PHAS3224: Nuclear and Particle Physics

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1 Preliminaries

1.1 How we got to Memphis

We need to know the following by heart:

- The quark composition of the following particles:
  - Proton - p - uud
  - Neutron - n - udd
  - Neutral Pion - π^0 - $u\bar{u}$ or $d\bar{d}$
  - Positive Pion - π^+ - $u\bar{d}$
  - Negative Pion - π^- - $d\bar{u}$
- The charges of all fundamental particles (anti-particles have opposite charges) in units of the fundamental electric charge $e \approx 1.6 \times 10^{-19}$:
  - $u, c, t = +2/3$
  - $d, s, b = -1/3$
  - $e^-, \mu^-, \tau^- = -1$
  - $\nu_e, \nu_\mu, \nu_\tau = 0$
  - $\gamma, g, Z^0, H = 0$
  - $W^\pm = \pm 1$
- The masses of the following particles[1] (anti-particles have the same mass) in electron volts and where $c = 1$:
  - Neutron - n - 940 GeV
  - Proton - p - 938 GeV
  - Top quark - t - 170 GeV
  - Bottom quark - b - 4.2 GeV
  - Electron - e - 0.511 MeV
  - Muon - μ - 106 MeV
  - Photon - γ - 0
  - Charged Weak Boson - $W^\pm$ - 80 GeV
  - Neutral Weak Boson - Z^0 - 91 GeV
  - Gluon - g - 0
  - Higgs Boson - H - 125 GeV
- The mass ordering of particles (implied by the above table), for example:

$$M_n > M_p > M_\mu > M_e > M_\gamma$$

1.2 Natural Units

To convert from natural units ($c = 1, \hbar = 1$) to S.I. units, just get the quantity in terms of eV^n and just multiply by $\hbar$ and c as shown below:

$$\mathrm{kg}^\alpha \mathrm{m}^\beta \mathrm{s}^\gamma \to \left(\hbar^{\beta+\gamma} c^{\beta-2\alpha}\right)^n$$

A useful conversion is:

$$\hbar c = 0.197 \mathrm{fm\ GeV}$$

[1] Quark masses are approximate as they change depending on how they're bound.

1.3 Uncertainty Relations

$$\Delta E \Delta t \geqslant \hbar$$

$$\Delta p \Delta x \geqslant \hbar/2$$

From the energy relation, one can retrieve the approximate lifetime and (by $\Delta s = c\Delta t$) range of virtual particles/forces:

$$\Delta t \approx \frac{\hbar}{\Delta E} \leqslant \frac{\hbar}{M_X c^2} \to R_X \leqslant \frac{\hbar}{M_X c}$$

Where R_X is the range of a force, and M_X is the mass of it's force carrier.

1.4 Invariant Mass

The S.R. line element is:

$$\mathrm{d}s^2 = c^2 \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$$

Such that the inner product of two four-vectors is defined as:

$$\mathbf{P_1} \cdot \mathbf{P_2} = (E_1, \mathbf{p}_1) \cdot (E_2, \mathbf{p}_2) = E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2 = W^2 c^4$$

Where $W^2 c^4$ is a quantity invariant under Lorentz boosts. This basically means you can calculate W (the invariant mass) in the rest frame of a particle before decay, and the square of the sum of the four momenta of it's products must also yield $W^2 c^4$:

$$W^2 c^4 = \left(\sum_i E_i\right)^2 - \left(\sum_i \mathbf{p}_i\right)^2$$

Momentum must be conserved on-shell, and for fast moving particles:

$$E \approx |\mathbf{p}|c$$

1.5 The Klein-Gordon Equation

Derived from the energy-momentum relation:

$$E^2 = p^2 c^2 + m^2 c^4$$

And subbing in the appropriate operators $\hat{p} = -i\hbar\nabla$ and $\hat{E} = i\hbar\frac{\partial}{\partial t}$.

1.6 Feynman Diagrams

complete

1.7 Particle Zoo

complete

Table 1: Interactions.

Force	Boson	Couples To (Linear)	Relative Strength (to EM)
Gravity	???	Mass/Energy	??? Tiny ???
???	Higgs	Mass	???
Electromagnetism	Photon (γ)	Electric Charge	1
Weak Force	Weak Bosons ($W^{\pm}, Z^0$)	Flavour (Anything Really)	$10^{-4}, 10^{-7}$ [2]
Strong Force	Gluon (g)	Colour Charge	60

1.8 Scattering/Yukawa

complete

Table 2: Conservation Laws for the Standard Model.

