PHAS 3224, 2015 ~~X X 3 4 5 6 7 8 9 10~~ ①

Q1

charge	1st generation	2nd generation	3rd generation	
+2/3	u	c	t	} quarks
−1/3	d	s	b	
~~+1~~ −1/2	e^-	μ^-	τ^-	} leptons
0	ν_e	ν_μ	ν_τ	

Q2 a) The mass of ~~the~~ a decaying particle must be at least the sum of the rest masses of the produced particles to satisfy mass-energy conservation; the additional mass of the decaying particles goes into the momentum of the produced particles to satisfy momentum conservation

b) p: uud

c) i) $E = 6500$ GeV

~~$E_{CM} = \sqrt{2mE}$ (for two counter propogating beams)~~

$$E_{CM} = \sqrt{(E_1+E_2)^2-(E_1-E_2)^2}$$ for two beams of energies E_1 & E_2 with $\theta = 180°$

since $E_1 = E_2 = E$

$$E_{CM} = 2E = 13000 \text{ GeV}$$

ii) The centre-of-mass energy of each parton parton interaction is less than this since the colliding protons consist of three quarks & a sea of gluons all of which carry a fraction of the protons momentum but which the sum of gives the center-of-mass energy

Q3 a) $E^2 = m^2c^4 + p^2c^2$

E: particle total energy [GeV]

m: particle rest mass [GeV/c^2]

p: particle momentum [GeV/c]

c: speed of light

* the charge,

b) ~~E~~ From the einstein mass-energy equation we see that −ve energy solutions are possible (i.e. $E = \pm(m^2c^4+p^2c^2)^{1/2}$) In classical ~~ph~~ mechanics, these negative energy solutions can be ignored. However, in quantum mechanics we must consider them. These negative energy solutions ~~th~~ that arose in the development of the relativistic quantum mechanical model of the electron were interpreted by P.A.M. Dirac to be 'anti particles'

c) Positron by Carl Andersen in 1932 ~~after observing~~ in cloud chamber experiments at Caltech

d) ~~There~~ The track that a particle makes can be photographed in cloud chamber experiments. If a magnetic field is applied to the interaction region, & the magnitude of this applied field is known then, from the radius of curvature & direction of the curved path ~~of the~~, one can determine the mass & charge of the particle ($F_{lorentz} = q\underline{v} \times \underline{B}$). Andersen discovered a particle of similar mass to the electron curving in the opposite direction – the positron.

Q4 [SEMF gives mass, increasing binding energy (all other things being equal) reduces mass
⇒ +ve terms in the SEMF reduce the binding energy & vice-versa]

a) ~~$M(Z,A) = Zm_p +$~~ The a_s term is the surface term that arises from the liquid drop model of the nucleus. It reduces the binding energy of the nucleus and ~~it intu~~ arises because ~~the~~ nucleons on the 'surface' of the nucleus have less neighbouring nuclei with which they can ~~less~~ strongly bind to & hence reducing the binding energy of the nucleus.

b) On average heavy nuclei contain fewer protons than neutrons. This is explained by the Coulomb term of the SEMF ($a_c Z^2 A^{-1/3}$) which ~~reduces the binding energy~~ arises from proton-proton repulsion (& hence is independent of the # of neutrons) and reduces the binding energy of the ~~the~~ nucleus.

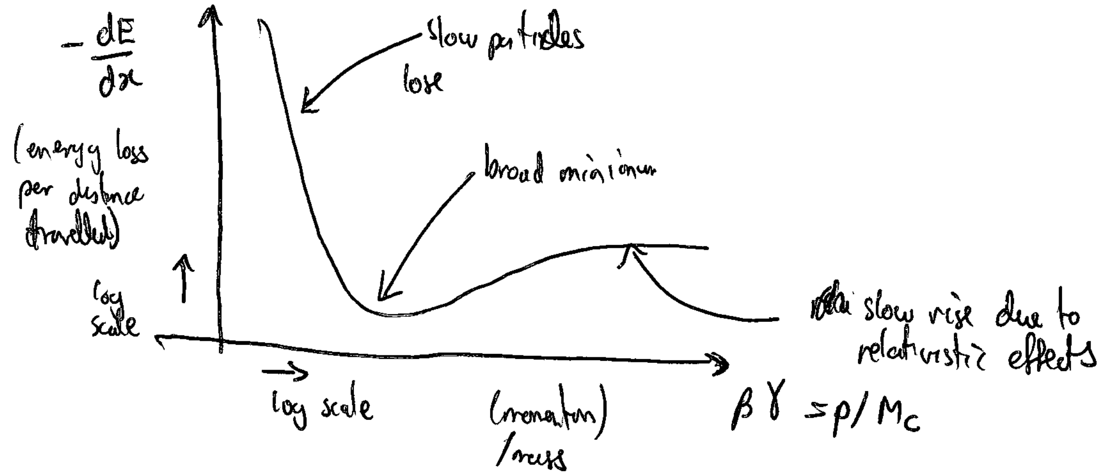
Q5 a) 1) ~~electron~~ electromagnetic calorimeter
2) hadronic calorimeter
3) muon spectrometer

