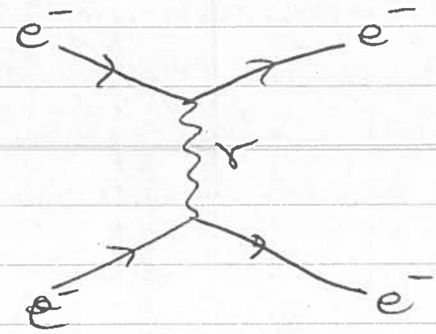
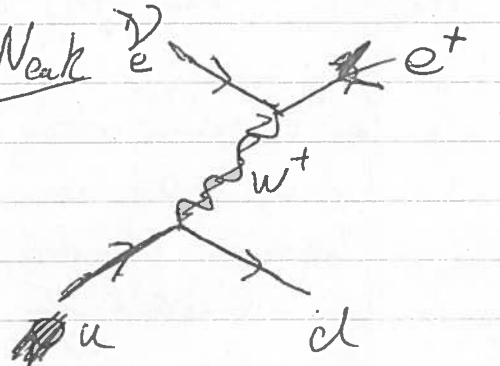
NPP - 2014

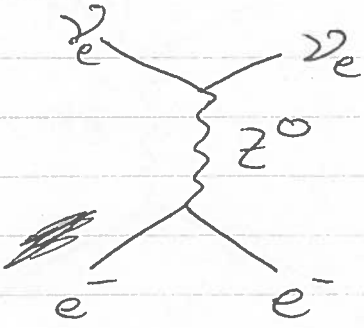
Q1. EM

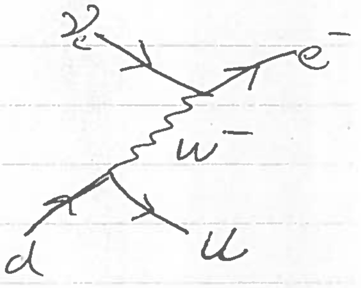


$M_\gamma = 0$

Weak







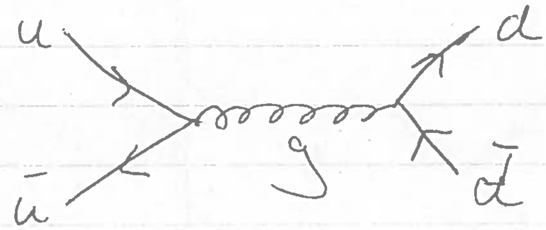
$\frac{2}{3} \rightarrow +1 \; -\frac{1}{3}$ ✓

$-1 \rightarrow -1$ ✓

$M_{W^\pm} \simeq 80\,GeV/c^2$ ~~$M_{W^\pm} \simeq 91\,MeV$~~

$M_{Z^0} \simeq 91\,GeV/c^2$ ~~$M_{Z^0} \simeq 80\,MeV$~~

Strong

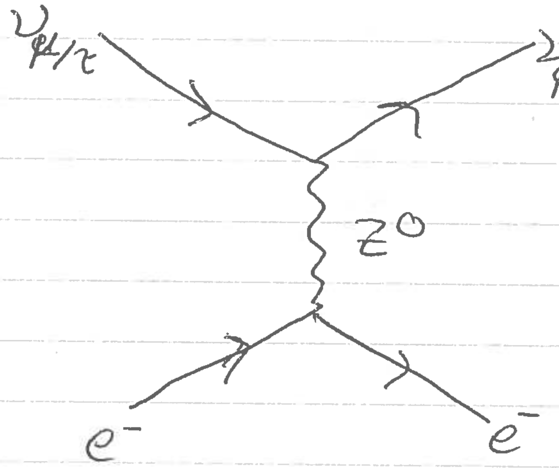


$M_g \simeq 0$

Q2. a) Fermi-gas model assumptions:
- Simple 3D potential (Spherical)
- Nucleons move freely inside nucleus (like a gas)
- Nucleons fill ups energy levels up to E_F, the fermi energy
- Potential wells for protons and neutrons can be different. [generally deeper for neutrons]
- 2 protons/2 neutrons per energy level...

5) Protons are less strongly bound than neutrons due to the coulomb interaction between protons. For ~~th~~ Heavy nuclei, the protons need to be "spaced out" by neutrons so that coulomb interaction between protons is low enough for the neucleus to stay together.

3.

