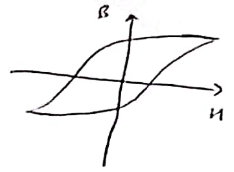
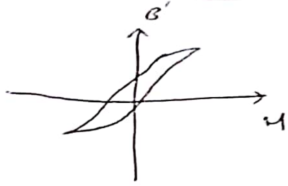
1a)

EMT 2013



(a)



(b)

(a) is a ~~Soft~~ Hard hard magnet and (b) is a soft magnet.

This is because ~~the~~ a ~~ferm~~ Hard magnet has a higher value for <u>coercivity</u> and <u>Remenence</u> - pronounced ~~ated~~ saturation value

b) soft - easier to reduce magnetization, lower remenence

b) $r = 2222.5 \times 10^3$

$m = 5.5 \times 10^{30}$

number of neutrons : $\frac{5.5 \times 10^{30}}{1.6 \times 10^{-27}} = 3.4 \times 10^{57}$ ✓

$M = \frac{n}{v} \times \mu$

Density $= \frac{3.4 \times 10^{57}}{\frac{4}{3}\pi (22.5 \times 10^3)^3} = 1.6 \times 10^{48}$ ✗)

$1.6 \times 10^{48} \times (9.6 \times 10^{-27}) = 1.5 \times 10^{22}\ Am^{-1}$

$= 6.62 \times 10^{17}$

2) $\sigma_p = P.n$

$\rho_p = -\nabla.P$

$j_m = M \times n$

$J_m = \nabla \times M$

electric dipole moment pe unit volume

J_m= Volume magnetization charge density

a) P is (electric) polarization ✓

M is magnitization ✓

n is unit vector ~~perpen~~ normal to surface

σ_p is surface polarization charge density in an electric feild

~~ρ_p it Surface current (charge)~~ density

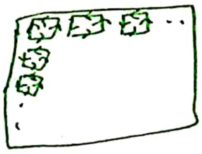
~~m is volume~~

is the volume polarization charge density ✓

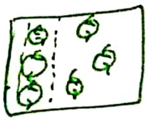
is the volume ~~charge density~~ density

surface magnetization current charge of a magnetic feild

b) j_M, the surface current arises
Inside dipoles cancel out thus left with a surface current



J_m volume current arises
← Non uniform ~~magnets~~ dipoles,
thus they don't cancel like the surface currents



Find better answer

3) a) $\omega^2 = \omega_p^2 + k^2c^2$

i) k is the wave ~~vector~~ vector number } some mini
ω is the angular frequency of the (incoming) wave
ω_p is the angular frequency of plasma → oscillation of plasma when the free charge are displaced by a small amount
c is the speed of light

ii) $V_{phase} = \frac{\omega}{k}$ → $\cancel{\frac{\omega^2}{k^2} = \frac{\omega_p^2}{k^2} + c^2}$ $\frac{\omega}{k} = \frac{\sqrt{\omega_p^2+k^2c^2}}{k}$

$\cancel{\frac{\omega}{k} = \sqrt{\frac{\omega_p^2}{k^2}}}$

$V_{group} = \frac{\partial \omega}{\partial k}$

$$\frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k}\left(\omega_p^2 + k^2c^2\right)^{1/2} = \frac{1}{2}\left(2kc^2\right)\left(\omega_p^2+k^2c^2\right)^{-1/2}$$

$$= \frac{kc^2}{\left(\omega_p^2+k^2c^2\right)^{1/2}}$$

$$V_p \times V_g = \frac{kc^2}{\left(\omega_p^2+k^2c^2\right)^{1/2}} \times \frac{\left(\omega_p^2+k^2c^2\right)}{k} = \underline{c^2}$$

