EMT 2013

$M_s = -\mu_B N$ $\qquad$ $N = \frac{\rho N_A}{A}$

)(b) $M = \chi H$

$\underline{B} = \mu_0 (\underline{H} + \underline{M})$

$M = \frac{m}{V}$

magnetic moment, $\underline{m} = 9.6 \times 10^{-27}$ Am^2

$= \frac{9.6 \times 10^{-27}}{\frac{4}{3}\pi (22.5 \times 10^{3})^3}$

(3) (a) $\omega^2 = \omega_p^2 + k^2 c^2$

(i)

(5) (a) Expression for the physical electric field of a plane wave.

$\omega = 2\pi f$

$\underline{C}(\underline{r}, t) = D \exp i(\underline{k} \cdot \underline{r} - \omega t)$

$\underline{E} = E_0 \exp i\left(k \cdot \left(\frac{\hat{x} + \hat{y} + \hat{z}}{3}\right) - 2\pi f t\right) \frac{(\hat{x} - \hat{y})}{\sqrt{2}}$

$\boxed{\text{SECTION B}}$

(7) (a) $\underline{E} = E_0 \exp i(\underline{k} \cdot \underline{r} - \omega t)$

$\nabla \cdot \underline{C} = i\underline{k} \cdot \underline{C}$

$\nabla \times \underline{C} = i\underline{k} \times \underline{C}$

$\frac{\partial \underline{C}}{\partial t} = -i\omega \underline{C}$

Faraday's law:

$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$

$i\underline{k} \times \underline{E} = +i\omega \underline{B}$

$\underline{B} = \frac{\underline{k} \times \underline{E}}{\omega}$

MAXWELL EQUATIONS

$\nabla \cdot \underline{E} = \frac{\rho}{\epsilon_0}$

$\nabla \cdot \underline{B} = 0$

$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$

$\nabla \times \underline{B} = \mu_0 \underline{J} + \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t}$

(b) Poynting vector $\Rightarrow$ $N = \frac{1}{\mu_0} \underline{E} \times \underline{B}$

$= \frac{1}{\mu_0 \omega} \underline{E} \times \underline{k} \times \underline{E}$

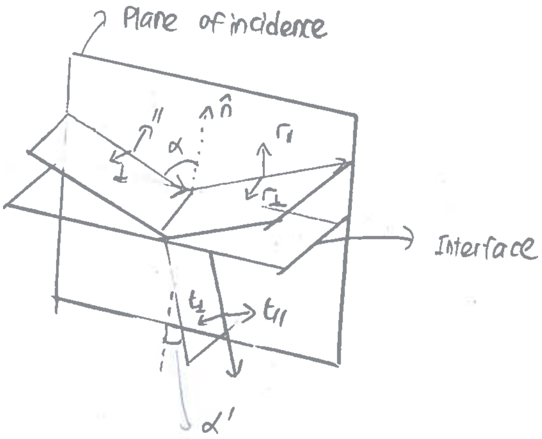
$= \frac{1}{\mu_0 \omega} |\underline{E}||\underline{k}||\underline{E}|$

$= \frac{k E_0^2}{\omega \mu_0} \Rightarrow \frac{n E_0^2}{c \mu_0} \Rightarrow$ ✓

$\boxed{\frac{\omega}{k} = \frac{c}{n}}$

$c = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$

(c)



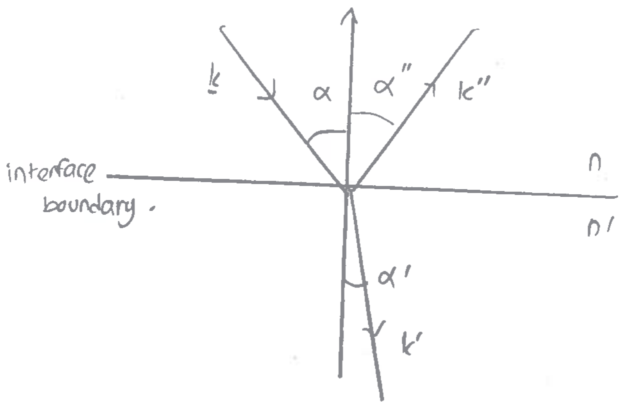
n: refractive index of the vacuum.

n': refractive index of the dielectric.

α: incident angle.

α': transmitted angle

$r_{\parallel}$: Ratio of the reflected wave's electric field amplitude to that of the incident wave. (Parallel component)



$r_{\perp}$: Ratio of the reflected wave's electric field amplitude to that of the incident wave (perpendicular component).