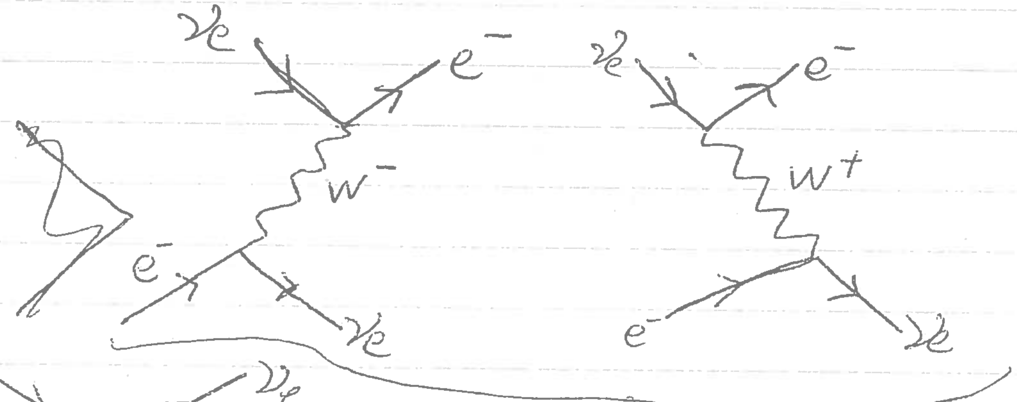


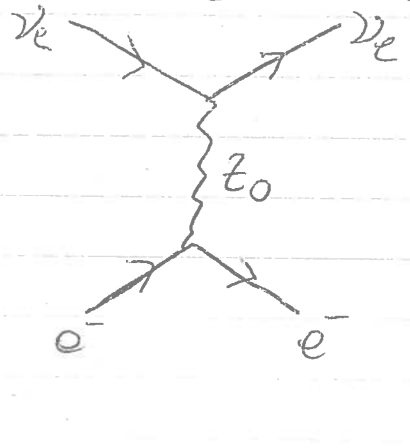
$x = \mu, \tau$

an e^- cannot decay into a ν_μ or ν_τ via a W boson $\therefore$ can only occur via a Z boson when the ν's are on the same vertex (Same for e^-)

for charges to be conserved at the verticies the process must us a Z boson.

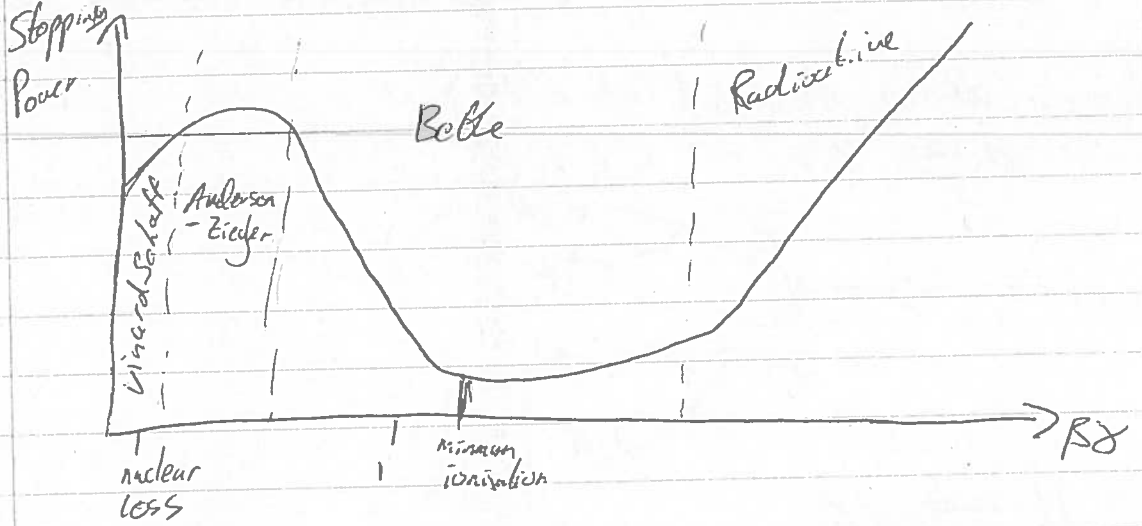
$x = e$





Here charges are conserved because a W Boson can decay into a ν_e, e^- pair

4. a) Bethe - Bloch Eq:



b) At low energies $\frac{dE}{dx} \sim \frac{1}{\beta^2}$ and slower particles loose more energy. ~~At the Bragg Peak~~ particles loose the most energy when they are slowest, this means that a particle will deposite most of its energy in a medium at the end of its path. This increase in energy deposition is the Bragg peak.

5. a) ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n + 17\,MeV$

b) Two methods of confinement:

1) Magnetic confinement - particles contained in a magnetic field, usually a Tokamak

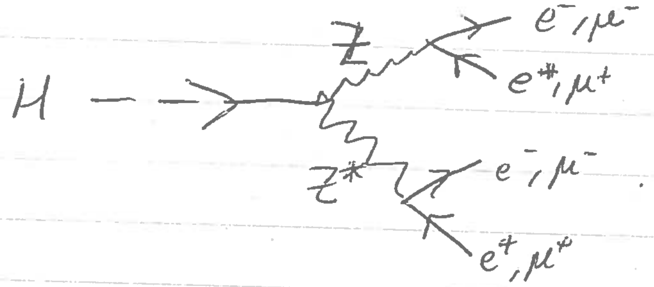
2) Inertial confinement - pulsed lasers bombard pellets of tritium-deuterium mix in many directions at very high energies.

6. a) Higgs mechanism ~~Spontaneously~~ Spontaneously breaks ~~symmetry of~~ the electroweak symmetry which allows gauge bosons to have mass. The Higgs field exists at all points in the universe, interactions with the field break the symmetry and give mass, but interactions with this field remain gauge invariant.

