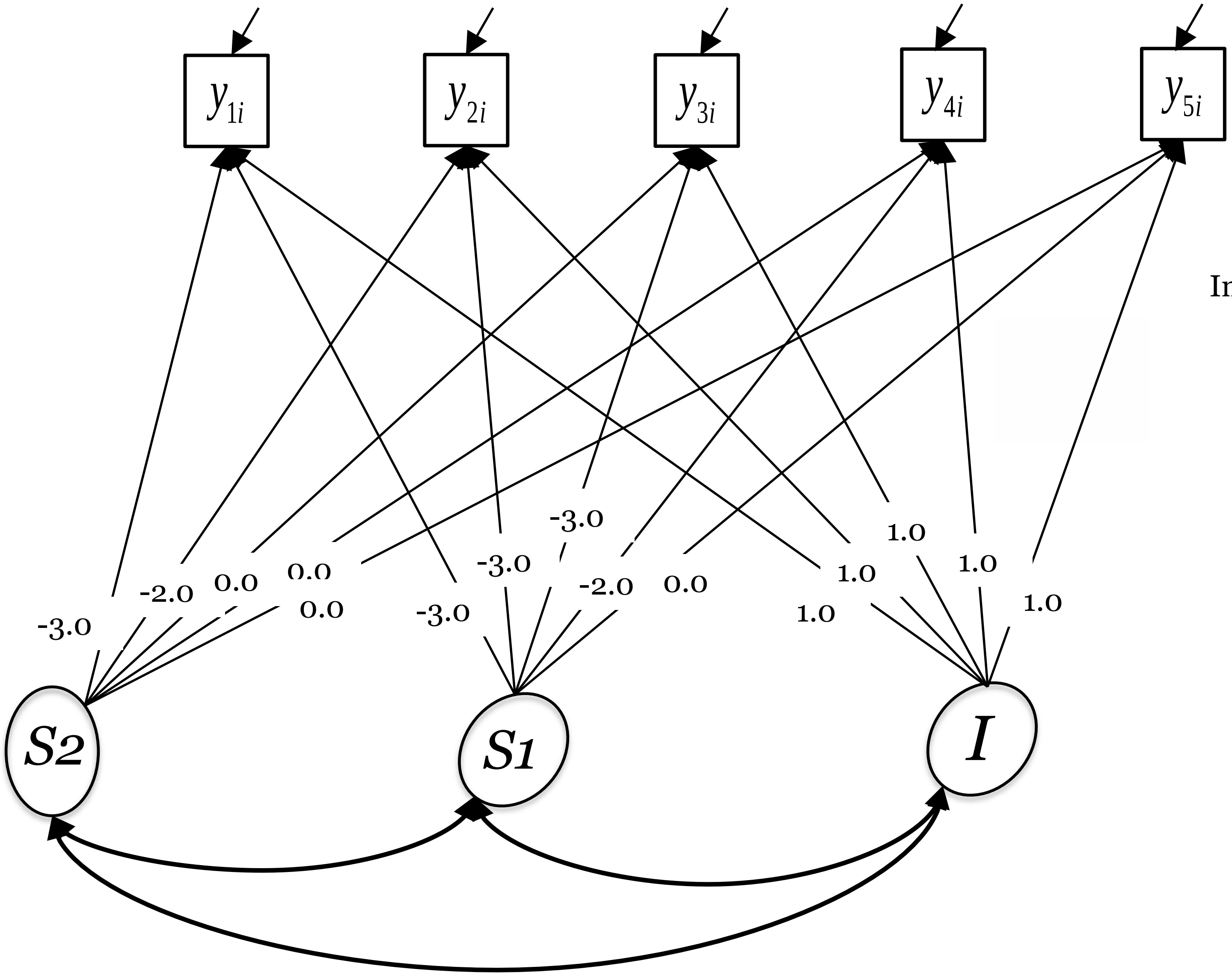


Intercept at Time 3, knot at Time 3

-3	1	0	Time 1, 2015
-2	1	0	Time 2, 2016
0	1	0	Time 3, 2018
0	1	1	Time 4, 2019
0	1	3	Time 5, 2021

I = Predicted average value of outcome in 2018
S1 = Average rate of change in outcome prior to 2018
S2 = Average rate of change in outcome from 2018 to 2021

Spline/Piecewise/Multiphase LGCM - 3



Intercept at Time 5, knot at Time 3

-3	-3	1	Time 1, 2015
-2	-3	1	Time 2, 2016
0	-3	1	Time 3, 2018
0	-2	1	Time 4, 2019
0	0	1	Time 5, 2021

I = Predicted value of outcome at end of observation, 2021
S1 = Average change in outcome from 2018 through to end of observation
S2 = Average change in outcome prior to 2018

Model comparisons and selection

Model		Compared with	χ^2	*df*	*RMSEA*	*CFI*	*TLI*
M1	No growth		193.118	13	.178[0.157, 0.201]	.679	.753
M2	Linear growth	M1	70.819	10	.118[0.093, 0.145]	.892	.892
M3	Quadratic growth	M2	42.671	6	.118[0.086, 0.153]	.935	.891
M4	Bilinear spline growth	NON-NESTED!	31.360	6	.098[0.066, 0.134]	.955	.925

Model Comparison

	Model M1 No growth	Model M2 Linear growth	Model M3 Quadratic growth	Model M4 Bilinear spline
Parameters	7	10	14	
-2LL	2276.104	2153.804	2125.656	
AIC	2290.103	2173.804	2153.656	2142.345
BIC	2318.647	2214.581	2210.743	2199.432
Δ parameters	-	3	4	
Δ -2LL	-	122.300***	28.148***	

- Do not compare nested models whose fit indices do not reach acceptable, e.g. M1 and M2
- Bilinear is a non-nested model
  - Use AIC/BIC

Objectives of longitudinal research

1. Identification of intraindividual (within-person) changes (and stability)
    - Is there change over time on average?
        - ✓ Growth models can address various linear and nonlinear patterns of intraindividual change

2. Analysis of causes (determinants) of intraindividual change
    - What factors/mechanisms – time-varying/dynamical predictors – drive change?
        - ✓ Inclusion of time-varying (dynamical) predictors in growth models

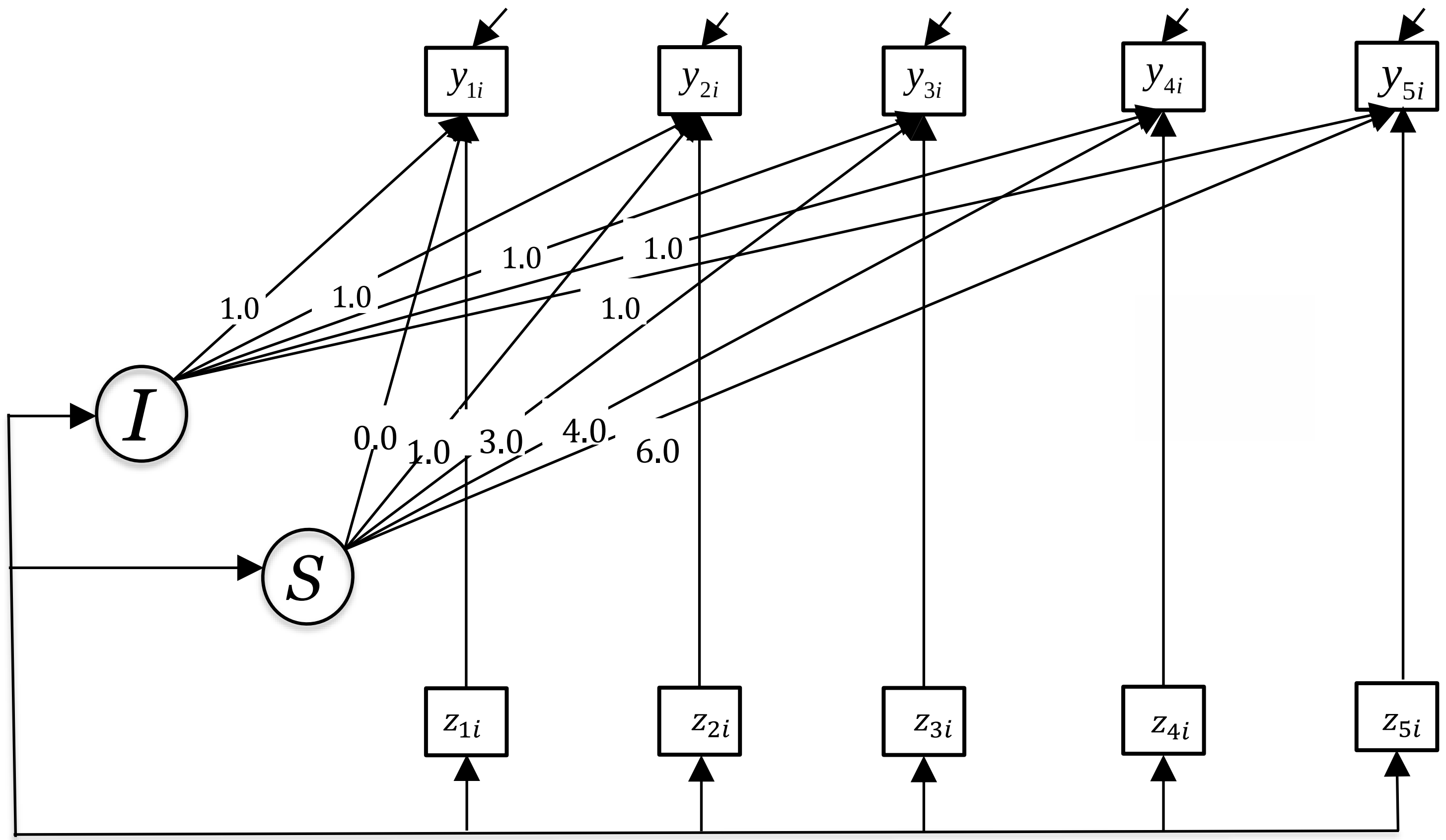
3. Identification of interindividual differences (or similarity) in intraindividual change
    - Do different individuals change in different ways?
        - ✓ Growth models are structured to address interindividual differences in the growth factors

4. Analysis of causes (determinants) of interindividual differences in intraindividual change
    - What factors explain between-person differences in change?
        - ✓ Inclusion of time-invariant covariates in growth models, multigroup LGCM, Growth mixture models

5. Interrelationships in change
    - Does change in one variable relate (*precede, covary and/or follow*) to change in another variable?
    - Correlating slopes of X and Y could provide evidence that one variable is changing in the same people as another variable – correlated changes! Or common changes!
        - ✓ Multi (parallel & sequentially contingent) process LGCM

- What dynamics or factors drive within-person changes?
  - Does hostility affect the trajectory of loneliness?
    - Loneliness is more elevated than usual when someone is more hostile than usual?