semiconductor trackers

b) Muons ~~& pions~~ traverse the whole detector (due to their high energies) & are measured in the outer most muon spectrometers. Pions are charged and ~~leak~~ leave a track in the trackers before being deposited in the hadronic calorimeters.

c) b-jets – identified by reconstructing the displaced secondary vertex.
$c\tau \simeq 350$ mm

Q6 a) Bethe-Bloche formula ← gives energy loss with distance travelled as a function of particle momentum



b) $p = 1\ GeV/c$

Proton will lose more momentum, its larger rest mass means that its $\beta\gamma$ is smaller. The proton is moving slower than the pion & so it loses more energy ($\beta\gamma$ is below the relativistic region).

Section B

$m_p = 938$ MeV
$m_n = 940$ MeV
$m_\tau = 178$ MeV
$\boxed{m_\pi = 130 \text{ MeV}}$

17) a) i) $p \rightarrow n + e^+ + \nu_e$

Not allowed due to energy conservation. & $m_n + m_e (+ m_{\nu_e}) > m_p$
But we also know that the proton does not decay.

ii) $p + p \rightarrow p + \bar{p} + \pi^0$

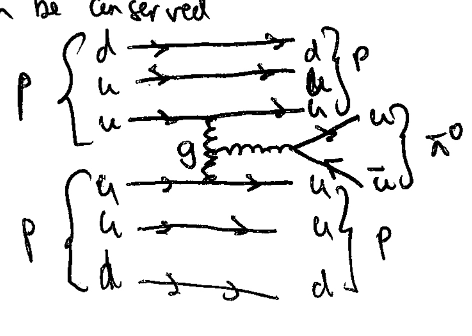
Baryon # violation!

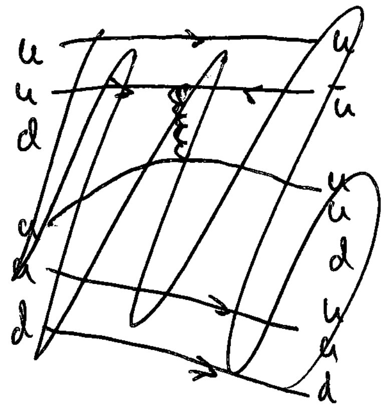
LHS: baryon # = 1+1 = 2
RHS: baryon # = 1 + (-1) + 0 = 0

Not allowed

iii) $p + p \rightarrow p + p + \pi^0$

- charge is conserved
- baryon # is conserved
- mass-energy & momentum can be conserved





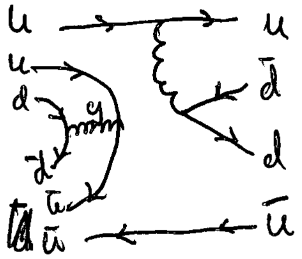
iv) $p + p \rightarrow \pi^0 + \pi^+ + \pi^+$

baryon # not conserved $\Rightarrow$ Not allowed

v) $p + \bar{p} \rightarrow \pi^+ + \pi^-$

$\pi^+ : u\bar{d}$
$\pi^- : \bar{u}d$

- baryon # conserved
- Charge conserved
- ~~charge~~ energy-momentum can be conserved