[illegible]

he maximum value both v_{group} and v_{phase} can reach is c. thus $v_p, v_g \le c$ v_p can be greater than c

v_g can't as information cannot ~~[illegible]~~ travel faster than c (v_g carries information) v_p does not so can travel faster than

b) Non propagating wave → $v_g = 0$ → $\frac{\partial \omega}{\partial k} = 0$

$\omega_p^2 = k$ k must be ~~[illegible]~~ wrong

$\omega_p^2 + k^2c^2 = 0$

$v_g = \frac{kc^2}{(\omega_p^2 + k^2c^2)^{1/2}} = 0$

$\omega_p^2 = kc$

$\omega_p = \omega$

4) a) "The permanent dipole moments of the atoms in a paramagnetic material are of similar magnitude to those in a ferromagnetic material" → that is not true. Ferromagnets have greater magnitudes. ✓ [illegible]

b) "When a large magnetic field is applied to a ferromagnetic material, then saturated [illegible]. It goes to zero" ✓

c) Incorrect. the ~~[illegible]~~ induced dipole will tend to align with the applied electric field ✓

5a) $E = E_0 e^{i(k\cdot r - \omega t)}$

$= E_0 \left(e^{i(k \cdot \frac{1}{3}(\hat{x}+\hat{y}+\hat{z}) - \omega t)} \right) \left(\frac{\hat{x}-\hat{y}}{\sqrt{2}} \right)$

b) One can confirm that a transverse wave by taking the dot product with a plane ~~[illegible]~~ normal to the direction of oscillation!

$k \cdot E = 0$

$k \cdot E$ = perpendicular plane   If the transverse

$k \cdot E = -\omega B$

6) $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

The conditinuity equation ensures that there ~~is a~~ ~~so~~ J is conservative when charge density ~~remain~~ is constant. ~~That cu~~rrent and changes by the rate of charge of ~~de~~ charge density.

current going in is same as current leaving a the point

$E = E_0 e^{i(k.r - \omega t)}$

Faradays law

$\nabla \times E = -\frac{\partial B}{\partial t}$

~~5)~~

5) i) continuity equation can be written as the scalar product between two four vectors

$J^\mu = (c\rho, J_x, J_y, J_z)$ $\frac{\partial}{\partial x^\mu} \left(\left(\frac{\partial}{\partial t}\frac{1}{c}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{dz} \right) \right)$

ii)

$E = E_0 e^{i(k\cdot r - \omega t)}$

faradays law

$\nabla \times E = -\frac{\partial B}{\partial t}$

$\nabla \times E = ik\times E$ $\qquad -\frac{\partial B}{\partial t} = +iB\omega$

$\cancel{i}k\times \overset{E}{k} = \cancel{i}B\omega$

$k\times E = B\omega$

$B = \frac{k\times E}{\omega}$

b) Poynting vector!

$N = \frac{1}{\mu_0}(E\times B)$

$= \frac{1}{\mu_0}\left(E\times \frac{(k\times E)}{\omega}\right)$

$= \frac{1}{\mu_0}\left(E^2\times \frac{k}{\omega}E_0^2\right)$

$\frac{\omega}{k} = \frac{c}{n}$

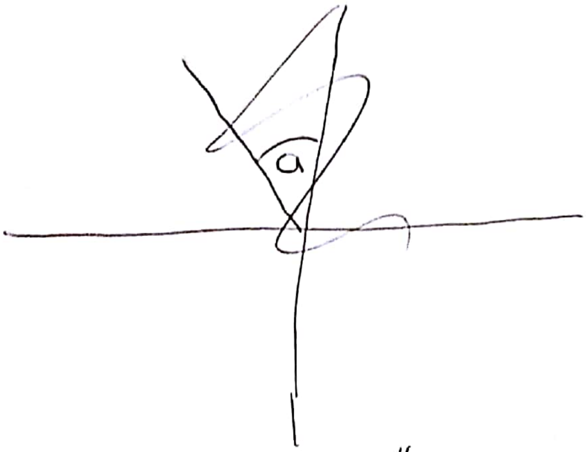
$\frac{k}{\omega} = \frac{n}{c}$

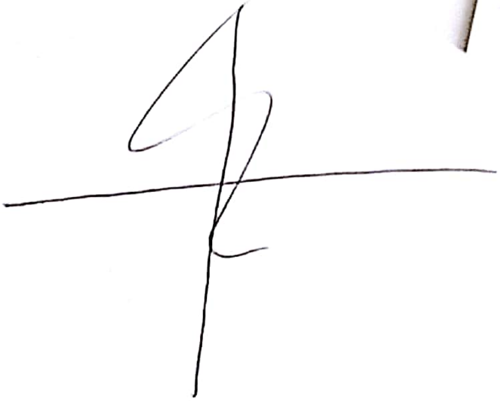
$= \frac{1}{\mu_0}\left(\frac{n}{c}E_0^2\right)$

