2011

(1)

4) A Ferromagnet at temperatures above the Curie temperature have no net magnetisation as it is more favourable for their moments to be randomly aligned than it is to be in aligned domains. As these begin to cool, the short range interaction between ferromagnetic moments results in their magnetic moments aligning. Thus the moments have ~~exe~~ exchange force between them. As these cool, these aligned moments have a lot of energy so form domains at the cost of exchange energy, domain walls form which ~~steady~~ slowly twist from one alignment to the next to minimise the loss in exchange energy.

Energy reduced by domains. opposite alignment.

$T > T_c$ $T < T_c$

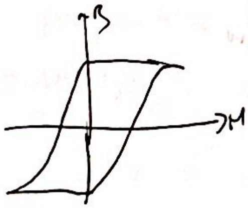
These domains have aligned spins within but are randomly orientated with respect to each other.

The domain walls twist to minimise the loss in exchange energy.

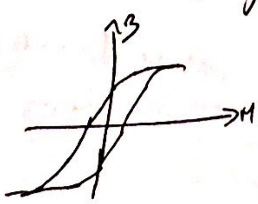
↑↗↗→→→↘↘↘↓↓↓↓

This energy is stored in the exchange force and kinetic energy dominate. Domain states are more energetically favourable as this is a lower energy state than if all moments were aligned without domains (would have a very large moment).

5) Hard magnet: High remanence & coercivity → Emits strong field in the absence of applied field and hard to magnetise & demagnetise → Harder to move domain walls through magnet



Soft ferromagnet: Low remanence & coercivity → Emits weak field & easily magnetised & demagnetised.



2a) ρ is the charge density and is defined as the charge distributed over a unit volume.

ρ_f is the free charge density and is the charge for free electrons with no bound currents.

In this, we have no free currents.

$\nabla \cdot E$ also accounts for the currents in a material but where $\nabla \cdot D$ only is for linear, homogeneous mediums only.

$\sigma_p = \underline{P} \cdot \hat{n}$

unit vector in the direction considered

surface polarisation charge Density: ~~Rate~~ Polarisation charge density due to bound currents in an electric field.

$\rho_p = -\nabla \cdot P$

volume polarisation charge density: Polarisation charge density due to non-uniformly polarised material.

3a) $N = \frac{1}{\mu_0}(\underline{E} \times \underline{B}) = (\underline{E} \times \underline{H})$

$V m^{-1} \cdot A m^{-1} = W m^{-2} = kg m s^{-2} \cdot m s^{-1} \cdot m^{-2} = \cancel{kg s^{-3}}\ kg\ s^{-3}$

~~Energy per unit time per unit area.~~

~~$kg\ m^2 s^{-2} \cdot s\ m^{-2}$~~

b) The poynting vector describes the energy flow ~~&~~ out of ~~E~~EM fields due to EM radiative emission. This applies for B + H so $\hat{\underline{\theta}} \times \hat{\underline{\phi}} = \hat{r}$ and the radiation flows radially outwards. This means that energy is subsequently dissipated over time from a material holding electric and magnetic fields.

~~This captures~~ ↳ Represents the directional flux density, accounts no changing electric fields over time.

$\boxed{4a)}$ $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

Absence of free charges, $\frac{\partial E}{\partial t} = 0$ & $J = 0$ (no free currents).

$\oint B \cdot dl = 0$

$B_{\parallel 1} - B_{\parallel 2} = 0$

$B_{\parallel 1} = B_{\parallel 2}$

Consider EM wave through material with no free charges

$\nabla \cdot D = \rho_f = 0$ in absence of free charges

so $\int_V \nabla \cdot D\ dv$

$= \int_S D\ ds \cdot n\ ds$ $D_{\perp 1} = D_{\perp 2}$

~~$\oint S0 = 0$~~

$-D_{2z} = \rho_f$

B is conserved at all interfaces.

5a) $\underline{B} = \underline{\nabla} \times \underline{A}$

b) $\nabla \cdot B = 0$

$\nabla \cdot (\nabla \times A) = 0$

$\therefore \nabla \cdot A = 0$

B so no magnetic monopoles can exist in nature

c) $\nabla \times B = \mu_0 J$

$\nabla \times \nabla \times A = \mu_0 J$

$\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J$

$\nabla \cdot A = 0$

$-\nabla^2 A = \mu_0 J$

$\nabla^2 A = -\mu_0 J$

6a) Polarisation of an electric field occurs when an electric field that oscillates in only one plane or a few planes and alternates between the two.

