PHAS3225: Solid State Physics

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Contents

1 General Mathematica

1.1 Simple Fundaments

The phase and group velocity of a wave is given by the dispersion relation $\omega \equiv \omega(k)$ by the following equations:

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{\partial \omega}{\partial k}$$

Bragg's Law is as follows:

$$2d \sin(\theta) = n\lambda$$

1.2 k-Space

1.3 Thermodynamics

Classical equipartition theorem is just:

$$C_v = \frac{Nq}{2} k_B T$$

Where N is the number of particles, and q is the number of degrees of freedom (terms in the Hamiltonian that are quadratic w.r.t. canonical coordinates). Also:

$$C_v = C_{\text{phonons}} + C_{\text{electrons}} + C_{\text{magnetic}}$$

For a non-metallic non-magnetic material:

$$C_v = C_{\text{phonons}} = \frac{\partial U}{\partial T}_V$$

Where U is the total energy. Sometimes it helps to use energy density and heat capacity density instead.

2 Lattice Stuff

A structure is made up of a lattice (an abstract set of points), onto which a basis (a collection of atoms in a certain shape) is superimposed at each point of the lattice.

2.1 Primitives and k-Space

A primitive cell is the smallest cell that can be tessellated to fill the structure, each cell always contains a single lattice point. It is not uniquely defined, but it has a unique volume:

$$V_p = |\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)|$$

Where the vectors $\mathbf{a}_i$ are the primitive lattice vectors. The lattice looks the same under translation by any integer combination of these vectors. We call these translations 'lattice vectors':

$$\mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3 \tag{1}$$

The Wigner-Seitz cell is the primitive cell constructed by taking the perpendicular bisecting planes of near neighbors. It plays an important role in the theory of crystal structures. In k-Space the Wigner-Seitz cell becomes the First Brouillin Zone.

We will only use a cubic lattice in this course, onto which we will typically impose one of three bases. Figure 1 shows the conventional cells for all three structures that these lattice/basis combinations can yield. These cells are characterized by the lattice constant a (length of each side). Since the primitive cell of a structure contains a single atom, the volume of these conventional cells is:

$$V_{sc} = V_p \quad V_{fcc} = 2V_p \quad V_{bcc} = 4V_p$$

The lattice vectors for fcc are:

$$\mathbf{a}_1 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}}) \quad \mathbf{a}_2 = \frac{a}{2}(\hat{\mathbf{y}} + \hat{\mathbf{z}}) \quad \mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{z}} + \hat{\mathbf{x}})$$

The lattice vectors for bcc are:

$$\mathbf{a}_1 = \frac{a}{2}\left(\hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}}\right) \quad \mathbf{a}_2 = \frac{a}{2}\left(-\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}\right) \quad \mathbf{a}_3 = \frac{a}{2}\left(\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}}\right)$$

The positions of the atoms w.r.t. to the origin should be clear.

We should know how to calculate packing fractions, and understand the difference between hexagonal close packing and fcc packing.
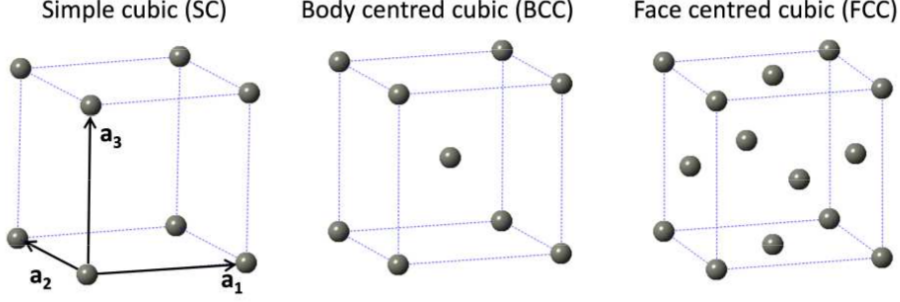

Figure 1: The conventional unit cell for each of the three important structures.

