CSC 343 Assignment 3

6 Pages

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Assignment 3

Dependencies, Decompositions, Normal Forms

1. (a) We start by splitting the right-hand side (RHS) of each functional dependency (FD):

$$S_P = \{IJ \rightarrow K, J \rightarrow I, J \rightarrow L, JN \rightarrow K, JN \rightarrow M, K \rightarrow I, K \rightarrow J,$$
$$K \rightarrow L, KLN \rightarrow M, M \rightarrow I, M \rightarrow J, M \rightarrow J, M \rightarrow L\}$$

It is clear by observation that there are two $M \rightarrow J$s, so we can remove one of them to eliminate this redundancy.

Next, we look at all the FDs whose left-hand side (LHS) contains at least two attributes in order to determine if any attributes are unnecessary and can be removed. The FDs with a single attribute on the LHS are not considered because it is not possible to simplify them any further.

- $JN \rightarrow K$:
  - $J^+ = \{I, J, \mathbf{K}, L\}$

  Therefore $JN \rightarrow K$ can be reduced to $J \rightarrow K$.
- $JN \rightarrow M$:
  - $J^+ = \{I, J, K, L\}$
  - $N^+ = \{N\}$

  Therefore $JN \rightarrow M$ cannot be reduced.
- $KLN \rightarrow M$:
  - $K^+ = \{I, J, K, L\}$
  - $L^+ = \{L\}$
  - $N^+ = \{N\}$
  - $KL^+ = \{I, J, K, L\}$
  - $KN^+ = \{I, J, K, L, \mathbf{M}, N\}$

  Therefore $KLN \rightarrow M$ can be reduced to $KN \rightarrow M$.
- $IJ \rightarrow K$:
  - $I^+ = \{I\}$
  - $J^+ = \{I, J, \mathbf{K}, L\}$

  Therefore $IJ \rightarrow K$ can be reduced to $J \rightarrow K$.

Our new set of FDs, $S1$, is:

$$S1 = \{J \rightarrow I, J \rightarrow K, J \rightarrow K, J \rightarrow L, JN \rightarrow M, K \rightarrow I, K \rightarrow J,$$
$$K \rightarrow L, KN \rightarrow M, M \rightarrow I, M \rightarrow J, M \rightarrow L\}$$

Similar to before, we can replace the two instances of $J \rightarrow K$ with just one in order to eliminate the redundancy.

The following notation will be used to reference each FD:

(i) $M \to I$
(ii) $M \to J$
(iii) $M \to L$
(iv) $J \to L$
(v) $J \to I$
(vi) $J \to K$
(vii) $JN \to M$
(viii) $KN \to M$
(ix) $K \to I$
(x) $K \to J$
(xi) $K \to L$

We now examine each FD to determine if it is necessary.

(i) $M \to I$: $M^+_{S1-\{\text{(i)}\}} = \{\mathbf{I}, J, K, L, M\}$
Therefore we do not need $M \to I$.

(ii) $M \to J$: $M^+_{S1-\{\text{(i)},\text{(ii)}\}} = \{M, L\}$
Therefore we need $M \to J$.

(iii) $M \to L$: $M^+_{S1-\{\text{(i)},\text{(iii)}\}} = \{I, J, K, \mathbf{L}, M\}$
Therefore we do not need $M \to J$.

(iv) $J \to L$: $J^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)}\}} = \{I, J, K, \mathbf{L}\}$
Therefore we do not need $J \to L$.

(v) $J \to I$: $J^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)},\text{(v)}\}} = \{\mathbf{I}, J, K, L\}$
Therefore we do not need $J \to I$.

(vi) $J \to K$: $J^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)},\text{(v)},\text{(vi)}\}} = \{J\}$
Therefore we need $J \to K$.

(vii) $JN \to M$: $JN^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)},\text{(v)},\text{(vii)}\}} = \{I, J, K, L, \mathbf{M}, N\}$
Therefore we do not need $JN \to M$.

(viii) $KN \to M$: $KN^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)},\text{(v)},\text{(vii)},\text{(viii)}\}} = \{I, J, K, L, N\}$
Therefore we need $KN \to M$.