6/7

↳ there is a phase difference between Ex and Ey components, where they propagate independently (amplitude and phase,

$$E(r,t) = e^{i(kz-\omega t)}\left(\bar{E}_{0x}\underline{i} + \bar{E}_{0y}e^{i\delta}\underline{j}\right)$$

where δ is the phase difference

With different types of polarisation

i) Circular: $\delta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

Elliptical: $\delta \neq 0$ and $\bar{E}_{0x} \neq \bar{E}_{0y}$

Linear: $\delta = 0$ or π

7a) Retarded time is defined as $t' = t - r/c$ and recognises that it takes time $\frac{r}{c}$ to for a signal at r and time t to reach an observer at point r' and later time t'.

This implies the signal has a finite speed and takes time to propagate so the signal does not ~~convey~~ ~~instantaneously~~ instantaneously thus the source of radiation is at different time t than observed t'.

b) $t' = t - r/c$

$$\frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'} \cdot \frac{\partial t'}{\partial t} = \frac{\partial f(t')}{\partial t'} \cdot 1 = \frac{\partial f(t')}{\partial t'}$$

$$\frac{\partial f(t')}{\partial r} = \frac{\partial f(t')}{\partial t'} \cdot \frac{\partial t'}{\partial r} = \frac{\partial f(t')}{\partial t'} \cdot \frac{-1}{c} = \frac{-1}{c}\frac{\partial f(t')}{\partial t'}$$

c) $M(t) = M_0 \cos(\omega t)\,\hat{\underline{i}}_z$

$V(\underline{r}, t) = 0$

$$A(\underline{r}, t) = \frac{-\mu_0 M_0 \omega}{4\pi c}\frac{\sin\theta}{r}\sin\omega t'\,\underline{i}_\phi$$

$$\underline{E} = -\nabla V - \frac{\partial A}{\partial t}$$

$$\nabla V = 0$$

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial t'} = \frac{-\mu_0 M_0 \omega}{4\pi c}\frac{\sin\theta}{r}\frac{\partial}{\partial t'}\sin(\omega t')\,\hat{\underline{i}}_\phi$$

$$= \frac{-\mu_0 M_0 \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos(\omega t')\,\hat{\underline{i}}_\phi$$

$$\therefore \underline{E} = \frac{\mu_0 M_0 \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos(\omega t')\,\hat{\underline{i}}_\phi$$

$$\underline{B} = \nabla \times \underline{A} = \frac{\partial A}{\partial r}\,\hat{i}_r \times \hat{\underline{i}}_\phi = \frac{-1}{c}\frac{\partial A}{\partial t'}\,\hat{i}_\theta = \frac{-1}{c}\cdot\frac{\mu_0 M_0 \omega^2}{4\pi c}\frac{\sin\theta}{r}\frac{\partial}{\partial t'}\sin(\omega t)\,\hat{\underline{i}}_\theta$$

$$= -\frac{\mu_0 M_0 \omega^2}{4\pi c^2}\frac{\sin\theta}{r}\cos(\omega t')\,\hat{\underline{i}}_\theta$$

$$= \frac{1}{\mu_0}(E \times B)$$

$$= \frac{1}{\mu_0}\left(\frac{\mu_0 m_0 \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos\omega t'\right)\left(-\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\frac{\sin\theta}{r}\cos(\omega t')\right)(\hat{i}_\phi \times \hat{i}_\theta)$$

$$= -\frac{\mu_0 m_0^2 \omega^4}{16\pi^2 c^3}\frac{\sin^2\theta}{r^2}\cos^2(\omega t')\,\hat{i}_r$$

Yes, there is radiation from this system in the $\hat{i}_r$ direction (radially)

e) rotation diple: $i\hat{z} \times \hat{r}$

magnetic diple: i, r?

$$P_{loop} = \frac{1}{\mu_0}\left(\frac{\mu_0 I_0 d\omega \sin\theta}{4\pi\ r}\cos\omega t'\right)\left(\frac{\mu_0 I_0 d\omega}{4\pi c}\frac{\sin\theta}{r}\cos\omega t'\right)$$

$$= \frac{\mu_0 I_0^2 (d\omega)^2 \sin^2\theta}{16\pi^2 c\ r^2}\cos^2(\omega t')$$

$$\frac{P_{loop}}{P_{rotation}} = \frac{\mu_0 I_0^2 (d\omega)^2 \sin^2\theta}{16\pi^2 c\, r^2}\cos^2(\omega t')\cdot\frac{16\pi^2 c^2 r^2}{\mu_0 m_0^2 \omega^4 \sin^2\theta \cos^2(\omega t')}$$

$$= \frac{I_0^2 (d\omega)^2 c^2}{m_0^2 \omega^4}$$

$$\boxed{I_0 = q_0 \omega}$$

$$\frac{I_0^2 (d\omega)^2}{\omega^4} = \frac{q_0^2 (d\omega)^2}{\omega^2} = q_0^2 \frac{d^2 \omega^2}{\omega^2}$$