2.2 Miller Indices

There are different families of lattice planes

2.3 Reciprocal Space

For a given lattice (equation 1), we generate the reciprocal space by:

$$\mathbf{b}_1 = \frac{2\pi}{V_p}\left(\mathbf{a}_2 \times \mathbf{a}_3\right)$$

$$\mathbf{b}_2 = \frac{2\pi}{V_p}\left(\mathbf{a}_3 \times \mathbf{a}_1\right)$$

$$\mathbf{b}_3 = \frac{2\pi}{V_p}\left(\mathbf{a}_1 \times \mathbf{a}_2\right)$$

Such that we can define reciprocal lattice vectors like so:

$$\mathbf{G} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3$$

Such that:

$$\mathbf{G} \cdot \mathbf{R} = 2\pi n$$

Where n is an integer.

The reciprocal lattice of a Bravais lattice is Bravais. Taking the reciprocal of a Bravais lattice twice yields the original lattice. fcc and bcc are (scaled) reciprocals of each other.

When h, k, and l share no common factors then $\mathbf{G}$ is perpendicular to lattice planes (hkl) and the interplane spacing is given by:

$$d_{hkl} = \frac{2\pi}{|\mathbf{G}|}$$

When h, k, and l do share common factors, there will be a set of lattice planes with interplane spacing:

$$d = n\frac{2\pi}{|\mathbf{G}|} = nd_{hkl}$$

Where n is an integer. Note that for cubic lattices:

$$d_{hkl} = \frac{2\pi}{\frac{2\pi}{a}\left(h^2 + k^2 + l^2\right)^{1/2}} = \frac{a}{\left(h^2 + k^2 + l^2\right)^{1/2}}$$

This lets us rewrite Bragg's Law as:

$$2\frac{d}{n}\sin\theta = \lambda = 2d_{hkl}\sin\theta \tag{2}$$

2.4 Laue Condition

2.5 Structure Factor

The structure factor $\mathbf{S}_{hkl}$ is a vector describing the amplitude and phase of the resulting wave from diffraction between lattice planes. We will only consider it's amplitude S_{hkl}. It is defined as:

$$S_{hkl} = \sum_i f_i \exp\left[-i\mathbf{G} \cdot \mathbf{r}_i\right]$$

Where $\mathbf{r}_i$ is the position of the i-th atom w.r.t. the conventional unit cell origin and $\mathbf{G}$ is the reciprocal lattice vector defined by h, k, l:

$$G = \frac{2\pi}{a}(h, k, l)$$

The function f_i encapsulates the structure of the point, and for our purposes is usually just equal to the charge Z at each point (assuming point-like atoms).

Calculating S_{hkl} for an arbitrary set of h, k, l will usually yield some nice rule. For example, for:

- fcc with structure functions $f_i = f$ at each point:

$$S_{hkl} = \begin{cases} 4f & \text{If } h, \, k, \, l, \text{ are all even or all odd} \\ 0 & \text{otherwise} \end{cases}$$

- bcc with structure functions $f_i = f$ at each point:

$$S_{hkl} = \begin{cases} 2f & \text{If } h + k + l, \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

The missing peaks in diffraction curves occur when $S_{hkl} = 0$ for certain lattice planes [hkl]. These are called 'missing orders'.

2.5.1 Measuring Stuff

Because we know the lowest values of h, k, l that yield a peak, we can easily measure this first peak and use it as a reference point (as we may not necessarily know the value of the lattice constant a). Dividing two versions of equation 2 for different h, k, l values for cubic lattices yields[1] an equation independent of a:

$$\left(\frac{\sin(\theta_n)}{\sin(\theta_1)}\right)^2 = \frac{(h^2 + k^2 + l^2)_n}{(h^2 + k^2 + l^2)_1}$$

By measuring θ_n and θ_1 and since we can figure out the value of $(h^2 + k^2 + l^2)_1$ from our structure factor, we can figure out the other h, k, l values. Conceptually what's going on is that certain planes result in destructive interference between the reflected and diffracted waves.

Since the allowed values of h, k, l are different for fcc and bcc, one could search for a value of $\frac{\sin(\theta_n)}{\sin(\theta_1)}$ that one has but the other doesn't, and thus experimentally determine what structure the sample has.