Quantity	**Symmetry**	**Broken By**
Mass-Energy	Time Translation	Off-Shell
Momentum	Spatial Translation	Off-Shell
Electric Charge	Gauge Symmetry	Never
Colour Charge	Magic	Never
Lepton Number	Magic Also	Never
Lepton Flavour	Magic Also Also	Flavour Oscillation (Long times/distances)
Baryon/Quark Number	Higher-Order Magic	Never
Quark Flavour	Witchcraft and/or Voodoo	Weak Force

1.9 Conservation Laws

2 Leptons and Quarks

2.1 Neutrino Oscillation

By the expansion postulate, time-independent flavour eigenstates $|\nu_l\rangle$ of neutrinos can be expanded in terms of the mass/energy eigenstates $|\nu_n\rangle$. For two mixing neutrinos:

$$|\nu_e, 0\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu, 0\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

Such that θ (the mixing angle) is a constant that defines how much they overlap, and the flavour basis vectors can be thought of as a rotated superposition of the mass/energy eigenstates.

We now make each mass/energy eigenstate time-dependent by multiplying each by a time-dependent plane wave:

$$|\nu_e, t\rangle = \cos\theta e^{-iE_1 t/\hbar}\,|\nu_1\rangle + \sin\theta e^{-iE_2 t/\hbar}\,|\nu_2\rangle$$

$$|\nu_\mu, t\rangle = -\sin\theta e^{-iE_1 t/\hbar}\,|\nu_1\rangle + \cos\theta e^{-iE_2 t/\hbar}\,|\nu_2\rangle$$

This basically means that at some time t, the electron flavour eigenstate $|\nu_e, t\rangle$ will look like the pure muon flavour eigenstate $|\nu_e, 0\rangle$. The probability that this happens is:

$$P\left(e \to \mu\right) = |\langle\nu_\mu, 0|\nu_e, t\rangle|^2 = \sin^2\left(2\theta\right)\sin\left[\frac{(E_2 - E_1)t}{2\hbar}\right]$$

Since these things travel at basically c (ridiculously low mass), and since we have an appropriate uncertainty relation between $\Delta E = E_2 - E_1$ and ΔL, we can effectively recast the above equation as a function of distance travelled rather than time[3].

The strength of this mixing is parameterised by the mixing angle, and it can be generalised to three flavour eigenstates by use of the CKM matrix. Note that, if the mixing angle is an integer multiple of $\pi/2$, the flavours are not mixed, and there is no oscillation.

2.2 Colour

Since some particles have quark configurations naively forbidden by Pauli's exclusion principle (like $\Omega^- = sss$, there must be some other reason that these quarks occupy different states that aren't entirely parameterised by quark type, charge, and spin. This 'saving grace' quantity is called colour, and is comprised of two conserved charges: Isospin and Hypercharge. Every flavoured particle (quarks and gluons) can carry one of three colours, which themselves contribute to Isospin and Hypercharge (the colour charges).

There is also anti-red, anti-blue, and anti-green which carry opposite charges to their standard counterparts. These anti-colours are sometimes called cyan, yellow, and magenta respectively. The reason for this is irrelevant and dumb.

An important note to make is that gluons can directly couple to themselves.

[3] This distance is on the order of kilometers

Colour	**Isospin I_3^C**	**Hypercharge Y^C**
red (r)	1/2	1/3
green (g)	-1/2	1/3
blue (b	0	-2/3

2.2.1 Number of Colours

How do we know there are only three colours? Simple! Look at the ratio of EM pair production and Strong Force pair production. Fill this in

2.2.2 Colour Confinement

The strong force potential goes something like this:

$$V(r) = -\frac{\alpha_s \hbar c}{r} + \lambda r$$

Which is highly repulsive (coulomb-like) at low distances (this gives us the approximate equilibrium size of hadrons) and crazy attractive at high distances. This can be conceptually understood as more and more gluons (a gluon 'tube') travelling between separated colour charges. Thus colour charges are confined, and cannot exist freely in nature - they are bound up in hadrons.

2.2.3 Quark Jets

If quarks are produced at very high energies, they'll move apart rapidly. Eventually it becomes more energetically viable for the ever-increasing stream of gluons they're producing/interacting via to decay into quark pairs themselves. The same thing happens to these quark pairs and you get a quark jet.