$$\boxed{p_0 = q_0 d}$$

$$\therefore = p_0^2$$

$$\Rightarrow \frac{p_0^2 c^2}{m_0^2}$$

iii) $d = \pi a$

$$\frac{P_{loop}}{P_{electric}} = \frac{M_0^2}{P_0^2 c^2}$$

$$M_0 = I_0 A = \frac{I_0 \cdot \cancel{8}\pi a^2}{P_0^2 c^2} = \frac{I \cdot \cancel{4\pi}\left(\frac{d}{\pi}\right)^2}{\cancel{2}3 d^2 c^2} = \frac{I_0 d^2}{\pi q_0^2 d^2 c^2} = \frac{I_0}{\pi q_0^2 c^2}$$

$$= \frac{q_0 \omega}{\pi q_0^2 c^2} = \frac{\omega}{\pi q_0 c^2} = \frac{2\cancel{\pi} f}{\cancel{\pi} q_0 c^2} = \frac{2f}{q_0 c^2}$$

This tells us the relative magnitudes is proportional to the frequency.

5) spec. Rel. +

9a)
- The first integral on the right hand side tells us the ~~energy~~ Rate of change ~~energy dissipation~~ of stored ~~the~~ energy in EM fields with respect to time
- The second integral tells us the energy dissipation due to EM & radiation ~~leaving the medium~~
- The poynting vector is the rate of change of energy at a point. It is the energy ~~to~~ around the current area leaving a point.

b) $\cancel{\frac{k}{\omega} = \frac{c}{}}$ $\quad \cancel{\frac{\omega}{k} = c = \frac{\Omega}{\omega}}$ $\quad \omega = v k$

$\cancel{k}$ $$\frac{\omega}{k} = \frac{c}{n} = \frac{c}{\sqrt{\epsilon_r}}$$

c) $\cancel{E}$, $\underline{E} = E_0 e^{i(kr - \omega t)}$

$\cancel{\nabla \times E =}$ $\quad \nabla \times E = -\frac{\partial B}{\partial t}$

$$i\underline{k} \times \underline{E} = +_{\!\!/} i\omega \underline{B}$$

$$\cancel{i}\underline{k} \times \underline{E} = \cancel{\$}i\omega \mu_0 \underline{H}$$

$$\cancel{z}\,\frac{|\underline{k}|\,\hat{k} \times \underline{E}}{\omega \mu_0} = \underline{H}$$

$\cancel{H}$ $$\frac{|\underline{k}|}{\omega} = \frac{n}{c}$$

$$\underline{H} = \frac{n}{c\mu_0}\,\hat{k} \times \underline{E}$$

$\underline{E \times H}$

$$= \underline{E} \times \frac{n}{c\mu_0} \hat{\underline{k}} \times \underline{E} = \frac{n}{c\mu_0} \underline{E}_0^2$$

$$= \frac{1}{\mu_0} \frac{n}{c} E_0^2$$

e) $\oint B \cdot dl = \mu_0 I N$

$B \cdot L = \mu_0 I N$

$B = \frac{\mu_0 I N}{L} = \mu_0 I n$ — Assume infinitely long solenoid

f) $B = \mu_0 \mu_r H$

$\oint H dl = I N$

$H \cdot L = I N$

$\frac{B}{\mu_0 \mu_r} = \frac{IN}{L}$

$B = I n \mu_0 \mu_r$

$= I n \mu$

g) $B = I_0 n \mu \cos(\omega t)$

$\nabla \times E = -\frac{\partial B}{\partial t}$

$\nabla \times E = I_0 n \mu \omega \sin(\omega t)$

$E = \frac{I_0 n \mu \omega \sin(\omega t)}{2} \int dr = \frac{\omega \mu n I_0 r}{2} \sin(\omega t) \hat{\underline{i}}_\phi$

Direction changes as we take cross product.

$E_{\parallel}$ is continuous at the boundary.

$\therefore E_{outside} = E_{inside}(a)$

$= \frac{\omega \mu n I_0 a}{2} \sin(\omega t) \hat{i}_\phi$

h) $E = \frac{\omega \mu n I_0 a}{2} \sin(\omega t + \frac{\pi}{4}) \hat{\underline{i}}_\phi$

i) $U_m = \int B \cdot dH = \int B_0 \sin(\omega t + \frac{\pi}{4}) \cdot -H_0 \omega \sin(\omega t)\, dt$

$= -H_0 B_0 \omega \int \sin(\omega t + \frac{\pi}{4}) \sin(\omega t)\, dt$

with varying phase delay : $\frac{\pi}{4} \to \delta$

$= -H_0 B_0 \omega \int \sin(\omega t + \delta) \sin(\omega t)\, dt$

If $\delta = 0$, we have a constant energy density

If $\delta = \frac{\pi}{2}$, we have zero. This fluctuates.