2.6 Brillouin Zone

Taking the Wigner-Seitz cell in the reciprocal space yields the first Brillouin Zone, which has some important stuff going on. Basically, cause the lattice is made up of a bunch of these identical zones crammed up next to each other, we can shift any behavior we find into or out of this zone - it encapsulates the entire physics of the lattice.

If the wavevector $\mathbf{k}$ terminates on the Brillouin Zone boundary (BZB), it diffracts.

For a cubic lattice, the BZB exists at $k = \pm\frac{\pi}{a}$.

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[1] After squaring.

3 Lattice Dynamics

The interatomic interactions can be modelled as springs (first order Taylor expansion). So basically we're gonna solve for 1-D chains of atoms which interact harmonically.

- **1-D Monatomic** All masses m and all spring constants C are the same. Hence, for the n-th particle in the chain:

$$m\ddot{u}_n = -C(u_{n-1} - u_n - a) - (-C(u_n - n_{n+1} - a))$$

Note that the interatomic spacing a disappears, and the final equation of motion is:

$$\ddot{u}_n = -\frac{C}{m}(u_{n-1} - 2u_n + u_{n+1})$$

Which has wavelike solutions:

$$u_n = u_0 \exp\left[i(k(na) - \omega t)\right]$$

Where na is the equilibrium position of the n-th atom. Note that - as expected - displacement of k by a reciprocal lattice vector G (in 1-D these are both scalars with $G = 2\pi/a$) does not change the system. For this reason a high-frequency phonon $k + G$, and a low-frequency phonon k are indistinguishable (the particles move the same way even though the underlying physics may be different). The 1st Brillouin Zone contains all unique solutions.

When a Bragg condition is satisfied, $k = \pm\frac{\pi}{a}$ lies on the Brillouin Zone Boundary, subbing this into the above solution yields a standing wave (neighboring atoms opposite to each other). Conceptually this can be thought of as the wave scattering backwards, interfering with itself, and creating standing wave behaviour. For this reason a wave cannot travel in a lattice if the Bragg condition is satisfied ($k = \pm\pi/2$).

- **1-D Diatomic** Alternating masses m and M with all spring constants C and interatomic spacings a the same has - for the n-th of each atom type - similar (though now coupled) equations of motion to the Monatomic solutions:

$$M\ddot{u}_n = C(v_n - 2u_n + v_{n-1})$$

$$m\ddot{v}_n = C(u_{n+1} - 2v_n + u_n)$$

Which have the same wavelike solutions as the monatomic case. Subbing these solutions back into the above equations yields two simultaneous equations in u_0 and v_0 which only have solutions if the inverse of this matrix exists:

$$\begin{bmatrix} 2C - \omega^2 M & -C\left(1 + e^{-ika}\right) \\ -C\left(1 + e^{-ika}\right) & 2C - \omega^2 m \end{bmatrix}$$

Two atoms per primitive cell gives us two branching solutions for each limiting case:

- The first limiting case is small k where $1 - \cos ka \approx \frac{1}{2}k^2a^2$ such that:
- The second limiting case is the BZB $k = \pm\frac{\pi}{2}$ where $1 - \cos ka = 2$ such that:

3.1 Drawing This Crap

You gotta know how to draw repeated, reduced, and extended zone schemes for graphical representations of the dispersion relation within the first Brillouin Zone. You should also know how to draw stuff for optical and acoustic mode OPTICAL AND ACOUSTIC STUFF.

4 Phonons and Whatnot

Phonons are pseudo-bosons that represent vibrational modes in a material. They are effectively 'fake particles' that let us do particle physics with waves.

4.1 Thermal Properties

4.1.1 Heat Capacity

First of all, a rough approximation of heat capacity/internal energy comes from equipartition. For small samples/low temperatures, this classical approximation fails spectacularly. Assuming Bose-Einstein statistics:

$$\langle n \rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

And that the interaction is approximately the QHO for small perturbations:

$$U = \left(\langle n \rangle + \frac{1}{2}\right)\hbar\omega$$

We get a value for the heat capacity:

$$C_v = \frac{\partial U}{\partial T}_V = k_B \left(\frac{\Theta}{T}\right)^2 \frac{e^{\Theta/T}}{\left(e^{\Theta/T} - 1\right)^2}$$

Where $\Theta = \hbar\omega/k_B$ is the 'characteristic temperature'.

