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No: .... Date: 7/04/2017.

NUCLEAR AND PARTICLE

2016

(1) Fundamental fermions of the Standard Model

- Quarks
- Antiquarks
- Leptons
- Antileptons.

Quarks : $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$ Antiquarks : $\begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \begin{pmatrix} \bar{c} \\ \bar{s} \end{pmatrix} \begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix}$

Generation: (I) (II) (III)

u, c, t : $+\frac{2}{3}$ charge $\bar{u}\,\bar{c}\,\bar{t}$: $-\frac{2}{3}$ charge

d, s, b : $-\frac{1}{3}$ charge $\bar{d}, \bar{s}, \bar{b}$: $+\frac{1}{3}$ charge

leptons : $\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$ antileptons : $\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix} \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix} \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix}$

e^-, μ^-, τ^- : -1 charge e^+, μ^+, τ^+ : $+1$ charge

ν_e, ν_μ, ν_τ : 0 charge $\bar{\nu}_e, \bar{\nu}_\mu, \nu_\tau$: 0 charge.

(2) ~~(a)~~ Conservation of energy must not be violated. ~~Assuming the particle is at rest before the decay,~~ Energy before must be equal to energy after the decay.

$$E^2 = m^2c^4 + p^2c^2$$

(b) Quark content of the proton : uud [2 mark?]

(c) (I) • Invariant mass, $W \Rightarrow$ $W^2c^4 = \left(\sum_i^N E_i\right)^2 - \left(c \sum_i^N \vec{p}_i\right)^2$

• In COM frame, $W^2c^4 = p^2 = E_{cm}^2$ NOTE

• $E_{cm}^2 = (6500 + 6500)^2 - (0)$

$E_{cm} = 13000$ GeV.

(II) Why is the centre-of-mass energy of each parton-parton interaction less than this?

ONLINE ANSWER! In the proton-proton collision, the partons annihilate (not the whole proton), so the effective centre-of-mass energy is smaller than energy of the machine.

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(3) (a) $E^2 = m_0^2c^4 + p^2c^2$

m_0 : rest mass

E : Particle's energy

p : momentum.

(b) Relativistic particles have $E^2 = m_0^2c^4 + p^2c^2$
which has solutions : $E = \pm\sqrt{m_0^2c^4 + p^2c^2}$

Quantum Mechanics requires us to consider the negative solutions as well.

We interpret the negative energy solution as particles propagating backwards in time, which is equivalent to positive energy antiparticles propagating forward in time.

(c) Positron.

(d) Opposite curvature to that expected for an electron.
Positron has ~~positive~~ +1 charge.

(4) (a) SEMF: $M(Z,A) = Zm_p + (A-Z)m_n - a_V A + a_S A^{2/3} + a_C Z^2 A^{-1/3} + a_a\left(Z - \frac{A}{2}\right)^2 A^{-1} \pm \delta a_p f(A)$

Explain the origin of the a_S term: $\downarrow$ Surface term

Liquid-drop model.

Nucleons near surface of nucleus surrounded by fewer nucleons and will therefore experience less attractive potential energy than those inside the nucleus. Compensate with a reduction in binding energy proportional to no. of nucleons in the nuclear surface.

(b) Heavy nuclei contain ~~more~~ fewer protons than neutrons.

- Elements that have atomic no. from 20 to 83 are heavy elements, They have $\frac{\text{neutron}}{\text{proton}}$ ratio of 1.5:1. It is due to the repulsive force between protons, stronger the repulsive force, the more neutrons are needed to stablize the nuclei.

- Assymmetry term.

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Components of a general purpose detector at the LHC

(5) (a) · Vertex and tracking detectors (inside a magnetic field)
· electromagnetic calorimeter
· hadronic calorimeter
· muon detectors.