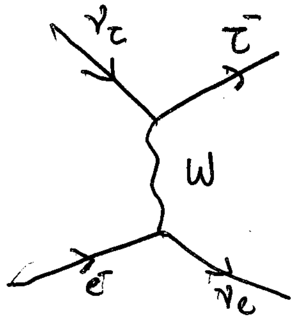


vi) $\tau^- \rightarrow \mu^- + \nu_\tau$

- lepton # violation, not allowed

vii) $\nu_\tau + e^- \rightarrow \nu_e + \tau^-$

- Lepton # conserved
  - tau & electron lepton # conserved
- charge conserved
- energy-momentum can be conserved



b) ^{18}F, undergoes β^+ decay

$t_{1/2} = 109.8$ mins

i) $^{18}_{9}F \rightarrow {}^{18}_{8}X + e^- + \bar{\nu}_e$

^{18}O produced in β^+ decay of ^{18}F

ii) # of emitted positrons $\equiv$ # of decayed ^{18}F

$N(t) = N_0 \exp(t/\tau)$, τ is the lifetime

~~$= [(1\,mol \times 6.022 \times 10^{23}\,mol^{-1}) \exp(5/109.8)]$~~

$$N(t_{1/2}) = N_0/2 = N_0 \exp(-t_{1/2}/\tau)$$

$$-\ln 2 = -t_{1/2}/\tau$$

$$\tau = t_{1/2}/\ln 2$$

$N(t = 5\,mins) = (6.022 \times 10^{23}) \exp(-5\ln 2/109.8) = 5.834 \times 10^{23}$ (atoms remaining)

b) ii) Number of positrons emitted $\equiv$ Number of ^{18}F decay $= N_A - 5.834 \times 10^{23}$

$= 1.87 \times 10^{22}$

iii) energy released in decay

$$\Delta M = \left[\left(17.9991610\cancel{7}u \cancel{+ 0.511 MeV}\right) - \left(18.0009380u\right)\right] / c^2$$

$\{u = 931.5 MeV\}$ ↑ no e^-

$E = \Delta M c^2 = 0.633 MeV$

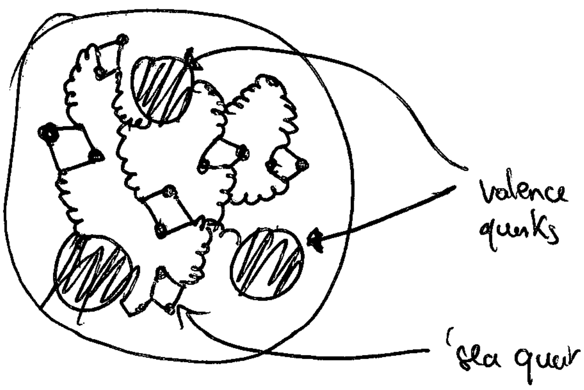
iv) • α-emission : emitted particles: α particles (^4He particles)

• γ-ray emission : γ rays...

Q8

a) ~~debroigle relation~~ To probe small distances we need a small / short wavelength to resolve these distances. The de Broigle relation gives the wavelength as a function of momentum as, $\lambda_{dB} = \frac{h}{p}$. Since $\lambda_{dB} \propto 1/p$ then large momenta & hence high-energies are required to probe small distances.

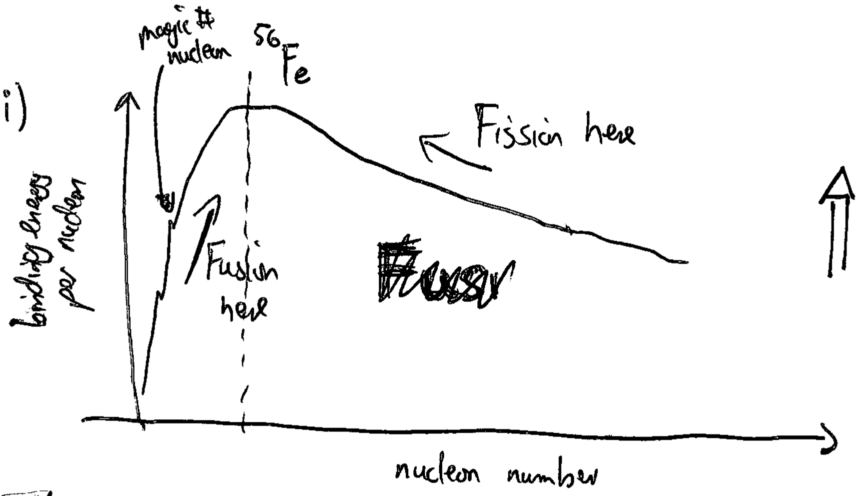
b) If the proton consisted of ~~3~~ only 3 quarks then we would expect that the momentum of each of these quarks to be 1/3 of the total momentum. However, what was observed in scattering ~~m~~ experiments was a continuous spectrum of momentum. This was explained by postulating that ~~the proton also consisted of quarks~~ the quarks that held the proton together can spontaneously create a quark-antiquark pair ($g \to \bar{q}q \to g$).