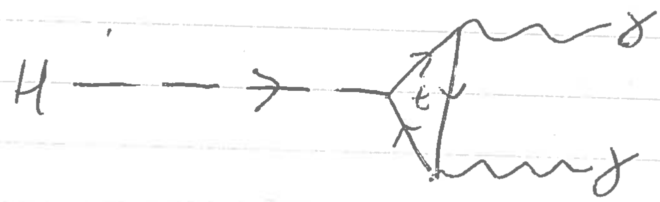
Masses occur because the expectation value of the Higgs field in a vacuum is non-zero (all other fields have $\langle\ \rangle = 0$ in a vacuum)

b) Higgs decay modes (at LHC):

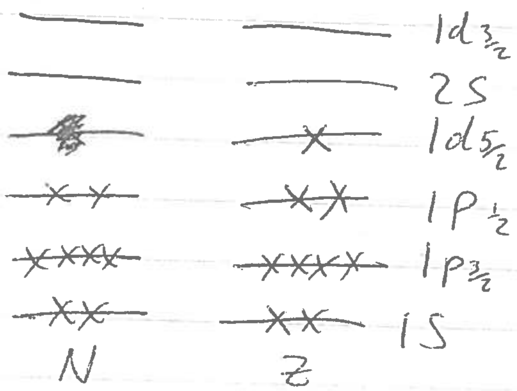
- $H \rightarrow ZZ^* \rightarrow l^+l^-l^+l^-$



- $H \rightarrow \gamma\gamma$



7. a) $^{17}_{9}F \Rightarrow$



$Z=9$
$N=8$

$s=0$
$p=1$
$d=2$

$P_{nucleus} = (-1)^1 \times (-1)^2 = -1$

~~$J = \frac{5}{2} \Rightarrow J^P = \left(\frac{5}{2}\right)^{-1} = \frac{2}{5}$~~

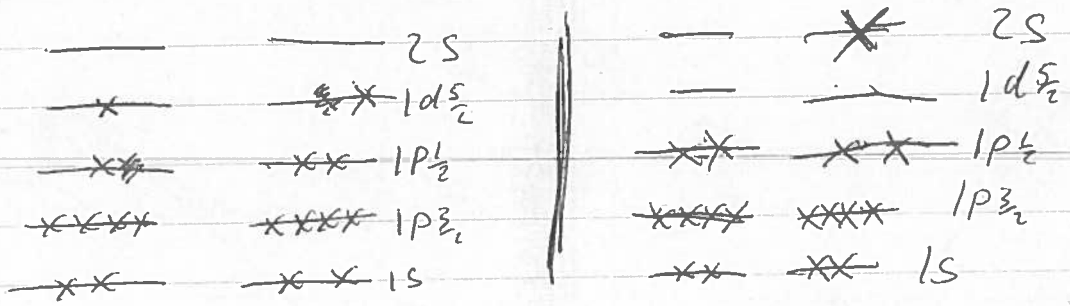
~~Spin $J = \frac{5}{2}$~~

$J = L + \frac{1}{2}$

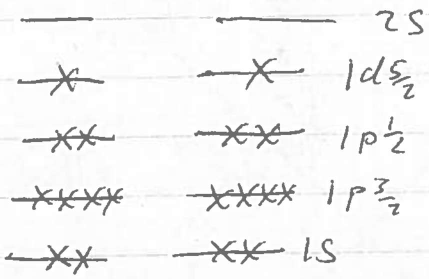
$\Rightarrow J = \frac{5}{2} + \frac{1}{2} = 3$

$\Rightarrow J^P = (3)^{-1} = -3$

b) First two excited states



c) $^{18}_{9}F \Rightarrow$



$j_p = \frac{5}{2} + \frac{1}{2} = 3$

$j_n = \frac{5}{2} + \frac{1}{2} = 3$

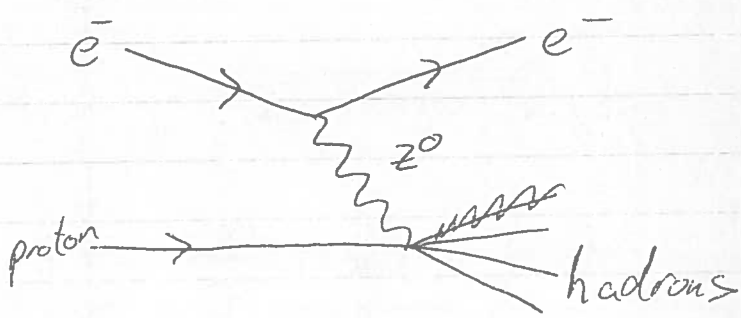
$J = |j_p - j_n| \rightarrow (j_p + j_n)$

$= \cancel{0 \rightarrow 5}$

$= 0 \rightarrow 6$?

d) 

DIS for e^- on proton



$E_{cm}^2 = (P_1 + P_2)\cdot(P_1 + P_2)$

$= (E_1 + E_2)^2 - (cp_1 + cp_2)^2$

$= (E_e + E_p)^2 - \left(\sqrt{E_e^2 - m_e^2 c^2} + \sqrt{E_p^2 - m_p^2 c^2}\right)^2$