$t_{\parallel,\perp}$: ratio of the transmitted wave's electric field amplitude to that of the incident wave. (parallel and perpendicular components)

(I) Derive the fresnel relation for $t_{\parallel}$.

Brewster $\rightarrow$ $r_{\parallel} \rightarrow 0$.

$$r_{\parallel} = \frac{n'\cos\alpha - n\cos\alpha'}{n'\cos\alpha + n\cos\alpha'}$$

$\alpha_B = \tan^{-1}\left(\frac{n'}{n}\right)$

$\alpha \rightarrow \alpha_B$

refractive index of air
$n = 1$

$$r_{\perp} = \frac{n\cos\alpha - n'\cos\alpha'}{n\cos\alpha + n'\cos\alpha'}$$

$\alpha_B = 58^{0}$

refractive index of polycarbonate
$n' = 1.6$

$r_{\perp} = 0.23076...$

23%? Check.

(8) (a) retarded time : $\boxed{t' = t - \frac{r}{c}}$

A wave takes time $\frac{r}{c}$ to reach observer due to finite speed.

Therefore, an observer experiences a charge distribution at a time retarded by $\frac{r}{c}$.

(b) $t' = t - \frac{r}{c}$

$$\frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'} \qquad\qquad \frac{\partial f(t')}{\partial r} = -\frac{1}{c}\frac{\partial f(t')}{\partial t'}$$

$$\frac{\partial f(t')}{\partial t'}\frac{\partial t'}{\partial t} \qquad\qquad \frac{\partial f(t')}{\partial r} = \frac{\partial f(t')}{\partial t'}\frac{\partial t'}{\partial r}$$

using $t' = t - \frac{r}{c}$ $\qquad$ using $t' = t - \frac{r}{c}$

$$\frac{\partial t'}{\partial t} = 1 \qquad\qquad \frac{\partial t'}{\partial r} = -\frac{1}{c}$$

$$\therefore \cancel{\frac{\partial f(t')}{\partial t}} \quad \frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'} \qquad\qquad \therefore \frac{\partial f(t')}{\partial r} = -\frac{1}{c}\frac{\partial f(t')}{\partial t'}.$$

(c) magnetic dipole

dipole moment, $\underline{m}(t) = m_0 \cos \omega t \, \underline{\hat{z}}$

It has potentials which, at large distances, can be written in spherical polar coordinates a

$$V(\underline{r}, t') = 0$$

$$A(\underline{r}, t') = -\frac{\mu_0 m_0 \omega}{4\pi c}\frac{\sin\theta}{r}\sin\omega t' \, \hat{\phi}$$

$$\nabla \times E = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r\sin\theta F \end{vmatrix}$$

① $\boxed{\underline{B} = \nabla \times \underline{A}}$

② $\boxed{\underline{E} = -\nabla\phi - \frac{\partial \underline{A}}{\partial t}}$

$\underline{B} = \nabla \times \underline{A}$

$$\nabla \times \underline{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & -\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r} \sin\omega t' \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \left\{ \hat{r}\left(-\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\cos\theta}{r} \sin\omega t' \right) - r\hat{\theta}\left(+\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r^2} \sin\omega t' \right) \right\}$$

$$= \frac{1}{r^2 \sin\theta} \left\{ \hat{r}\left(-\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\cos\theta}{r} \sin\omega t' \right) + \hat{\theta}\left(-\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r} \sin\omega t' \right) \right\}$$

$$\nabla \times \quad \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & -\frac{\mu_0 m_0 \omega}{4\pi c} \sin^2\theta \sin\omega t' \end{vmatrix}$$

$$-r\hat{\theta}\left(\frac{\partial}{\partial r}\left(-\frac{\mu_0 m_0 \omega}{4\pi c} \sin^2\theta \sin\omega\left(t - \frac{r}{c}\right) \right)\right)$$

$$= \frac{1}{r^2 \sin\theta} \left\{ \hat{r}\left(\frac{\partial}{\partial \theta}\left(-\frac{\mu_0 m_0 \omega}{4\pi c} \sin^2\theta \sin\omega t' \right)\right) - r\hat{\theta}\left(\frac{\partial}{\partial r}\left(-\frac{\mu_0 m_0 \omega}{4\pi c} \sin^2\theta \sin\omega t' \right)\right) \right\}$$

$$= \frac{1}{r^2 \sin\theta} \left\{ -\frac{\mu_0 m_0 \omega}{4\pi c} 2\sin\theta\cos\theta \sin\omega t' \right\} \hat{r} \rightarrow -\frac{2\mu_0 m_0 \omega \cos\theta \sin\omega t'}{4\pi c r^2} \hat{r}$$

$$\underline{E} = -\nabla\phi - \frac{\partial \underline{A}}{\partial t} \qquad \phi = V(\underline{r}, t') = 0$$