Those quark-antiquark pairs created by quantum fluctuations also carry a fraction of the protons momentum explaining the spectrum seen

Q8

e) i)



⇑ tightness of binding

ii) ~~Nuclear Fission~~

The principle ~~between~~ behind nuclear fission and fusion is to take advantage of the difference in binding energy per nucleon between nuclei with different nucleon numbers

Nuclear Fission

For nuclei with nucleon number >100 it may be energetically favorable for the massive nuclei to split into ~~two~~ ~~lig~~ less massive daughter nuclei that are more tightly bound & hence more stable. In doing so the excess energy is carried off ~~as~~ the kinetic energy of the fission products

Nuclear Fusion

In fusion, two light nuclei combine/fuse to form a heavier nucleus which has a larger binding energy per nucleon & hence is more tightly bound. The mass of the fused nuclei is less than the sum of the mass of the two nuclei being fused, & this ~~mass~~ difference in mass manifests itself as kinetic energy of the fused nuclei.

iii)
$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}H + p$$

~~Binding energy of~~ $\Delta E = (8.482 - 2(2.224))\,MeV$

$= 4.033\ MeV$

8

Q9

i) $B_{s2}^* \to B^+ + K^-$

ii) $B_c^+ \to B_s^0 + \pi^+$

d) $B_{s2}^* : \bar{b}s$, $m_{B_{s2}^*} = 5840\ \mathrm{MeV}/c^2$

$B_s^0 : \bar{b}s$, $m_{B_s^0} = 5367\ \mathrm{MeV}/c^2$

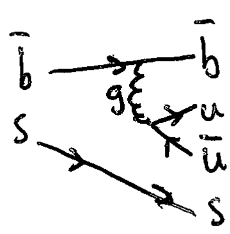
$B_c^+ : \bar{b}c$, $m_{B_c^+} = 6276\ \mathrm{MeV}/c^2$

$B^+ = \bar{b}u$, $m_{B^+} = 5279\ \mathrm{MeV}/c^2$

$K^- = s\bar{u}$, $m_{K^-} = 494\ \mathrm{MeV}/c^2$

i) $B_{s2}^* \to B^+ + K^-$

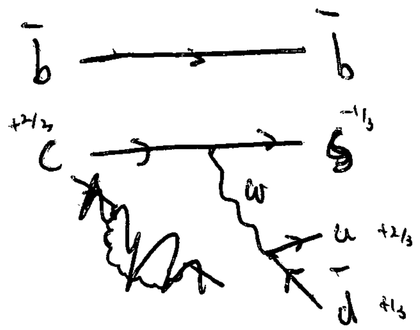
$\bar{b}s \quad \bar{b}u \quad s\bar{u}$



ii) $B_c^+ \to B_s^0 + \pi^+$

$\bar{b}c \quad \bar{b}s \quad u\bar{d}$

$1/3 \ +2/3 \quad 1/3 \ (-1/3) \quad (2/3) \ +1/3$



b) i) Strong interaction

ii) Weak interaction

c) B^*_{s2} decays faster because it decays via strong force.

d) B^+ , $c\tau \sim 450\mu m$ $\quad (B^*_{s2} \rightarrow B^+ + K^-)$

B^*_{s2} had 10 GeV/c at time of decay

$$P_{B^*_{s2}} \simeq P_{B^+}$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

time dilation

$$\tau' = \gamma\tau = \frac{1}{\sqrt{1-\beta^2}}\tau, \quad \beta = v/c$$

$\beta = ?$

$$P_{B^*_{s2}} = P_{B^+} = \gamma m_{B^+} v = \overbrace{(10E9)(5.344 \cdot 10^{-28})}^{num} \text{ Kg m s}^{-1}$$

$$\frac{\beta}{\sqrt{1-\beta^2}} m_B = num/c$$

$$\frac{\beta^2}{1-\beta^2} = \left(\frac{num}{m_B c}\right)^2 \Rightarrow \beta^2 = \left(\frac{num}{m_B c}\right)^2 - \left(\frac{num}{m_B c}\right)^2 \beta^2$$