$E_{cm} = \sqrt{(E_e + E_p)^2 - \left(\sqrt{E_e^2 - m_e^2 c^2} + \sqrt{E_p^2 - m_p^2 c^2}\right)^2}$

$= \sqrt{(7070)^2 - (70 + 7000)^2}$

$E_{cm} = \sqrt{s} = 2\sqrt{E_e E_p} = 2 \cdot \sqrt{7000 \times 70} = 1400\,\text{GeV}$

? f) $\lambda = \frac{h}{p} = \frac{h}{mv}$

$\lambda = \frac{hc}{E}$

$\lambda = \frac{hc}{E} = \frac{2\pi\hbar c}{E} \simeq \frac{2\pi}{E}$ in natural units $\hbar = 1, c = 1$

$m_e \approx 0.5\, MeV/c^2$

$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$

$E_{cm} = \sqrt{s}$

$E_{cm}^2 = s = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$

$\sqrt{E_{cm}^2 + (E_1 + E_2)^2} = p_1 + p_2$

$s = m_e^2 + m_p^2 + 2(E_e E_p - |\vec{p}_e||\vec{p}_p| \underbrace{\cos\theta}_{=-1,\ \theta = 180})$

$= m_e^2 + m_p^2 + 2(E_e E_p + |\vec{p}_e||\vec{p}_p|)$

$E >> m$

$\simeq \sqrt{4 E_e E_p}$

$|p_e| = E_e$

$E = \frac{1}{2}mv^2$

$\Rightarrow p = \sqrt{2Em}$

$\lambda = \frac{2\pi}{\sqrt{2 \times 70 \times (0.5 \times 10^{-3})}}$

$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{2\pi\hbar}{p}$

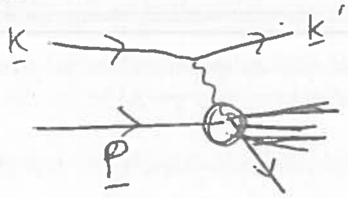
$E_{cm} = 1400\ GeV$

$p = 700\ GeV/c$

$\Rightarrow \lambda = \frac{2\pi\hbar}{700\ GeV/c} = \frac{2\pi\hbar c}{700\ GeV} = \frac{2\pi \cdot 0.197\ GeV fm}{700\ GeV}$

$\Rightarrow \lambda = 1.7\cancel{8}7 \times 10^{-3}\ fm$

$\simeq 1.8 \times 10^{-18}\ m$



7 g) i) $y = \frac{P \cdot q}{k \cdot P}$ $q = k - k'$, P is incoming proton 4 momentum

k is incoming electron 4 momentum

~~u~~ $\underline{k} = \underline{k}' + \underline{k}_y \Rightarrow \underline{k}' = \underline{k} + \underline{k}_y$.

$\cancel{q = \underline{k} - \underline{k}'}$
$\cancel{= \underline{k} - \underline{k} - \underline{k}_y}$
$\cancel{= -\underline{k}_y}$

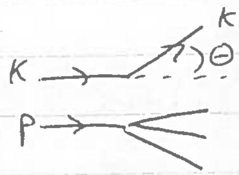
K·P is energy before
+ P·q is energy

$\cancel{(P+q)\cdot(P+q) = 0}$
$\cancel{P^2 + q^2 + 2P\cdot q = 0}$

$$P\cdot q = \underline{P}\cdot(\underline{k}-\underline{k}') = \underline{P}\cdot\underline{k} - \underline{P}\cdot\underline{k}'$$

$$y = \frac{P\cdot q}{k\cdot P} = \frac{\underline{P}\cdot\underline{k} - \underline{P}\cdot\underline{k}'}{\underline{P}\cdot\underline{k}} = 1 - \frac{P\cdot k'}{P\cdot k}$$

ii) $x = \frac{-q^2}{2P\cdot q}$



$\underline{k} = (E, 0, 0, E)$
$\underline{P} = (E, 0, 0, -E)$
$\underline{k}' = (E, E\sin\theta, 0, E\cos\theta)$

$\Rightarrow y = \frac{1}{2}(1 - \cos\theta)$

iii) if $\theta = 90^\circ \Rightarrow \cos\theta = 0$

$\Rightarrow y = \frac{1}{2}$

∴ half of the incident electrons ~~[illegible]~~ energy was transferred to the hadronic system in the protons rest frame..

8. a) i) $p + e^- \rightarrow n + \nu_e$

ii) $^{51}_{24}Cr$ has 24 protons, 27 neutrons
∴ an even-P, odd-N nuclei

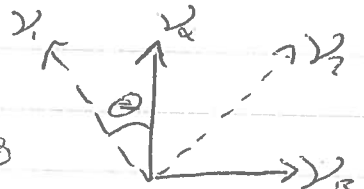
for β^+ decay $M(Z,A) > M(Z-1,A) + 2m_e$

$$Q = M(Z,A) - M(Z-1,A)$$

$\Rightarrow$ for β^+ to be possible:
$Q > 2m_e \simeq 1 MeV$

clearly $753 keV < 1 MeV$
∴ β^+ decay is forbidden

iii) 753 keV ?