These quarks can of course bind with each other into hadrons which decay further. These quark jets rapidly become hadronic cascades which are varied, constantly fluctuating, and pretty fucking cool in general.

When the cascade finally stops, the produced hadrons are rather slow-moving compared to the original quarks - so studying the rest mass of products lets us reconstruct the energies of the original quarks produced.

2.3 Coupling Constants

The relative strength of a force is dictated by it's coupling constant α which is - of course - not a constant. At higher momenta, higher-order feynman diagrams become more frequent/accessible, which screens the relative strength of the force. Some forces increase in strength at higher momenta, some decrease. We're gonna study the strong and EM forces now:

2.3.1 Asymptotic Freedom

The low-momentum coupling constants for the strong force α_s and EM force α_{EM} are important:

$$\alpha_s \approx 0.1$$

$$\alpha_{EM} \approx 1/137$$

At higher momenta, the strong force weakens, and the EM force strengthens:

$$\alpha_s \to 0$$

$$\alpha_{EM} \to \infty$$

Eventually these values get so messed up that QCD/QED fall apart entirely. At these momenta, General Relativity starts to rear is geometrically ugly head and physics as we know it becomes high fantasy and/or sci-fi.

Since the strong force weakens as the momentum of interacting particles increases, it is 'asymptotically free' while EM is 'asymptotically bound'. In a different universe, this may not be the case. The strong force coupling constant as a function of four-momentum $\mathbf{Q}$ is:

$$\alpha_s(\mathbf{Q}^2) = \frac{\alpha_s(\mu^2)}{1 + B\alpha_s(\mu^2)\ln\left(\frac{\mathbf{Q}^2}{\mu^2}\right)}$$

Where μ is a reference four-momentum which we arbitrarily measure α_s for, and $B = (33 - 2N_f)/12\pi$ where N_f is the number of quark flavours. In our universe $N_f = 6$ and $B > 0$, which means that α_s increases with $|\mathbf{Q}|$. In a world with enough quark flavours, $B < 0$ and the strong force becomes asymptotically bound, similarly to the EM force.

3 Experimental Particle Physics

3.1 Detector Types

There are two kinds of detector, those that measure position and those that measure energy.

3.1.1 Position Detection

Position detection falls into Fill in all this crap

3.1.2 Energy Detection

The most common energy detector is a calorimeter. These things basically slow down or stop particles entirely to measure their energy. As particles are quite 'rare' compared to the rest of the infinite void of existence and reality, they're governed by the poisson distribution, yielding a resolution proportional to $1/\sqrt{E}$ or $1/\sqrt{N}$:

- **EM Calorimeters** stop charged particles and photons. These particles have very low masses, and hence lower absolute energy fluctuations. These calorimeters are much more precise than hadronic ones.
- **Hadronic Calorimeters** stop hadrons. Hadrons are heavier which results in smaller proportional energy fluctuations, but the absolute fluctuations are insanely high. Low precision.

4 Deep Inelastic Scattering

Deep inelastic scattering is 'deep' because it uses high-energy lepton beams to probe the internal structure of nucleons, and 'inelastic' because these beams typically smash the nucleon apart. The lepton has an incoming four-momentum $\mathbf{P}$ and a scattered four-momentum $\mathbf{P'}$ such that:

$$\mathbf{P} = (E, \mathbf{k})$$

$$\mathbf{P'} = \left(E', \mathbf{k'}\right)$$

We can then define a quantity $\mathbf{q} = \mathbf{k} - \mathbf{k'}$ such that:

$$Q^2 = -\mathbf{q}^2$$

Is an invariant quantity for fast-moving incoming/outgoing leptons with $\mathbf{k} \approx E/c$ and $\mathbf{k'} \approx E'/c$.

The nucleon has a momentum $\mathbf{p}$, and a given single parton (constituent of the nucleon) has momentum $x\mathbf{p}$ where x is just a fraction (the fraction of the total momentum carried by the parton).

In the rest frame of the parton, this value is zero:

$$(x\mathbf{p})^2 = 0$$

We can then get:

$$(x\mathbf{p} + \mathbf{q})^2 = 0$$

Which is just a statement of conservation of momentum. Rearranging and using $(x\mathbf{p})^2 = 0$ yields:

$$x = \frac{-\mathbf{q}^2}{2\mathbf{p}\cdot\mathbf{q}}$$

Which shows that x is another invariant quantity.

