CHAPTER 1

→ LAST YEAR'S NOTES
- Biot Savart

→ magnetism summary table.

- Divergence theorem

$$\int_S \underline{F} \cdot \underline{n}\, da = \int_V \nabla \cdot \underline{F}\, dV$$

- Stokes theorem

$$\oint_C \underline{F} \cdot d\underline{l} = \int_S \nabla \times \underline{E} \cdot \underline{n}\, da$$

- Conservative fields →

- $\underline{D} = \epsilon_0 \underline{E} + \underline{P}$

  $\underline{B} = \mu_0 (\underline{H} + \underline{M})$

- Maxwell :

$$\nabla \cdot \underline{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{B} = \mu_0 \underline{J} + \mu_0 \epsilon_0 \frac{d\underline{E}}{dt}$$

- Lorentz force

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

- Flux , $\Phi = \int_S \underline{B} \cdot \underline{n}\, da$

- Emf , $\epsilon = -\frac{d\Phi}{dt} = \oint_C \underline{E} \cdot d\underline{l}$

- Biot-Savart $\quad d\underline{B}(\underline{r}) = \frac{\mu_0 I}{4\pi} \frac{d\underline{l} \times (\underline{r} - \underline{r}')}{|\underline{r} - \underline{r}'|^3}$

$$\underline{B} = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\underline{l} \times (\underline{r} - \underline{r}')}{|\underline{r} - \underline{r}'|^3}$$

CHAPTER 2

- Dielectric → Electric insulator that can be polarized by an applied electric field.

- $C_{dielectric} = k\, C_{vacuum}$

  $\downarrow$

  $k = 1 + \chi_e$

- Dipole moment

  $\underline{p} = q\,\underline{d}$

- Polarization $\underline{P} = \epsilon_0 \chi_e \underline{E}$

- Surface polarization charge density → $\boxed{\sigma_p = \underline{P} \cdot \underline{n}}$

  Volume polarization charge density → $\boxed{\rho_p = -\nabla \cdot \underline{P}}$

- Uniform polarization → $\nabla \cdot \underline{P} = 0$

  There is no bound charge within the material, but there will be bound charge on the surface.

- Free charge → free to move, carry electric current

  Bound charge → charge within a material, unable to move freely through the material.

  Small displacement of bound charge causes polarization.

- Linear → χ_e is independent of $\underline{E}$ (or χ_m of $\underline{B}$)

  Isotropic → $\underline{P}$ is parallel to $\underline{E}$ (or $\underline{M}$ to $\underline{H}$)

  Homogenous → χ_e is position independent.

- Energy density of an Electric field → $U_e = \frac{1}{2} \underline{D} \cdot \underline{E}$.

  magnetic field → $U_m = \frac{1}{2} \underline{B} \cdot \underline{H}$

- Coulomb gauge → $\nabla \cdot \underline{A} = 0$

  Lorentz gauge → $\nabla \cdot \underline{A} = -\mu_0 \epsilon_0 \left(\frac{\partial V}{\partial t}\right)$

- Surface magnetization current density → $\boxed{J_M = \underline{M} \times \underline{n}}$

  Volume magnetization current density → $\boxed{J_M = \nabla \times \underline{M}}$

- For linear, isotropic material

$$\underline{M} = \chi_m \underline{H}$$

$$\underline{B} = \mu_0 (1 + \chi_m) \underline{H}$$

$\chi_m > 0$ paramagnetic

$\chi_m < 0$ diamagnetic

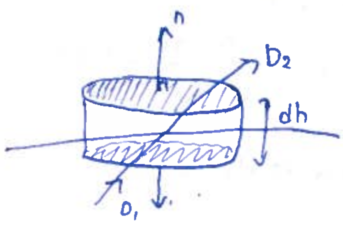
- Boundary conditions [DERIVATION]

  → $\underline{D}$ and $\underline{B}$ together

  → $\underline{E}$ and $\underline{H}$ together

$\nabla \cdot \underline{D} = \rho_f$

$\nabla \cdot \underline{B} = 0$



consider an interface with no free charges.

$$\int_V \nabla \cdot \underline{D} \, dv = \int_V \rho_f \, dv$$

$$\int_S \underline{D} \cdot \underline{n} \, da = \int_V \rho_f \, dv \quad - ①$$

Divergence

$$\int_S \underline{F} \cdot \underline{n} \, da = \int_V \nabla \cdot \underline{F} \, dv.$$

similarly for magnetic field,

$$\int_V \nabla \cdot \underline{B} \, dv = 0$$

$$\int_S \underline{B} \cdot \underline{n} \, da = 0 \quad - ②$$

$$\int_S \underline{D}\cdot\underline{n}\,da = \underline{D}_2\cdot\underline{n}\,da - \underline{D}_1\cdot\underline{n}\,da = 0$$

① $\int_V \rho_f\, dv \doteq 0$

Because interface with no free charge.

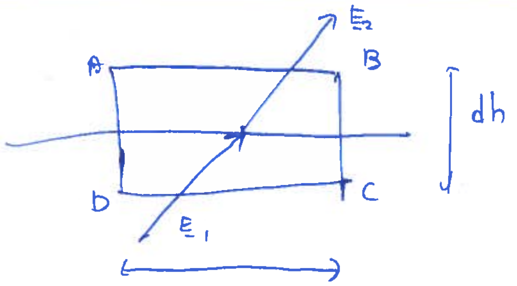
$$\underline{D}_2\cdot n = \underline{D}_1\cdot n$$

② limit $dh \to 0$

$$\boxed{\begin{aligned} D_{1\perp} &= D_{2\perp} \\ B_{1\perp} &= B_{2\perp} \end{aligned}}$$

NOTE: If the interface has free charge

$$\boxed{D_{2\perp} - D_{1\perp} = \sigma_f}$$



$$\nabla\times\underline{E} = -\frac{\partial B}{\partial t}$$

$$\nabla\times\underline{H} = \underline{J}$$

Interface with no free current

$$\int_S \nabla\times\underline{E}\cdot\underline{n}\,da = -\int_S \frac{\partial \underline{B}}{\partial t}\cdot\underline{n}\,da$$