- Einstein Solid: assume all phonons have the same frequency ω_E such that $\Theta = \hbar\omega_E/k_b$, sub this into above equation and we get a good approximation for temperatures Θ_E and above. Since we haven't considered low-momentum phonons (as they all have the same, relatively high, frequency) it fails for temperatures below Θ_E and doesn't decay as quickly as it needs to.

- Debye Solid: assume phonons occupy all frequencies up to a maximum one ω_D. The total energy of the solid is then:

$$U = \int_0^{\omega_D} g(\omega)\left(n(\omega) + \frac{1}{2}\right)\hbar\omega \mathrm{d}\omega$$

Where ω_D is defined such that the number of states is correct:

$$N = \int_0^{\omega_D} g(\omega)\mathrm{d}\omega$$

And where we retrieve $g(\omega)\mathrm{d}\omega$ from $g(k)\mathrm{d}k$ by assuming a linear dependence on k such that: $\omega = v_s k$. We get $g(k)\mathrm{d}k$ by considering a volume in k-space. For $\Delta k = 2\pi/L$, we have (in 3-D):

$$g(k)\mathrm{d}k = \underbrace{\frac{1}{\Delta k}^3}_{\text{Density of k-States}} \overbrace{4\pi k^2}^{\text{Area of spherical shell at } k} \mathrm{d}k$$

Subbing in correctly for $\omega = v_s k$ yields:

$$g(\omega) = \frac{V}{2\pi^2}\frac{\omega^2}{v_s^3}$$

Now that we have the density of states $g(\omega)$ in terms of ω, and ω_D in terms of N, we can sub these back into the equation for the energy:

$$U = \underbrace{N_m \int_0^{\omega_D} \frac{1}{2}\hbar\omega g(\omega)\mathrm{d}\omega}_{\text{Zero point energy } U_z} + N_m \int_0^{\omega_D} g(\omega)n(\omega)\hbar\omega \mathrm{d}\omega$$

Where $n(\omega)$ is just the Bose-Einstein statistic for occupancy number, and N_m is the number of acoustic modes. Crunching through the integrals eventually yields:

$$C_v = 9Nk_B \left(\frac{T}{\Theta_D}\right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x}{\left(e^x - 1\right)^2}\mathrm{d}x$$

Where $x = \hbar\omega/k_B T$ and $\Theta_D = \hbar\omega_D/k_B$. This yields equipartition in the high-temperature regime (where x gets small and we can approximate e^x with Taylor expansions), and the correct proportionality $C_v \propto 1/T^3$ in the low-temperature regime (where $x \to \infty$ and the integral becomes a standard integral).

4.2   Anharmonicity and Thermal Conductivity

Some interatomic bonds do not approximate well to the harmonic oscillator. We call materials that exhibit this behaviour 'anharmonic'. Anharmonic materials typically expend less energy expanding than contracting, and the acoustic modes are no longer normal/independent. Phonons scatter off each other which determine the thermal conductivity of the solid.

Phonon collisions obey conservation of momentum and energy:

$$\hbar \mathbf{k}_1 + \hbar \mathbf{k}_2 = \hbar \mathbf{k}_3$$

$$\hbar \omega_1 + \hbar \omega_2 = \hbar \omega_3$$

Phonon momentum is defined modulo a reciprocal lattice vector, so we can distinguish between two distinct types of scattering event:

- Normal (N) Events: If we put the collision at the origin, the resulting wavevector $\mathbf{k}_3$ is within the 1st Brillouin Zone Boundary.

- Umklapp (U) Events: If we put the collision at the origin, the resulting wavevector $\mathbf{k}_3$ is outside the 1st Brillouin Zone Boundary and must be brought back inside by translation by a reciprocal lattice vector.

Heat flow carried by phonons is unaffected by N events, but impeded by U events. Since U events occur at higher phonon momenta, they are more likely in high-temperature regimes. In general, thermal conductivity is defined as:

$$\kappa = \frac{1}{3} v l C_v$$

Where l is the mean free length, and v is the mean velocity of phonons. In general v is fixed, l is inversely proportional to the number of phonons, and C_v has different regimes depending on T/Θ_D:

- Low Temp: U events disappear entirely. l is governed by size and shape of crystal.