5 Symmetries and How I Learned to Stop Worrying and Love the Higgs.

5.1 Space Inversion and Parity Symmetry

The space inversion/parity operator $\hat{P}$ just reverses the two fundamental canonical quantities:

$$\mathbf{r} \to -\mathbf{r}$$

$$\mathbf{p} \to -\mathbf{p}$$

Note that, by definition of the operator:

$$\hat{P}\hat{P}\psi_\alpha = P_\alpha^2\psi_\alpha = \psi_\alpha$$

So eigenvalues of the parity operator $P_\alpha = \pm 1$ for parity eigenstates. Parity is a multiplicative quantum number such that:

$$P_{TOT} = P_a P_b P_c ... \qquad L_{TOT} = P_{TOT}(-1)^L$$

Fermion-antifermion have opposite parities. Boson-antiboson have same parities.

5.1.1 Parity Breaking

Parity is conserved in all but the weak interaction. This is the $\tau - \theta$ and β-decay stuff. fill this in

5.2 Charge Conjugation

The charge conjugation operator $\hat{C}$ changes all particles a to antiparticles $\bar{a}$ with a phase shift $e^{i\theta}$ (this is only relevant in multi-particle systems):

$$\hat{C}\psi_\alpha = C_\alpha\psi_{\bar{\alpha}}$$

- For Majorana particles (γ, π^0, etc.), their wavefunctions are eigenstates of the charge conjugation operator:

$$\begin{aligned} \psi_\alpha &= \psi_{\bar{\alpha}} \\ \hat{C}\hat{C}\psi_\alpha &= C_\alpha^2\psi_\alpha \\ &\to C_\alpha = \pm 1 \end{aligned}$$

- For Dirac particles (q, π^+, etc.), their wavefunctions are superpositions of eigenstates of the charge conjugation operator:

$$\psi = \frac{1}{\sqrt{2}}\left(\psi_\alpha + \psi_{\hat{\alpha}}\right)$$

5.2.1 Charge Conjugation Breaking

Consider muon decay:

$$\mu^- \to e^- + \bar{\nu}_e + \nu \qquad \text{or} \qquad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

The rate of this decay with respect to the angle θ between muon spin and electron direction is:

$$R_{e^\pm}(\theta) = \frac{1}{2}\Gamma_{\mu^\pm}\left(1 \pm \frac{1}{3}\cos\theta\right)$$

Note that this function changes under charge conjugation ($e^\pm \to e^\mp$), which implies that charge conjugation conservation is broken in this reaction (alignment reverses under the change). However, parity change ($\theta \to \pi - \theta$) in conjunction with charge conjugation yields the same formula. This implies CP-conservation.

5.3 CP-Symmetry

First we need to talk about helicity H (the projection of particle spin onto it's velocity):

$$H = \frac{\mathbf{s} \cdot \mathbf{p}}{|\mathbf{p}|}$$

This is $\pm 1/2$ for fermions. Positive H is called 'right-handedness', negative H is called 'left-handedness'. For some reason, only left handed neutrinos and right-handed anti-neutrinos interact. Because they can only do so through the weak force, only left-handed neutrinos and right-handed antineutrinos can be found in Nature:

$$\nu_l \quad \text{are all left-handed}$$

$$\bar{\nu}_l \quad \text{are all right-handed}$$

For massive particles, we can always travel at a speed $v < c$ such that their helicity reverses, for this reason we cannot say with 100% certainty whether or not a massive particle is left or right handed. This is suppressed by a factor of $\left(\frac{mc^2}{E}\right)^2$.

Charge conjugation of a neutrino would make a left-handed antineutrino. The parity operator applied to a neutrino would make it right-handed. Both of these are impossible *but by applying both operators together makes it work.* CP transformations conserve particle helicity.

5.3.1 CP Breaking

wu experiment, more helicity stuff, etc.

5.4 Higgs Boson

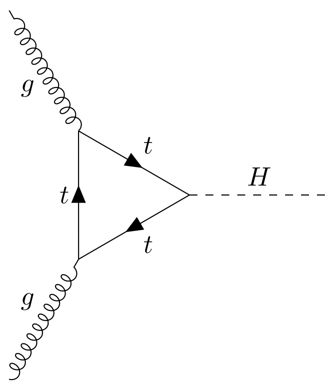
5.4.1 Global Symmetry Breaking

Mexican hat potential: at high energies the Higgs is going crazy all over the place, but at low energies it has to fall into one of the the minima of the potential. This locally looks symmetric, but is globally asymmetric. This is an example of spontaneous symmetry breaking.