Stoke's theorem

$$\oint_C \underline{E}\cdot d\underline{l} = \int_S (\nabla\times\underline{E})\cdot\underline{n}\,da$$

$$\oint_C \underline{E}\cdot d\underline{l} = -\int_S \frac{\partial \underline{B}}{\partial t}\cdot\underline{n}\,da$$

$dh \to 0$ $\qquad da = dl\,dh \to 0$

$$\int_S \nabla\times\underline{E}\cdot\underline{n}\,da = \oint_C \underline{E}\cdot d\underline{l} = 0$$

$$\underline{E}_1\cdot d\underline{l} = \underline{E}_2\cdot d\underline{l}$$

$$\boxed{E_{1\parallel} = E_{2\parallel}}$$

$$\underline{H}_1\cdot d\underline{l} = \underline{H}_2\cdot d\underline{l}$$

$$\boxed{H_{1\parallel} = H_{2\parallel}}$$

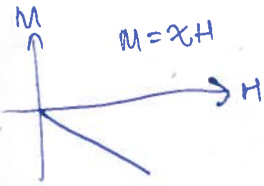
NOTE: If the interface has free current

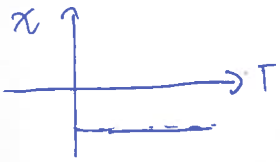
$$H_{2\parallel} - H_{1\parallel} = I_{fenc}$$

- Continuous across an interface
  - $B_\perp$
  - $\underline{D}_\perp$ when no free charges
  - $E_\parallel$
  - $H_\parallel$ when no free currents.

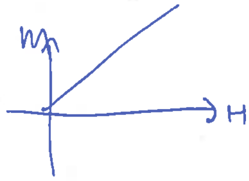
CHAPTER 3

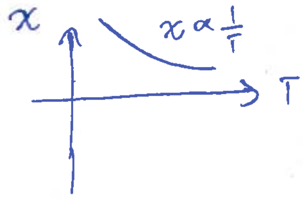
- Diamagnetism





Paramagnetism





CHAPTER 4

→ B vs H curve

→ Hard vs soft magnetic material.

CHAPTER 5

Divergence theorem (GAUSS LAW): $\int_S \underline{F}.d\underline{S} = \int_V \nabla.\underline{F}\,dV$

Stokes theorem: $\oint_C \underline{F}.d\underline{l} = \int_S \nabla\times\underline{F}.d\underline{S}$

- Current density in a general volume of space

$$\underline{J} = \sum_i N_i q_i \underline{v}_i$$

N_i : no. of charge carriers of type i per unit volume

- Current through an arbitrary surface

$$\underline{I} = -\oint_S \underline{J}.d\underline{S} = -\int_V \nabla.\underline{J}\,dV$$

Gauss law.

DERIVATION OF CONTINUITY EQUATION!

$$I = \frac{dQ}{dt} \qquad Q = \int_V \rho\,dV$$

$$I = \int_V \frac{\partial\rho}{\partial t}\,dv$$

$$\underline{I} = -\oint_S \underline{J}.d\underline{S} = -\int_V \nabla.\underline{J}\,dV$$

$$\int_V \frac{\partial\rho}{\partial t}\,dV = -\int_V \nabla.\underline{J}\,dV$$

$$\int_V \left(\nabla.\underline{J} + \frac{\partial\rho}{\partial t}\right) dV = 0$$

CONTINUITY EQUATION $\Rightarrow$ $\boxed{\nabla.\underline{J} + \frac{\partial\rho}{\partial t} = 0}$

- Deriving Ampére - Maxwell equation

$\nabla.\underline{D} = \rho_f$

using continuity equation:

$$\nabla.\underline{J} + \frac{\partial\rho}{\partial t} = 0$$

$$\nabla.\underline{J}_f + \frac{\partial\rho_f}{\partial t}$$

$$\nabla.\underline{J}_f + \frac{\partial}{\partial t}\nabla.\underline{D} = 0$$

$$\nabla.\left(\underline{J}_f + \frac{\partial\underline{D}}{\partial t}\right) = 0$$

Ampére's law,

$$\nabla\times\underline{H} = \underline{J}_f$$

$$\nabla.(\nabla\times\underline{H}) = \nabla.\underline{J}_f$$

Because diver[...] of any curl is [...]

compare

$$\nabla\times\underline{H} = \underline{J}_f + \ldots$$

- MAXWELL'S EQUATIONS (ABSENCE OF DIELECTRIC OR POLARIZABLE MEDIA).

	DIFFERENTIAL FORM	INTEGRAL FORM
Gauss law in electrostatic	$\nabla \cdot \underline{E} = \dfrac{\rho}{\epsilon_0}$	$\int_S \underline{E} \cdot d\underline{S} = \dfrac{Q}{\epsilon_0}$
Gauss law in magneostatic	$\nabla \cdot \underline{B} = 0$	$\int_S \underline{B} \cdot d\underline{S} = 0$
Faraday's law	$\nabla \times \underline{E} = -\dfrac{\partial B}{\partial t}$	$\oint \underline{E} \cdot d\underline{\ell} = -\dfrac{d\Phi_B}{dt}$
Ampére-Maxwell law	$\nabla \times \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \dfrac{\partial \underline{E}}{\partial t}$	$\oint \underline{B} \cdot d\underline{\ell} = \mu_0 I + \epsilon_0 \mu_0 \dfrac{d\Phi_E}{dt}$

- $\boxed{\text{Wave equation for } \underline{H}}$ (We need two ME → Ampere-maxwell law : $\nabla \times \underline{H} = \underline{J}_f + \dfrac{\partial \underline{D}}{\partial t}$; → Faraday's law : $\nabla \times E = -\dfrac{\partial B}{\partial t}$

- Assume uniform, linear, isotropic medium $\Rightarrow$ $\underline{D} = \epsilon \underline{E}$ $\quad \underline{B} = \mu \underline{H}$

- Assume that the medium has uniform conductivity, g $\Rightarrow$ $\underline{J}_f = g \underline{E}$

- Start with Ampére-Maxwell law:

$$\nabla \times \underline{H} = \underline{J}_f + \frac{\partial \underline{D}}{\partial t}$$

$$\nabla \times \nabla \times H = \nabla \times \left(\underline{J}_f + \frac{\partial \underline{D}}{\partial t} \right)$$