- Medium Temp: $N_{\mathrm{phonons}} \propto \exp\left[-\Theta_D/T\right]$, l is inversely proportional to it.

- High Temp: Phonon number is linearly proportional to internal energy $k_b T$, hence: $l \propto 1/T$.
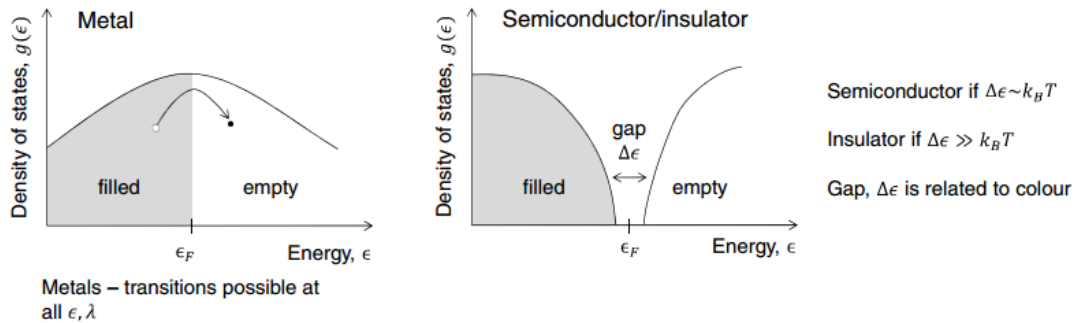
4.3   Free Electron Theory

Figure 2:   Important density of states diagrams showing the difference between metals and semiconductors/insulators.

The free electron theory of materials is pretty straightforward. We assume that the material is a collection of fixed positive charges with unbound 'free' electrons moving through it (no interactions between each other or the ion cores).

Using the same idea as the Debye solid, we can define a 'Fermi Energy' ϵ_F up to which all states are filled:

$$N = \int_0^{\epsilon_F} g(\epsilon) \mathrm{d}\epsilon$$

Where $\epsilon = (\hbar k)^2 / 2m$. Finding the Fermi Energy yields:

$$\epsilon_F = \frac{(\hbar k_F)^2}{2m}$$

With Fermi Wavevector:

$$k_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3}$$

This wavevector/energy defines a surface in k-Space such that:

$$g(\epsilon_F) = \frac{3}{2}\frac{N}{\epsilon_F}$$

THE REST OF THIS HALF OF THE COURSE

5   Magnets

Most of this is EMT. Know Dia/Paramagnets and all about linear magnetic susceptibility. The magnetic moment for a single-electron atom is:

$$\mu = -\mu_B \mathbf{l}$$

For many-electron atoms the magnetic moment is:

$$\mu = -g\mu_B \mathbf{s}$$

Where μ_B is the Bohr magneton, and $g \approx 2$. Derivation of magnetic moments of singly-charged/many electron atoms

5.1   Electromagnetic Momentum

The EM momentum is a conserved quantity defined as:

$$\mathbf{p} = m\mathbf{v} + q\mathbf{A}$$

For an electron $q = -e$.

- Classically:

$$\langle \mathbf{v} \rangle = 0$$

    Because at thermal equilibrium there is no mean current.

$$\langle \mathbf{p} \rangle = -e\langle \mathbf{A} \rangle$$

- Quantum Mechanically, when a small uniform field $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$ is applied (to first order in perturbation theory) the wavefunctions remain the same. This is called 'wavefunction rigidity'. Since momentum is a function of change in wavefunctions, this basically means that the EM momentum is zero:

$$\langle \mathbf{p} \rangle = 0 \qquad \Longrightarrow \qquad \langle \mathbf{v} \rangle = \frac{e}{m}\langle \mathbf{A} \rangle$$

    The kinetic energy of the field at this quantum level is:

$$\begin{aligned}
KE = \frac{\mathbf{p}^2}{2m} &= \frac{m^2\mathbf{v}^2}{2m} - \frac{e}{2m}m\mathbf{v}\cdot(\mathbf{B}\times\mathbf{r}) + \frac{e^2(\mathbf{B}\times\mathbf{r})^2}{8m} \\
&= \underbrace{\frac{m^2\mathbf{v}^2}{2m}}_{E_z} \underbrace{- \frac{e}{2m}m\mathbf{v}\cdot(\mathbf{B}\times\mathbf{r})}_{E_1} + \underbrace{\frac{e^2 B_z^2\left(x^2+y^2\right)}{8m}}_{E_2} \\
&= E_z + E_1 + E_2
\end{aligned}$$

    Where:

    − E_z is the zero-point field energy which is a feature of wavefunction rigidity.