$$\beta^2 = \frac{(num/m_B c)^2}{1 + (num/m_B c)^2}$$

$$\simeq 0.782$$

$$c\tau' = c\gamma\tau \simeq 722\ \mu m$$

$$\beta = \frac{v}{c}$$

distance $= v\tau'$

$*$ $= \beta c \tau' \simeq 638.56\ \mu m$

Q9 e) $m_{B_s} = 5840\ \text{MeV}/c^2$

$m_{B^+} = 5279\ \text{MeV}/c^2$

$m_{K^-} = 494\ \text{MeV}/c^2$

$\frac{1}{2} m v^2$

f) $\frac{mv^2}{r} = qBv$

$r = \frac{mv}{qB}$

$1/r = qB/mv = \frac{(1)(3)}{1\ \text{GeV}/c}$

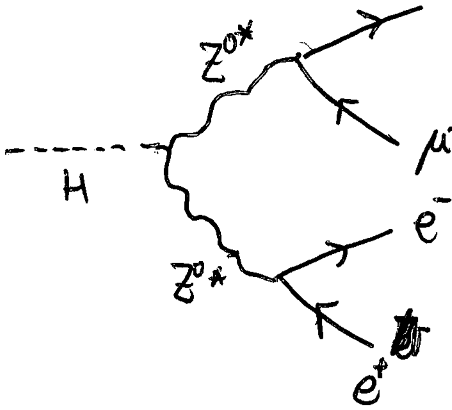
$= \frac{eC \quad 1\ \text{Kg}\ C^{-1} s^{-1}}{(1E9)(5.344 \times 10^{-28})\ \text{Kg}\ m^2 s^{-2}}$

$\simeq 0.3\ m^{-1}$

(10)

Q6

a) i)



ii) $m_H = 125\ GeV/c^2$

$m_{Z^0} = 91\ GeV/c^2$

$m_{Z^{0*}} = 34\ GeV/c^2$

b) $\sigma_{Higgs} = 20\,000\ fb$

$f_{Z \to e^- e^+} = 3.36\%$

$f_{Z \to \mu^- \mu^+} = 3.36\%$

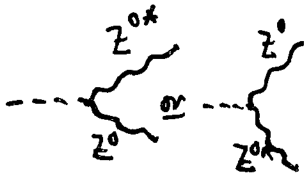
$f_{H \to ZZ} = 2.7\%$

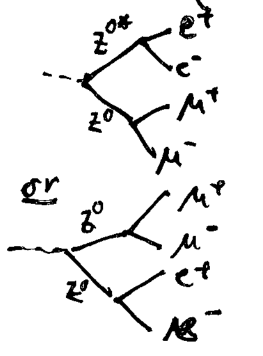
10 fb^{-1} of data

$\Rightarrow$ total H events $= (20000)(10)$
$= 2E5$

total $H \to ZZ$ events $= \left[(0.027)(2E5)\right] \times 2$
$=$ ~~5.4E3~~ $10.8\ E3$

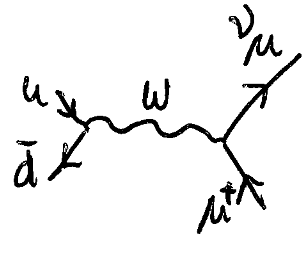
total $ZZ \to e^- e^+ \mu^- \mu^+$ events $= \left((10.8\ E3)(0.0336)^2\right) \times 2$
$= 24$



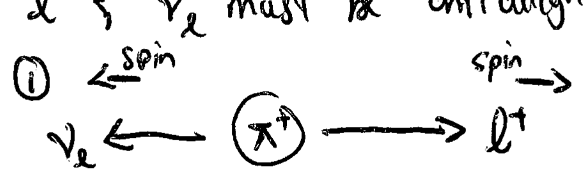


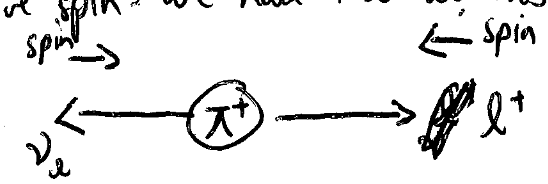
c) Higgs does not couple directly to photons because Higgs only couples to massive particles & photons are massless...

