UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **PHAS3224**

ASSESSMENT PATTERN : **PHAS3224A**

MODULE NAME : **Nuclear and Particle Physics**

DATE : **14-May-14**

TIME : **10:00**

TIME ALLOWED : **2 Hours 30 Minutes**

ANSWER ALL QUESTIONS IN SECTION A AND TWO QUESTIONS FROM SECTION B

The numbers in square brackets at the right-hand edge of the paper indicate the provisional allocation of maximum marks for each sub-section of a question.

SECTION A

Question 1

Draw example Feynman diagrams for the electromagnetic, weak (both neutral and charged current) and fundamental strong interactions. Indicate the mass of the gauge boson in each case. **[8]**

Question 2

Give a very brief description of the assumptions behind the Fermi Gas Model of the nucleus. **[4]**

Explain why the neutrons have a deeper potential well than the protons in the Fermi Gas Model, and what simple fact about the ratio of the number of neutrons to protons in heavy nuclei does this accommodate ? **[2]**

Question 3

One of the important reactions used by the SNO experiment is the elastic scattering of neutrinos on atomic electrons :

$$\nu_x + e^- \rightarrow \nu_x + e^-$$

Explain, with the aid of Feynman diagrams, why this reaction must proceed via the weak neutral current for $x = \mu, \tau$ but can involve the exchange of either W or Z bosons when $x = e$. **[6]**

Question 4

A simplified form of the Bethe-Bloch equation describing the average ionisation energy loss by charged particles is :

$$-\frac{dE}{dx} \propto \frac{1}{\beta^2}\left[\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right) - \beta^2\right] ,$$

where m_e is the mass of the electron and I is the ionization potential of the material. $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$ have their usual relativistic definitions.

Sketch the shape of $-dE/dx$ as a function of $\beta\gamma$, identifying the important regions. You do not need to indicate any specific values of dE/dx or $\beta\gamma$ on your sketch. **[6]**

With specific reference to the leading $1/\beta^2$ term in the expression for $-dE/dx$, explain the phenomenon of the *Bragg peak* whereby there is a large increase in the rate of ionisation at the end of a charged particle's trajectory. **[2]**

Question 5

Give an example of a reaction that could be used in a man-made fusion reactor. **[3]**

Briefly describe the two main methods of confinement which are being pursued in the development of fusion reactors. **[4]**

Question 6

Give a qualitative explanation of the concept of electroweak symmetry breaking. **[3]**

Write down two decay modes of the Higgs boson that played an important role in its discovery at the Large Hadron Collider. Feynman diagrams are not required. **[2]**

SECTION B

Question 7.

(a) Using the single-particle Shell Model, predict the spin and parity (J^P) of the ground state of ${}^{17}_{9}\mathrm{F}$. [4]

(b) Suggest, giving reasons, possible spins and parities (J^P) of the first two excited states of ${}^{17}_{9}\mathrm{F}$. [4]

(c) Suggest a range of possible nuclear spins (J) for the ground state of the isotope ${}^{18}_{9}\mathrm{F}$. [3]

(d) Draw a Feynman diagram for the neutral-current deep inelastic scattering (DIS) of an electron on a proton. You do not need to specify in detail the hadronic final state. [4]

(e) The proposed LHeC collider will collide 70 GeV electrons and $7,000$ GeV protons head on. What will the centre-of-mass energy of the collisions be ? [3]

(f) Calculate the De-Broglie wavelength of the electrons in the LHeC centre-of-mass system. Comment on the distance scales probed by this collider, compared with the size of the proton. [3]

(g) One of the variables that describes a DIS interaction, y, is defined as follows :

$$y = \frac{P \cdot q}{k \cdot P} ,$$

where P is the incoming proton 4-momentum, k is the incoming electron 4-momentum and $q = k - k'$ is the difference between the incoming and scattered electron 4-momenta.

Show that in the rest-frame of proton, y is the fraction of the incoming electron's energy lost in the interaction. [3]

In the electron-quark centre-of-mass frame, the electron does not lose energy but is scattered through an angle θ with respect to its initial direction. Neglecting particle masses and recalling that the quark 4-momentum is a fraction x of the incoming proton 4-momentum, write down a simple expression for y in terms of θ. [4]

In a DIS interaction, the electron is scattered through 90° in the electron-quark centre-of-mass frame. What fraction of the electron's energy was transferred to the hadronic system, in the proton's rest frame ? [2]

Question 8.

(a) The isotope ${}^{51}_{24}\mathrm{Cr}$ decays to ${}^{51}_{23}\mathrm{V}$ via electron capture. Write down an equation showing the nuclear transition involved in this decay. **[2]**

The Q-value for this decay, defined as the difference between the initial and final state *atomic* masses, is 753 keV. Explain why β^+-decay is forbidden for this isotope. **[2]**

Neglecting atomic excitation energies, what is the energy of the neutrino emitted in the electron capture decay to the ground state of the daughter nucleus ? **[1]**

(b) The probability for a neutrino of flavour α to oscillate to flavour β is given, in the 2-flavour approximation, by the following expression :

$$P\left(\nu_\alpha \to \nu_\beta\right) = \sin^2(2\theta)\sin^2\left[1.27\frac{\Delta m^2 L}{E}\right] ,$$

where Δm^2 is in units of eV^2, L is the path length in metres and E is the energy in MeV. Explain the meaning of the parameters θ and Δm^2. **[4]**