$\boxed{\text{EQUATION FOR } H}$

$$\nabla(\nabla \cdot \underline{H}) - \nabla^2 \underline{H} = g \, \nabla \times \underline{E} + \frac{\partial}{\partial t} \nabla \times \underline{D}$$

$$\nabla(\nabla \cdot \underline{H}) - \nabla^2 \underline{H} = g \, \nabla \times \underline{E} + \epsilon \frac{\partial}{\partial t} \nabla \times \underline{E}$$

- Using the Faraday's law $\left[\nabla \times \underline{E} = -\dfrac{\partial \underline{B}}{\partial t} \right]$

$$\nabla(\nabla \cdot \underline{H}) - \nabla^2 \underline{H} = -g \frac{\partial \underline{B}}{\partial t} - \epsilon \frac{\partial^2 \underline{B}}{\partial t^2}$$

- $\nabla \cdot H = 0$ and $\underline{B} = \mu \underline{H}$

$$-\nabla^2 \underline{H} = -g\mu \frac{\partial \underline{H}}{\partial t} - \epsilon\mu \frac{\partial^2 \underline{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \underline{H} - g\mu \frac{\partial \underline{H}}{\partial t} - \epsilon\mu \frac{\partial^2 \underline{H}}{\partial t^2} = 0}$$

- damping proportional to $g\mu$
- $g \to 0$ for non-conducting medium

Solution: $\underline{H}(\underline{r}, t) = H_0 e^{i(\underline{k}_H \cdot \underline{r} - \omega_H t)}$

- Wave equation of $\underline{E}$

  - Assume uniform, linear, isotropic medium $\underline{D} = \epsilon \underline{E}$ $\underline{B} = \mu \underline{H}$
  - Assume that the medium has uniform conductivity, g $\Rightarrow$ $\underline{J}_f = g \underline{E}$.
  - Start with Faraday's law

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \nabla \times \underline{H} = \underline{J}_f + \frac{\partial \underline{D}}{\partial t}$$

$$\nabla \times \nabla \times \underline{E} = -\frac{\partial}{\partial t} \nabla \times \underline{B}$$

$$\nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E} = -\mu \frac{\partial}{\partial t} \nabla \times \underline{H}$$

$$\nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E} = -\mu \frac{\partial}{\partial t}\left(\underline{J}_f + \frac{\partial \underline{D}}{\partial t}\right)$$

  - $\nabla \cdot \underline{E} = 0$ when there are no free charges

$$-\nabla^2 \underline{E} = -\mu \frac{\partial}{\partial t}\left(g \underline{E} + \epsilon \frac{\partial \underline{E}}{\partial t}\right)$$

$$-\nabla^2 \underline{E} = -\mu g \frac{\partial \underline{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \underline{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \underline{E} - \mu g \frac{\partial \underline{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \underline{E}}{\partial t^2} = 0}$$

Solution : $\underline{E} = \underline{E}_0 e^{i(k_E \cdot r - \omega_E t)}$

phase velocity, $\boxed{V_p = \frac{1}{\sqrt{\epsilon \mu}}}$ $\xrightarrow{\text{in vacuum}}$ $V_p = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$

- Link between electric and magnetic field

$$\boxed{\underline{k} \times \underline{E}_0 = \omega \underline{B}_0}$$

Can obtain this using Faraday's law

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t}$$

And the plane wave equation for $\underline{E}$ & $\underline{B}$.

- Polarization

→ Assume propagation along z-axis, $\underline{k} = (0, 0, k)$

→ E_x and E_y have independent amplitude and phase

$$\underline{E}_0 = E_{0x} e^{i\phi_x} \hat{\underline{i}} + E_{0y} e^{i\phi_y} \hat{\underline{j}}$$

→ $\underline{E} = \underline{E}_0 e^{i(kz - \omega t)}$ → Direction of propagation.

Sometimes, can also write $E_0 \Rightarrow E_0 \hat{n}$

unit vector $\hat{n}$ is the polarization.

→ $\underline{E} = e^{i(kz-\omega t)} \left[E_{0x} e^{i\phi_x} \underline{i} + E_{0y} e^{i\phi_y} \underline{j} \right]$

= Bring out $e^{i\phi_x}$ & Bring in $e^{i(-\omega t)}$

$$= e^{i(kz+\phi_x)} \left[E_{0x} e^{i(-\omega t)} \underline{i} + E_{0y} e^{i(\phi_y - \phi_x - \omega t)} \underline{j} \right]$$

$$\left[\cos(kz+\phi_x) + i\sin(kz+\phi_x)\right]\left[E_{0x}\{\cos(\omega t) + i\sin(\omega t)\}\underline{i} + E_{0y}\{\cos(\phi_y - \phi_x - \omega t) + i\sin(\phi_y - \phi_x - \omega t)\}\underline{j}\right.$$

$$\mathrm{Re}[\underline{E}(\underline{r},t)] = \cos(kz+\phi_x)\{ E_{0x}\cos(-\omega t)\,\underline{i} + E_{0y}\cos(\phi_y$$

- TYPE OF POLARISATION

phase difference between E_{0x} & E_{0y} is ϕ

$\phi = 0$ or π → PLANE / LINEAR POLARISATION

If $E_{0x} \neq E_{0y}$ for plane polarization, then the plane is at an angle $\theta = \tan^{-1}\left(\frac{E_{y0}}{E_{x0}}\right.$

$\phi = \frac{\pi}{2}$ or $\frac{3}{2}\pi$ with $E_{0x} = E_{0y}$ → CIRCULAR POLARISATION

$E_{0x} \neq E_{0y}$, $\phi \neq 0$ → ELLIPTICAL POLARISATION.

$$\bar{f}(z,t) = \bar{A} e^{i(kz - \omega t + \phi)} \hat{s}$$

$$\hat{s} = (\cos\theta)\hat{x} + (\sin\theta)\hat{y}$$

$$\therefore \bar{f}(z,t) = \bar{A}\cos\theta\, e^{i(kz-\omega t+\phi)}\hat{x} + \bar{A}\sin\theta\, e^{i(kz-\omega t+\phi)}\hat{y}$$

CHAPTER 6

- Solution for the plane waves for 2 cases
  - non-conducting dielectric
  - A conducting system.