b) i) θ is the mixing angle
where $|\nu_\alpha\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta$
$|\nu_\beta\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta$

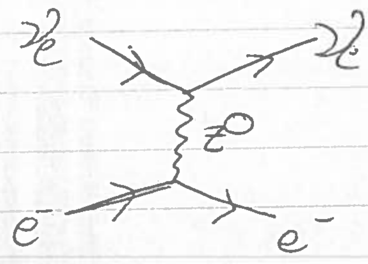
$\Delta m^2 = M_\alpha^2 - M_\beta^2$ is the diffence in the squared masses
means neutrinos have mass

? ii) call $P=1$ $\Rightarrow$ $\theta = 45°$ $\Rightarrow$ $\sin^2(2\theta) = 1$

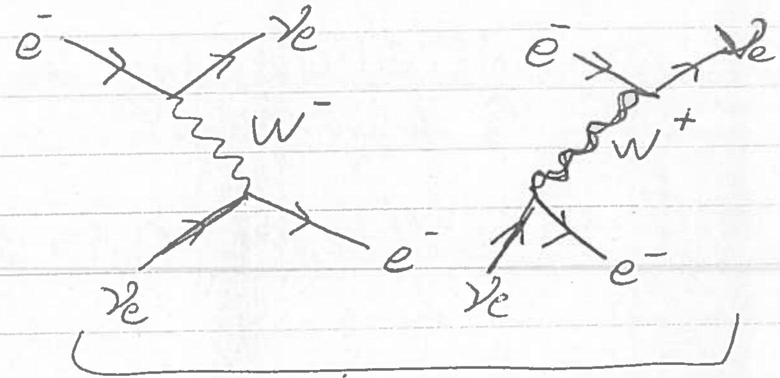
$$\Rightarrow 1 = \sin\left[1.27 \frac{0.1[eV^2] L}{753 keV}\right]$$

$$90 = 1.27 \frac{0.1 L}{753} \frac{eV}{10^3}$$

c)



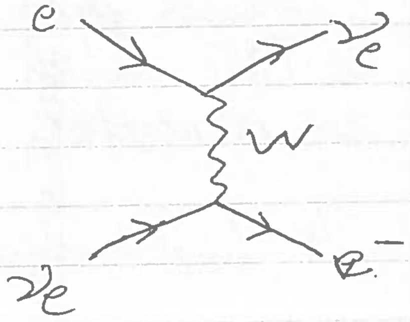
neutral current



charged

↓

Overall feynman:



d) $400\,PBq = 4\times10^{17}$ decays per second
270 tons of C_9H_{12}, average distance 8.25m from source
$\sigma(\nu_e e) = 0.72\times10^{-44}\,cm^2$

$$\cancel{\frac{dN}{dt} = \mathcal{L}\sigma \qquad N = \int \mathcal{L}\sigma\, dt}$$

$$\cancel{R = \sigma \times \mathcal{L} \Rightarrow \mathcal{L} = \frac{4\times10^{17}}{0.72\times10^{-44}\times10^{6}} = 5.56\times10^{55}\ \text{decays}\ s^{-1}\,m^{-2}}$$

$\frac{Z}{A}$ for C_9H_{12} : $\cancel{\frac{Z}{A} = \frac{9\times6+12}{9\times12}}$

$$\frac{Z}{A} = \frac{(9\times6)+12}{(9\times12)+12} = \frac{11}{20} = 0.55$$

$$\cancel{\text{Rate}} \Rightarrow N_e = \frac{270\times10^3}{(1.66\times10^{-27})}\times0.55 = 8.95\times10^{31}$$

$$\text{Rate} = \text{flux}\times\sigma\times N_e = \frac{\text{Activity}}{4\pi R^2}\times\sigma\times N_e = \frac{4\times10^{17}}{4\pi\times\underbrace{825^2}_{\text{converted to cm}}}\times(0.72\times10^{-44})\times(8.95\times10^{31})$$

$= 3.01\times10^{-2}\,Hz$ ~~3.01×10^{-2} Hz~~ $= 2604$ counts per day

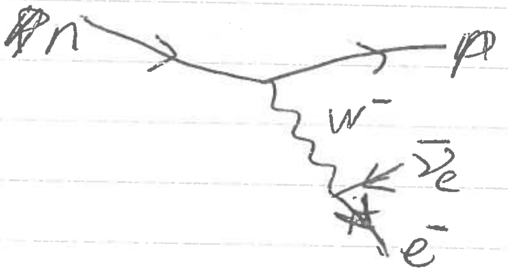
e) $\sin^2(2\theta)=0.1$ for $\nu_e \rightarrow \nu_s$
ν_s do not interact

∴ would expect $0.9 \times 2604 = 2343$ counts per day of ν_e

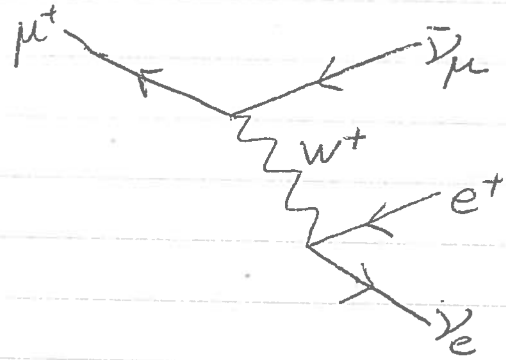
f) Scintillator material slow electrons, the ~~dta~~ energy dumped by the electron excites the scintillator material which produces photons. Thes photons are picked up by a PMT which amplifies the signal so that the electron can be recorded.