Experimental Methods - pg 21-22

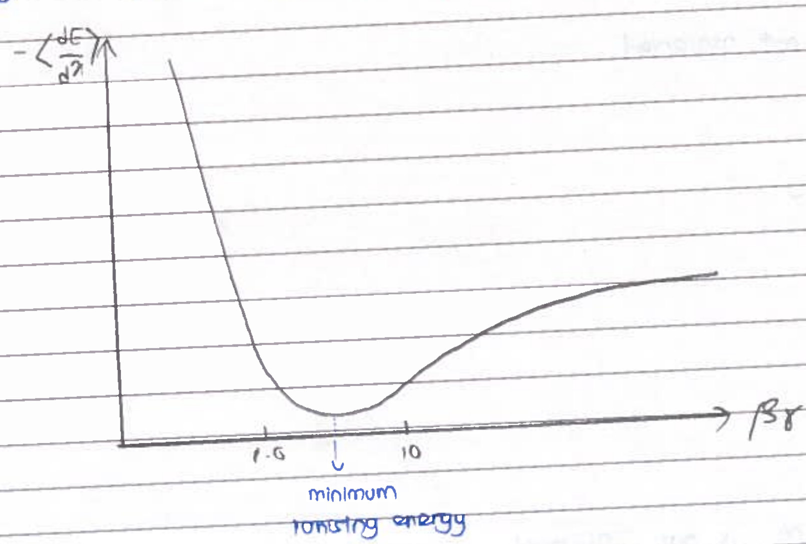
(b) Muons can transverse the whole detector and are measured in the outermost muon spectrometer.
Charged pions will deposit energy in the hadronic calorimeter and leave a track.

(c) Identify b-jets in the detector: b-hadrons are longer-lived. With a $c\tau \sim 450\,\mu m$, they decay a few mm away, due to boost. B-jets can be identified by reconstructing the displaced secondary vertex (a track crossing away from the collision point).

(6) (a) Ionisation energy loss → Bethe-Bloch formula

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{K z^2 n_e}{\beta^2} \left[\ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2}{I} \right) - \beta^2 - \frac{\delta(\gamma)}{2} \right]$$

The energy loss of a particle inside a given material, depends only on the speed and charge of the particle. [speed]



$\beta = \frac{v}{c}$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

· At low energies $\frac{dE}{dx} \sim \frac{1}{\beta^2}$ and slower particles loose more energy.

· Particle that loose the least energy around the minimum of $\beta\gamma \sim 4$ are called minimum ionising particles (MIPS).

(b) $p = 1\,GeV/c$ $\quad p = mv$ $\quad v = \frac{p}{m}$ $\quad$ mass of proton > mass of pion.

proton is slower. ∴ At lower energies, proton loose more energy.

A Captain's Product

Baryons 1

anti-Baryons : -1

No: Mesons : 0 . Date:

SECTION B

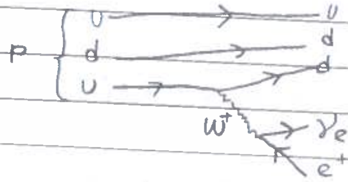
7 (a) (I) $p \rightarrow n + e^+ + \nu_e$

β^+-decay.

$\pi^0 = u\bar{u}$ $p = uud$

$\pi^+ = u\bar{d}$ $n = udd$

- lepton no. is conserved
- Baryon no. is conserved
- charge is conserved .



Allowed .

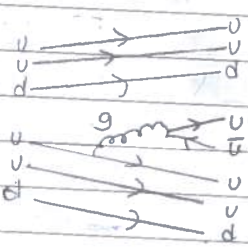
(II) $p + p \rightarrow p + \bar{p} + \pi^0$

- Baryon no. is not conserved .
- charge is not conserved .

Forbidden .

Feynman Diagram →

(III) $p + p \rightarrow p + p + \pi^0$



Allowed .

(IV) $p + p \rightarrow \pi^0 + \pi^+ + \pi^+$

- Baryon no. is not conserved .

Forbidden .

Feynman diagram .

(V) $p + \bar{p} \rightarrow \pi^+ + \pi^-$

Allowed .