- Start with Maxwell's equation [Ampére's law], in a linear conducting medium of conductivity g

$$\nabla \times \underline{H} = J + \frac{\partial D}{\partial t}$$

$$\underline{J} = g\,\underline{E}$$

- Write electric displacement and magnetic intensity as plane waves

$$\underline{D} = \underline{D}_0 \exp i(\underline{k}.\underline{r} - \omega t)$$

$$\underline{H} = \underline{H}_0 \exp i(\underline{k}.\underline{r} - \omega t + \phi)$$

- Subs. into Ampére's law

$$i\underline{k} \times \underline{H} = g E + (-i\omega)\,\underline{D}$$

$$i\underline{k} \times \underline{H}_0 = g\,\underline{E}_0 - i\omega\,\underline{D}_0$$

multiply by i,

$$-\underline{k} \times \underline{H}_0 = ig\,\underline{E}_0 + \omega\,\underline{D}_0$$

$$\underline{k} \times \underline{H}_0 = -\omega\,\underline{D}_0 - ig\,\underline{E}_0$$

$$\underline{k} \times \frac{\underline{B}_0}{\mu_0} = -\omega\epsilon_r\epsilon_0\underline{E}_0 - ig\,\underline{E}_0$$

$$\underline{k} \times \underline{B}_0 = -\omega\epsilon_0\mu_0\epsilon_r\underline{E}_0 - i\mu_0 g\,\underline{E}_0$$

$$\underline{k} \times \underline{B}_0 = -\omega\epsilon_0\mu_0\epsilon_r\underline{E}_0 - \frac{i\omega\epsilon_0\mu_0 g}{\omega\epsilon_0}\underline{E}_0$$

$$\underline{B} = \mu\underline{H}$$

$$\underline{B} = \mu_0\mu_r\underline{H} \qquad \text{set } \mu_r = 1$$

$$\underline{B}_0 = \mu_0\underline{H}_0$$

$$\underline{H}_0 = \frac{\underline{B}_0}{\mu_0}$$

$$\underline{D} = \epsilon_r\epsilon_0\underline{E}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\sqrt{\mu_0\epsilon_0} = \frac{1}{c}$$

$$\mu_0\epsilon_0 = \frac{1}{c^2}$$

$$\underline{k} \times \underline{B}_0 = -\frac{\omega}{c^2}\underbrace{\left(\epsilon_r - \frac{ig}{\epsilon_0\omega}\right)}\underline{E}_0$$

$$\tilde{\epsilon} = \epsilon_r - \frac{ig}{\epsilon_0\omega}$$

$$\boxed{\underline{k} \times \underline{B}_0 = -\frac{\omega}{c^2}\tilde{\epsilon}\,\underline{E}_0}$$

→ True for a dielectric with no free charge

OR

Low frequency wave in a conductor where free charges disperse rapidly.

- $\underline{k} \times \underline{B}_0 = -\frac{\omega}{c^2} \bar{\epsilon}\, \underline{E}_0$ $\qquad \bar{\epsilon} = \epsilon_r + i \frac{g}{\epsilon_0 \omega}$

$$\underline{k} \times \underline{E}_0 = \omega\, \underline{B}_0$$

$$\underline{k} \times \underline{k} \times \underline{E}_0 = \omega\, \underline{k} \times \underline{B}_0$$

$$\underline{k}(\underline{k} \cdot \underline{E}_0) - k^2 \underline{E}_0 = -\frac{\omega^2}{c^2} \bar{\epsilon}\, \underline{E}_0$$

$\underline{k} \cdot \underline{E}_0 = 0$ because we assume they are transverse.

$$-k^2 \underline{E}_0 = -\frac{\omega^2}{c^2} \bar{\epsilon}\, \underline{E}_0$$

$$k^2 = \frac{\omega^2}{c^2} \bar{\epsilon}$$

- Phase velocity $\rightarrow$ $v_p = \frac{\omega}{k} = \frac{c}{\sqrt{\bar{\epsilon}}} = \frac{c}{n}$

$$\boxed{n = \sqrt{\bar{\epsilon}}} \qquad \bar{\epsilon} = \epsilon_r + i \frac{g}{\epsilon_0 \omega}$$

(1) For dielectric $\Rightarrow$ $g = 0$

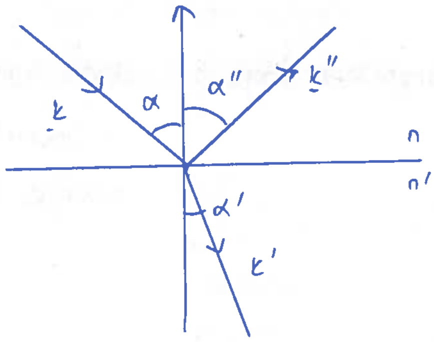
$$\therefore n = \sqrt{\bar{\epsilon}} = \sqrt{\epsilon_r}$$

(2) Conducting system $\Rightarrow$ $g \neq 0$

$$n = \sqrt{\bar{\epsilon}}$$

(has complex dielectric constant & complex refractive index).

- Reflection & refraction.