Conditional Linear LGCM, LGCM with Time-varying covariates



- TVC is exogenous to the developmental process
- The effect of TVC is a within-person effect which influences an individual's change trajectory
- Flexibility in SEM allows correlating TVC with I and S – growth factors.
  - Convergence problems
    - *Fix covariance between TVC with I and S to zero*

Objectives of longitudinal research

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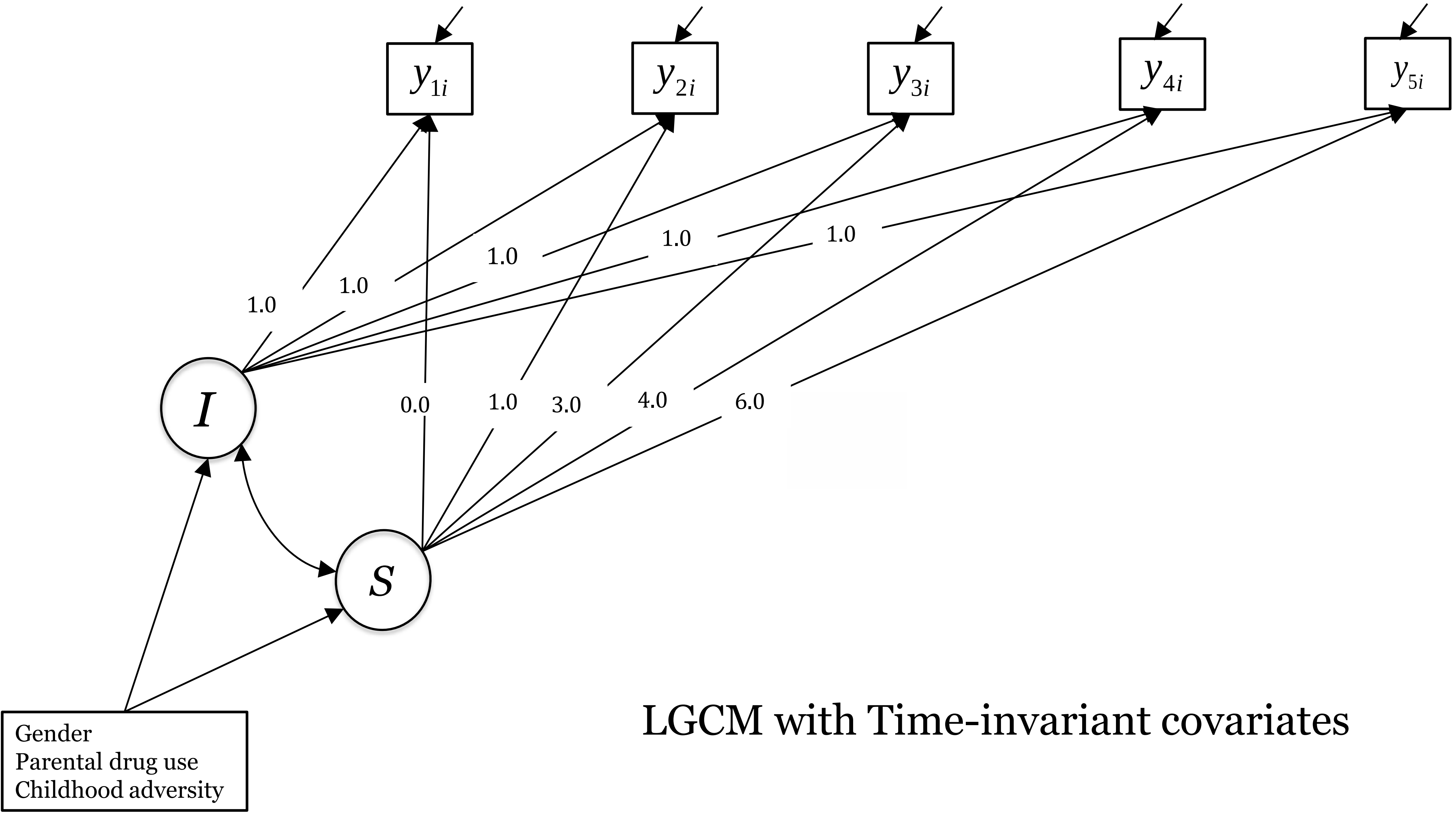
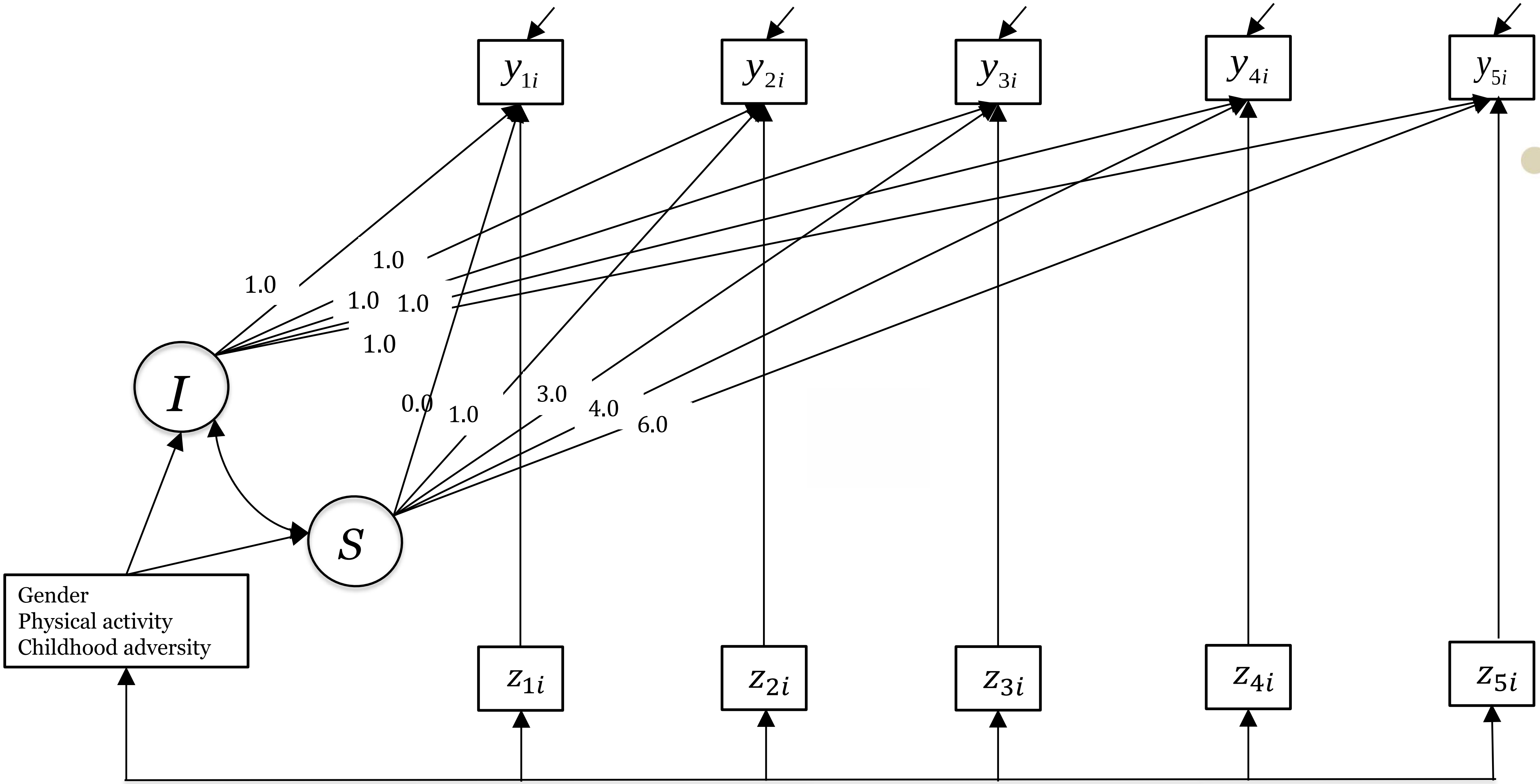
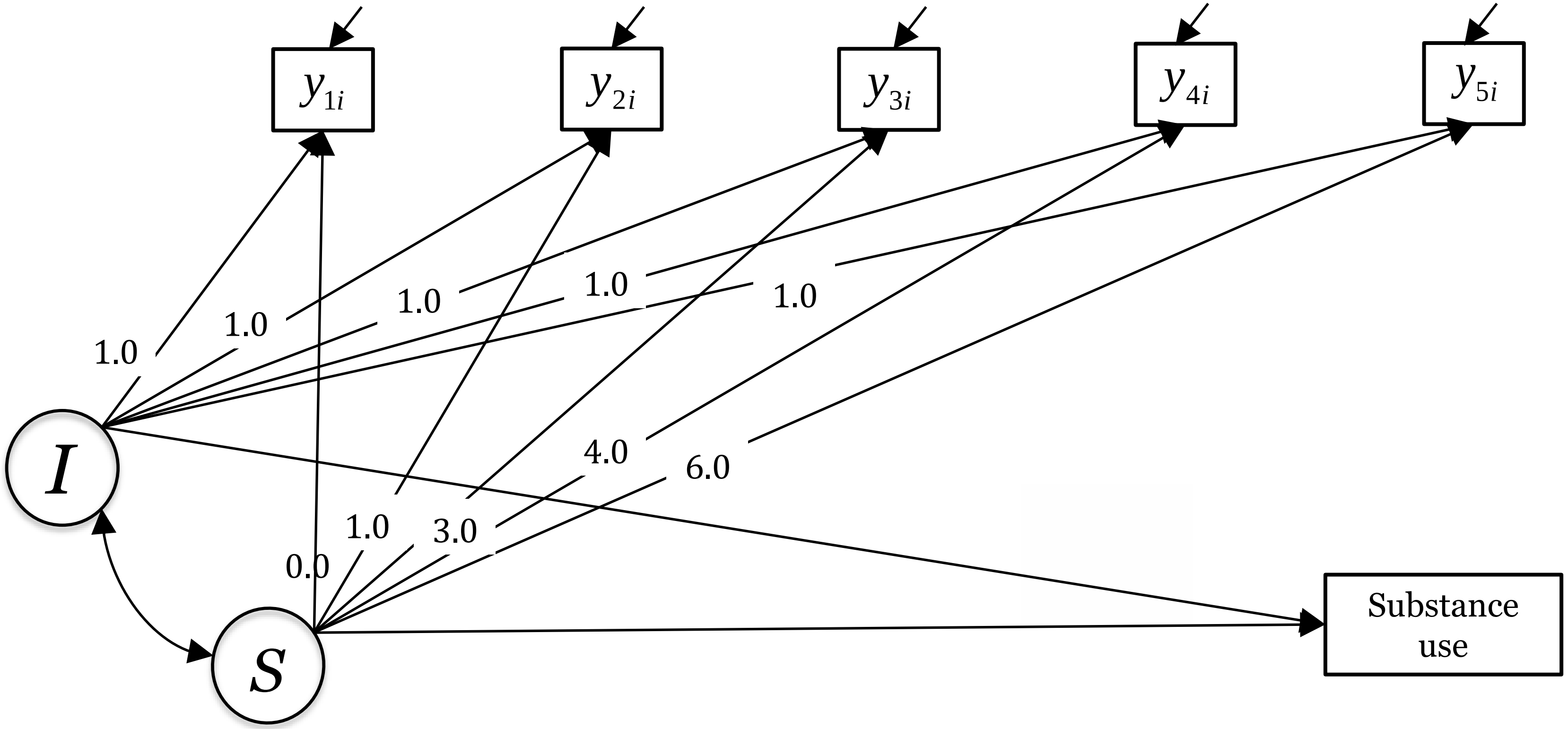
- What factors explain between-person difference in change?
  - Does gender influence between-person differences in the trajectory of loneliness?