For neutrinos from the decay of ${}^{51}_{24}\mathrm{Cr}$ and assuming that $\Delta m^2 = 1.0\ \mathrm{eV}^2$ for oscillations to a new neutrino species ν_s, calculate the distance between successive oscillation maxima. **[3]**

(c) Draw two labelled Feynman diagrams for the scattering of electron neutrinos from atomic electrons, indicating whether the interaction is charged- or neutral-current. **[4]**

(d) The SOX experiment proposes to directly search for oscillations of electron neutrinos from an intense ${}^{51}_{24}\mathrm{Cr}$ source located close to a large liquid-scintillator detector. Assuming a constant source activity of 400 PBq $= 4 \times 10^{17}$ decays per second, a detector comprising 270 tons of C_9H_{12} at an average distance of 8.25 m from the source and a cross-section for electron-neutrino scattering on atomic electons $\sigma(\nu_e e) = 0.72 \times 10^{-44}\ \mathrm{cm}^2$, calculate the number of events per day that would be observed in the absence of any oscillations. **[6]**

Atomic mass unit : 1.66×10^{-27} kg; carbon can be assumed to be 100% ${}^{12}_{6}\mathrm{C}$.

(e) If the SOX detector extends over several neutrino oscillation lengths, estimate the number of events that would be observed per day if $\nu_e \to \nu_s$ oscillations occur with $\sin^2(2\theta) = 0.1$, assuming that ν_s do not interact. Comment on the statistical significance of such a deficit of events. **[4]**

(f) How does a scintillation detector measure the energy of the scattered electrons ? **[2]**

(g) Are there any nuclear decays that produce a mono-energetic *anti*-neutrino spectrum ? **[2]**

Question 9.

(a) Draw and label a Feynman diagram for the decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. [2]

(b) Define the term *helicity*. [2]

(c) By considering the helicity of the decay products as well as angular momentum conservation, explain why high-energy positrons in the decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ are preferentially emitted in the direction of the muon's spin in its rest frame. [6]

Why is this evidence for parity-violation in weak interactions ? [2]

(d) Explain why muons from the decay $\pi^+ \to \mu^+ + \nu_\mu$ are produced with negative helicity. [3]

(e) Assuming that the muons produced in the decay $\pi^+ \to \mu^+ + \nu_\mu$ maintain their spin direction after being stopped in a target, will the subsequent high-energy decay positrons be forward- or backward-peaked with respect to the original muon direction of motion ? [2]

(f) By assuming CP-symmetry, or otherwise, state whether the angular distribution of the decay electron from the stopped muon can be used to identify whether it was a μ^- or μ^+. You can neglect any different interactions between the μ^- and μ^+ and the stopping target material. [3]

(g) Draw Feynman diagrams for the decays $\tau^- \to \nu_\tau + \pi^-$ and $\tau^- \to \nu_\tau + K^-$. Why is the kaon decay mode suppressed with respect to the pion decay mode ? You may neglect any effect due to the mass difference between the pion and kaon. [5]

(h) Assuming the τ^- decays at rest, calculate the energy of the ν_τ produced in the decays $\tau^- \to \nu_\tau + \pi^-$ and $\tau^- \to \nu_\tau + K^-$. [5]

Numerical data:

$m_\tau = 1.78$ GeV/c^2; $m_{K^-} = 494$ MeV/c^2; $m_{\pi^-} = 140$ MeV/c^2

Question 10.

(a) The semi-empirical mass formula (SEMF) for the binding energy of a nucleus with atomic number Z and mass number A is :

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(Z - A/2\right)^2 A^{-1} \pm \delta a_p f(A)$$

The term $-a_c Z^2 A^{-1/3}$ represents the reduction in binding energy due to the electrostatic Coulomb repulsion of the protons inside the nucleus. By considering the nucleus to be a uniformly charged sphere with total charge Ze, and integrating over thin spherical shells, estimate a value for the coefficient a_c. [6]

Numerical data:
$\alpha = e^2/4\pi\epsilon_0\hbar c = 1/137$; nuclear radius $R = R_0 A^{1/3}$ where $R_0 = 1.2$ fm

(b) Starting from the SEMF and neglecting the pairing term, derive an expression for the atomic number Z that maximises the binding energy for fixed mass number A. You can neglect the mass difference between protons and neutrons. [5]

(c) The most stable isotope with mass number 166 is the isotope $^{166}_{68}$Er. Use this fact to extract a value for the coefficient a_c in units of MeV. [2]

Given this estimate of a_c, use the result in part (a) above to derive a value for the fine structure constant α. [2]

Numerical data:
$a_a = 93.15$ MeV

(d) Which of the following reactions:

(i) $\gamma + \gamma \to \tau^+ + \tau^-$
(ii) $\nu_e \to e^- + \mu^+ + \nu_\mu$
(iii) $\nu_\mu + p \to \mu^- + \Delta^{++}$
(iv) $\Delta^{++} \to p + p$
(v) $\Delta^{++} \to p + \pi^+$
(vi) $K^- \to e^- + \gamma$

are allowed and which are forbidden ? Explain why and draw the lowest order Feynman diagrams for the allowed reactions. [15]

Particle data:
$\Delta^{++} = uuu$, $m_{\Delta^{++}} = 1232$ MeV/c^2

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