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} \qquad \boxed{t' = t - \frac{r}{c}}$$

$$A(\underline{r}, t') = -\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r} \sin\omega t' \,\hat{\phi}$$

$$A(\underline{r}, t) = -\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r} \sin\omega\left(t - \frac{r}{c}\right) \hat{\phi}$$

$$-\frac{\partial \underline{A}}{\partial t} = +\frac{\mu_0 m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos\omega\left(t - \frac{r}{c}\right) \hat{\phi}$$

$$\therefore \underline{E} = \frac{\mu_0 m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos\omega t' \,\hat{\phi}.$$

$$\underline{B} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2} \frac{\sin\theta}{r} \cos\omega t' \,\hat{\theta}$$

$N = \frac{1}{\mu_0}(\underline{E} \times \underline{B})$

$\boxed{\hat{\underline{\theta}} \times \hat{\underline{\phi}} = \hat{\underline{r}}}$ → REMEMBER!

$\cancel{\underline{E} = \frac{\mu_0 I_0 d\omega}{4\pi} \frac{\sin\theta}{r} \cos\omega t' \hat{\theta}}$

$\cancel{\underline{B} = \mu_0 I_0 d\omega}$

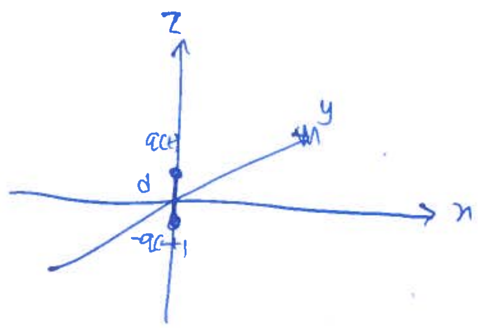
$$\underline{E} = \frac{\mu_0 m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos\omega t' \hat{\underline{\phi}}$$

$$\underline{B} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2} \frac{\sin\theta}{r} \cos\omega t' \hat{\underline{\theta}}.$$

$$N = +\frac{1}{\mu_0} \frac{\mu_0 m_0 \omega^2}{4\pi c} \frac{\mu_0 m_0 \omega^2}{4\pi c^2} \frac{\sin^2\theta}{r^2} \cos^2\omega t' \hat{\underline{r}}$$

$r \gg \frac{c}{\omega}$

(e)



$q(t) = q_0 \cos\omega t$

(I) $N_{Hertzian} = \frac{1}{\mu_0}(\underline{E} \times \underline{B})$

$$= \frac{1}{\mu_0} \frac{\mu_0 I_0 d\omega}{4\pi} \frac{\mu_0 I_0 d\omega}{4\pi c} \frac{\sin^2\theta}{r^2} \cos^2\omega t' \hat{\underline{r}}$$

∴ Same direction.

(II) $\cancel{\langle N \rangle = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}} E_0^2 \underline{\hat{z}}}$

$\langle \cos^2\omega t \rangle = \frac{1}{2}$

$$\boxed{\begin{aligned} I_0 &= -q_0\omega \\ P_0 &= q_0 d \end{aligned}}$$

$$N_{loop} = \frac{\mu_0 m_0^2 \omega^4}{(4\pi)^2 c^3} \frac{\sin^2\theta}{r^2} \cos^2\omega t' \hat{\underline{r}}$$

$$N_{Hertzian} = \frac{\mu_0 I_0^2 (d\omega)^2}{(4\pi)^2 c} \frac{\sin^2\theta}{r^2} \cos^2\omega t' \hat{\underline{r}}.$$

$$\frac{P_{loop}}{P_{Hertzian}} = \frac{m_0^2 \omega^{\cancel{4}2}}{c^2 I_0^2 (d^2\cancel{\omega^4})}$$