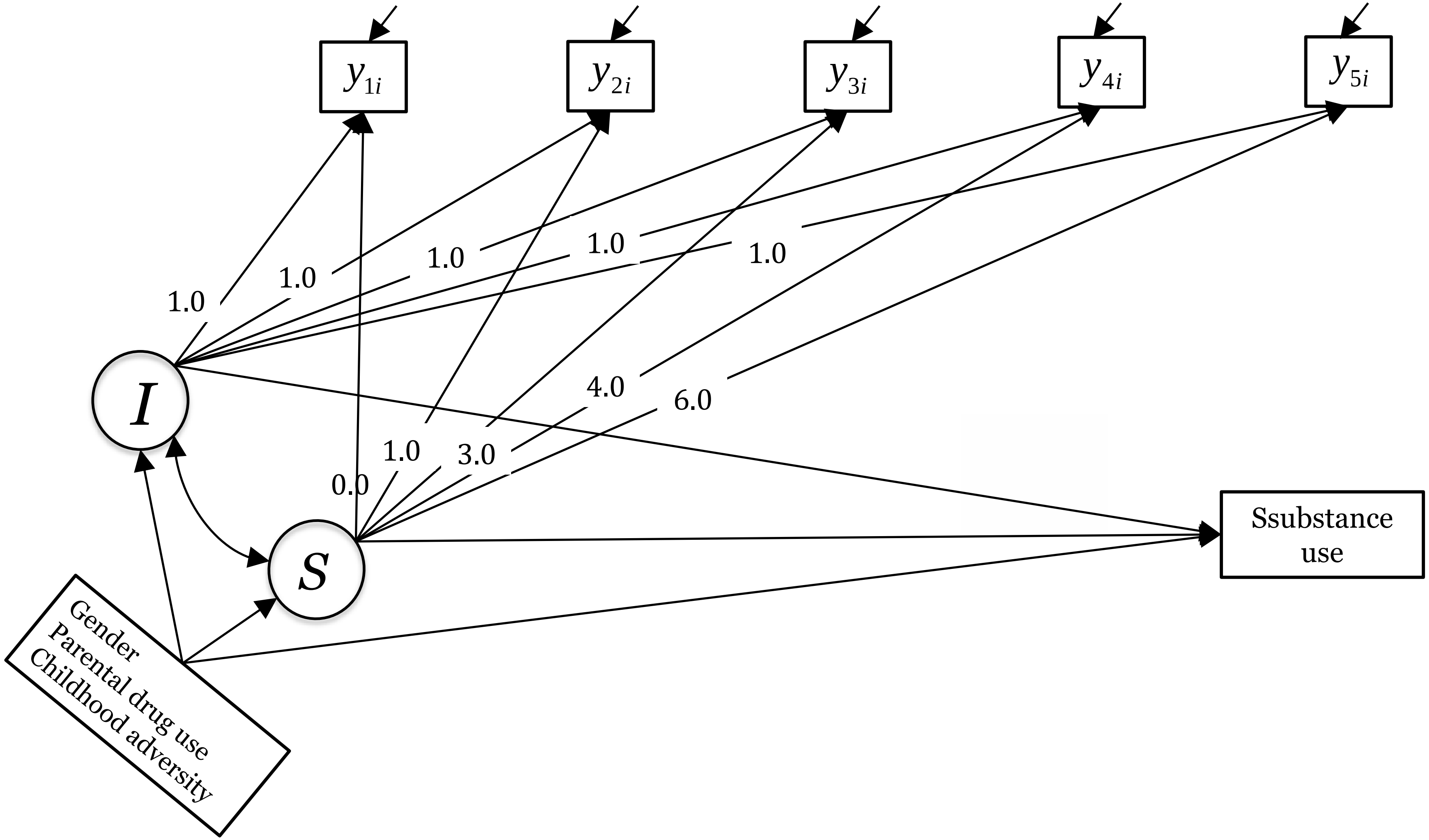
LGCM with Time-invariant covariates



LGCM with Time-invariant and time-varying covariates



LGCM growth factors predicting distal outcome



LGCM with Time-invariant covariates and distal outcome

Multigroup LGCM

- How is it different from using time-invariant covariate model?
  - Time-invariant covariate (TIC) models examine the difference in the average growth trajectory
  - TIC models do not examine aspects of the growth model related to
    - Group differences in the
      i. Average trajectory and invariance testing
      ii. Variability and covariability of growth trajectories
      iii. Unexplained within-person variability

- Multigroup LGCM models can examine differences in any aspect of growth trajectory for known, measured or predefined groups
  - Growth mixture models are used for unknown or unmeasured groups that may exist as subpopulations in the data

Multigroup LGCM

- What mechanisms or factors account for between-person differences in the trajectory of loneliness between boys and girls?
    i. Do boys and girls differ in their trajectories of loneliness?
    ii. What is the extent of between-person differences in the average trajectory of boys and girls?
    iii. What is the extent of variability around the individual trajectories of boys and girls?

Model comparisons

Model Comparison for the Multiple-Group LGCM

	Model M1 Invariance model	Model M2 Means model	Model M3 Means and covariances model	Model M4 Means, covariances and residual variances model
Parameters				
-2LL				
Δ parameters				
Δ -2LL				

Sequence of testing

- M1: Constrain all parameters (growth factor means, growth factor variances and covariances, and residual variances) to be identical across groups
- M2: Freely estimate growth factor means across groups, while keeping constraints on the growth factor variances and covariance, and residual variances
- M3: Freely estimate growth factor means, growth factor variances and covariances, while keeping the constraints on the residual variances
- M4: Freely estimate growth factor means, growth factor variances and covariances, and residual variances across the groups

Questions to answer

- M1 vs. M2: Tests whether the groups differ in their average trajectory
- M2 vs. M3: Tests the extent of between-person differences in the trajectories of groups
- M3 vs. M4: Tests the extent of within-person variability around individual trajectories
    - If all models fit similarly, choose the model with the smallest number of parameters, M1 – most parsimonious model as it is the most constrained/reduced form

Model comparisons

Model Comparison for the Multiple-Group LGCM

	Model M1 Invariance model	Model M2 Means model	Model M3 Means and covariances model	Model M4 Means, covariances and residual variances model
Parameters	6	8	11	20
-2LL	2024.982	2010.644	1992.510	1925.744
Δ parameters	-	2	3	9
Δ -2LL	-	14.338**	18.134***	66.766***

Sequence of testing

- M1: Constrain all parameters (growth factor means, growth factor variances and covariances, and residual variances) to be identical across groups
- M2: Freely estimate growth factor means across groups, while keeping constraints on the growth factor variances and covariance, and residual variances
- M3: Freely estimate growth factor means, growth factor variances and covariances, while keeping the constraints on the residual variances
- M4: Freely estimate growth factor means, growth factor variances and covariances, and residual variances across the groups

Questions to answer

- M1 vs. M2: Tests whether the groups differ in their average trajectory
- M2 vs. M3: Tests the extent of between-person differences in the trajectories of groups
- M3 vs. M4: Tests the extent of within-person variability around individual trajectories
    - If all models fit similarly, choose the model with the smallest number of parameters, M1 – most parsimonious model as it is the most constrained/reduced form

Males and females differ in their average growth trajectories of loneliness, the extent of between-person differences in those trajectories and the extent of variability around their individual trajectories

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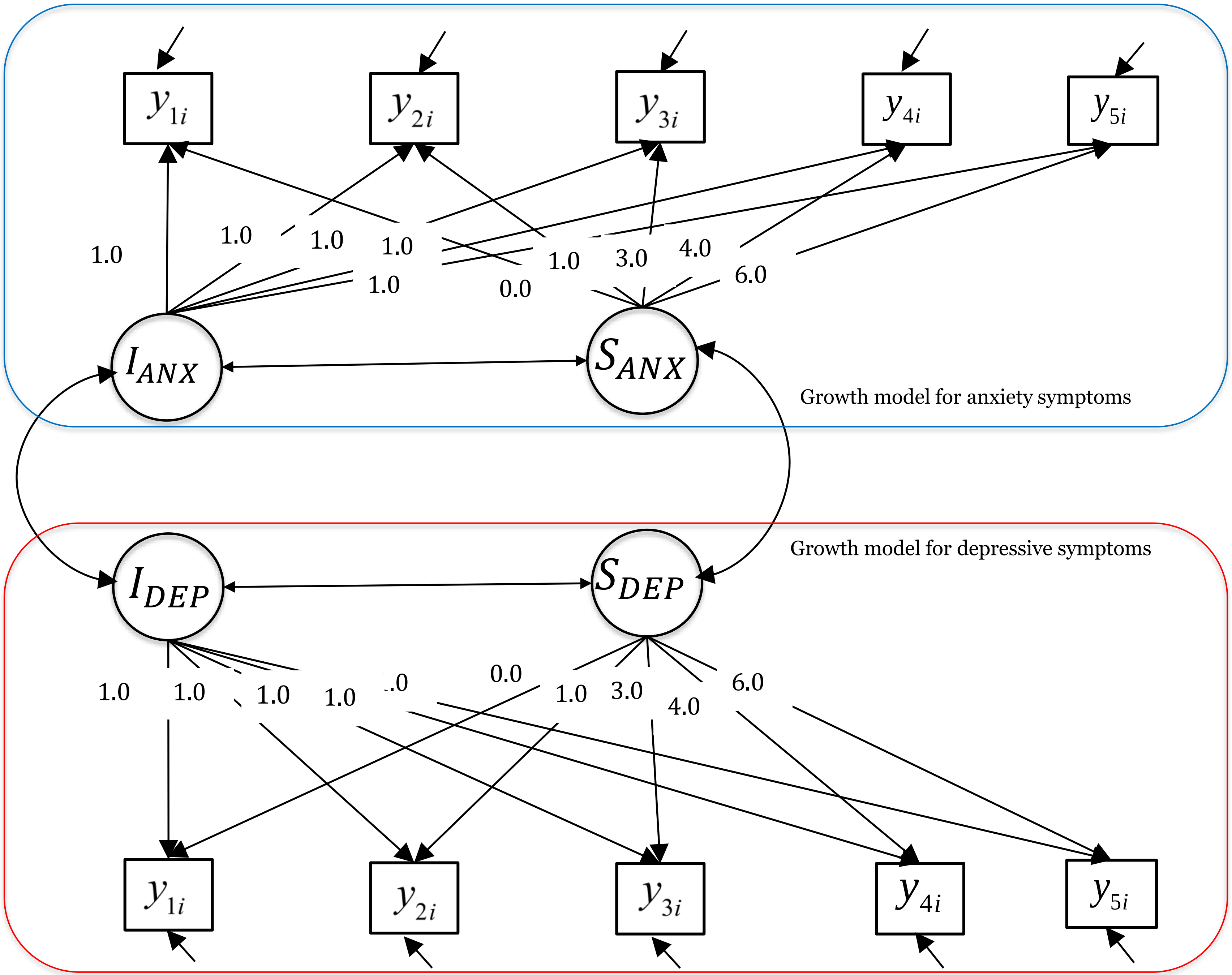
     - ✓ Multi (parallel & sequentially contingent) process LGCM

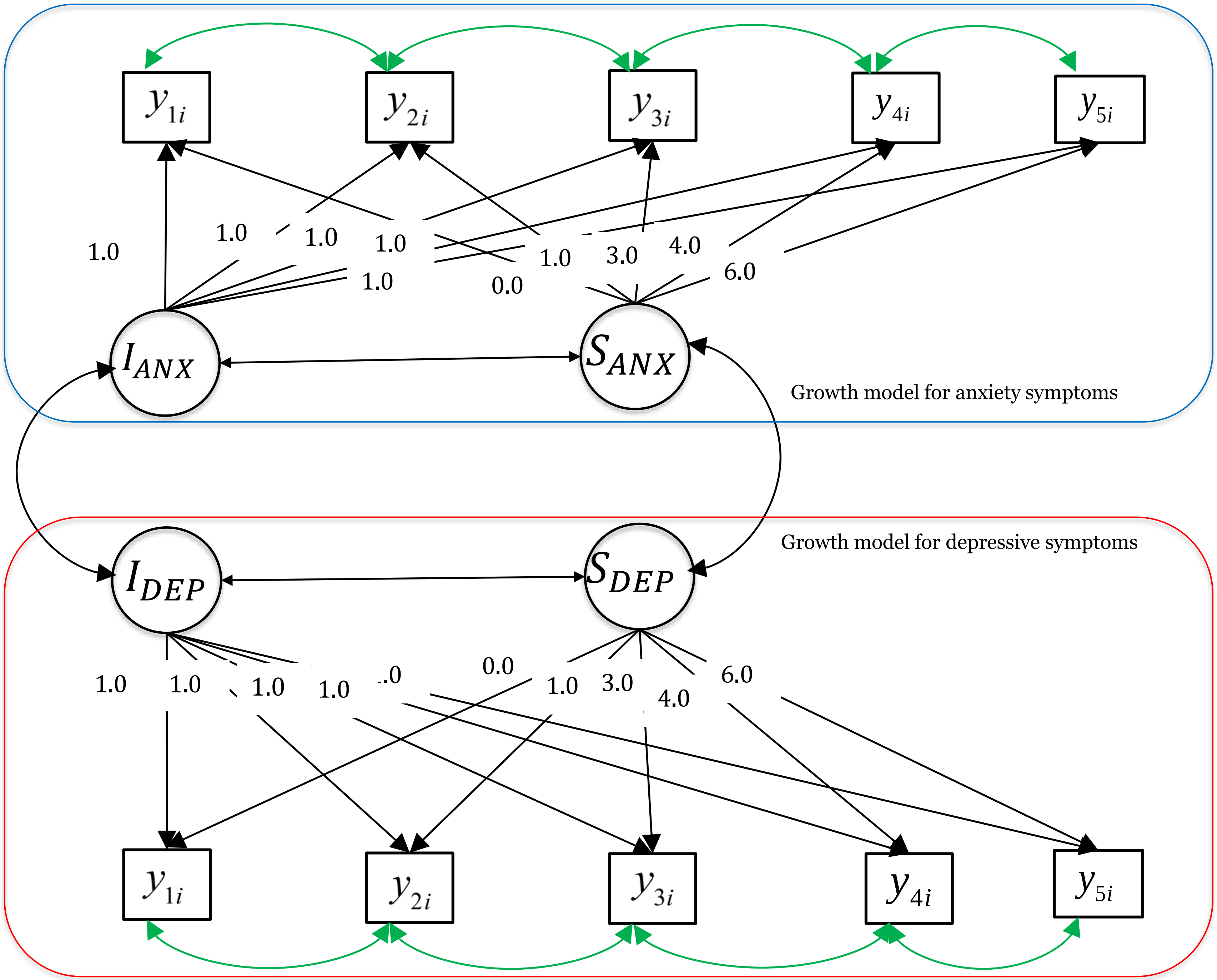
Multi Process LGCM

- Simultaneously model multiple outcomes
  1. Parallel LGCM
     - Covariance between two intercepts
       – Information about association between the two outcomes at the beginning of observation period
     - Covariance between two slopes
       – How rate of change in the two outcomes are associated with each other

