MATH3305 — Problem Sheet 4

Problems 4, 5 and 6 to be handed in **at the lecture** on Friday, 4 November 2016

1. Show that the line element $ds^2 = g_{ab}dX^a dX^b$ transforms like a scalar under general coordinate transformations.

2. Repeat in detail and explain both derivations of the Christoffel symbol (i) extremising the length of a curve (ii) uniqueness of the covariant derivative satisfying $\nabla_a g_{bc} = 0$.

3. Consider the metric $ds^2 = \Omega(x, y)^2(dx^2 + dy^2)$ and compute all Christoffel symbol components.

4. Compute the components of the Christoffel symbol for the following metric

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)\,.$$

   This is known as the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric and is of great importance in modern cosmology. (Hint: the number of non-vanishing components is 6).

5. Consider the metric

$$ds^2 = v^2du^2 + u^2dv^2\,.$$

   Compute R_{1212}.

6. Let W_{abcd} be a tensor satisfying (i) $W_{abcd} = -W_{bacd}$, (ii) $W_{abcd} = -W_{abdc}$ and (iii) $W_{abcd} + W_{cabd} + W_{bcad} = 0$. Show that this implies $W_{abcd} = W_{cdab}$.