– E_1 is an angular momentum energy term as follows:

$$E_1 = -\frac{e}{2m}m\mathbf{v} \cdot (\mathbf{B} \times \mathbf{r})$$

lecture 3 this makes no sense

– E_2 finish this

magnetic order stuff?!

5.2 Curie-Weiss Law

Derivation?! and MEAN FIELD THEORY The Curie-Weiss Law approximately describes the behaviour of the diamagnetic constant for a ferro/anti-ferro magnet w.r.t. temperature:

$$\chi = \frac{C}{T \pm \theta}$$

Where C is the Curie constant and θ is the Curie-Weiss constant, the positive branch is for ferromagnets, the negative one for anti-ferromagnets. The issue with this theory is that it gets some really important stuff wrong. We're gonna ignore it. summary of lecture 4 has some important looking stuff in it

6 Supermagnets

Supermagnets are perfect diamagnets that come in two distinct types. The main objectives of this section are to understand the behaviour of each type, and eventually understand why there are two types at all.

In general, superconductors have a (primary) critical temperature of about 1K.

6.1 Type I Supermagnets

Common in pure metals, they are perfect diamagnets and have $\chi_m = -1$. They fully exclude applied magnetic fields from their bulk, and the applied field only penetrates into a thin layer of the supermagnet's surface. This is called the 'Meissner Effect', and because of this there must be a thin layer of current of thickness λ in the surface of the supermagnet.

For a strong enough applied field, this supermagnetic state is destroyed (see figure 3.a) above a 'critical field' H_c or B_c.
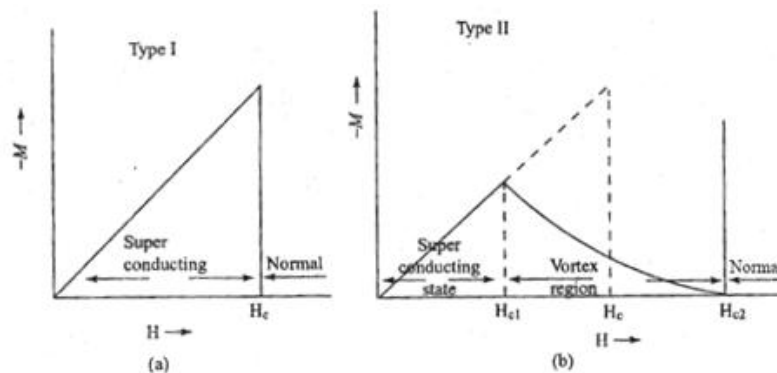

Figure 3: Magnetisation against applied field for **(a)** type I supermagnet **(b)** type II supermagnet

6.1.1 Spherical Supermagnet

spherical sample boundary stuff

6.1.2 Normal/Superconducting Boundary

In a rod which contains a transition between a normal and a superconducting state (see figure 4), Ampere's law yields:

$$I = 2\pi r H$$

Such that:

$$I_c = 2\pi r H_c$$

Where I_c is the critical current which destroys the supermagnetism in the rod. This is called the 'Silsbee hypothesis'.
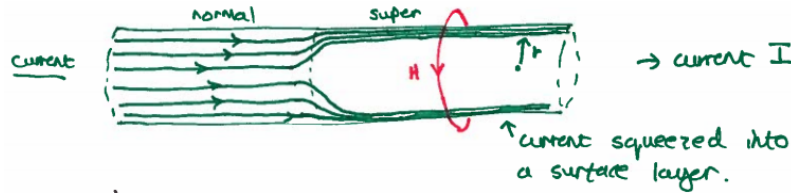