  2. Sequential LGCM
     - Change in one subdomain, e.g. Anxiety symptoms (T1-T5) predicting change in another subdomain, e.g. depressive symptoms (T6-T10)



- Study distinct but related subdomains
- Developmental associations (co-development) between anxiety and depressive symptoms
- Non-directional associations represent co-development/occurrence

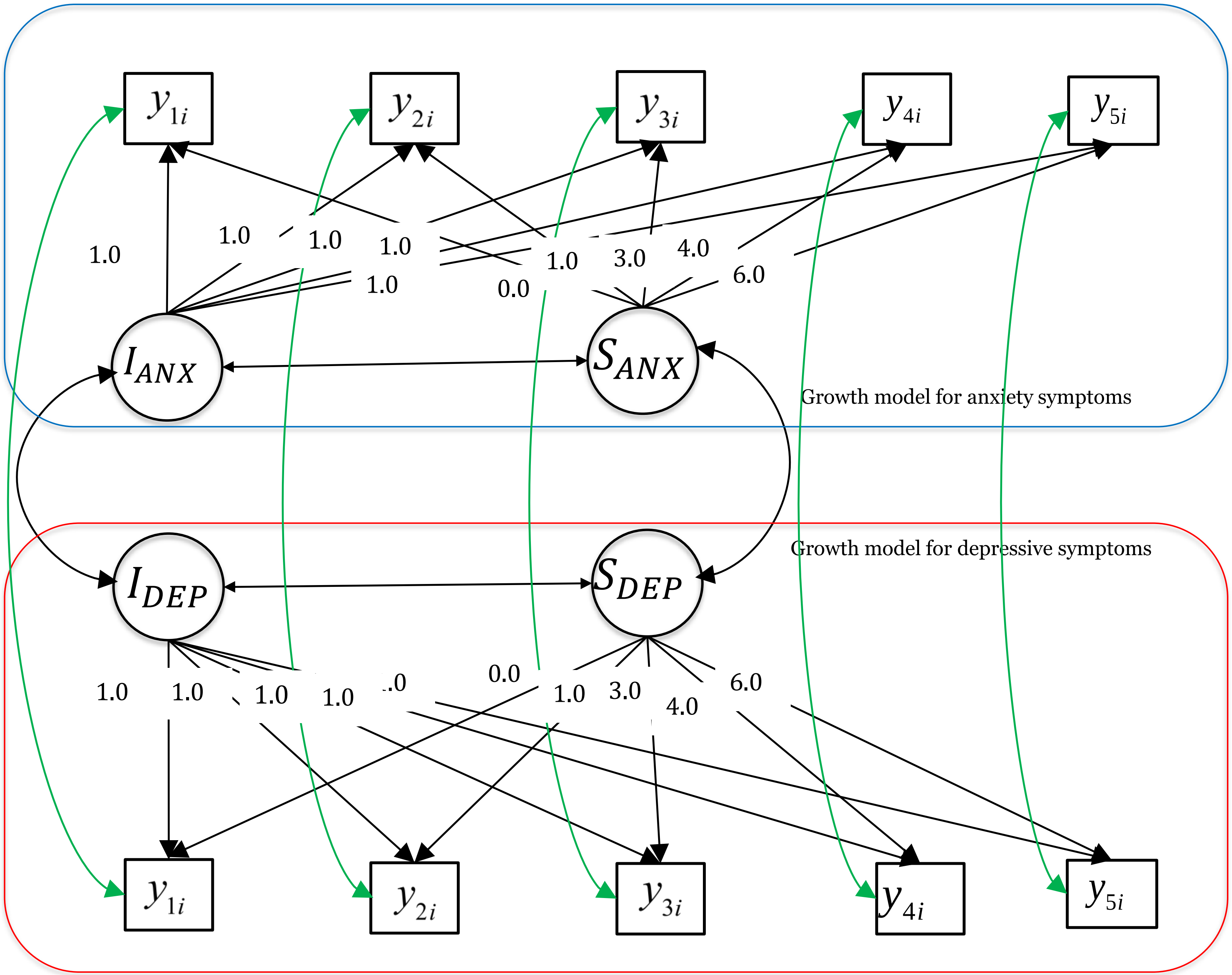


- Each measure consist of

  i. Item-specific variance component
  ii. Time-specific variance component

ii. Item-specific variance component = measurement error (within subdomain error correlations – autocorrelated errors)

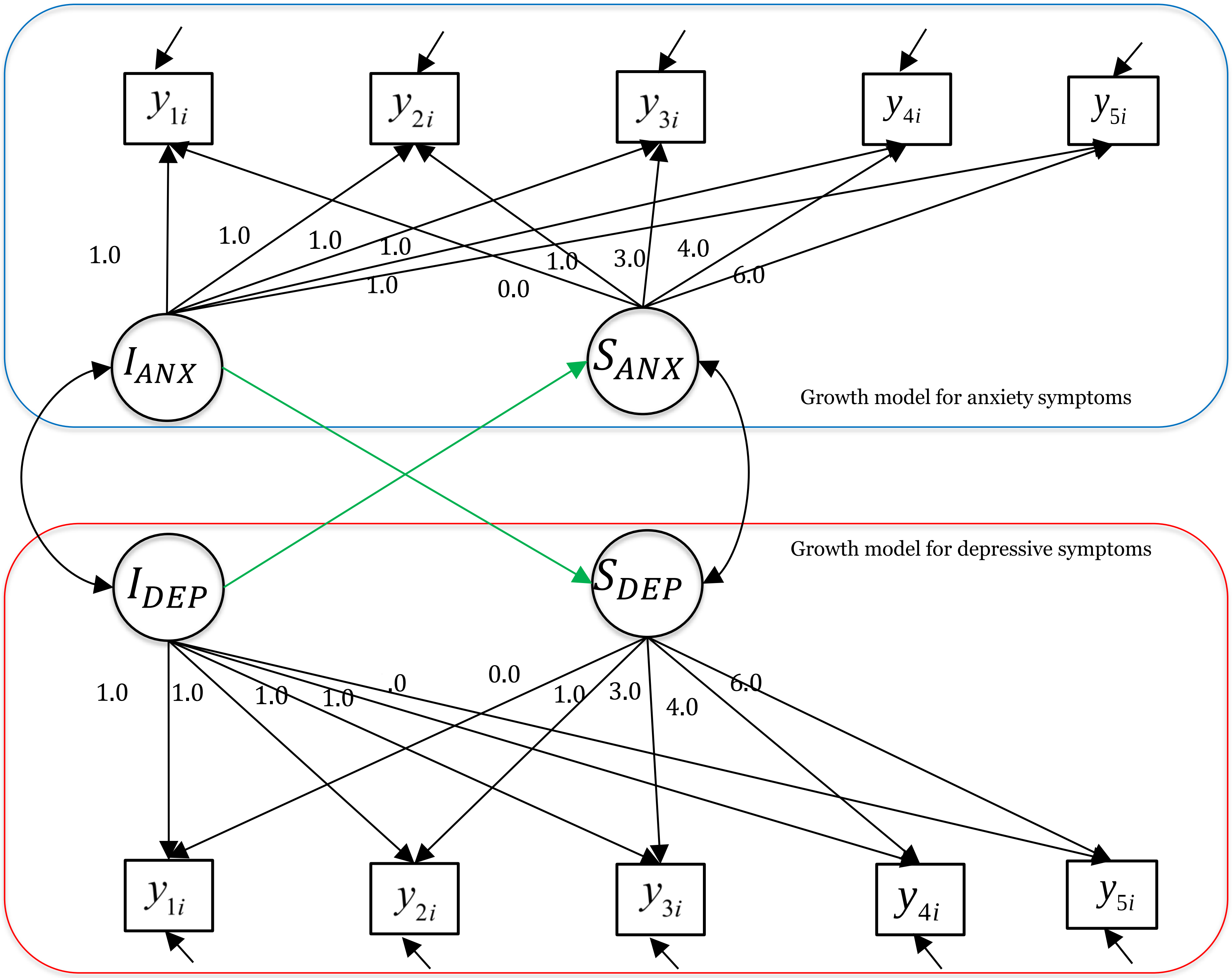
- Autocorrelated error capture unique and methodological biases within a measure (e.g., due to repeated administration, reporting biases)