$$I_0 = - q_0 \omega$$

$$P_0 = q_0 d$$

$$= \frac{m_0^2}{P_0^2 c^2} \qquad \frac{m_0^2 \omega^2}{c^2 I_0^2 d^2}$$

$$\frac{P_{loop}}{P_{Hertzian}} = \frac{m_0^2}{P_0^2 c^2}$$

$$m_0 = IA$$

$$= I_0 \pi a^2$$

$$= \frac{m_0^2 \omega^2}{c^2 I_0^2 d^2}$$

$$= \frac{\cancel{I_0^2} \cancel{\pi^2} a^4 \omega^2}{c^2 \cancel{I_0^2} \cancel{\pi^2} a^2}$$

$$= \frac{a^2 \omega^2}{c^2}$$

(9) (a) (I) $\underline{J} = g\,\underline{E}$

(II) Faraday's law: $\nabla \times \underline{E} = -\frac{\partial B}{\partial t}$ Ampére - Maxwell law: $\nabla \times \underline{H} = J_f + \frac{\partial D}{\partial t}$

$\underline{B} = \mu \underline{H}$

$\underline{D} = \epsilon \underline{E}$

$\frac{1}{\mu} \nabla \times \underline{B} = J_f + \frac{\partial D}{\partial t}$

$\nabla \times \underline{B} = \mu \left(J_f + \frac{\partial D}{\partial t} \right)$

$$\nabla \times \underline{E} = \frac{-\partial \underline{B}}{\partial t}$$

$$\nabla \times \nabla \times \underline{E} = -\frac{\partial}{\partial t} \nabla \times \underline{B}$$

$$\nabla \times \nabla \times \underline{E} = -\mu \frac{\partial}{\partial t}\left(J_f + \frac{\partial D}{\partial t} \right)$$

$$\nabla(\nabla \cdot \underline{E}) - \nabla^2 E = \text{"}$$

$\nabla \cdot \underline{E} = 0$

$$-\nabla^2 E = -\mu \frac{\partial}{\partial t}\left(g\,\underline{E} + \epsilon \frac{\partial \underline{E}}{\partial t} \right)$$

$$\nabla^2 E = \mu \frac{\partial}{\partial t}\left(g\,\underline{E} + \epsilon \frac{\partial \underline{E}}{\partial t} \right)$$

$$\nabla^2 E - g\mu \frac{\partial \underline{E}}{\partial t} - \epsilon\mu \frac{\partial^2 E}{\partial t^2} = 0 .$$

(III) Dispersion relation:

$\underline{E}(r, t) = E_0 \exp i(\underline{k} \cdot r - \omega t)$

$$-k^2 \underline{E} - g\mu(-i\omega \underline{E}) - \epsilon\mu(-\omega \underline{E}) = 0 .$$

$\frac{\partial E}{\partial t} = -i\omega\,\underline{E}$

$$-k^2 + ig\mu\omega + \epsilon\mu\omega = 0$$

$$k^2 = \mu\epsilon\omega^2\left(1 + \frac{ig}{\epsilon\omega}\right)$$

$\frac{\partial^2 \underline{E}}{\partial t^2} = -\omega\,\underline{E}$

$\nabla^2 \underline{E} = -k^2 \underline{E}$

(IV) Good conductor

$g >> \epsilon\omega$

$$k^2 = \mu\epsilon\omega^2\left(1 + \frac{ig}{\epsilon\omega}\right)$$

$$\simeq \mu\epsilon\omega^2\left(\frac{ig}{\epsilon\omega}\right)$$

$$\simeq \mu\omega(ig)$$

$$k = +\sqrt{i\mu g\omega}\ .$$

$$\sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$$

$$\therefore\ k = \frac{1}{\sqrt{2}}(1+i)\sqrt{\mu g\omega}$$

$$k = \sqrt{\frac{\mu g\omega}{2}} + i\sqrt{\frac{\mu g\omega}{2}}\ .$$

(b) (I) Electric current flows mainly at the "skin" of the conductor.

A pulse travelling along a good conductor is attenuated going into the conductor.

(II)

$$E_0(d) = E_0(0)\, e^{-\frac{d}{\delta}}$$

$$\delta = \frac{1}{k_i} = \sqrt{\frac{2}{\mu g\omega}}$$

(III)

$$k^2 = \mu\epsilon\omega^2\left(1 + \frac{ig}{\epsilon\omega}\right)$$

$$k = \sqrt{\mu\epsilon}\,\omega\left(1 + \frac{ig}{\epsilon\omega}\right)^{1/2}$$

$$= \sqrt{\mu\epsilon}\,\omega\left(1 + \frac{1}{2}\left(\frac{ig}{\epsilon\omega}\right) + \cdots\right)$$

$$k = \sqrt{\mu\epsilon}\,\omega + \frac{1}{2}\sqrt{\mu\epsilon}\,\omega\left(\frac{ig}{\epsilon\omega}\right)$$

$$\sqrt{\mu\epsilon}\,\omega + i\frac{g}{2}\sqrt{\frac{\mu}{\epsilon}}$$

Good conductor $\rightarrow$ $g >> \epsilon\omega$

Poor conductor $\rightarrow$ $g << \epsilon\omega$.