(VI) $\tau^- \rightarrow \mu^- + \nu_\tau$

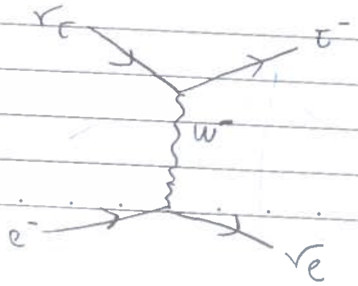
- Individual lepton no. is not conserved .

Forbidden

(VII) $\nu_\tau + e^- \rightarrow \nu_e + \tau^-$

Feynman diagram .

Allowed



Check whether W^- or not ?

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(II) 3 things the instantaneous luminosity of the machine will depend.

→ No. of colliding bunches
→ The no. of particles in each beam
→ the cross-sectional area of the beams and
→ frequency with which the bunches circulate the ring.

(d) $\pi^+ \rightarrow \mu^+ \nu_\mu$ $\qquad E^2 = m_0^2c^4 + p^2c^2$

Energy of π^+ before decay: $E^2 = m_{\pi^+}^2c^4 + p^2c^2$

(before decay π^+ will be at rest) $E^2 = m_{\pi^+}^2c^4$

$E = m_{\pi^+}c^2$

$E = 139.57\ \text{MeV}$

After decay, to conserve momentum: $\vec{P}_{\mu^+} = -\vec{P}_{\nu_\mu} =$

Energy before = energy after

$139.57\ \text{MeV} = \sqrt{m_\mu^2c^4 + |\vec{p}|^2c^2} + \sqrt{m_{\nu_\mu}^2c^4 + |-\vec{p}|^2c^2}$

Actual way!

- $0 \rightarrow 1+2$ decay in the rest frame of 0
- In the rest frame of the particle 0, the 4-momentum of the particles are related by $P = P_1 + P_2$ where $P = (M, \vec{0})$
- We can write $P_2 = P - P_1$

$$P_2^2 = (P-P_1)^2 = P^2 - 2P.P_1 + P_1^2$$

$$m_2^2 = M^2 - 2ME_1 + m_1^2$$

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

Initial four-momentum: $(E_{\pi^+}, 0)$ $\qquad (E_\pi, 0) = (E_\mu, -\vec{p}) + (E_{\nu_\mu}, \vec{p})$

(After decay) four-momentum: $(E_\mu, -\vec{p}), (E_{\nu_\mu}, \vec{p})$ $\qquad (E_\mu, -\vec{p}) = (E_\pi, 0) - (E_{\nu_\mu}, \vec{p})$

Let $P_2 = P - P_1$

$P_2^2 = (P-P_1)^2$

$P_2^2 = (P-P_1)(P-P_1)$

$P^2 = P.P = E^2 - \vec{p}^2c^2$ $\rightarrow P_2^2 = E_\mu^2 - \vec{p}^2c^2$ $P_1^2 = E_{\nu_\mu}^2 - \vec{p}^2c^2$

$P^2 = E_\pi^2$

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GET A PROPER WAY FROM BS.

$P_2^2 = (P - P_1)(P - P_1)$

$P_2^2 = P^2 - 2P.P_1 + P_1^2$

$E_\mu^2 - \vec{p}^2c^2 = E_\pi^2 - 2E_\pi(\sqrt{E_{\nu_\mu}^2 - \vec{p}^2c^2}) + E_{\nu_\mu}^2 - \vec{p}^2c^2$

$$\sqrt{E_{\nu_\mu}^2 - p^2c^2} = \frac{E_\pi^2 + E_{\nu_\mu}^2 - E_\mu^2}{2E_\pi}$$

similarly

$$\boxed{\sqrt{E_\mu^2 - p^2c^2} = \frac{E_\pi^2 + E_\mu^2 - E_{\nu_\mu}^2}{2E_\pi}}$$