10a) i) $\cdot k = \frac{2\pi}{\lambda}$; is the wavenumber and is the number of waves per unit length

$\cdot \omega = 2\pi f$ is the angular frequency and is the number of oscillations per unit time the wave makes

ii) $\underline{E}$ propagates in the direction of $\underline{k}$.

$E_0 \parallel$ to k

$$\nabla \cdot E = i\underline{k}\cdot\underline{E} = \frac{\rho}{\varepsilon_0}$$

If $\underline{k}$ & $\underline{E}$ are perpendicular, $\rho = 0$

iii) $$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = i\underline{k} \times \underline{E}$$

$$\frac{\partial B}{\partial t} = -i\omega\underline{B}$$

$$B \Rightarrow i\underline{k} \times \underline{E} = -(-i\omega\underline{B})$$

$$B(r) = \frac{k \times E}{\omega} = \frac{\underline{k} \times E_0}{\omega} e^{i(kr - \omega t)}$$

We assume the $\underline{B}$-field also propagates as a plane wave

b) i) $\omega = ck$

ii) $$\frac{\omega}{k} = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$\omega = \frac{ck}{\sqrt{\varepsilon_r \mu_r}}$$

$$v_s = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$ → The speed of light slows $\propto \frac{1}{\sqrt{\varepsilon_r \mu_r}}$ in a medium.

iii) . The plane wave velocity will increase to a speed greater than the speed of light (the phase velocity) ~~and~~ though the group velocity will decrease below the speed of light to compensate for this.

c) $n = \sqrt{\varepsilon_r \mu_r}$

ii) $$\varepsilon_r = 1 + \frac{\omega_a^2}{\omega_0^2 - \omega^2}$$

$$\omega \to 0, \ \varepsilon_r = 1 + \left(\frac{\omega_a}{\omega_0}\right)^2 \quad , \omega_a > \omega_0$$

. $\varepsilon_r > 1$, the EM wave ~~reflects~~ and travels through the material (propagates) and is essentially a travelling wave.

$= 1 + 0$

$\varepsilon_r = 1$

$n = 1$

There is no reflection, the wave propagates through entirely.

iii) $$\varepsilon_r = 1 + \frac{\omega_a^2(\omega_0^2-\omega^2) + i\gamma\omega\omega_a^2}{(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}$$

$\omega \approx \omega_0$

$$\varepsilon_r = 1 + \frac{i\cancel{\gamma}\cancel{\omega}\,\omega_a^2}{\cancel{\gamma}\omega^{\cancel{2}}} = 1 + \frac{i\omega_a^2}{\omega}$$

$$= 1 + \frac{i\omega_a^2}{\omega_0}$$

If $\omega_0 > \omega$, ~~$\varepsilon_r = 1 + \frac{i\omega_a^2}{\omega}$~~ ε_r has a lower value

$\omega_0 < \omega$, ε_r has a greater value.

iv) $$\hat{\sigma} = -i\omega\varepsilon_0\left(\frac{\omega_a^2(\omega_0^2-\omega^2) + i\gamma\omega\omega_a^2}{(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}\right)$$

$\omega_0 = 0$

$$= -i\omega\varepsilon_0\left(\frac{\omega_a^2(-\omega^2) + i\gamma\omega\omega_a^2}{(-\omega^2)^2 + \gamma^2\omega^2}\right)$$

$$= i\omega\varepsilon_0\left(\frac{i\omega_a^2\omega^2 + \gamma\omega\omega_a^2}{\omega^2(\omega^2+\gamma^2)}\right) = \varepsilon_0\left(\frac{i\omega_a^2\omega^{\cancel{3}} + \gamma\cancel{\omega^2}\omega_a^2}{\cancel{\omega^2}(\omega^2+\gamma^2)}\right)$$

$$= \varepsilon_0\left(\frac{i\omega_a^2 + \gamma\omega_a^2}{\omega^2+\gamma^2}\right) = \varepsilon_0\omega_a^2\left(\frac{i+\gamma}{\omega^2+\gamma^2}\right)$$

$\omega = 0$

$$\hat{\sigma} = \varepsilon_0\omega_a^2\left(\frac{i+\gamma}{\gamma^2}\right) \neq 0$$

$$\hat{\sigma}_{Re} = \frac{\varepsilon_0\omega_a^2\cancel{\gamma}}{\gamma^{\cancel{2}}} = \frac{\varepsilon_0\omega_a^2}{\gamma}$$

v) i) → Non zero ω_0, zero γ: Dielectric, strong attenuation of waves and no free current flows.

ii) → ~~Non~~ non zero γ, $\omega_0 = 0$: Metals, ~~non~~ there is a real component so the conducting and free current may flow.