① Angle of incidence = angle of reflection

$$\alpha = \alpha''$$

② Snell's law

$$n \sin\alpha = n' \sin\alpha'$$

- $r = \dfrac{|\underline{E}''|}{|\underline{E}|}$

$t = \dfrac{|\underline{E}'|}{|\underline{E}|}$

- $\underline{E} = \underline{E}_0 \exp i(\underline{k}.\underline{r} - \omega t)$

$\underline{k} \times \underline{E}_0 = \omega \underline{B}_0$ // $\omega \underline{B}_0 = \underline{k} \times \underline{E}_0$

Write $\underline{k} \to k\hat{\underline{k}}$

$$k = \frac{\omega}{c'} = \omega\sqrt{\epsilon\mu}$$

$$\omega \underline{B}_0 = \omega\sqrt{\epsilon\mu}\ \hat{\underline{k}} \times \underline{E}_0$$

$$\underline{B} = \sqrt{\epsilon\mu}\ \hat{\underline{k}} \times \underline{E}$$

$\underline{B} = \mu \underline{H}$

$\underline{H} = \dfrac{\underline{B}}{\mu}$

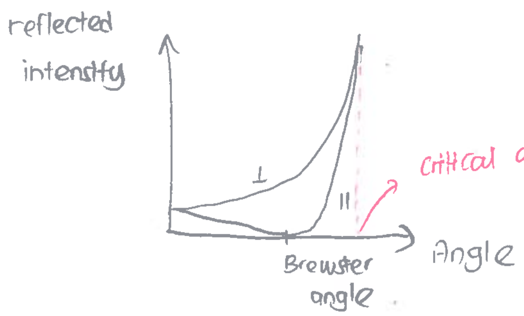
$$\boxed{\underline{H} = \sqrt{\frac{\epsilon}{\mu}}\ \hat{\underline{k}} \times \underline{E}}$$

- Derivation of Fresnal equations!

✓

- Brewster angle derivation !

Brewster angle : When light is incident at the Brewster angle, the reflected light is linearly polarised because the reflection coefficient for the parallel (∥) component is 0.



$$r_{\parallel} = \frac{\tan(\alpha-\alpha')}{\tan(\alpha+\alpha')} \qquad r_{\perp} = \frac{\sin(\alpha-\alpha')}{\sin(\alpha+\alpha')}$$

$$r_{\parallel} = \frac{n'\cos\alpha - n\cos\alpha'}{n'\cos\alpha + n\cos\alpha'}$$

- If $r_{\parallel} = 0$, $n'\cos\alpha_B = n\cos\alpha'$ — ①

- Snell's law . $n\sin\alpha = n'\sin\alpha'$

  $n\sin\alpha_B = n'\sin\alpha'$ — ②

- Using $r_{\parallel} = \dfrac{\tan(\alpha-\alpha')}{\tan(\alpha+\alpha')}$

  $r_{\parallel} \to 0$ if $\tan(\alpha+\alpha') \to \infty$

  ↓

  This will happen if $\alpha+\alpha' = \frac{\pi}{2}$

  $\alpha' = \frac{\pi}{2} - \alpha_B$ — ③

- $n\sin\alpha_B = n'\sin\alpha'$

  $n\sin\alpha_B = n'\sin\left(\frac{\pi}{2} - \alpha_B\right)$

  $n\sin\alpha_B = n'\cos(\alpha_B)$

  $\tan\alpha_B = \frac{n'}{n}$

  $$\boxed{\alpha_B = \tan^{-1}\left(\frac{n'}{n}\right)}$$

- Critical angle

- $n > n'$
- using snell's law:

$$n' \sin \alpha' = n \sin \alpha$$

$$\sin \alpha' = \frac{n}{n'} \sin \alpha$$

$$\sin \alpha' > 1 \quad \text{(This is unphysical)}$$

critical angle $\boxed{\alpha_c = \sin^{-1}\left(\frac{n'}{n}\right)}$

CHAPTER 7

$$\nabla^2 \underline{E} - g\mu \frac{\partial \underline{E}}{\partial t} - \epsilon\mu \frac{\partial^2 \underline{E}}{\partial t^2} = 0 \rightarrow$$ In a conducting medium, $\underline{J} = g\underline{E}$

conductance, g

$$\underline{E} = \underline{E}_0 \exp i(\underline{k} \cdot \underline{r} - \omega t)$$

get dispersion relation

$$\boxed{k^2 = \mu\epsilon\omega^2\left(1 + \frac{ig}{\epsilon\omega}\right)}$$

group velocity, $v_g = \frac{d\omega}{dk}$

phase velocity, $v_p = \frac{\omega}{k}$

Poor conductor $\rightarrow$ $\boxed{g \rightarrow 0}$

$$k^2 = \mu\epsilon\omega^2$$

Good conductor $\rightarrow$ $\boxed{g \gg \epsilon\omega}$

$$k^2 \approx \mu\epsilon\omega^2\left(\frac{ig}{\epsilon\omega}\right)$$

$$k^2 \approx i\mu\omega g$$

$$k = +\sqrt{i\mu\omega g}$$

$$\sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$$

$$k = \frac{1}{\sqrt{2}}(1+i)\sqrt{\mu\omega g}$$

$$k = \underbrace{\sqrt{\frac{\mu\omega g}{2}}}_{k_r} + i\underbrace{\sqrt{\frac{\mu\omega g}{2}}}_{k_i}$$

- Skin depth

$$\underline{E} = \underline{E}_0 \exp i(\underline{k}\cdot\underline{r} - \omega t)$$

$$\underline{k} = \underline{k}_r + i\,\underline{k}_i$$

$$\underline{E} = \underline{E}_0\, e^{i((\underline{k}_r + i\,\underline{k}_i)\cdot\underline{r} - \omega t)}$$

$$= \underbrace{\underline{E}_0\, e^{-k_i}}\; e^{i(\underline{k}_r\cdot\underline{r} - \omega t)}$$

Normal travelling wave is exponentially damped in the direction of $\underline{k}$.

$$E_0(d) = E_0(0)\, e^{-\frac{d}{\delta}}$$

Skin depth : $\boxed{\delta = \frac{1}{k_i}}$ $\quad \delta = \sqrt{\frac{2}{\mu\omega g}}$

★ The point where the fields are reduced by a factor of $\frac{1}{e} \approx \frac{1}{2.71}$ is call SKIN DEPTH

- Plasma → • A group of (massive), (slowly moving) positive ions with a cloud of free electrons surrounding it so that the whole system is neutral.
  - The system is homogenous on macroscopic length scales.
  - If there is a local fluctuation, so that the electrons are displaced by x, there is a resulting polarization, $\boxed{\underline{P} = -N_e e \underline{x}}$, leading to a restoring force on the electrons.