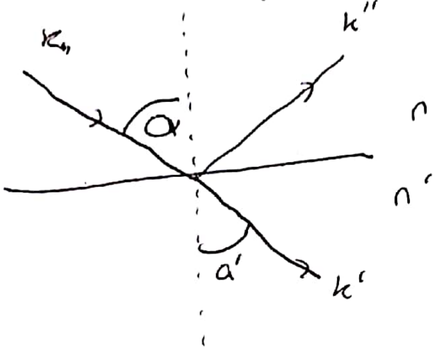
$= \frac{1}{\mu_0}\left(n\sqrt{\varepsilon_0\mu_0}\,E_0^2\right)$

$N = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,nE_0^2$

c)







n is the refractive index of the initial medium (vaccum in this case thus n=1)

n' is the refractive index of the dielectric

α → ~~[illegible]~~ angle of incidence normal to surface

a' → angle of transmitted wave normal to surface (refraction)

$r_{\parallel}$ → ~~or~~ parrellel component of wave reflected ~~[illegible]~~

$r_{\perp}$ → perpendicular component of wave reflected

$t_{\parallel}$ → Parrellel component of wave transmitted

$t_{\perp}$ → perpendicular component of wave transmitted.

ii) Boundary conditions: $E_{\parallel} \begin{cases} H_0 + H_0'' = H_0'' \\ E\cos a + E''\cos a = E'\cos a' \end{cases}$

$$H_0 = \frac{1}{\mu} B = \sqrt{\frac{\varepsilon}{\mu}}\, E$$

$$\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E + \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E'' = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E'$$

$$n E + nE'' = nE'$$

$$nE'' = \frac{nE' - nE}{n}$$

$J = \omega_0 C \varepsilon_r$

$= \frac{n'}{n} E' - E$

$E\cos a - \frac{n'}{n} E' - E = E' \cos a'$

$nE\cos a - (n'E' - nE)\cos a = nE'\cos a'$ $\qquad t_{\parallel} = \frac{|E'|}{|E|}$

divide by E

$n\cos a - n' t_{\parallel}\cos a + n\cos a = n t'' \cos a'$

$2n\cos a = t_{\parallel}(n'\cos a + n\cos a')$

$$t_{\parallel} = \frac{2n\cos a}{(n'\cos a + n\cos a')}$$

d) ii $r_{\parallel}^2 = \frac{(n'\cos a - n\cos a')}{(n'\cos a + n\cos a')} \frac{(n'\cos a - n\cos a')}{(n'\cos a + n\cos a')}$

$$= \frac{(n')^2\cos^2 a + n^2(\cos a')^2 - nn'\cos a\cos a'}{(n')^2\cos^2 a + n^2(\cos^2 a') + nn'\cos a\cos a'}$$

n' ~~sin~~

$$\left.\begin{aligned} R &= \left(\frac{n-n'}{n+n'}\right)^2 = r^2 \\ T &= 1 - r^2 = 1 - R = \frac{(n+n')^2 - (n+n')^2}{(n+n')^2} \end{aligned}\right\} \quad a = 0$$

i)

Brewsters angle (quick derivation)

$$r_{\parallel} = \frac{n'\cos a - n'\cos a^*}{n'\cos a + n\cos a} \quad \times \quad \frac{n'\cos a - n\cos a'}{n'\cos a + n\cos a'}$$

$$n' = \frac{n\sin a}{\sin a'}$$

$$r_{\parallel} = \frac{n'\sin a'\cos a^* - n^*\cos a\sin a}{n^*\cos a\sin a + n\cos a\sin a} = \frac{n}{n}\frac{\tan(a^* - a')}{\tan(a' + a_B^*)} = r_{\parallel}$$

$$a^* + a_B = \pi/2$$

$$a' = ~~a_B~~ a_B - \pi/2$$

$$\cos a' = n'\cos a_B$$

$$n\cos(a_B - \pi/2) = n'\cos a_B$$

$$\tan a_B = \frac{n'}{n}$$

$$a_B = \tan^{-1}\left(\frac{n'}{n}\right)$$

$$a_B = \tan^{-1}(1.6) = 57.99^{\circ}$$

$$a' = \overset{\text{rad}}{(57.99} - \pi/2$$

$$r_{\parallel} = 0$$

$$a' = 1.01^{\circ} - 0.558$$

$$r_{\perp} = 0$$

$$a_B = 1.01$$

$$a_c = -0.558$$

$$r_{\perp} = \left(\frac{\cos(1.01) - 1.6\overset{\cos}{(}-0.558)}{\cos(1.01) + 1.6\cos(0.558)}\right)^2 = R_{\perp} = 0\ 19\%$$