$$\delta = \frac{1}{k_i} = \frac{2}{g}\sqrt{\frac{\epsilon}{\mu}}$$

(IV) $V_p = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}$

$\delta = 2.1\ \mu m$

$f = 10^{11}\ Hz.$

$\omega = 2\pi f = 2\pi \times 10^{11}$

$$k^2 = \mu\epsilon\omega^2\left(1 + \frac{ig}{\epsilon\omega}\right)$$

Non magnetic material. $\rightarrow$ $\left.\begin{array}{l}\mu = \mu_0\mu_r \\ \mu_r \approx 1\end{array}\right\} \rightarrow \therefore \boxed{\mu = \mu_0}$

Using ~~poor~~ good conductor approx.

$$\delta = \sqrt{\frac{2}{\mu g \omega}}$$

$$\delta = \sqrt{\frac{2}{\mu_0 g \omega}}$$

Get g :

Then use poor conduction approx

$$\delta = \frac{2}{g}\sqrt{\frac{\epsilon}{\mu}}$$

get ϵ

Then $\boxed{|V_p| = \frac{1}{\sqrt{\epsilon\mu}}}$

(10)

(a) current density 4-vector : $j_\mu = (J_1, J_2, J_3, ic\rho)$

EM potential 4-vector : $a_\mu = (a_1, a_2, a_3, \frac{i\phi}{c})$

c:⇒ speed of light in vacuum.

J_1, J_2, J_3 ⇒ the components of the current density vector, $\underline{J}$

ρ: free charge density.

A_1, A_2, A_3 ⇒ the components of the magnetic vector potential $\underline{A}$.

ϕ : Electric potential.

(b) (I) S' moving with speed v in the x_3 direction.

$x_1' = x_1$

$x_2' = x_2$

$x_3' = \gamma x_3 + \beta\gamma x_4$

$x_4' = \gamma x_4 - i\beta\gamma x_3$

$\beta = \frac{v}{c}$ $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

$x_1 = x$

$x_2 = y$

$x_3 = z$

$x_4 = ict$

NOT JUST t

BUT $\boxed{ict}$ ☆

(II) Inverse Lorentz transform in terms of β and γ.

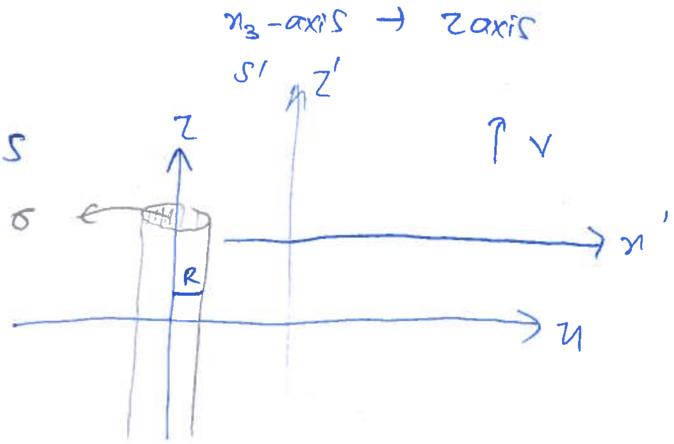
$x_1 = x_1'$

$x_2 = x_2'$

$x_3 = \gamma x_3' - \beta\gamma x_4'$

$x_4 = \gamma x_4' + i\beta\gamma x_3'$

(c)



long straight cylindrical insulating rod

External electric field in this reference frame is

$$\underline{E} = \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{\underline{r}}.$$

r: perpendicular distance from the rod's axis.

The electric potential relative to the surface of the rod is

$$\phi(r) = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)$$

R: radius of the rod.

$$j_\mu = (J_1, J_2, J_3, ic\rho) \qquad a_\mu = \left(a_1, a_2, a_3, \frac{i\phi}{c}\right)$$

The rod is at rest so $v=0$ $\quad J = v\rho \quad \rho = \frac{\lambda}{\sigma}$ → free charge density.

$$j_\mu = \left(0,0,0, ic\frac{\lambda}{\sigma}\right)$$

The potential is only outside the surface of the rod. ∴ Inside the rod,

$$a_\mu = \left(0,0,0,\frac{i}{c}\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)\right)$$

λ: $\frac{\text{charge}}{\text{length}}$.