d) $\pi^+ \to \mu^+ \nu_\mu$
$u\bar{d}$



e) We consider the decay $\pi^+ \to l^+ \nu_e$, where l is the lepton in question. We can consider the decay in the rest frame of the π^+, & so the l^+ & ν_e must travel back to back to conserve momentum. We know that π^+ is a spin-0 particle & so the spins of the products l^+ & ν_e must be antialligned to conserve spin. We have two scenarios



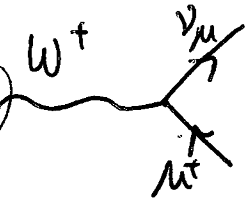


Q10 e) We know that only left handed neutrinos occur in nature & so ① is not allowed. However this would imply a left handed anti-lepton. If this particle were massless & hence travelling at the speed of light then ② too would be disallowed. But since the charged lepton is massive, its helicity cannot be exactly defined & so ② is allowed but suppressed by $\left(\frac{mc^2}{E}\right)^2 \rightarrow \left(\frac{2m_\pi m_\ell}{m_\ell^2+m_\pi^2}\right)^2$

$\Rightarrow$ This explains why the branching ratio to the more massive anti-muon is less suppressed than to the anti-electron.

(This can also be considered at a more superficial level, the larger mass of the muon implies that its velocity is smaller in the rest frame of π^+. This means that its helicity is less well defined than the e^- that moves at relativistic speeds)

g) 4 marks



In a collider detector, the muon would be detected by the muon spectrometers at the outermost layer of the collider. The neutrino however, being only weakly interacting would not be detected. However, its existence can be inferred by reconstructing the missing momentum.

h) $\boxed{\text{Scattering Amplitude} \quad M(p^2) = \frac{g^2\hbar^2c^2}{p^2 - M_x^2c^4}}$ $\longrightarrow$ at low energies this becomes $M \sim \frac{-g^2\hbar^2}{M_x^2c^4}$

Since we assume that $g_\gamma \simeq g_W$, then it is easy to see that since the weak force is mediated by Weak boson which is massive while the electromagnetic force is mediated by the photon (massless) then the weak force is <u>much weaker</u> at lower energies.

f) $W^+ \rightarrow \mu^+ \nu_\mu$

<u>assuming masses are negligible</u>

$W^+ \rightarrow \ell^+ \nu_\mu$ (3 lepton generations)

$W^+ \rightarrow u\bar{d}, u\bar{s}, c\bar{d}, c\bar{s}, u\bar{b}, c\bar{b}$

} Branching fraction $= 1/9$

c) i) electron beam energy : 70 GeV $= E_1$

proton beam energy : 7000 GeV $= E_2$

centre of mass energy : $E_{CM} = \sqrt{(E_1+E_2)^2 - (E_1-E_2)^2}$

$= 1400$ GeV

ii) $\mathcal{L}$ depends on
- # of colliding bunches
- # of particles in each beam ~~thru~~
- cross sectional area of the beam
- frequency ~~f~~ with which th bunches circulate.

↑ instantaneous luminosity

d) $\pi^+ \to \mu^+ \nu_\mu$

total energy of μ^+ in rest frame of π^+

$m_\pi = 139.57\ MeV/c^2$, $m_\mu = 105.7\ MeV/c^2$

$\cancel{p_\pi^2 = p_{\mu^+\nu_\mu}^2}$

$\cancel{m_\pi^2 c^4 = m_\mu^2 c^4 + (2pc)^2}$

$\cancel{p^2c^2 =}$

$W^2 = m_\pi^2 c^4 = \cancel{(E_\pi+E_\nu)^2}\ (E_\mu + E_\nu)^2 - (p_\mu c + p_\nu c)^2$

but by conservation of momentum

$p_\mu = -p_\nu$

$\Rightarrow m_\pi^2 c^4 = (E_\mu + E_\nu)^2$

but we assume massless neutrinos $\Rightarrow E_\nu = p_\nu c = p_\mu c$

$\cancel{m_\pi c^2 = m_\mu c^2 + p_\mu c}$

$m_\pi^2 c^4 = (m_\mu c^2 + 2 p_\mu c)^2$

$p_\mu c = \cancel{\frac{1}{2}(m_\pi^2 c^4 - m_\mu^2 c^4)} = \frac{1}{2}(m_\pi c^2 - m_\mu c^2)$

$p_\mu c = 16.935\ MeV$

$E_\mu = (m_\mu^2 c^4 + p_\mu^2 c^2)^{1/2} = 107.0\ MeV$