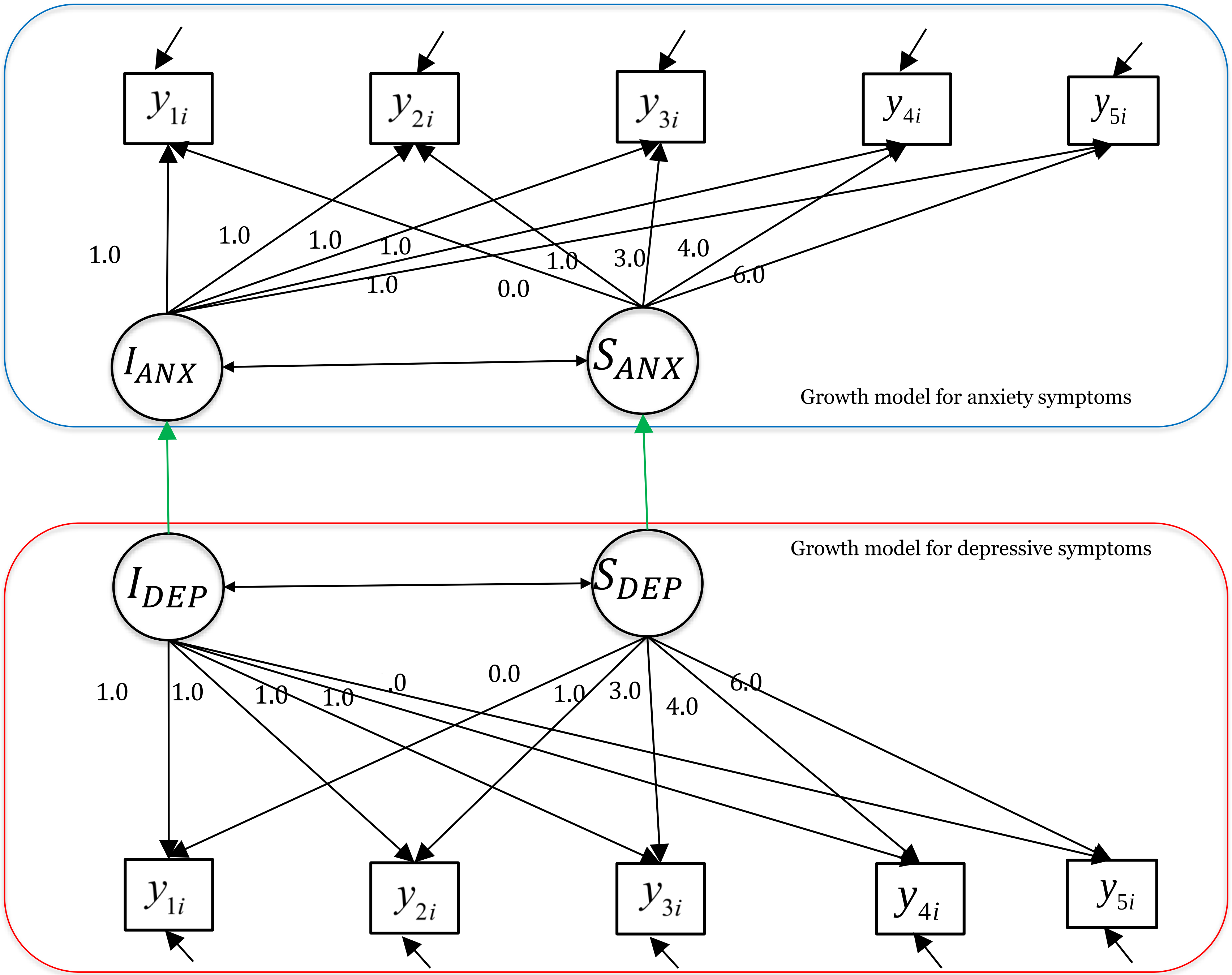
- Each measure consist of

  i. Item-specific variance component
  ii. Time-specific variance component

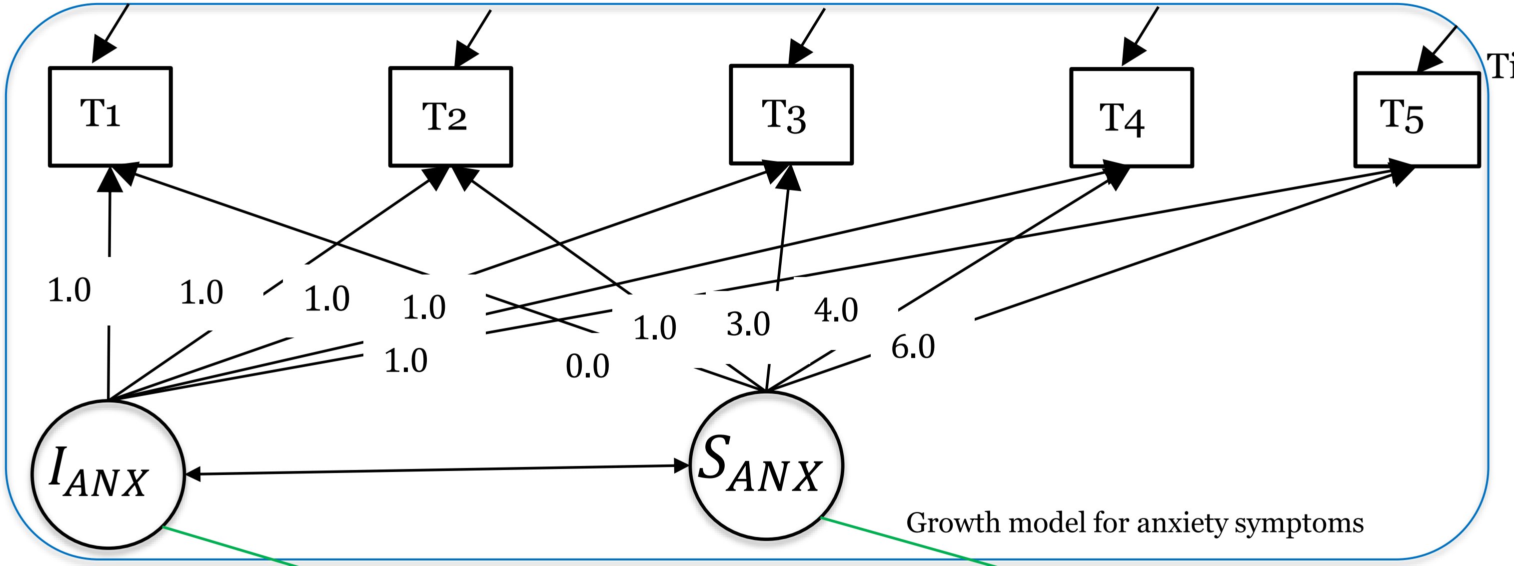
i. Time-specific variance component = between subdomain error correlations
   - Attributed to time-specific trends/patterns reporting biases (e.g., reporting lower level of symptoms at first timepoint)



- Parallel growth process with regressions of the slope growth factor
  - Higher levels of anxiety symptoms influence the growth rate of depressive symptoms?



- Parallel growth process with uni-directional effects
  - Evidence of co-development
  - Influence of growth factors of one subdomain on the growth factors of another subdomain



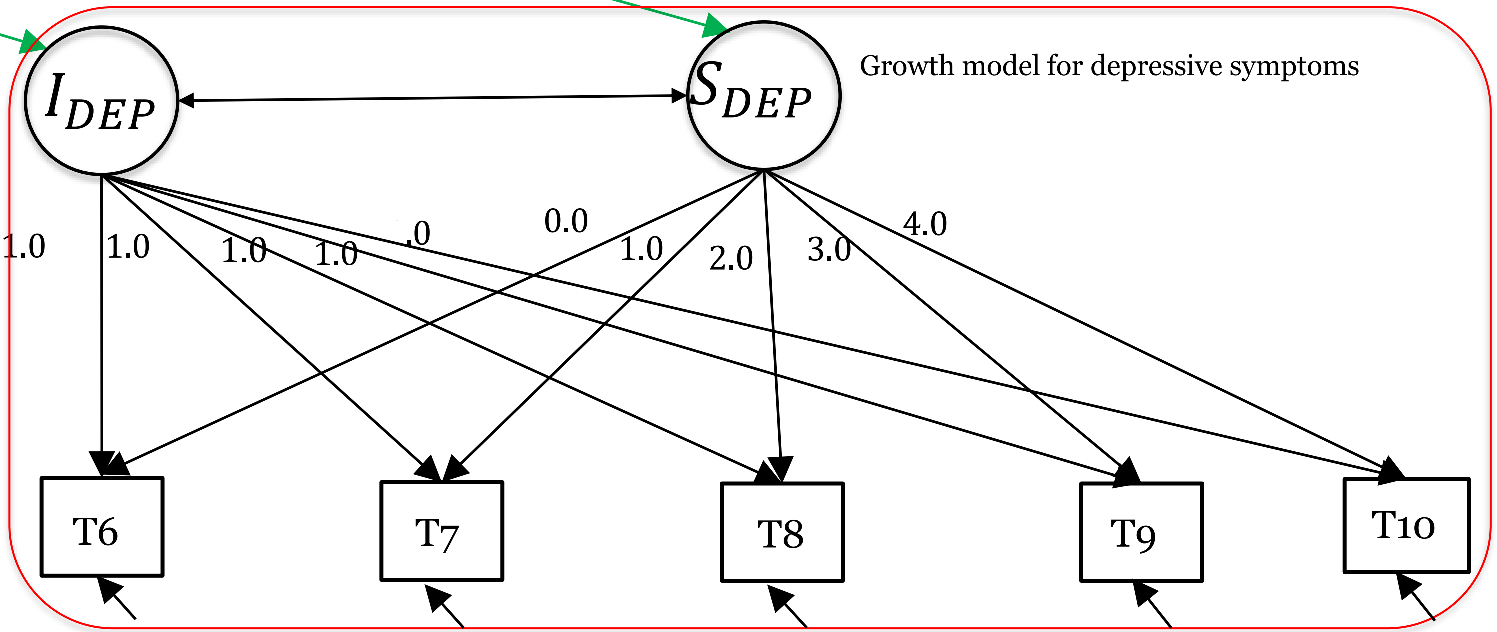
Time scores/factor loading matrix

1	0	Time 1, 2010
1	1	Time 2, 2011
1	3	Time 3, 2013
1	4	Time 4, 2014
1	6	Time 5, 2016

Time scores/factor loading matrix

1	0	Time 1, 2017
1	1	Time 2, 2018
1	2	Time 3, 2019
1	3	Time 4, 2020
1	4	Time 5, 2021

- Sequentially contingent process over time
  - One growth trajectory influencing a later growth trajectory
  - Change in one subdomain (T1-T5) predicting change in another subdomain(T6-T10)
  - Life-course studies



Growth Mixture Models (GMM)

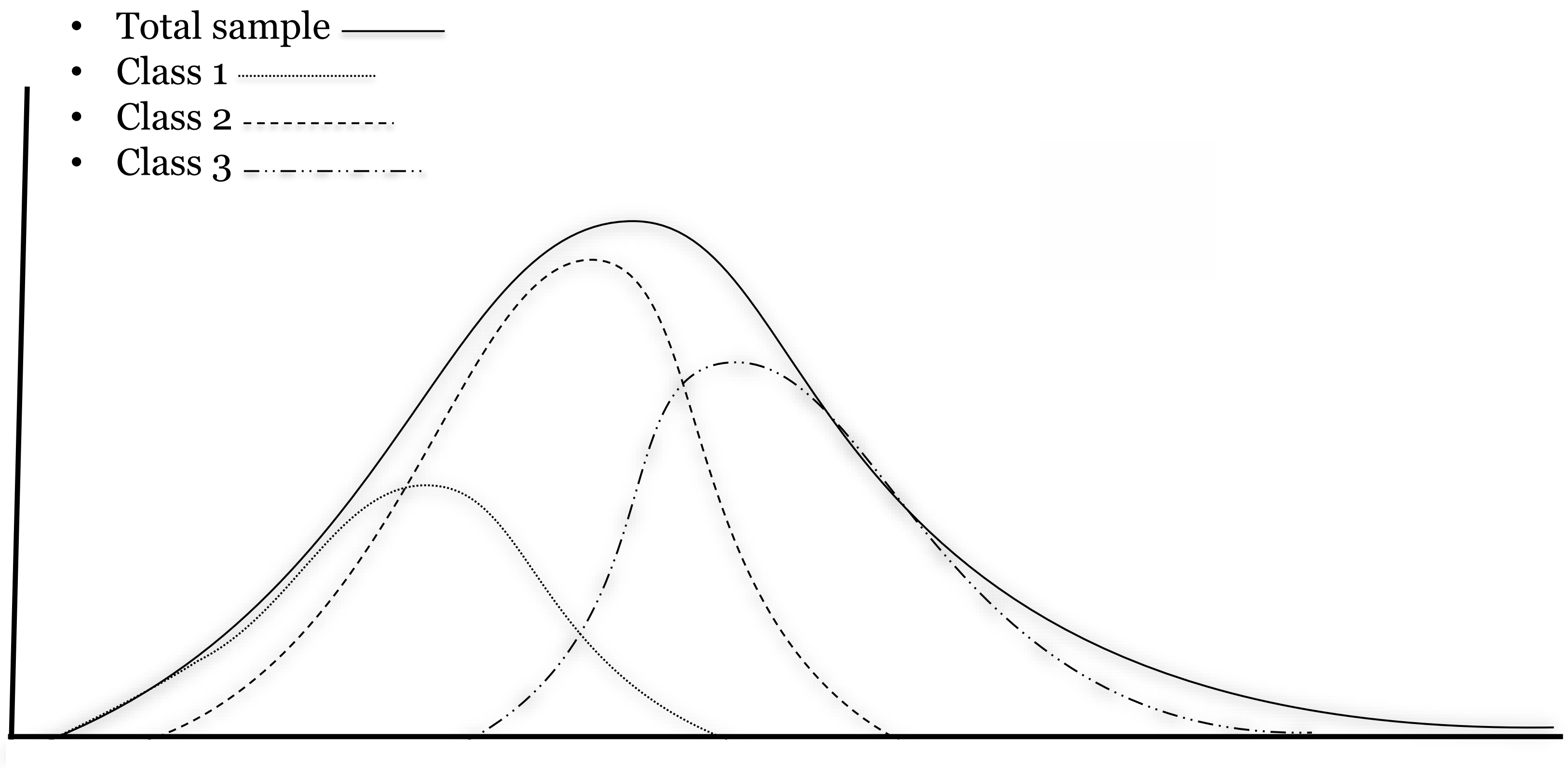
- Limitations of LGCM in the context of heterogenous subpopulations
  - LGCM assumes all individuals have a common trajectory shape/growth
  - Effect of covariates on growth factors is the same for all individuals

- Growth mixture model
  - A latent class extension of LGCM
    - A "mixture" of latent class trajectories
  - Where trajectory heterogeneity exists
    - GMM searches for distinct subsamples/class in the dataset with heterogeneous intercept, slopes and variance components
      - Individuals in the same subsample/class have similar trajectories
      - Individuals in different subsample/class have different trajectories

      - Example based on rate of change
        - » Escalating, Stable, Recovering level of anxiety symptoms