DERIVING PLASMA FREQUENCY

$$\underline{D} = \epsilon \underline{E}$$

$$\underline{E} = \frac{\underline{D}}{\epsilon}$$

The electric displacement, $D = \frac{Q}{A}$ for a capacitor

$$E = \frac{Q}{\epsilon A}$$

$$Q = N_e e x A$$

$$E = \frac{N_e e x \cancel{A}}{\epsilon \cancel{A}}$$

since $\boxed{\underline{P} = -N_e e \underline{x}}$

$$\underline{E} = \frac{N_e e x}{\epsilon} \xrightarrow{\text{assume } \epsilon_r = 1} \underline{E} = \frac{N_e e x}{\epsilon_0} \longrightarrow \boxed{\underline{E} = -\frac{\underline{P}}{\epsilon_0}}$$

$\boxed{\underline{P} = -N_e e \underline{x}}$

$\boxed{\underline{E} = -\frac{\underline{P}}{\epsilon_0}}$

$\underline{F} = -e\underline{E}$

$\boxed{\underline{F} = -\frac{N_e e^2 \underline{x}}{\epsilon_0}}$

$$m_e \frac{d^2\underline{x}}{dt^2} = -\frac{N_e e^2}{\epsilon_0}\underline{x}$$

$$\boxed{\omega_p = \sqrt{\frac{N_e e^2}{m_e \epsilon_0}}}$$

- Derivation of dispersion relation for plasma.

$$\underline{E}(\underline{r},t) = \underline{E}_0 e^{i(\underline{k}.\underline{r} - \omega t)}$$

Force on an electron in the plasma

$$F = m\frac{d\underline{v}}{dt} = -e\left(E(\underline{r},t) + \underline{v} \times \underline{B}(\underline{r},t)\right)$$

neglect the contribution from the magnetic field in a non-relativistic plasma

★ must know why ($v B_0 \ll E_0$) for non-relativ

$$m\frac{d\underline{v}}{dt} = -e\,\underline{E}(\underline{r},t)$$

$$\frac{d\underline{v}}{dt} = -\frac{e}{m}\underline{E}(\underline{r},t)$$

$$\int \frac{d\underline{v}}{dt}\,dt = -\frac{e}{m}\underline{E}_0 e^{i(\underline{k}.\underline{r})} \int e^{-i\omega t}\,dt$$

$$\boxed{\underline{v} = -\frac{ei}{\omega m}\underline{E}}$$

Electrons moving together as a group with this velocity gives a current, with a current density

$$\underline{J} = -N_e e \underline{v}$$

$$\boxed{\underline{J} = i\left(\frac{N_e e^2}{m\omega}\right)\underline{E}}$$

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P}$$

$$\underline{D} = \epsilon_r \epsilon_0 \underline{E}$$

$$\boxed{\underline{P} = -N_e e \underline{x}}$$

$$\boxed{\underline{J} = -N_e e \underline{v}}$$

$$\underline{J} = -N_e e \underline{v}$$

$$= -N_e e \frac{d\underline{x}}{\partial t}$$

$$\boxed{\underline{J} = \frac{\partial \underline{P}}{\partial t}}$$

$$\boxed{\underline{D} = \epsilon_0 \underline{E} + \underline{P}}$$

$$\frac{\partial \underline{D}}{\partial t} = \epsilon_0 \frac{\partial \underline{E}}{\partial t} + \frac{\partial \underline{P}}{\partial t}$$

since $\boxed{\underline{D} = \epsilon_r \epsilon_0 \underline{E}}$

$$\epsilon_r \epsilon_0 \frac{\partial \underline{E}}{\partial t} = \epsilon_0 \frac{\partial \underline{E}}{\partial t} + \frac{\partial \underline{P}}{\partial t}$$

since $\boxed{\frac{\partial \underline{P}}{\partial t} = \underline{J}}$

$$\epsilon_r \epsilon_0 \frac{\partial \underline{E}}{\partial t} = \epsilon_0 \frac{\partial \underline{E}}{\partial t} + \underline{J}$$

$$\underline{J} = \epsilon_0 (\epsilon_r - 1) \frac{\partial \underline{E}}{\partial t}$$

Recall, $\boxed{\underline{E} = \underline{E}_0 e^{i(\underline{k}\cdot\underline{r} - \omega t)}}$

$$\boxed{\underline{J} = i\left(\frac{N_e e^2}{m\omega}\right)\underline{E}}$$

$$\frac{\partial \underline{E}}{\partial t} = -i\omega \underline{E}$$

$$i\left(\frac{N_e e^2}{m\omega}\right)\underline{E} = \epsilon_0 (\epsilon_r - 1)(-i\omega \underline{E})$$

REMEMBER THE YELLOW BOX!

$$\epsilon_r = 1 - \frac{1}{\omega^2}\frac{N_e e^2}{\epsilon_0 m}$$

$$\boxed{\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}}$$

$$v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0 \mu_r}}$$

for plasma $\mu_r = 1$

$$v_p = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0}}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \qquad c^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0}} \Rightarrow \frac{\omega^2}{k^2} = \frac{1}{\epsilon_0 \epsilon_r \mu_0}$$

$$\frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_r} \Rightarrow \boxed{k^2 = \frac{\omega^2}{c^2}\epsilon_r}$$

$$\boxed{k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)}$$

→ If $\omega > \omega_p$, then $k^2 > 0$, so k is real and there is no attenuation.

If $\omega < \omega_p$, then $k^2 < 0$, we have absorption of energy and damping over some attenuation length, L.

→ $v_p = \dfrac{\omega}{k}$

$v_g = \dfrac{d\omega}{dk}$

$$k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$2kc^2 = 2\omega \frac{d\omega}{dk}$$

$$\frac{kc^2}{\omega} = \frac{d\omega}{dk}$$

$$\frac{c^2}{v_p} = v_g$$

$$\boxed{v_g v_p = c^2}$$

CONDITION → • Both velocities are equal to c

OR

• $v_p > c$

CHAPTER 8

- Energy densities in static fields

$$U_e = \frac{1}{2} \underline{E} \cdot \underline{D}$$

$$U_m = \frac{1}{2} \underline{B} \cdot \underline{H}$$

- $\underline{F} = q\underline{E}$

Rate of work done (Power, P) :
$$\underline{F} \cdot \underline{v} = q\,\underline{E} \cdot \underline{v}$$
$$= \underline{E} \cdot q\underline{v} \qquad \underline{J} = q\underline{v}$$
$$= \underline{E} \cdot \underline{J}$$
$\downarrow$

This is work done per unit volume.