×

The retarded time is the time at which the source emmitted radiation

$$t_r = r\,\cancel{+\frac{t}{c}}\; t - \frac{r}{c}$$

$$\cancel{t}' = \cancel{\ } \; t - \frac{r}{c}$$

5)

$$\frac{\partial f(t')}{\partial t} = 1$$

$$\frac{\partial f(t')}{\partial t'} = 1 \qquad \text{thus} \qquad \frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'}$$

$$\frac{\partial f(t')}{\partial r} = -\frac{1}{c}$$

$$\frac{\partial f(t')}{\partial t'} = 1 \qquad \text{thus} \qquad \frac{\partial f(t')}{\partial r} = -\frac{1}{c}\,\frac{\partial f(t')}{\partial t'}$$

c) $B = \nabla \times A$

$$B = \nabla \times \left(-\frac{\mu_0 M_0 \omega}{4\pi c}\,\frac{\sin\theta}{r}\,\sin\omega t'\right)\hat{\phi}$$

$$\cancel{\nabla \times f = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \end{vmatrix}}$$

$$\nabla \times A = \frac{-\frac{\partial}{\partial r}\left(r\sin\theta\left(-\frac{\mu_0 M_0 \omega}{4\pi c}\,\frac{\sin\theta}{r}\,\sin\omega t'\right)\right) r\hat{\theta}}{r^2 \sin\theta}$$

$$\nabla \times f = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ f_r & r f_\theta & r\sin\theta\, f_\phi \end{vmatrix}$$

$$\frac{\partial A(r,t')}{\partial r} = -\frac{1}{c}\,\frac{\partial f(t')}{\partial t'}$$

$$= \cancel{-\frac{\mu_0 M_0 \omega}{4\pi c}} \; -\frac{1}{c}\,\frac{\partial}{\partial t'}\left(\frac{\mu_0 M_0 \omega}{4\pi c}\,\frac{\sin\theta}{r}\,\sin\omega t'\right)$$

$$B = -\frac{\mu_0 M_0 \omega}{4\pi c}\,\frac{\sin\theta}{r}\,\frac{1}{c}\cdot\omega\cos\omega t'$$

$$= -\frac{\mu_0 M_0 \omega^2}{4\pi c^2}\,\frac{\sin\theta}{r}\,\cos\omega t'$$

$$E = \cancel{-}\frac{1}{c}B_0 \qquad = -$$

$$E = \cancel{B}c$$

Brewsters angle [illegible]

$\cancel{\nabla\times B} =$ $\boxed{E = -\nabla V + \frac{\partial A}{\partial t}}$ $\boxed{E = -\nabla V + \frac{\partial A}{\partial t}}$

$\cancel{\nabla}$ $\cancel{E}=0$ $\cancel{\nabla\cdot B}=$ $\nabla\times A = B$

$\nabla V =$

↳ $\nabla\times E = -\frac{\partial B}{\partial t}$ $\nabla\cdot B = 0$

↳ $\nabla\times A = B$

$\nabla\times E = -\frac{\partial}{\partial t}(\nabla\times A)$

$\nabla\times E + \frac{\partial}{\partial t}\nabla\times A = 0$

$\nabla\times\left(E + \frac{\partial A}{\partial t}\right) = 0$

$E + \frac{\partial A}{\partial t} = -\nabla V$

$\boxed{E = -\nabla V - \frac{\partial A}{\partial t}}$

$E = -\frac{\partial A}{\partial t^*}$ → $\frac{\partial f(t')}{\partial t^*} = \frac{\partial f(t')}{\partial t'}$

$B = +\left(\frac{\mu_0 m_0 \omega}{4\pi c}\frac{\sin\theta}{r}\omega \cos(\omega t')\right)$