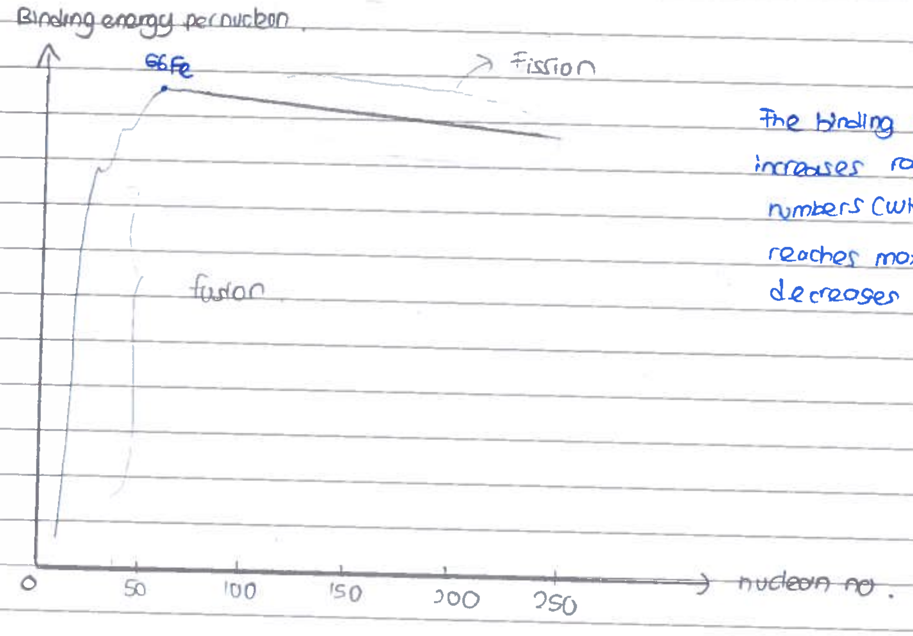
using this $\rightarrow$

$$= \frac{139.57^2 + 105.7^2 - E_{\nu_\mu}^2}{2(139.57)}$$

$\rightarrow$ negligible.

$E = 109.8096...$ MeV

(e) (i)



The binding energy per nucleon vs A increases rapidly for the lowest nucleon numbers (with some spikes for magic nuclei) reaches max. at ^{56}Fe, the slowly decreases again.

(ii) Fission: Splitting one of the heaviest nuclei into two lighter nuclei results in a larger total binding energy

Fusion: Two of the lightest nuclei can be fused together to form a heavier nucleus, the total (negative) binding energy will be larger and hence the total energy will be smaller, releasing some KE.

(iii) $^2_1H + ^2_1H \rightarrow ^3_1H + p$

4.034 MeV (why +ve?)

$K^- = s\bar{u}$, $m_{K^-} = 494\ MeV/c^2$

$B^*_{s2} = \bar{b}s$, $m_{B^*_{s2}} = 5840\ MeV/c^2$

$B_s^0 = \bar{b}s$, $m_{B_s^0} = 5367\ MeV/c^2$

$\pi^+ = u\bar{d}$

$B_c^+ = \bar{b}c$, $m_{B_c^+} = 6276\ MeV/c^2$

$B^+ = \bar{b}u$, $m_{B^+} = 5279\ MeV/c^2$

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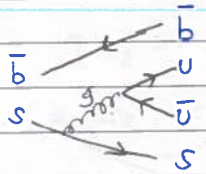
(9) (I) $B^*_{s2} \to B^+ + K^-$

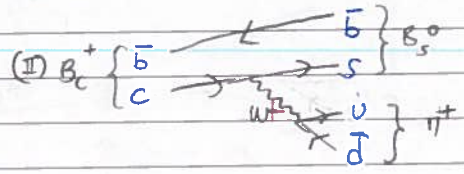
(II) $B_c^+ \to B_s^0 + \pi^+$

$\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix}$

(a) Lowest order feynman diagram.

(I)





$W^{+/-}$ depends on the product being positive or negative. Since π^+ is positive, it W^+

(b) (I) proceeds through strong decay

(II) " " weak decay.