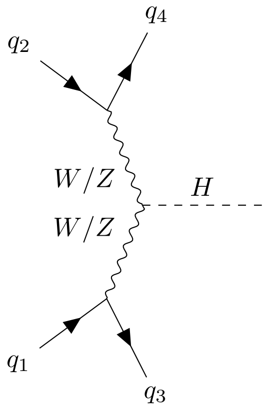
5.4.2 Production, Decay, and Detection

There are two main production modes of the Higgs boson:

- **Gluon Loop:**

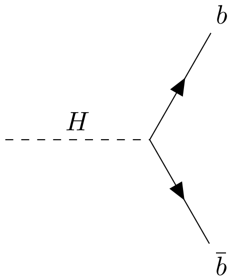


- **Vector Boson:**

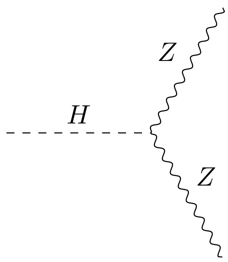


There are two main decay modes of the Higgs boson (the two heaviest particles it can possibly decay into because it couples to mass):

- $H \to b\bar{b}$



- $H \to ZZ$



Of these two decay channels, the $H \to b\bar{b}$ channel is far more likely than any other channel (branching ratio of 57.7%). However, there is a lot of background noise due to very common $Z \to b\bar{b}$ decay, and the energy fluctuations for hadronic jets are incredibly high. For this reason the Higgs was first discovered via the $H \to ZZ$ channel at CERN.

Note that the mass of the Higgs is about 125 GeV, whilst the mass of two top quarks is about 340 GeV, it is for this reason that the Higgs cannot decay *at all* into the most massive fundamental particle - the top quark (even though a rudimentary understanding dictates that it should most strongly couple to the particle with the most mass).

6 Nuclear Physics

6.1 SEMF

The semi-empirical mass formula is a long and scary-looking formula for the mass of a nucleus in terms of the atomic number A, the proton number Z (equivalent to the charge of nucleus in units of $e = 1$), and several constants labelled $a_{\text{subscript}}$. It looks like this:

$$M(A,Z) = \underbrace{Am_{\text{amu}} + Zm_e}_{\text{Actual Mass}} \underbrace{+a_v A}_{\text{Volume}} \underbrace{-a_s A^{2/3}}_{\text{Surface}} \underbrace{+a_c \frac{Z^2}{A^{\frac{1}{3}}}}_{\text{Coulomb}} \underbrace{+a_f \frac{\left(Z - \frac{A}{2}\right)^2}{A^{\frac{1}{3}}}}_{\text{Symmetry}} \underbrace{+a_p \frac{1}{2}\left((-1)^Z + (-1)^A\right)}_{\text{Pairing}}$$

It has a straightforward mass term, and five correction terms of (typically) decreasing magnitude. These correction terms are conceptually related to three different ways of modelling the nucleus:

- **The Tear Drop Model:**

  The tear drop model basically says that we pretend the nucleus is a ball of particles that behaves like a fluid. This model gives us three of the terms in the SEMF:

  - **Volume Term:** Since the binding energies between nucleons are roughly the same, we can just construct a constant which is the average binding energy per nucleon. This binding energy is equivalent to extra nucleus mass by Einstein's mass-energy equivalence, hence we can just add an extra constant unit of mass a_v per nucleon. This gives rise to the volume term:

    $$M_v = a_v A$$

    Which - by some horrific betrayal of unit homogeneity - implies that $A \to r^3$, and $r \to A^{1/3}$. This relationship will give us a conceptual understanding of the use of the atomic number in the other terms of the SEMF.

  - **Surface Term:** Since the nucleons on the surface of the nucleus are bound to fewer neighbours, they have a lower average binding energy than the nucleons in the core. For this reason we make a small negative correction to the volume term which is proportional to the surface area ($r^2 \implies A^{2/3}$):

    $$M_s = -a_s A^{2/3}$$

  - **Coulomb Term:** The protons repel each other, and hence there's some extra energy wrapped up in the electromagnetic interactions within the nucleus. From the form of the electric potential energy:

    $$U = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{\text{Some Trash Constant}} \frac{Q^2}{r}$$

    We get the form of the coulomb term by taking $r^2 \to A^{2/3}$ and $Q^2 \to Z^2$:

    $$M_c = a_c \frac{Z^2}{A^{2/3}}$$

- **The Fermi Gas Model:**

  The Fermi gas model basically says that the nucleons are non-interacting particles occupying energy levels in a potential well. By Pauli exclusion no two particles can occupy the same quantum state, and so identical particles fill up an energy 'ladder'. Since protons and neutrons are fundamentally different, they're effectively filling two different ladders.