σ: ~~[illegible]~~ length × length

ρ: $\frac{\text{charge}}{\text{length}}$; ⊥

(d) Obtain expressions for the cross sectional area of the rod σ' and the components of the 4-current density j'_μ and 4-potential a'_μ.

$x_1' = x_1$

$x_2' = x_2$

$x_3' = \gamma x_3 + \beta\gamma x_4$

$x_4' = \gamma x_4 - i\beta\gamma x_3$

similarly,

$j_1' = j_1$

$j_2' = j_2$

$j_3' = \gamma j_3 + \beta\gamma j_4$

$j_4' = \gamma j_4 - i\beta\gamma j_3$

Should have i?

$j_1' = j_2' = 0$

$j_3' = \gamma j_3 + i\beta\gamma j_4$

$= 0 + i\beta\gamma\left(ic\frac{\lambda}{\sigma}\right)$

$j_3' = -\cancel{i}\beta\gamma c\frac{\lambda}{\sigma}$

$j_4' = i\gamma c\frac{\lambda}{\sigma}$

$$\begin{pmatrix} j_1' \\ j_2' \\ j_3' \\ j_4' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\cancel{i}\beta\gamma c\frac{\lambda}{\sigma} \\ i\gamma c\frac{\lambda}{\sigma} \end{pmatrix}$$

$a_1' = a_1$

$a_2' = a_2$

$a_3' = \gamma a_3 + i\beta\gamma a_4$

$a_4' = \gamma a_4 - i\beta\gamma a_3$

$a_1' = a_2' = 0$

$a_3' = \gamma(0) + i\beta\gamma\left(\frac{i}{c}\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)\right)$

$= -\frac{\beta\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right)$

$a_4' = \gamma\left(\frac{i}{c}\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)\right) - i\beta\gamma(0)$

$= i\frac{\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right).$

$a_\mu' = \left(0, 0, -\frac{\beta\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right), \frac{i\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right)\right).$

(e) (I) $j_\mu = \left(0, 0, 0, ic\frac{\lambda}{\sigma}\right)$

$$a_\mu = \left(0, 0, 0, \frac{i}{c}\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)\right)$$

$$j'_\mu = \left(0, 0, -\beta\gamma c\frac{\lambda}{\sigma}, i\gamma c\frac{\lambda}{\sigma}\right).$$

$$a'_\mu = \left(0, 0, -\frac{\beta\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right), \frac{i\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right)\right)$$

Cylindrical polar coordinates

$\boxed{z = \hat{z}}$

considering only the spatial parts for j'_μ and a'_μ

$$\underset{\sim}{J}' = -\beta\gamma c\frac{\lambda}{\sigma}\hat{z}$$

$$\underset{\sim}{A}' = -\frac{\beta\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right)\hat{z}.$$

$$\underset{\sim}{J}' = -\beta\gamma c\frac{\lambda}{\sigma}\hat{z}$$

$\underset{\sim}{I}' = \underset{\sim}{J}'A$ $A = \sigma$

$$\underset{\sim}{I}' = -\frac{v}{c}\gamma c\frac{\lambda}{\sigma}\cdot\sigma\hat{z}$$

$$\boxed{\underset{\sim}{I}' = -v\gamma\lambda}$$

$$\boxed{\underset{\sim}{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$$

charge per unit length $\Rightarrow$

$$ic\rho = i\gamma c\rho'$$

$$\rho = \gamma\rho'$$

$$\frac{\lambda}{\sigma} = \gamma\frac{\lambda'}{\sigma}$$

$$\boxed{\lambda = \gamma\lambda'}$$

$$\underset{\sim}{E}' = \frac{\lambda'}{2\pi\epsilon_0 r}\hat{r}$$

$$\underset{\sim}{E}' = \frac{\gamma\lambda'}{2\pi\epsilon_0 r}\hat{r}.$$

$$\underset{\sim}{B}' = \nabla\times a'_z$$

$$= \nabla\times\left(-\frac{\beta\gamma\lambda}{2\pi\epsilon_0 c}\ln\left(\frac{R}{r}\right)\right)$$

$$B' = -\frac{\beta\lambda\gamma}{2\pi\epsilon_0 c}\frac{\hat{\phi}}{r}$$