(c) Strong decays faster $\to$ B^*_{s2} will decay faster.

Weak decay depends on Q-value $\sim Q^5$

Decay time is to first order inversely proportional to the coupling constant squared (from ~~the~~ a first order Feynman diagram with only a vertex).

The lifetime of a particle is proportional to the inverse square of the coupling constant of the force which causes the decay.

(d) B^+ meson has a lifetime of $c\tau \sim 450\ \mu m$.

$B^*_{s2} \to B^+ \longrightarrow$
K^-

$\longrightarrow$
$\beta c\tau$

distance is proportional to $\sim \beta\gamma c\tau$

$c\tau$ is given. Need to find $\beta\gamma$. The momentum $p \sim \beta\gamma m$

$$\beta\gamma = \frac{10\ GeV/c}{5.8\ GeV/c^2}$$

Once we find $\beta\gamma$, multiply by $c\tau$ to find distance.

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(e) $B^*_{s2} \rightarrow B^+ + K^-$

$\vec{P}_{B^*_{s2}} \sim 10\,GeV$

4-vector invariant

p^2

$$P^2_{B^*_{s2}} = (P_{B^+} + P_{K^-})^2$$

$$m^2_{B^*_{s2}} = P^2_{B^+} + P^2_{K^-} + 2P_{B^+} \cdot P_{K^-}$$

3 momentum vector.

valid in all frame

$$= m^2_{B^+} + m^2_{K} + 2(E_{B^+}E_{K^-} - \vec{P}_{B^+} \cdot \vec{P}_{K^-})$$

MOMENTUM CONSERV

valid in rest frame

In B^*_{s2} rest frame, $\vec{P}_{B^*_{s2}} = 0 = \vec{P}_{B^+} + \vec{P}_{K^-}$

$$\boxed{P_{B^+} = -P_{K^-} = P}$$

Frame that we can use to solve problems

- Rest frame : $E=m$ (used for decays)

$P(m_{B_{s2}}, 0)$

$p^2 \rightarrow m^2_{B_{s2}} - 0$

$E_{B^*_{s2}} = m_{B^*_{s2}} = E_{B^+} + E_{K^-}$ ENERGY CONSERVATION

$$E^2_{K^-} = m^2_{K^-} + \vec{P}^2_{K^-} = m^2_{K^-} + \vec{P}^2$$

$$E_{B^+} = m_{B^*_{s2}} - E_{K^-}$$

$$2(E_{B^+}E_{K^-} - \vec{P}_{B^+} \cdot \vec{P}_{K^-})$$

$$2((m_{B^*_{s2}} - E_{K^-})E_{K^-} + p^2)$$

$$2(m_{B^*_{s2}}E_{K^-} - E^2_{K^-} + E^2_{K^-} - m^2_{K^-})$$

$$\boxed{2m_{B^*_{s2}}E_{K^-} - 2m^2_{K^-}}$$

$$m^2_{B^*_{s2}} - m^2_{B^+} - m^2_{K^-} = 2(EE - \vec{p}\,\vec{p})$$

$$= 2m_{B^*_{s2}}E_{K^-} - 2m^2_{K^-}$$

$$E_{K^-} = \frac{m^2_{B^*_{s2}} - m^2_{B^+} - m^2_{K^-} + 2m^2_{K}}{2m_{B^*_{s2}}}$$

$$\boxed{E_{K^-} = \frac{m^2_{B^*_{s2}} - m^2_{B^+} + m^2_{K^-}}{2m_{B^*_{s2}}}}$$

This is in rest frame. We need to use Lorentz transformation to convert to Lab frame.