$= \frac{\mu_0 m \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos(\omega t')$

d) $\bar{S} = \frac{1}{\mu_0}(E\times B)$

$+ \frac{1}{\mu_0}\left(\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\right)^2 \cdot \frac{1}{c^2}\frac{\sin^2\theta}{r^2}\cos^2\omega t'\,\hat{r}$

$$\begin{vmatrix} r & \hat\theta & \phi \\ E_r & E_\theta & E_\phi \\ B_r & B_\theta & B_\phi \end{vmatrix}$$

$= \sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\right)^2 \frac{\sin^2\theta}{r^2}\cos^2\omega t'\,\hat{r}$

There is radiation from this ~~direction~~ system unless $\theta = 0$ or $\omega t' = 0$?

e) ~~They~~ ~~Bo~~ for Both ~~the~~ Hertzian dipole and magnetic ~~dipole~~

~~E is in $\hat\theta$ and B is on $\hat\phi$~~

$\hat\theta \times \cancel{\hat\phi} = -\hat{r}$

for magnetic dipole

$-\hat\phi \times \hat\theta = \hat{r}$

for Hertzian dipole

$\hat\theta \times \hat\phi = \hat{r}$

~~Fo~~ $P_{\text{Hertian}} = \frac{1}{\mu_0} I_0^2 \underbrace{\left(\frac{\mu_0 \, \partial \omega}{4\pi d}\right)^2}_{P} \cdot \frac{1}{c} \, \frac{\sin^2\theta}{r^2} \cos^2 \omega t' \, \hat{r}$

$$\frac{P_{\text{loop}}}{P_{\text{Hertian}}} = \frac{\frac{\cancel{\mu_0^2} m_0^2 \omega^4}{\cancel{\mu_0} \cancel{4^2} \cancel{\pi^2} \; c^{\cancel{3}2}} \; \frac{\cancel{\sin^2\theta}}{\cancel{r^2}} \; \cos^2(\cancel{\omega t'})}{I_0^2 \; \frac{\cancel{\mu_0^2} \, \partial \omega^2}{\cancel{\mu_0} 4^2 \pi^2} \cdot \frac{1}{\cancel{c}} \; \frac{\cancel{\sin^2\theta}}{\cancel{r^2}} \; \cancel{\cos^2} \omega t')}$$

$$= \frac{\frac{m_0^2 \omega^4}{c^2}}{I_0^2 \; \partial \omega^2} = \frac{m_0^2}{c^2} \times \underbrace{\frac{\cancel{\omega^4}}{I_0^2 \partial \cancel{\omega^2}}}_{} \; \frac{\omega^2}{I_0^2 d^2}$$

$\frac{1}{p}? \, \varepsilon_c \frac{\partial a}{dt} = \Phi$

$$= \frac{M_0^2}{c^2 P_0^2}$$

P_0 = magnitude of Hertzian dipole

$$\boxed{\begin{aligned} I_0 &= -q_0 \omega \\ P_0 &= q_0 d \end{aligned}}$$

$$\frac{m_0^2 \omega^2}{c^2 I_0^2 d^2} = \frac{m_0^2}{c^2 P_0^2}$$

iii)

$$\frac{m_0^2}{c^2} \; \frac{\omega^2}{I_0^2 (\pi a)^2} = \frac{m_0^2}{c^2} \cdot \frac{\omega^2}{(I_0 a)^2 \pi^2} = \frac{\omega^2}{c^2 \pi^2}$$

$$\frac{m_0^2 \omega^2}{c^2 I_0^2 d^2}$$

$d = \pi a$

$$\frac{m_0^2 \omega^2}{c^2 I_0^2 (\pi a)^2} = \frac{m_0^2 \omega^2}{c^2 I_0^2 a^2 \pi^2} - \quad \frac{\omega^2}{c^2 \pi^2}$$

9a i) $J = gE$ ✓

ii) faraday's law:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Ampere maxwell

$$\nabla \times B = \cancel{\mu_0 J + \mu_0 \varepsilon_0}$$
$$\mu J + \mu \varepsilon_0 \frac{\partial E}{\partial t}$$ ✓

$$\nabla \times (\nabla \times E) = \frac{-\partial}{\partial t}(\nabla \times B)$$

$$\underbrace{\nabla \cdot (\nabla \cdot E)}_{=0} - \nabla^2 E = \frac{-\partial}{\partial t}\left(\mu J + \mu_0 \varepsilon \frac{\partial E}{\partial t}\right)$$