The change in momentum of the e^- can be used to find it's energy

g) β^- decay produces a mono-energetic anti-neutrino spectrum



9. a)



b) helicity: the projection of a particles spin on its direction of motion.

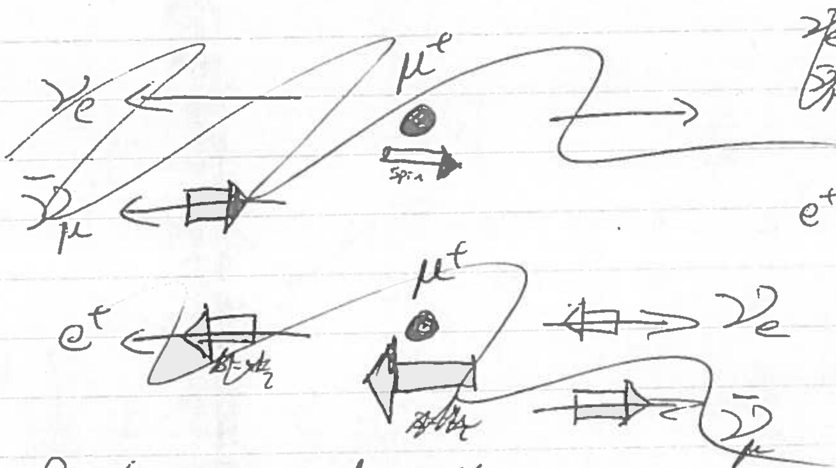
$$H = \frac{S \cdot p}{|p|}$$

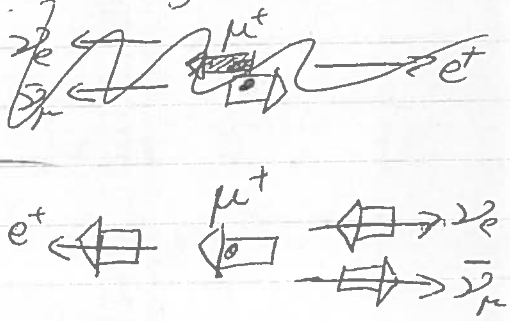
? c) Relativistic fermions with mass, a state with negative helicity is dominantly left-handed, but with a small right-handed component.

Only left-handed fermions couple ~~with~~ to the W boson
($\therefore$ only right handed anti particles couple to W boson)

$\mu^+_R \xrightarrow{W} e^+_R + \nu_{e_L} + \bar{\nu}_{\mu_R}$ only this can occur so that they interact with the W Boson

if the anti-muon is in its rest frame: for the highest energy e^+:



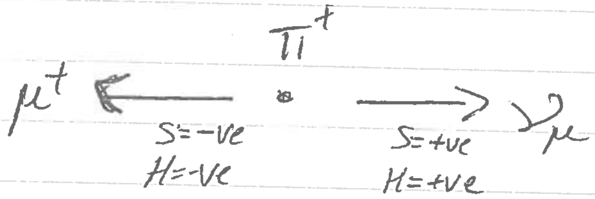


Parity is violated in this because ~~ν~~ ν_e is right handed and $\bar{\nu}_\mu$ is left handed

? d) π^+ has 0 spin. ~~Therefore~~ If π^+ is in the rest frame the μ^+ and ν_μ are produced ~~back~~ back to back. In order for spin to be to conserve angular momentum the spins of the μ^+ and ν_μ must be antialligned [this is because they ~~have~~ are spin ½ particles].

Since the ν_μ is left handed $\Rightarrow$ ν_μ Spin is positive and its helicity is negative.
μ^+ has a negative spin $\therefore$ has a negative helicity.

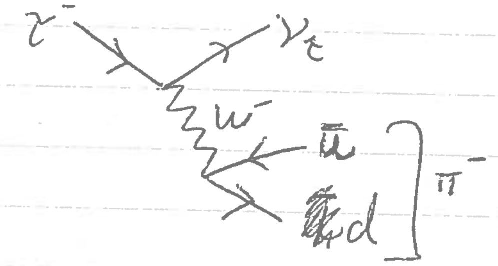
? e)

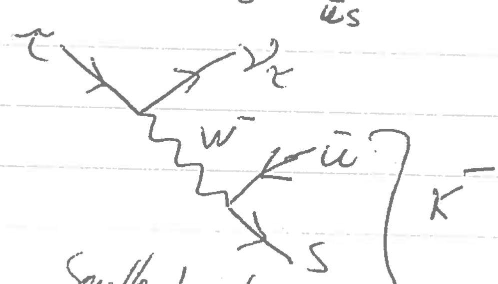


forward-peaked

? f)

g) $\tau^- \rightarrow \nu_\tau + \underset{\bar{u}d}{\pi^-}$ | $\tau^- \rightarrow \nu_\tau + \underset{\bar{u}s}{K^-}$





Smaller branching ratio

($\tau \rightarrow \nu_\tau K^-$) is surpressed because u and d quarks pair and in order to produce an $\bar{u}$ ans s quarks it requires quark mixing, which is less likely to occur.