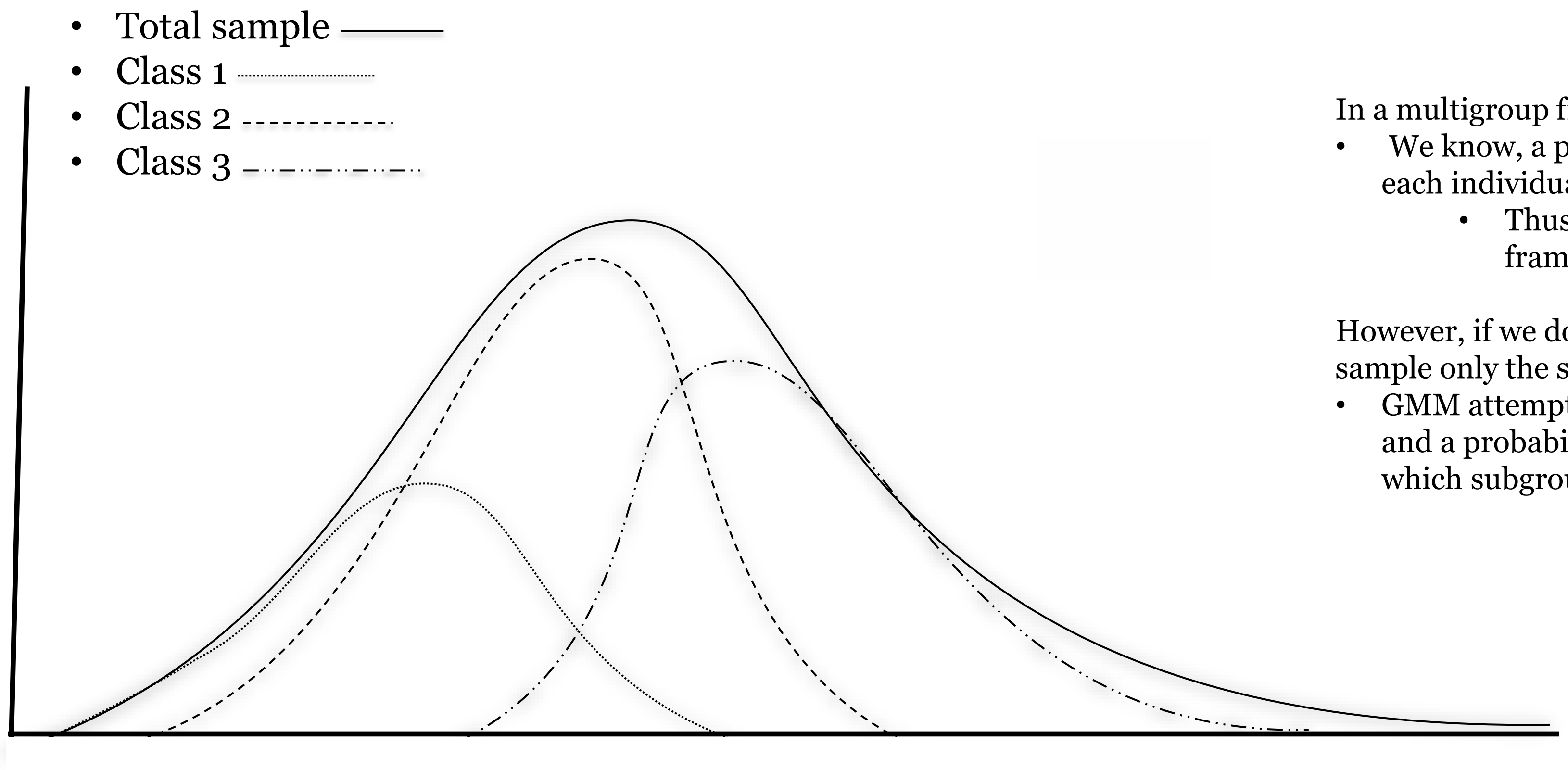
The general idea of GMM

The distribution of an intercept
or slope for 3-classes



The general idea of GMM

How different from Multigroup LGCM



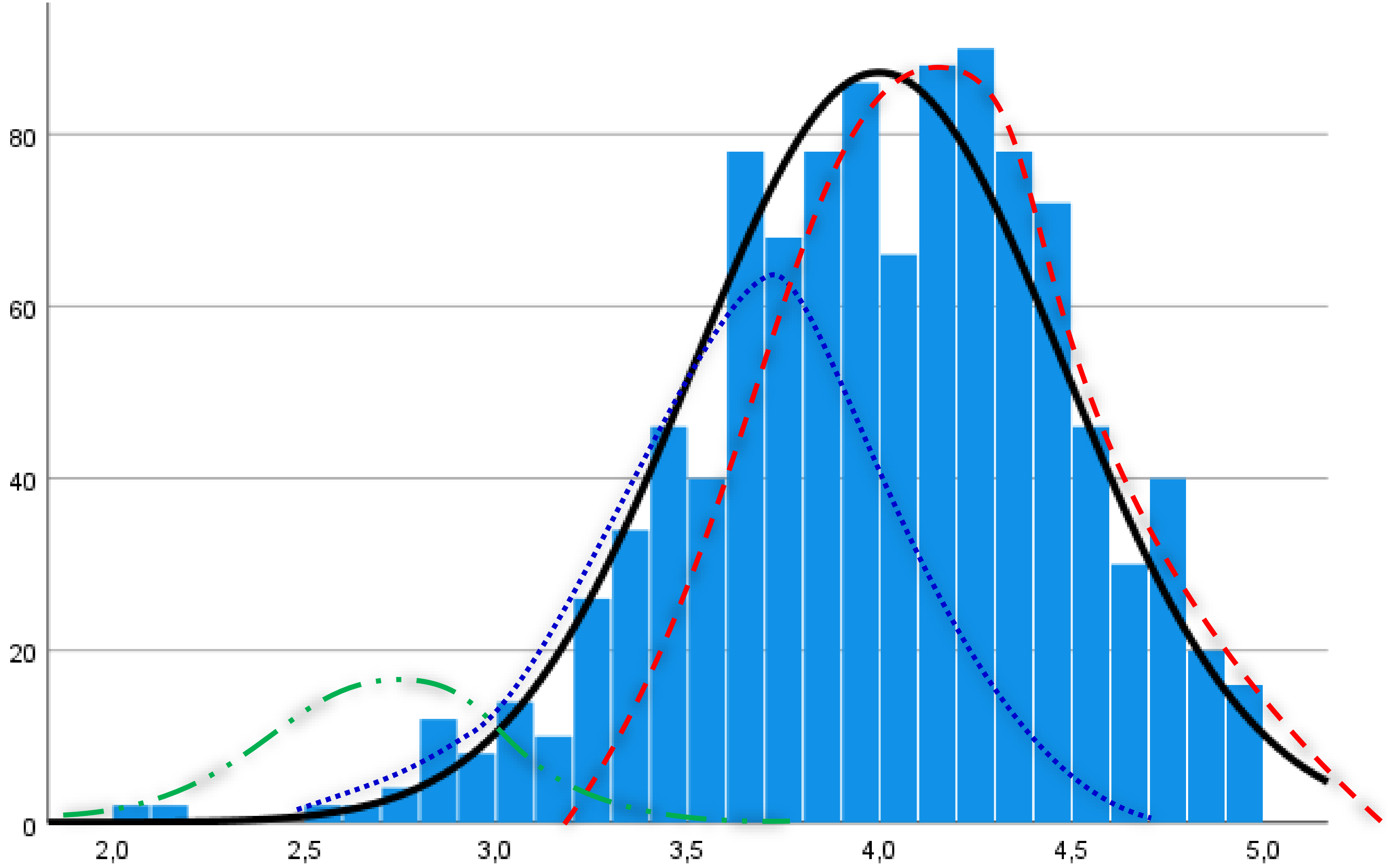
In a multigroup framework

- We know, a priori, that there are three groups and the group that each individual belongs to
    - Thus dashed lines will be observed in a multigroup framework

However, if we do not know in advance, what groups exist in the total sample only the solid line will be observed

- GMM attempts to uncover the three underlying normal distributions and a probabilistic determination of which individuals belong to which subgroups

The general idea of GMM



- Total sample
- Class 1
- Class 2
- Class 3

LGCM before GMM/LCGA

- Examine overall fit of the data to a trajectory with a single growth shape before taking heterogeneity into account
  - Evidence for potential heterogeneity in growth trajectory
    - Model fit indices
    - Variance of growth factors
      - Poor fit indices and significant variances of growth factors may suggest heterogeneous trajectories or subgroups with distinct trajectories

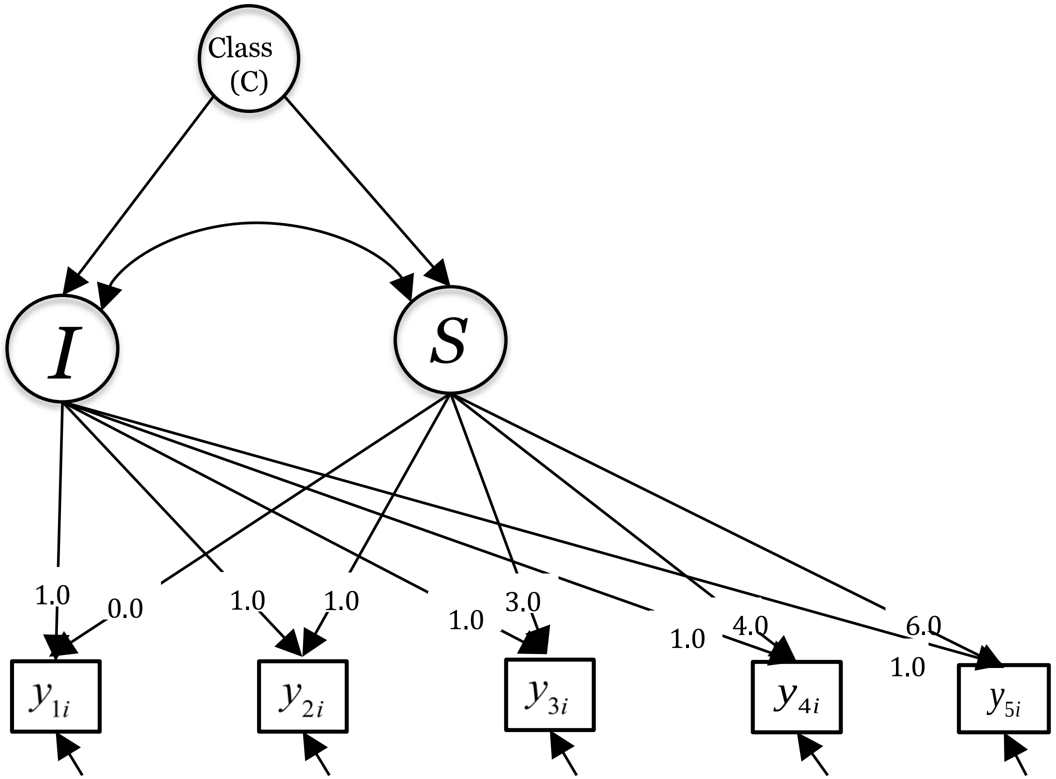
Growth Mixture Model (GMM) or Latent Class Growth Analysis (LCGA)

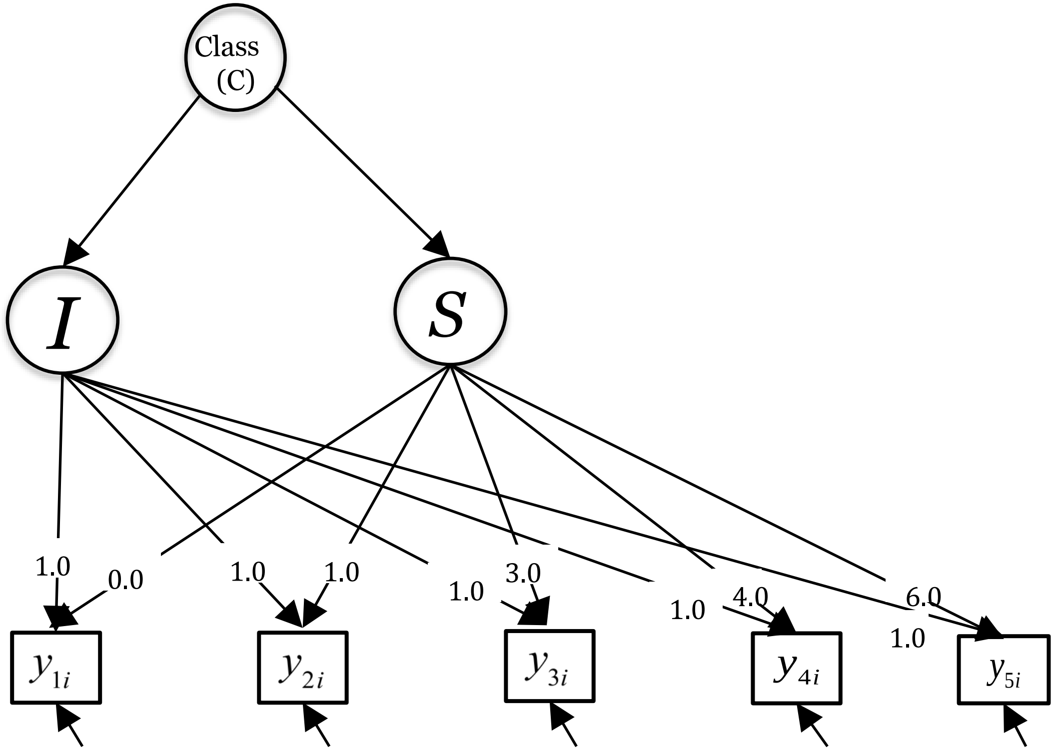
GMM

- Full model
- Allows within-class variation and between-class variation
- Covariates can thus, explain both within- and between-class variations