(ix) $K \to I$: $K^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)},\text{(v)},\text{(vii)},\text{(ix)}\}} = \{J, K, L\}$
Therefore we need $K \to I$.

(x) $K \to J$: $K^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)},\text{(v)},\text{(vii)},\text{(x)}\}} = \{I, K, L\}$
Therefore we need $K \to J$.

(xi) $K \to L$: $K^+_{S1-\{\text{(i)},\text{(iii)},\text{(iv)},\text{(v)},\text{(vii)},\text{(xi)}\}} = \{I, J, K\}$
Therefore we need $K \to L$.

Removing the unnecessary FDs gives us the minimal basis for relation R:

$$\boxed{\{J \to K, K \to I, K \to J, K \to L, KN \to M, M \to J\}}$$

(b) By observation, we note that any key must contain O and P since no FDs from our result in 1a contain those attributes. Additionally, our key must contain N because it is not on the RHS of any FDs. As such, we perform a systematic examination of closures for attributes containing N, O, and P.

- $NOP^+ = \{N, O, P\}$
- $INOP^+ = \{I, N, O, P\}$
- $JNOP^+ = \{I, J, K, L, M, N, O, P\}$
- $KNOP^+ = \{I, J, K, L, M, N, O, P\}$
- $LNOP^+ = \{L, N, O, P\}$
- $MNOP^+ = \{I, J, K, L, M, N, O, P\}$

Therefore, $\boxed{JNOP}$, $\boxed{KNOP}$, and $\boxed{MNOP}$ are keys for R.

(c) We begin with our minimal basis from 1a:

$$\{J \to K, K \to IJL, KN \to M, M \to J\}$$

We then construct the set of relations that would contain these attributes:

- $R1(J, K)$
- $R2(K, I, J, L)$
- $R3(K, N, M)$
- $R4(M, J)$

$R1$ is contained within $R2$, so we can remove $R1$.

Since attributes O and P are not contained within any FDs, we must add one of the keys found in 1b. We choose $JNOP$ and create an additional relation: $R5(J, N, O, P)$.

This leaves us with our final set of relations in 3NF:

$$\boxed{R2(K, I, J, L)}$$
$$\boxed{R3(K, N, M)}$$
$$\boxed{R4(M, J)}$$
$$\boxed{R5(J, N, O, P)}$$

(d) The schema in 1c allows redundancy because of FD BCNF violations. Each relation in the final 3NF set above was formed from functional dependencies, with the LHS of each FD being a superkey. This is fine, but we may find that other FDs do not satisfy BCNF. For example, if we compute a projection of the given set of FDs onto $R3(K, N, M)$, one of the FDs we infer is $M \to K$. We compute the closure of M as $M^+ = KM$. Clearly M is not a superkey of $R3$, since M^+ does not contain N. This is a BCNF violation, since M is on the LHS of an $R3$ FD; thus the schema allows redundancy.

2. (a) We compute the closure of each FD's LHS to determine whether each one is a superkey:

- $C^+ = \{C, D, E, F, G, H, I, J\}$
- $DEI^+ = \{D, E, F, I\}$
- $F^+ = \{D, F\}$
- $EH^+ = \{C, D, E, F, G, H, I, J\}$
- $J^+ = \{D, F, G, I, J\}$

Therefore, FDs $\boxed{DEI \to F}$, $\boxed{F \to D}$, and $\boxed{J \to FGI}$ violate BCNF since they are not superkeys.

(b) We begin by selecting one of the BCNF-violating FDs and decomposing. In our case, we chose $DEI \to F$. Note that the closure is computed with respect to the current relation (may not be the original).

- $DEI^+ = \{D, E, F, I\}$

We now split the relation T into $T1 = DEI^+$ and $T2 = T - (DEI^+ - DEI)$.