~~h) for $\tau^- \rightarrow \nu_\tau + \pi^-$~~

~~$E_\tau = m_\tau c^2 = 1.78\,GeV$ Since at rest~~

~~$E_\tau = E_\nu + \sqrt{E_{\pi}^2 + p_{\pi}^2c^2} = E_\nu + \sqrt{m_\pi^2 c^4 + E_\nu^2}$~~

~~$p_\nu = \frac{1}{c}\sqrt{E_\nu^2 - m_\nu^2 c^4} = \frac{E_\nu}{c}$~~

~~$(m_\tau c^2)^2 = E_\nu^2 + m_\pi^2 c^4 + E_\nu^2$~~

~~$\Rightarrow E_\nu^2 = \frac{1}{2}(m_\tau^2 c^4 - m_\pi^2 c^4) =$~~

h) i) $\tau^- \rightarrow \nu_\tau + \pi^-$ $E^2 = m^2c^4 + p^2c^2$

momentum $0 \Rightarrow p_\nu + p_\pi$ $\Rightarrow p = \frac{1}{c}\sqrt{E^2 - m^2c^4}$

energy $m_\tau c^2 = E_\nu + E_\pi$

$$\Rightarrow 0 = \frac{1}{c}\sqrt{E_\nu^2 - m_\nu^2c^4} - \frac{1}{c}\sqrt{E_\pi^2 - m_\pi^2c^4}$$

$$0 = \frac{1}{c}E_\nu - \frac{1}{c}\sqrt{(m_\tau c^2 - E_\nu)^2 - m_\pi^2c^4}$$

$$\therefore E_\nu = \sqrt{(m_\tau c^2 - E_\nu)^2 - m_\pi^2 c^4}$$

$$E_\nu^2 = m_\tau^2c^4 + E_\nu^2 - 2E_\nu m_\tau c^2 - m_\pi^2c^4$$

$$\Rightarrow 2E_\nu m_\tau c^2 = m_\tau^2c^4 - m_\pi^2c^4$$

$$\therefore E_\nu = \frac{m_\tau^2c^4 - m_\pi^2c^4}{2m_\tau c^2} = \frac{m_\tau^2 - m_\pi^2}{2m_\tau}c^2$$

$$\Rightarrow E_\nu = \frac{1780^2 - 140^2}{2 \cdot 1780}c^2 = 885\ MeV/c^2$$

ii) $\tau^- \rightarrow \nu_\tau + K^-$

$$\Rightarrow E_\nu = \frac{m_\tau^2 - m_K^2}{2m_\tau}c^2$$

$$= \frac{1780^2 - 494^2}{2 \cdot 1780}c^2$$

$$= 821\ MeV/c^2$$

10. a) ?

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} a_c Z^2 A^{-\frac{1}{3}} \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= 2\pi \left[-\cos\theta\right]_0^{\pi} \, a_c Z^3 A^{-\frac{1}{3}} \frac{r^3}{3}$$

$$\frac{Ze^2(Z-1)}{\frac{4}{3}\pi R^3 \varepsilon_0 \hbar c} =$$

b) $\cancel{\frac{\partial B}{\partial A} = 0 = a_v - \frac{2}{3} a_s A^{-\frac{1}{3}} + \frac{1}{3} a_c Z^2 A^{-\frac{4}{3}}}$

$\cancel{- \frac{\partial}{\partial A} a_a \left(Z^2 A^{-1} - \frac{A}{4} - Z\right)}$

$\cancel{0 = a_v - \frac{2}{3} a_s A^{-\frac{1}{3}} + \frac{1}{3} a_c Z^2 A^{-\frac{4}{3}} - a_a\left(-Z^2 A^{-2} - \frac{1}{4}\right)}$

$\left(Z - \frac{A}{2}\right)^2 A^{-1}$
$\Rightarrow \left(Z^2 + \frac{A^2}{4} - AZ\right) A^{-1}$
$= Z^2 A^{-1} + \frac{A}{4} - Z$

$$\frac{\partial B}{\partial Z} = -2a_c Z A^{-\frac{1}{3}} - a_a\left(2ZA^{-1} - 1\right) = 0$$

$$-2a_c Z A^{-\frac{1}{3}} - a_a 2 Z A^{-1} + a_a = 0$$

$$Z\left(2a_c A^{-\frac{1}{3}} + a_a 2 A^{-1}\right) - a_a = 0$$

$$Z = \frac{+a_a}{\left(2a_c A^{-\frac{1}{3}} + 2a_a A^{-1}\right)}$$

c) $A = 166$
$Z = 68 \Rightarrow 68 = \frac{+93.15\,MeV}{2a_c \cdot 166^{-\frac{1}{3}} + 2 \cdot 93.15 \cdot 166^{-1}}$