  When there are more protons than neutrons (or vice versa), the ladders have different maximum filled energy levels and it would be energetically favourable for one of the protons to become a neutron (or vice versa). Due to this imbalance, the nucleus has more energy in it than it would have if there were the same number of protons and neutrons.

  The number of protons is Z, and perfect symmetry would correspond to $Z = A/2$. Since the energy of a level n are proportional to n^2, the additive correction to the SEMF is proportional to $\left(Z - \frac{A}{2}\right)^2$:

  $$M_f = a_f \frac{\left(Z - \frac{A}{2}\right)^2}{A^{1/3}}$$

  Where I have used the subscript f to remind you that it's due to the Fermi gas model interpretation. The factor of $A^{1/3}$ in the denominator confuses me, and hence I have ignored it.

- **The Shell Model:**

  The shell model basically says that the nucleons form shells by occupying lowest-energy states and by obeying the Pauli exclusion principle. This is essentially electron shell stuff, but with a different ordering of states w.r.t. energy. Once again, since protons and neutrons are fundamentally different, we consider them independently.

Each level is denoted by a symbol:

$$({}^{n}\mathrm{l}_j)^p$$

Where n is the energy level (principal quantum number), l is the orbital angular momentum quantum number, j is the total angular momentum quantum number, and p is the occupancy of the level. These obey typical rules for quantum numbers, but we'll just concern ourselves with this gem:

$$0 \leqslant p \leqslant 2j+1$$

And the fact that l follows the naming convention of:

$$l = 0, 1, 2, ... \to l = s, p, d...$$

The following is a list of shells in increasing energy order (this is basically random and wholly empirical at our level). Note that $p = p_{\mathrm{MAX}} = 2j+1$ for all of these shells:

$$\underbrace{\underbrace{\underbrace{\left({}^{1}\mathrm{s}_{1/2}\right)^{2}}_{N=2}\left({}^{1}\mathrm{p}_{3/2}\right)^{4}\left({}^{1}\mathrm{p}_{1/2}\right)^{2}}_{N=8}\left({}^{1}\mathrm{d}_{5/2}\right)^{6}\left({}^{2}\mathrm{s}_{1/2}\right)^{2}\left({}^{2}\mathrm{d}_{3/2}\right)^{4}}_{N=22}$$

Where N is the maximum total occupancy number of all shells within three groups. *We must memorise these first six shells and their groupings.* For some reason the energy gaps between the grouped shells are big and so the maximum occupancy number of these groups are 'magic numbers': stable numbers of nucleons, adding another nucleon costs a lot of energy.

We find the ground state of a nucleus by filling up these levels with protons and neutrons *independently.*

Nuclei with 'magic numbers' of protons/neutrons are stable. For example, oxygen-16 has 8 protons and 8 neutrons, both of which fill the second grouping (8 is the second magic number), and hence oxygen-16 is crazy stable. Helium-4 is another (even stabler) example of this.

In the ground state, everything is paired. So, for odd numbers of protons/neutrons:

$$J = \sum j_{\text{unpaired}} = j_{\text{extra-proton}} + j_{\text{extra-neutron}}$$

Protons and neutrons have intrinsic parity -1, and hence the parity of a nucleus is (again considering only unpaired protons and neutrons):

$$P = (-1)^{l_{\text{extra-proton}}}(-1)^{l_{\text{extra-neutron}}}$$

Finally we get to the final term in the SEMF. Nobody knows where this comes from, just know that it comes from the shell model. It's called the pairing term and has something to do with the pairing of neutrons and the pairing of protons.

- If Z and A are both even then $M_p = a_p$.
- If Z and A are both odd then $M_p = -a_p$.
- Otherwise $a_p = 0$.

The term is typically given by:

$$M_p = \pm a_p \delta_A^Z$$

But this makes no sense to anyone who has ever used the Kronecker delta. Hence I constructed a different form for it in the SEMF above.

6.2 Decays

fill in β decay and electron/neutrino capture stuff

6.3 Fission

Releases more energy per nucleus than fusion.

6.4 Fusion

Releases more energy per nucleon than fission. list common fusion reactions