- $T1 = \{D, E, F, I\}$
- $T2 = \{C, D, E, G, H, I, J\}$

We project our FDs onto the new relations. If we encounter a BCNF violation, we can stop the projection immediately, since we will discard the relation anyway and continue the decomposition:

- We begin by checking $T1$. Note that, with respect to the original relation, $F^+ = \{D, F\}$. Thus $F \to D$ is an applicable FD, but violates BCNF as F is not a superkey. Thus, we must decompose $T1$ into two new relations, using the same process as previously described. F^+ with respect to $T1$ is $\{D, F\}$.
  - $T3 = \{D, F\}$
  - $T4 = \{E, F, I\}$
- We now check $T2$. Note that, with respect to the original relation, $J^+ = \{D, F, G, I, J\}$. Thus $J \to DGI$ is an applicable FD, but violates BCNF as J is not a superkey. We decompose $T2$ accordingly. J^+ with respect to $T2$ is $\{D, G, I, J\}$.
  - $T5 = \{D, G, I, J\}$
  - $T6 = \{C, E, H, J\}$

We again project our FDs onto the new relations:

Table 1: Projection of FDs onto $T3$

Take all subsets X of Attributes: D F	**Closure of the subset**	**FDs Inferred**
D	$D^+ = D$	Nothing
F	$F^+ = DF$	$F \rightarrow D$

Table 2: Projection of FDs onto $T4$

Take all subsets X of Attributes: E F I	**Closure of the subset**	**FDs Inferred**
E	$E^+ = E$	Nothing
F	$F^+ = DF$	Nothing
I	$I^+ = I$	Nothing
EF	$EF^+ = DEF$	Nothing
EI	$EI^+ = EI$	Nothing
FI	$FI^+ = DFI$	Nothing

Table 3: Projection of FDs onto $T5$

Take all subsets X of Attributes: D G I J	**Closure of the subset**	**FDs Inferred**
D	$D^+ = D$	Nothing
G	$G^+ = G$	Nothing
I	$I^+ = I$	Nothing
J	$J^+ = DFGIJ$	$J \rightarrow DGI$
DG	$DG^+ = DG$	Nothing
DI	$DI^+ = DI$	Nothing
DJ	No need to consider; since J is a key the resulting FDs will be weaker	
GI	$GI^+ = GI$	Nothing
GJ	No need to consider; since J is a key the resulting FDs will be weaker	
IJ	No need to consider; since J is a key the resulting FDs will be weaker	
DGI	$DGI^+ = DGI$	Nothing
Subsequent supersets will only yield weaker FDs as previous subsets include keys		

Table 4: Projection of FDs onto $T6$

Take all subsets X of Attributes: C E H J	**Closure of the subset**	**FDs Inferred**
C	$C^+ = CDEFGHIJ$	$C \rightarrow EHJ$
E	$E^+ = E$	Nothing
H	$H^+ = H$	Nothing
J	$J^+ = DFGIJ$	Nothing
CE	No need to consider; since C is a key the resulting FDs will be weaker	
CH	No need to consider; since C is a key the resulting FDs will be weaker	
CJ	No need to consider; since C is a key the resulting FDs will be weaker	
EH	$EH^+ = CDEFGHIJ$	$EH \rightarrow CJ$
EJ	$EJ^+ = DEFGIJ$	Nothing
HJ	$HJ^+ = DFGHIJ$	Nothing
Subsequent supersets will only yield weaker FDs as previous subsets include keys		

We continue by checking for BCNF violations in each relation:

- Check $T3 = \{D, F\}$, which has a single FD $F \to D$. F is a superkey, so this satisfies BCNF.
- Check $T4 = \{E, F, I\}$, which has no applicable FDs. This satisfies BCNF.
- Check $T5 = \{D, G, I, J\}$, which has a single FD $J \to DGI$. J is a superkey, so this satisfies BCNF.
- Check $T6 = \{C, E, H, J\}$, which has two FDs $EH \to CJ$ and $C \to EHJ$. EH and C are both superkeys, so this satisfies BCNF.

The decomposition is now complete. Below are the final relations and their respective FDs.

Table 5: Final Relations With Associated FDs

Relations	**Functional Dependencies**
{C,E,H,J}	$C \to EHJ, EH \to CJ$
{D,F}	$F \to D$
{D,G,I,J}	$J \to DGI$
{E,F,I}	None