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rest $\to$ Lab (+)

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$$\begin{pmatrix} E_{K^-}' \\ \vec{p}_{K^-}' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E_{K^-} \\ \vec{p}_{K^-} \end{pmatrix}$$

$$\beta\gamma = \frac{p}{m} = \frac{10\,GeV}{5.8\,GeV}$$

↗ four momentum squared is always m^2

Ex: $P_{B'}^2$ → $p^2 = E^2 - |p|^2$

$= (m^2 + |\vec{p}|^2) - |\vec{p}|^2$

$= m^2$

(f) $\frac{p}{[GeV]} \sim 0.3 \frac{B}{[T]} \frac{r}{[m]}$

$F = q\,\underline{v} \times \underline{B} = \frac{mv^2}{r}$

$q|\underline{v}||\underline{B}|\sin\theta = \frac{mv^2}{r}$

$\frac{q|B|\sin\theta}{mv} = \frac{1}{r}$

$\frac{q|B|}{p} = \frac{1}{r}$

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(P) $\frac{1}{r} \propto \frac{QB}{p}$ $1\ GeV/c$ $1T = 1\frac{kg}{C\cdot s}$ $e = 1.602 \cdot 10^{-19} C$

$3T$

$$= \frac{(1.602\times10^{-19}\,C)(3\,\frac{kg}{Cs})}{1\times10^9\,(5.344\times10^{-28}\,\frac{kg\cdot m}{s})}$$

$= 0.8993..\ m^{-1}$

$H \to ZZ$

$\downarrow$ ee $\quad$ $\downarrow$ ee

0.036 $\quad$ 0.036

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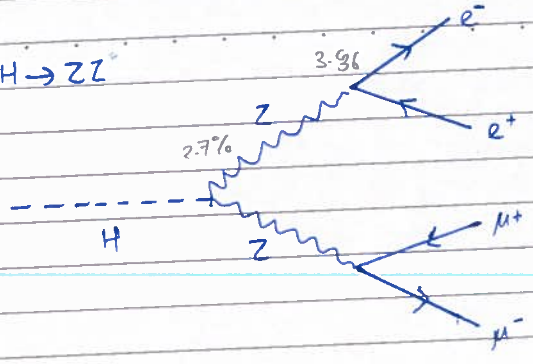
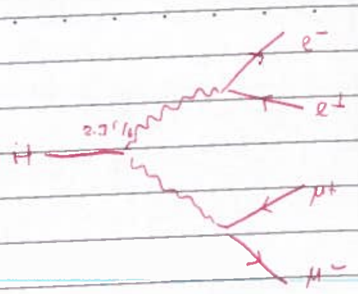
(10)(a)(I) $H \to ZZ$





$Br(H \to ZZ) = 2.7\%$

$200000 \times (0.027) = 5400 \quad H \to ZZ$

$ZZ \to eeee \quad (0.036)^2$

~~$ZZ \to \mu\mu\mu\mu \quad (0.036)^2$~~

$ZZ \to ee\mu\mu \quad (0.036)^2$

$\to \mu\mu ee \quad (0.036)^2$

~~add all.~~

(II) 34 GeV/c^2

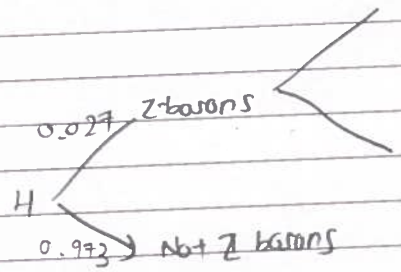
(b) Total Higgs production cross section at LHC $\to$ 20,000 fb.

How many $H \to ZZ$ events, where both Z bosons decay to either a e^+e^- pair or a $\mu^+\mu^-$ pair are expected in an LHC detector after collecting 10 fb^{-1} of data?

$Br(Z \to e^+e^-) = 3.36\%$.

$Br(H \to ZZ) = 2.7\%$

$Br(Z \to \mu^+\mu^-) = 3.36$



$N = 20000 \text{ fb} \times 10 \text{ fb}^{-1}$

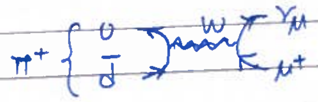
$N = 200000$ Higgs are produced.

$N = \sigma L$

5400 events of $H \to ZZ$

(c) No. massless particle do not couple to Higgs.

(d)



(e)

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