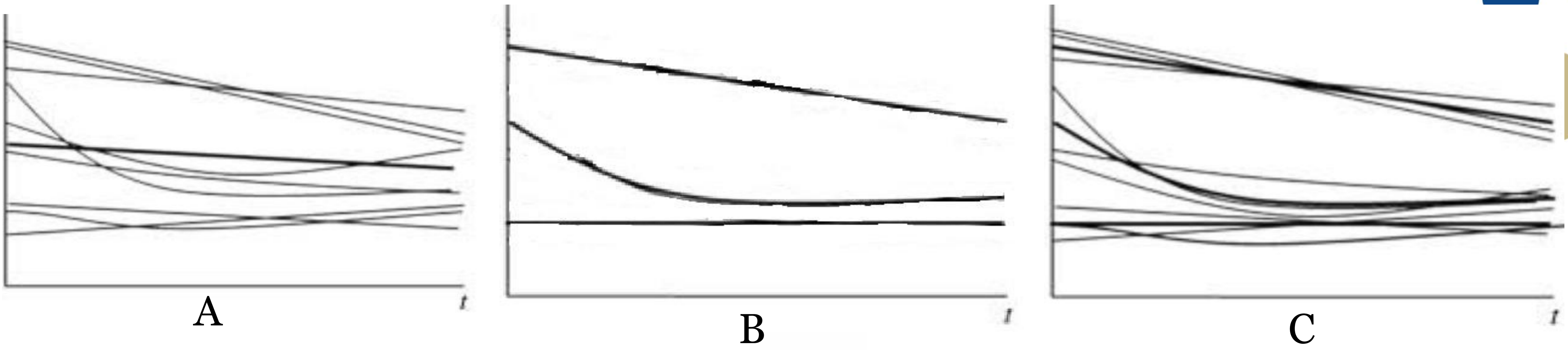
LCGA

- Reduced nested model of GMM or simplified GMM
- Allows between-class variation
  - Within-class variation is constrained to zero
- Covariates can thus, explain only between-class variations
  - Because LCGA assumes within-class individuals share the same trajectory





LGCM vs. LCGA vs. GMM



A = Conventional LGCM

- Assumes that individuals in the sample come from a single population
- Outcome growth trajectories vary around the overall average growth trajectory

B = LCGA

- Heterogeneity is only between classes – assumes within class homogeneity
- Within-class variation (inter-individual differences within class) constrained to zero

C = GMM

- Heterogeneity is both within and between classes
- Allows for different parameterizations of within-class variations across different classes

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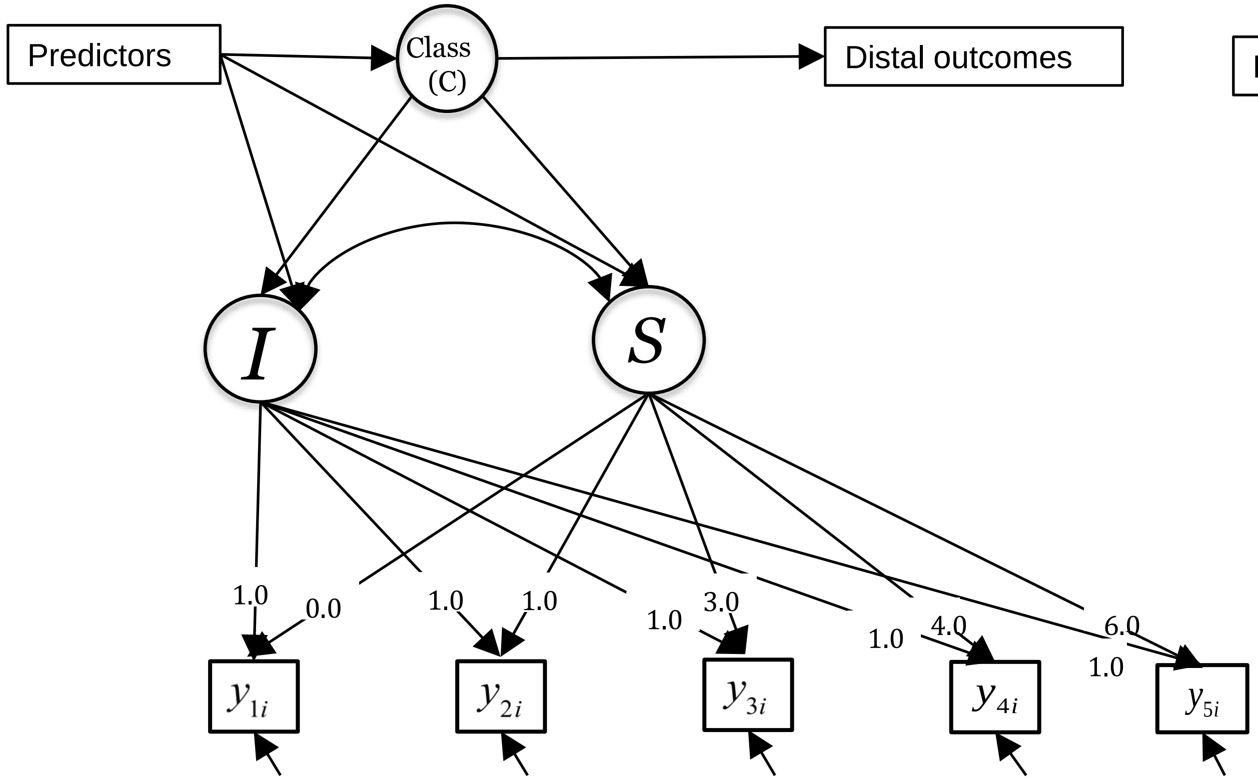

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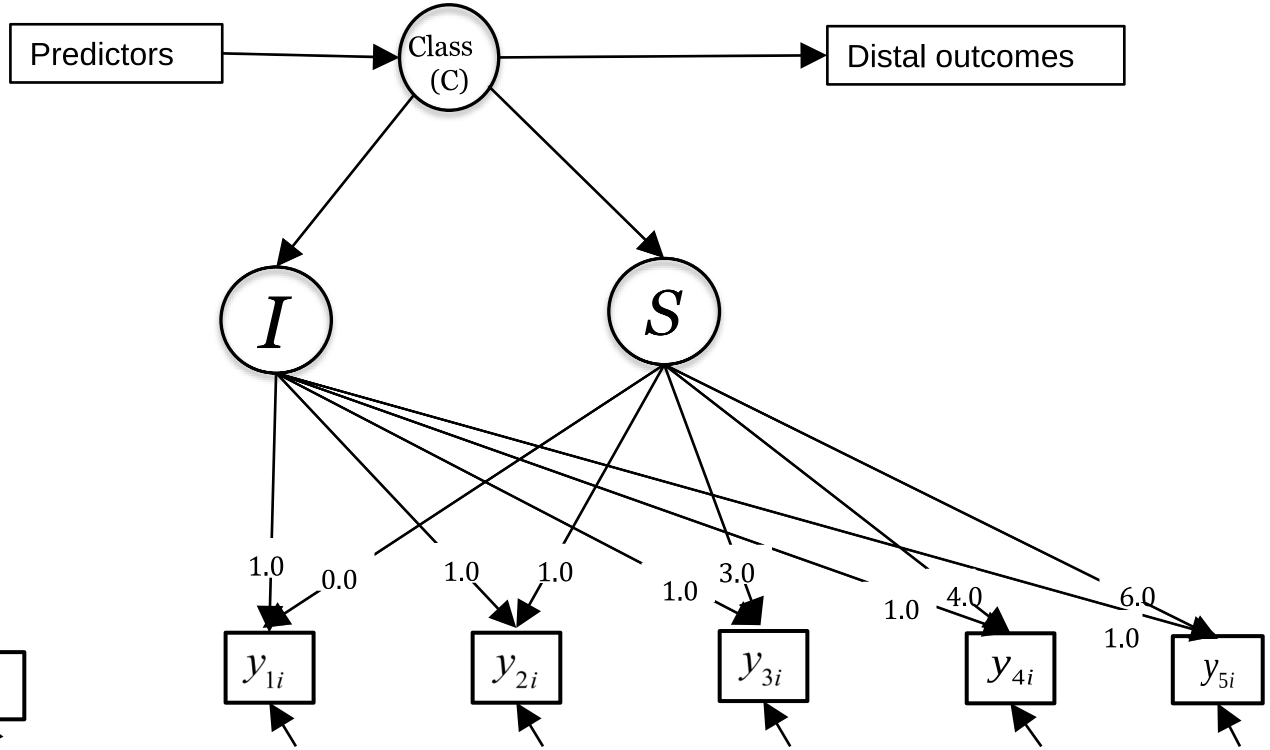


```
TITLE: 2-Class LCGA
DATA:       FILE IS data.dat;

VARIABLE:
NAMES ARE
            id female pdu1 ace1 worry1
            lone1 lone2 lone3 lone4 lone5
            anx1 anx2 anx3 anx4 anx5
            dep1 dep2 dep3 dep4 dep5
            host1 host2 host3 host4 host5
            percon5 percon5b subs5 subs5b sleep5 sleep5b;

USEVARIABLES ARE
            lone1 lone2 lone3 lone4 lone5;
            IDVARIABLE = id; !Use with SAVEDATA to identify the participant ID
            CLASSES = C(2); !Number of latent classes specified. Change to specify more than 2 classes
             MISSING ARE ALL (-999);
ANALYSIS:
            TYPE = MIXTURE; !Required for mixture models
            STARTS = 500 10; !500 random sets of starting values for initial stage and 50 final  optimizations
                            !Increase STARTS values when loglikelihood has not been replicated
            STITERATIONS = 20; !Maximum number of iterations in the initial stage
            LRTBOOTSTRAP = 50; !Number of bootstrap draws for TECH14
            !LRTSTARTS  = 0 0 40 8 number of initial stage starts and number of final stage optimizations for
                                  TECH14. Increase when TECH14 shows warning message
            PROCESSOR = 4; !Number of processors to use
MODEL:
            %OVERALL% !Specifies an overall mixture model
            I S|lone1@0 lone2@1 lone3@3 lone4@4 lone5@6;
            I-S@0; !Fix variance of Intercept and Slope growth factors to zero

OUTPUT:
            TECH7 !Sample statistics for each class
            TECH11 !Lo-Mendell-Rubin likelihood ratio test comparing the k-1 class model to the K class model
            TECH14; !Bootstrapped likelihood ratio test comparing the k-1 class model to the K class model
PLOT:
            TYPE = PLOT3;
            SERIES = anx1-anx5(S);
SAVEDATA: !You can save a text file that contains classification information (i.e., individual posterior
probabilities and everyone's latent class number to use in post hoc analysis)
            SAVE = CPROB;
            FILE = 2_CLASS.txt
```

```
TITLE: 2-Class GMM
DATA:       FILE IS data.dat;

VARIABLE:
NAMES ARE
            id female pdu1 ace1 worry1
            lone1 lone2 lone3 lone4 lone5
            anx1 anx2 anx3 anx4 anx5
            dep1 dep2 dep3 dep4 dep5
            host1 host2 host3 host4 host5
            percon5 percon5b subs5 subs5b sleep5 sleep5b;

USEVARIABLES ARE
            lone1 lone2 lone3 lone4 lone5;
            IDVARIABLE = id;
            CLASSES = C(2);
             MISSING ARE ALL (-999);
ANALYSIS:
            TYPE = MIXTURE;
            STARTS = 500 10;
            STITERATIONS = 20;
            LRTBOOTSTRAP = 50;
            !LRTSTARTS  = 0 0 40 8
            PROCESSOR = 4;
MODEL:
            %OVERALL% !Specifies an overall mixture model. However, Mplus defaults to equality of
variances/covariances of growth factors across class. Hence, the need to add class-specific syntax
            I S|lone1@0 lone2@1 lone3@3 lone4@4 lone5@6;

            %C#1% !Class-specific syntax for Class 1
            I-S; !Estimate the variance for the Intercept and Slope growth factor

            %C#2%
             I-S; !Estimate the variance for the Intercept and Slope growth factor
OUTPUT:
            TECH7
            TECH11
            TECH14;
PLOT:
            TYPE = PLOT3;
            SERIES = SERIES = anx1-anx5(S);
SAVEDATA:
            SAVE = CPROB;
            FILE = 2_CLASS.txt
```