Figure 4: Boundary of Normal/Superconducting Rod

6.1.3 Temperature Dependence of Critical Field B_c

Experimentally, B_c is determined to behave approximately as:

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

Where T_c is the superconducting transition temperature (below which the material exhibits superconducting properties).
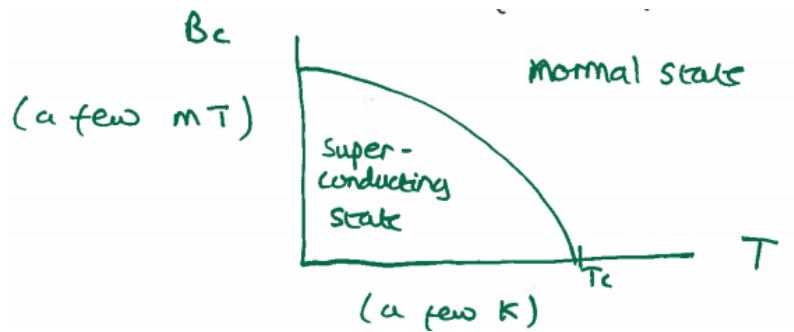

Figure 5: Graph of B_c against T

6.1.4 Measuring and Using λ

More How to get resistivity from λ
  How to measure λ from inductance
  One can use λ to compute the resistivity of a material:

$$\rho = \frac{RA}{L}$$

R is a property of the material and is measurable, L is a geometric property that is calculable/measurable, and since all the current exists in a thin sheet of width λ, you just have to find the length of that sheet l and set $A = \lambda l$.

6.1.5 Thermodynamics of Supermagnets

Replace p with B_0 and $\mathrm{d}V$ with $\mathrm{d}M$ such that the fundamental relation (internal energy) of a superconducting state is:
$$\mathrm{d}U = T\mathrm{d}S - B_0\mathrm{d}M$$

Where we're operating in terms of density (hence the use of M rather than m).

Finish and also learn this
Condensation Energy

6.2 Type II Supermagnets

More common in alloys, has two 'critical' fields H_{c_1} and H_{c_2}, though the area under the type II curve is roughly the same as the area under the type I curve. Between these two critical fields, the superconductor is in a mixed state where the supermagnetic behaviour isn't broken, but flux lines are not fully excluded from the bulk of the material.

Type II superconductors also exhibit some hysteresis around H_{c_1}. Pauli and G_n/Clogston limit

6.3 General Supermagnetic Stuff

6.3.1 London Equation

For a given current density:
$$\langle \mathbf{j} \rangle = -ne \langle \mathbf{v} \rangle$$

And from our expression (quantum-scale EM momentum for rigid wavefunctions) $\langle \mathbf{v} \rangle = (e/m)\langle \mathbf{A} \rangle$, after dropping the means, we retrieve the London Equations:

$$\mathbf{j} = -\frac{n_s e^2}{m}\mathbf{A}$$

$$\nabla \times \mathbf{j} = -\frac{n_s e^2}{m}\mathbf{B}$$

Where $\mathbf{A}$ satisfies the Coulomb gauge, and where n_s is the number of superconducting electrons per unit volume. This paints a picture of a 'two-fluid' material:

$$n_T = n_s + n_c$$

Where n_T is total number of free electrons, and n_c are those doing normal conduction. Below H_{c_1} all electrons are in n_s, above H_{c_2} all electrons are in n_c, and between them the electrons occupy both superconducting and normal conducting states.

6.3.2 Penetration Depth λ

MUST KNOW DERIVATION FOR THIS

Plugging the above London equation for $\mathbf{j}$ in terms of $\mathbf{B}$ into Maxwell's yields:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

$$\nabla \times \nabla \times \mathbf{B} = -\nabla^2 \mathbf{B} = \mu_0 \nabla \times \mathbf{j}$$

$$\nabla^2 \mathbf{B} = \frac{\mu_0 n_s e^2}{m}\mathbf{B} \tag{3}$$

Which gives us a description of how $\mathbf{B}$ behaves in space. By dimensional analysis we get a definition for λ:

$$\lambda = \sqrt{\frac{m}{n_s \mu_0 e^2}}$$

Which clearly becomes a measure of penetration depth for solutions to 1-D versions of equation 3. Solutions of the form:

$$B = B_0 e^{-x/\lambda}$$

In which, penetrations $x > \lambda$ yield exponentially diminishing field strengths.