Rate of energy transfer from EM field to the current in a volume, V $\rightarrow$ $\boxed{P_V = \int_V \underline{J} \cdot \underline{E}\, dv}$

$\rightarrow$ If the medium obeys $\underline{J} = g\underline{E}$

then $P_V = \int_V gE^2 dv$ ✓

using Ampére-Maxwell :
$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$$
$$\underline{J} = \nabla \times \underline{H} - \frac{\partial \underline{D}}{\partial t}$$
$$P_V = \int_V \left(\nabla \times \underline{H} - \frac{\partial \underline{D}}{\partial t}\right) \cdot \underline{E}\, dv$$
$$P_V = \int_V \left\{ \underline{E} \cdot \nabla \times \underline{H} - \underline{E} \cdot \frac{\partial \underline{D}}{\partial t} \right\} dv$$

- Poynting vector $\Rightarrow$ $\boxed{\underline{N} = \underline{E} \times \underline{H}}$
- Poynting's theorem $\Rightarrow$ $\int_V \nabla \cdot \underline{N}\, dv = \oint_S \underline{N} \cdot \underline{n}\, da$

$\rightarrow$ rate of flow of energy through the surface S as EM waves

- Time Average ~~flux~~ : $\boxed{\langle N \rangle = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\, E_0^2\, \hat{\underline{k}}}$

- momentum, $\boxed{P = \frac{E}{c} = \frac{\langle N \rangle}{c}}$

CHAPTER 9

- Radiation : Irreversible flow of energy which occurs from an accelerated charge away from the source to infinity.
- Retarded Potential → Scalor or Vector potential for the electromagnetic field generated by time varying ~~ele~~ current or charge distribution.

→ MEMORISE !

- $$\boxed{\begin{aligned} \underline{B} &= \nabla \times \underline{A} \\ \underline{E} &= -\nabla\phi - \frac{\partial \underline{A}}{\partial t} \end{aligned}}$$

$\xrightarrow{\text{derivation}}$

Start with Faraday's law

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = 0$$

$$\nabla \times E + \frac{\partial}{\partial t} \nabla \times \underline{A} = 0$$

$$\nabla \times \left(\underline{E} + \frac{\partial \underline{A}}{\partial t} \right) = 0$$

Curl of a gradient is always zero. Therefore,

Let $\underline{E} + \frac{\partial \underline{A}}{\partial t} = -\nabla\phi$

$$\therefore \underline{E} = -\nabla\phi - \frac{\partial \underline{A}}{\partial t}$$

- Start with Ampere Maxwell

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$$

Assuming LIH : $\underline{B} = \mu \underline{H}$, $\underline{D} = \epsilon \underline{E}$

$$\frac{1}{\mu} \nabla \times \underline{B} = \underline{J} + \epsilon \frac{\partial \underline{E}}{\partial t}$$

using $B = \nabla \times A$

$$\frac{1}{\mu} \nabla \times \nabla \times A = \underline{J} + \epsilon \frac{\partial \underline{E}}{\partial t}$$

$$\frac{1}{\mu}\nabla\times\nabla\times\underline{A} = \underline{J} + \epsilon\frac{\partial \underline{E}}{\partial t}$$

since $\underline{E} = -\nabla\phi - \frac{\partial \underline{A}}{\partial t}$

$$\frac{1}{\mu}\nabla\times\nabla\times\underline{A} = \underline{J} + \epsilon\frac{\partial}{\partial t}\left(-\nabla\phi - \frac{\partial \underline{A}}{\partial t}\right)$$

$$\nabla\times\nabla\times\underline{A} = \nabla(\nabla\cdot\underline{A}) - \nabla^2\underline{A}$$

$$\frac{1}{\mu}\left[\nabla(\nabla\cdot\underline{A}) - \nabla^2\underline{A}\right] = \underline{J} + \epsilon\frac{\partial}{\partial t}\left(-\nabla\phi - \frac{\partial \underline{A}}{\partial t}\right)$$

$$\nabla(\nabla\cdot\underline{A}) - \nabla^2\underline{A} + \epsilon\mu\frac{\partial}{\partial t}\left(\nabla\phi + \frac{\partial \underline{A}}{\partial t}\right) = \mu\underline{J}$$

USE LORENTZ CONDITION

$$\nabla\cdot\underline{A} + \epsilon\mu\frac{\partial\phi}{\partial t} = 0$$

$$-\nabla^2\underline{A} + \epsilon\mu\frac{\partial^2\underline{A}}{\partial t^2} + \nabla(\nabla\cdot\underline{A}) + \epsilon\mu\nabla\frac{\partial\phi}{\partial t} = \mu\underline{J}$$

$$-\nabla^2\underline{A} + \epsilon\mu\frac{\partial^2\underline{A}}{\partial t^2} + \nabla\left(-\epsilon\mu\frac{\partial\phi}{\partial t}\right) + \epsilon\mu\nabla\frac{\partial\phi}{\partial t} = \mu\underline{J}$$

$$-\nabla^2\underline{A} + \epsilon\mu\frac{\partial^2\underline{A}}{\partial t^2} = \mu\underline{J}$$

Vector potential ~~sa~~ satisfies

$$\boxed{-\nabla^2\underline{A} + \epsilon\mu\frac{\partial^2\underline{A}}{\partial t^2} = \mu\underline{J}}$$

Scalar potential satisfies

$$\boxed{-\nabla^2\phi + \epsilon\mu\frac{\partial^2\phi}{\partial t^2} = \frac{\rho}{\epsilon}}$$

- Retarded time : $\boxed{t' = t - \frac{r}{c}}$

→ A wave takes time $\frac{r}{c}$ to reach observer due to finite speed, therefore, an observer experiences a charge distribution at a time retarded by $\frac{r}{c}$.

- Retarded scalar potential

$$\boxed{\phi(\underline{r},t) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\underline{r}',t')}{|\underline{r}-\underline{r}'|}\,dv'}$$

- Retarded vector potential

$$\boxed{\underline{A}(\underline{r},t) = \frac{\mu_0}{4\pi}\int_V \frac{\underline{J}(\underline{r}',t')}{|\underline{r}-\underline{r}'|}\,dv'}$$

• Hertzian dipole.