- Is there any evidence for multiple subsamples based on the observed trajectories of loneliness?
  - Distinct subsample represented by different profiles of loneliness
    - Which individuals are likely to be in which profile
    - What are the characteristics of the groups and potential mechanisms

  - Growth mixture models
    1. LCGA: No within-class variation
    2. GMM – CV: Freely estimated co/variances – class-varying
    3. GMM – CI: Equal co/variances across classes

Selecting optimal class solution

1. Theory and meaningful interpretation of classes
   - Distinct and separate class trajectories

2. Information criteria (e.g., AIC, BIC, SSABIC)
   - Lower values indicate better model fit (class solution)

3. Entropy values
   - Measure of "clear class separation"
     - 0 = no class separation
     - 1 = perfect class separation
       - .40, .60, .80 =low, medium and high class separation
   - Classification accuracy
     - Average Latent Class Probabilities for Most Likely Latent Class Membership (Row) by Latent Class (Column)
       - Average posterior probability of members in each class/percentage of class members accurately assigned (> .80)

4. Likelihood ratio test (e.g., LMR-RT, ALMR-RT, BLRT)
   - Sig *p*-values indicate the current class solution (k-class) provides better fit than the solution with one less class (k-1 class)

Adding covariates

- The 1-Step Approach (i.e., Direct specification)
  - Results in different class classification solution from the unconditional model
    - Latent class classification not based solely on the indicator variables
  - Precise estimation of covariate’s effect on class solution
    - Only use when you want the covariate’s effect to influence the class classifications

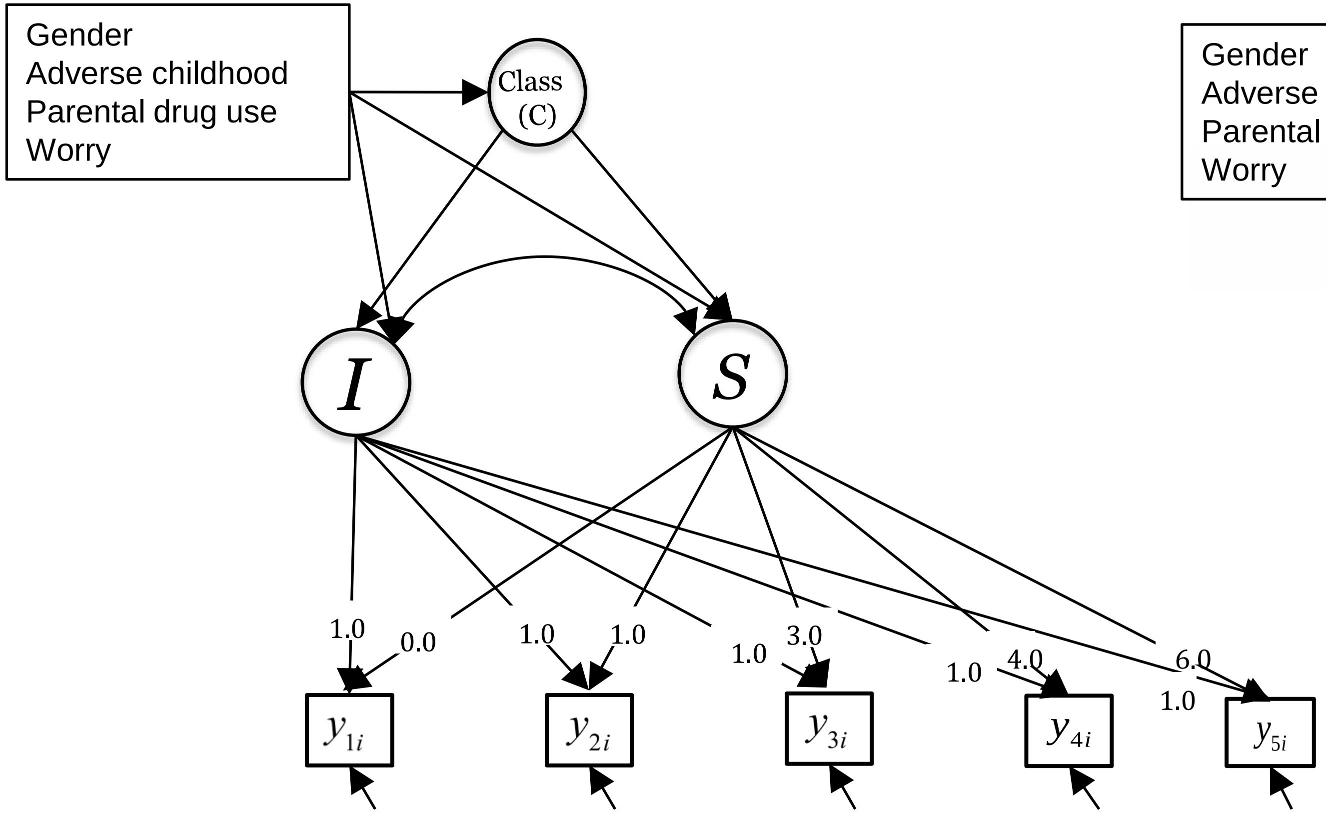
- The 3-Step Approach
  - Preserves class classification solution from the unconditional model

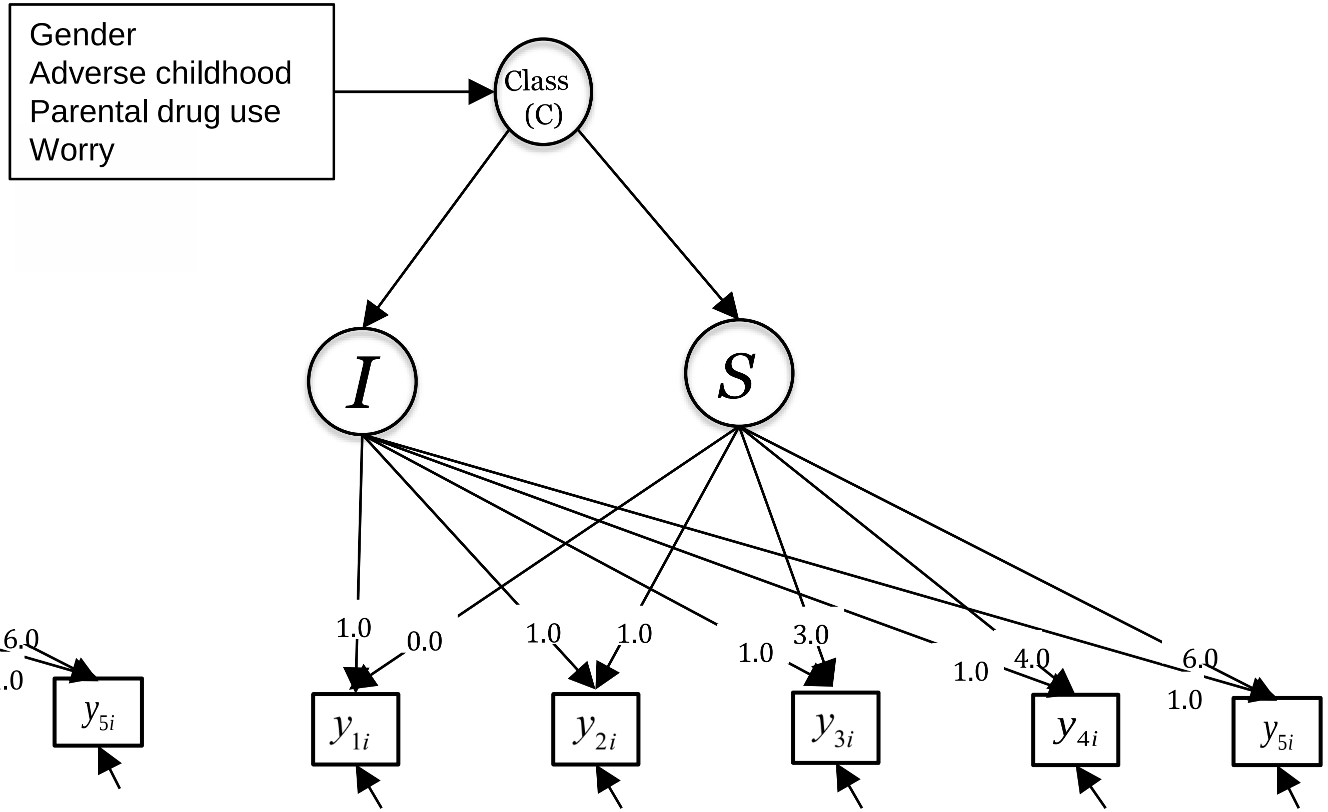
  i. Manual option

  ii. Auxiliary option

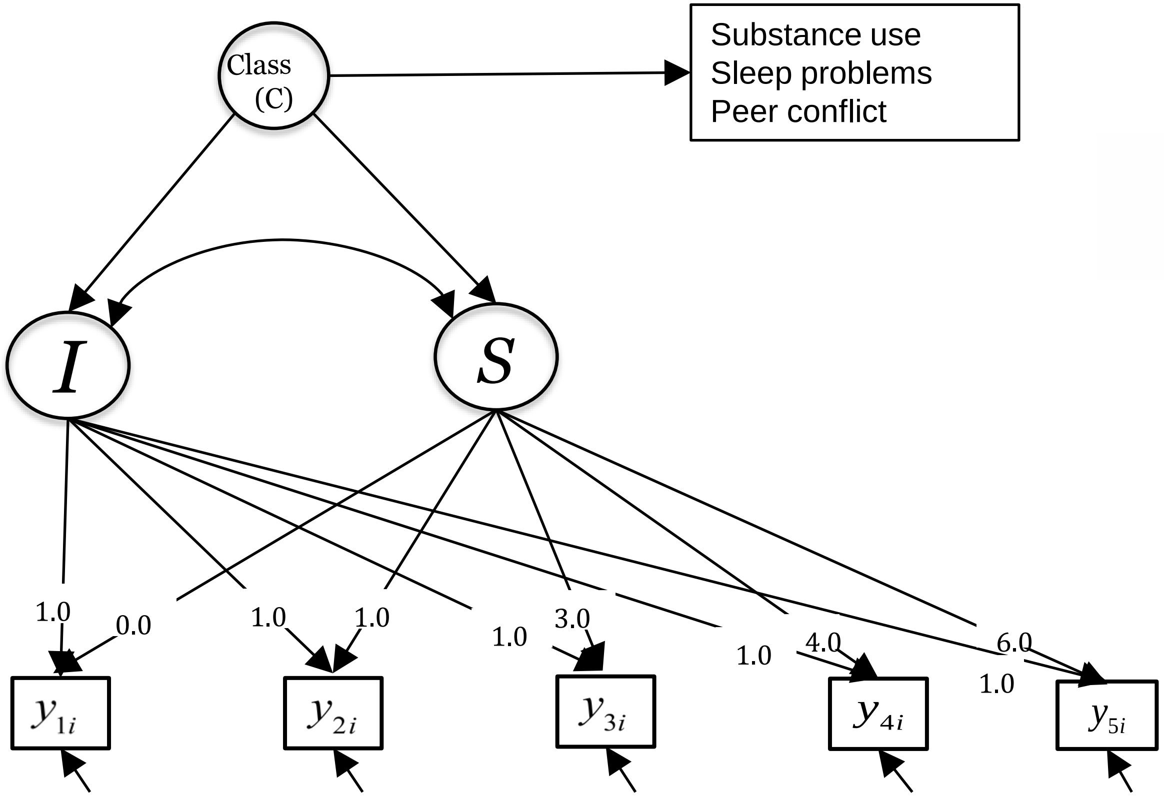
     - R3STEP = Covariates are included as predictors
     - BCH or DU3STEP = Covariates are included as outcomes

- Do the following covariates (i.e., gender, parental drug use, childhood adverse environment and worry) influence the trajectories of adolescents' experience of loneliness?





Do the different profiles of adolescents' experience of loneliness differ in the prediction of substance use, sleep problems and peer conflict?



Evaluations and Suggestions

- Kindly respond to these 7 questions to help improve the course
- You will also have the option to suggest additional quantitative methods/analyses, which will be considered for subsequent workshops

- Use the link below

- https://forms.gle/NZGMJe66eRiCiDup7