$$\left(2a_c A^{-\frac{1}{3}} + 2a_a A^{-1}\right) Z = a_a$$

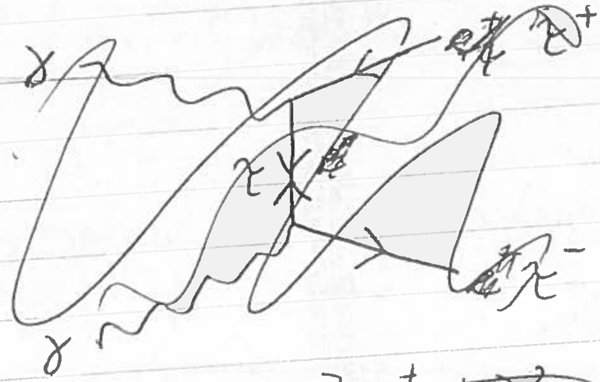
$$2a_c A^{-\frac{1}{3}} = \frac{a_a}{Z} - 2a_a A^{-1}$$

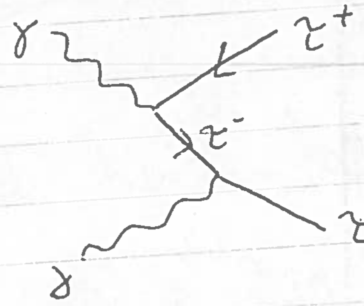
$$a_c = \frac{\left(\frac{a_a}{Z} - 2a_a A^{-1}\right)}{\left(2A^{-\frac{1}{3}}\right)} = \frac{\frac{93.15}{68} - \left(2 \cdot 93.15 \cdot \frac{1}{166}\right)}{2 \cdot 166^{-\frac{1}{3}}}$$

$$= 0.680\,MeV$$

d) i) $\gamma + \gamma \rightarrow \tau^+ + \tau^-$

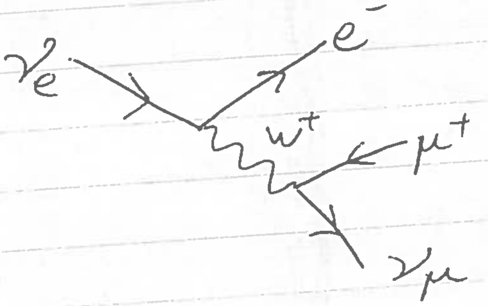
allowed only if the phontons have enough energy



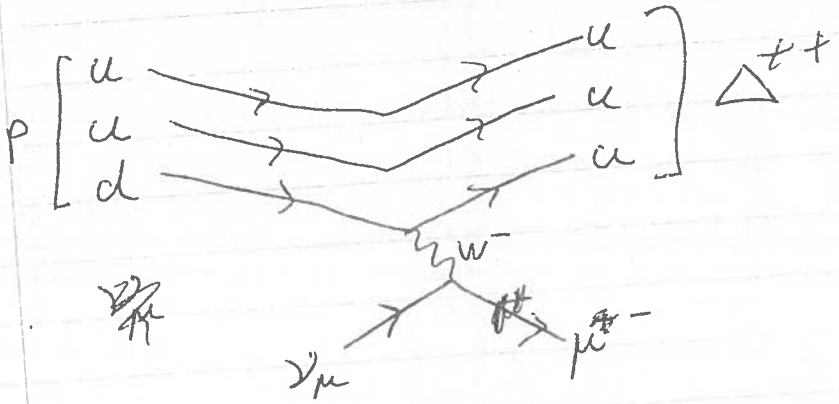


ii) ~~$\nu_e + e \rightarrow \mu^+ + \nu_\mu$~~

$\nu_e \rightarrow e^- + \mu^+ + \nu_\mu$ Allowed



iii) $\underset{uud}{\nu_\mu + p} \rightarrow \mu^- + \underset{uuu}{\Delta^{++}}$

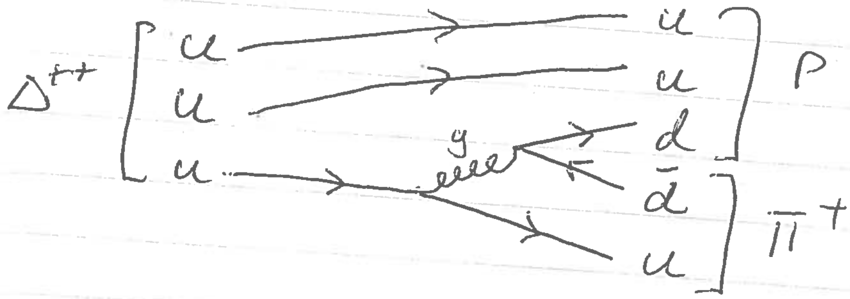


iv) $\underset{uuu}{\Delta^{++}} \rightarrow \underset{uud}{p} + \underset{uud}{p}$

not allowed Baryon number not conserved

$m_{\Delta^{++}} < \cancel{2m_{\pi}}\ 2m_p$

v) $\underset{uuu}{\Delta^{++}} \rightarrow \underset{uud}{p} + \underset{u\bar{d}}{\pi^+}$



vi) $K^- \rightarrow e^- + \gamma$

not allowed because electron lepton number is not conserved.