$J = gE$

$$-\nabla^2 E = -\left(\frac{\partial}{\partial t} E(g\mu) + \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2}\right)$$

$$\nabla^2 E - g\mu \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

iii) $$E = E_0 e^{i(k \cdot r - \omega t)}$$

$$\nabla^2 E = -k^2 E_0, \quad \frac{\partial E}{\partial t} = -i\omega E \quad \frac{\partial^2 E}{\partial t^2} = -\omega^2 E$$

$$-k^2 \cancel{E} - g\mu(-i\omega \cancel{E}) - \varepsilon\mu(-\omega^2 \cancel{E}) = 0$$

$$k^2 = ig\mu\omega + \varepsilon\mu\omega^2 = 0$$

$$k^2 = \varepsilon\mu\omega^2\left(1 + \frac{ig}{\varepsilon\omega}\right)$$ ✓

[illegible] $(1 + [illegible])$

[illegible]

$k = [illegible] (1 + [illegible])^{1/2}$

[illegible]

$k = [illegible] (1 + [illegible])$

[illegible]

$\frac{1}{\delta} = [illegible]$

$\frac{1}{\delta} = [illegible] \frac{1}{2} [illegible]$

$= \sqrt{\frac{\mu}{\epsilon}} \frac{1}{2} [illegible]$

$\delta = [illegible] \sqrt{\frac{\epsilon}{\mu}}$

(b)

$\delta = 8.1\text{mm}$ [illegible]

$f = 10^{11}\,\text{Hz}$

σ no
magnet → bad conductor
non-magnet → good conductor

In this case non-magnetic material is good conductor

good conductor
$\boxed{\mu = \mu_0}$

thus

$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$

$\frac{1}{\sigma} = \frac{\delta^2}{2} \mu\omega$

$\frac{1}{\sigma} = \frac{(8.1\times10^{-3})^2}{2} \mu_0 (2\pi\times10^{11})$ ✓

Re

Answer be [illegible]!

$\sigma = 5.74\times10^7$

(c)

$V_g = \frac{\omega}{k}$

$k = [illegible] \sqrt{\frac{\omega\mu\sigma}{2}}$ $\quad \therefore V_p = \frac{\omega}{k} = \sqrt{\frac{2\omega}{\mu\sigma}}$

$V_p = 1.32\times10^6$

or [illegible]
[illegible]

$V_p = \omega\delta$

(d)

iv) For a poor caductor $g \to 0$ thus

$$k^2 = \mu \epsilon w^2$$

For a good conductor $g \to \infty$

$$k^2 = \frac{i \mu \epsilon w^2 g}{\epsilon w} = i \mu g w$$

b) i) Skin depth is $(\delta = \frac{1}{k})$ is the distance ~~inside~~ before it a wave propogates inside a dielectric e^{ikz} x amplitudes reaches

Skin depth is a measure of how closely electric current flows along the surface of a metarial

~~iii~~ ii) $$k^2 = \mu \epsilon w^2 \left(1 + \frac{ig}{\epsilon w}\right)$$

$g \to \infty$ for good conduction so ~~$\mu = \mu_0$~~ $\to$ good conductor

$$k^2 = i \mu g w$$

$$\sqrt{i} = \frac{1+i}{\sqrt{2}}$$

$$k = \sqrt{i \mu g w} =$$

$\Downarrow$

$$k = \sqrt{\frac{\mu g w}{2}}$$

take imaginary part

$$\delta = 1/k$$

$$\delta = \sqrt{\frac{2}{\mu g w}}$$

(m2x

10a)

$$J_\mu = (J_1, J_2, J_3, ic\rho)$$

$$a_\mu = (a_1\, a_2, a_3\; i\phi/c)$$

$$(\;(\mu = 1, 2, 3, 4)$$

C = Speed of light in a vacuum

J_1, J_2, J_3 = component of current density vector

A_1, A_2, A_3 = component of the magnetic vector potential

ϕ = electric potential