λ is usually on the order of a hundred angstroms or so, and experimentally varies w.r.t. temperature (approximately) as:

$$\lambda(T) = \frac{\lambda(0)}{\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{1/2}}$$

Where T_c is the critical temperature of the superconductor. This is only valid for low temperatures, and diverges rapidly as $T \to T_c$.

6.3.3 Perfect superconductor lecture 6 p7 stuff

6.4 B-Cooper-S (BCS) Theory

write up coherence length and fermi energy gap stuff conceptually The coherence length is conceptually the rough distance between two parts of a cooper pair. It can also be thought of as the range beyond which the cooper pair dissolves and joins the great wavefunction in the sky[2].

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta(0)}$$

Where v_F is the Fermi velocity, and $\Delta(0)$ is the Fermi energy gap.

6.4.1 Macroscopic Superconductivity

If we treat a cooper pair in BCS theory as a single particle at position $\mathbf{r}$ that can be described by a single wavefunction $\psi(\mathbf{r})$, then (since there are half as many pairs as there are electrons):

$$|\psi(\mathbf{r})|^2 = \frac{n_s}{2} = \text{Cooper Pair Density} \implies \psi(\mathbf{r}) = \sqrt{\frac{n_s}{2}} e^{i\theta(\mathbf{r})}$$

Application of the momentum operator to this wavefunction yields an eigenvalue of $p = \hbar\nabla\theta(\mathbf{r})$, which can be subbed into the EM momentum equation (with $q = -2e$ for a Cooper pair) such that:

$$m\mathbf{v} = \hbar\nabla\theta\mathbf{r} + 2e\mathbf{A}$$

Finish this by setting v=0 for interior solution and solving for flux
All of lecture 7

7 Semiconductors

A semiconductor is a band solid (see figures 2 and 6) with a band gap on the order of $\tilde{k}_B T$. This means that electrons can jump the gap due to thermal fluctuations.

Electrons below the energy gap include 'valence electrons' and can be liberated to become 'conductance electrons' above the energy gap, these form the valence and conductance bands respectively. There are two types of semiconductor: *intrinsic* semiconductors which naturally have a band gap on the order of $k_B T$ and *extrinsic* semiconductors which must be doped to yield this effect.

Doping is the process of mixing the material with some other material so that the band gap changes/ shifts.

Note that intrinsic semiconductors have the same number of conductance electrons n as they have holes p (missing electrons) in the valence band such that $n = p$.

There are two main types of semiconductor: direct and indirect band gap semiconductors.
FINISH THIS

7.1 Summary

summary of two types and why they are what they are

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[2]It's where it's gonna go when it dies, when it dies and they lay it to rest it's gonna go to the wavefunction that's the best.
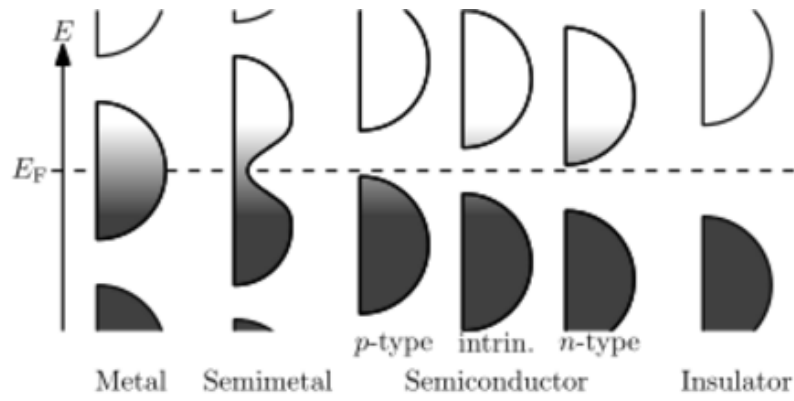
Figure 6: Graph of energy w.r.t. density of energy states for different types of material.

8 Semiconductors

9 STILL TO DO

Pauli Paramagnetism