- Electric dipole has its charges oscillating with frequency ω

$$q\left(t-\frac{r}{c}\right) = q_0 \cos \omega\left(t-\frac{r}{c}\right)$$

$$p = qd$$

$$p = \underbrace{q_0 l}_{p_0} \cos \omega\left(t-\frac{r}{c}\right)$$

$$\boxed{\begin{aligned} I &= \frac{dq}{dt} \\ I &= -\omega q_0 \sin \omega\left(t-\frac{r}{c}\right) \\ I &= I_0 \sin \omega\left(t-\frac{r}{c}\right) \end{aligned}}$$

$$\boxed{\begin{aligned} I_0 &= -q_0 \omega \\ p_0 &= q_0 l \end{aligned}}$$

- The changes in Q and I are propagated as EM waves radiated outwards from the centre of the dipole.
(EM waves are produced by oscillating charges and currents)

- there is a retarded magnetic vector potential parallel to the current, which is along the direction of the dipole:

$$A_z(r,t) = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2} \frac{I\left(r',\ t-\frac{r}{c}\right)}{r}\, dz'$$

Taking far field limit $\boxed{(r \gg l)}$ and ignore the variation in I along the dipole

$\equiv$ Equivalent to assumin $\boxed{\lambda \gg l}$

$$\boxed{A_z(r,t) = \frac{\mu_0 l}{4\pi}\left(\frac{I\left(t-\frac{r}{c}\right)}{r}\right)}$$

- The time variation of the scalar potential can be obtained using Lorentz condition.

$$\boxed{\phi(r,t) = \frac{l}{4\pi\epsilon_0}\,\frac{z}{r^2}\left(\frac{q\left(t-\frac{r}{c}\right)}{r} + \frac{I(t-r/c)}{c}\right)}$$

Vector potential in spherical polar coordinates.

- $$A_r = \frac{\mu_0}{4\pi} \frac{I_0 l}{r} \cos\theta \sin \omega \left(t - \frac{r}{c}\right)$$

$$A_\theta = \frac{\mu_0}{4\pi} I_0$$

$$A_\phi = 0$$

using $\underline{B} = \nabla \times \underline{A}$

- $B_r = 0$

$B_\theta = 0$

$$B_\phi (r,t) = \frac{\mu_0}{4\pi} \frac{I_0 l}{r} \sin\theta \left(\frac{\omega}{c} \cos \omega \left(t - \frac{r}{c}\right)\right)$$

- using $\underline{E} = -\nabla\phi - \frac{\partial A}{\partial t}$

- $E_r = 0$

$$E_\theta = \frac{I_0 l}{4\pi\epsilon_0} \sin\theta \; \frac{\omega \cos \omega (t - r/c)}{rc^2}$$

$E_\phi = 0$

$\Rightarrow$ $\boxed{\underline{E} \times \underline{B} \Rightarrow \underline{\hat{\theta}} \times \underline{\hat{\phi}} = \underline{\hat{r}}}$ ★

CHAPTER 10

- Consider a flash of light from origin of S at $t = t' = 0$
- The location of a point on the wavefront in S at dt is

$$dx^2 + dy^2 + dz^2 - c^2 dt^2 = 0$$

- This point on the wavefront in S' at later time dt' is

$$(dx')^2 + (dy')^2 + (dz')^2 - c^2 (dt')^2 = 0 .$$

- Define an arbitrary space-time ~~time~~ interval

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$$

$ds^2 < 0$ time-like

$ds^2 > 0$ space-like

$ds^2 = 0$ light-like.

- Time dilation $\rightarrow$ $\Delta t' = \gamma \Delta t$

Length contraction $\rightarrow$ $L = \dfrac{L'}{\gamma}$

$L = x_2 - x_1$

$L' = x_2' - x_1'$

- $x_\mu = (x, y, z, ict)$

$\mu = 1 \ldots 4$

$x_\mu x_\mu = x^2 + y^2 + z^2 - c^2 t^2 = ds^2$

$\downarrow$

This interval is invariant under Lorentz transformation.

- $$R = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$\boxed{x'_\mu = R_{\mu\nu} x_\nu}$$

$x_1' = R_{11} x_1 + R_{12} x_2 + R_{13}x$

$+ R_{14} x$

$= \gamma x_1 + 0 + 0 + i\beta\gamma$

$\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$ $\quad \beta = \dfrac{v}{c}$

- $P_\mu = \left(p_x, p_y, p_z, i\frac{E}{c} \right)$

- $\partial_\mu = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4} \right)$

  $= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, -\frac{i}{c}\frac{\partial}{\partial t} \right)$

Also invariant under Lorentz transformation.

- continuity equation

$$\nabla \cdot \underline{J} + \frac{\partial \rho}{\partial t} = 0$$

- Current density 4-vector

$$j_\mu = (J_1, J_2, J_3, ic\rho)$$

- $\boxed{\partial_\mu j_\mu = 0}$ → Also represent the continuity equation.

- $a_\mu = \left(A_1, A_2, A_3, i\frac{\phi}{c} \right)$

$$\partial_\mu \partial_\mu = \Box$$

$$\Box a_\mu = -\mu_0 J$$

$\frac{\partial^2 A_1}{\partial x_1^2} + \frac{\partial^2}{\partial x}(A_1 + A_2 + A_3) - \frac{i}{c^2}\frac{\partial^2}{\partial t^2}\frac{\phi}{c}$

$-\frac{i}{c^3}\frac{\partial^2 \phi}{\partial t^2}$

- Lorentz condition

$$\nabla \cdot \underline{A} + \epsilon \mu \frac{\partial \phi}{\partial t} = 0$$

↓

$$\boxed{\partial_\mu a_\mu = 0}$$

- Electromagnetic field tensor

$$F_{\mu\nu} = \frac{\partial a_\nu}{\partial x_\mu} - \frac{\partial a_\mu}{\partial x_\nu}$$

$\nabla^2 \underline{A} - \frac{1}{c^2}\frac{\partial^2 \underline{A}}{\partial t^2} = -\mu_0 \underline{J}$

$\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$