EMT 2016

1) $\vec{E} = \frac{1}{2}(x^3\hat{x} + y^3\hat{y} + z^3\hat{z})$

a) For electrostatic feild

$$\nabla\cdot E = \frac{1}{2}(3x^2\hat{x} + 3y^2\hat{y} + 3z^2\hat{z})$$

$\boxed{\nabla\times E = 0}$ → conservative feild
↳ electro static feild are conservative

b) $\rho = \frac{1}{2}(3x^2\hat{x} + 3y^2\hat{y} + 3z^2\hat{z})\varepsilon_0 \rightarrow \underbrace{\frac{3}{2}(\hat{x}+\hat{y}+\hat{z})}_{\nabla\cdot E}$

$\int_v \nabla\cdot E\, dv = \frac{1}{\varepsilon_0}\int_v \rho\, dv$

$\Phi = \frac{3}{2}\int_0^\pi d\theta \int_0^{2\pi} d\phi \int_0^a r^4 \sin\theta\, dr$

$\frac{3}{2}\int_0^\pi d\theta\, 2\pi\left[\frac{r^5}{5}\right]_0^a \sin\theta$

$\frac{3\pi a^5}{5}\varepsilon_0\left[-\cos\theta\right]_0^\pi$

$\Phi = \frac{6\varepsilon_0\pi a^5}{5}$

2) a) An invarient quantity is one that does not change under transormation

i) e^- charge ii) Speed of light in vaccum iv) $E\cdot B$ } All invarient

b) $(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - c^2(dt)^2$

i) for spacelike $(ds)^2 > 0$ ✓

ii) for timelike $(ds)^2 < 0$ ✓

3) Gauss's law

$$\nabla\cdot E = \frac{\rho}{\varepsilon_0}$$

$\int \rho\, dv = Q$

Div theorem:

$\boxed{\int \nabla\cdot F\, dv = \oint_s F\cdot n\, da}$

Stokes theorem

$\boxed{\int \nabla\times F\cdot n\, da = \oint F\cdot dL}$

$\int \nabla\cdot E\, dv = \oint E\cdot n\, da$

$$\boxed{\oint E\cdot n\, da = \int \frac{\rho}{\varepsilon_0} dv = \frac{Q}{\varepsilon_0}}$$

$\nabla\times B = -\frac{\partial E}{\partial t}$ ✗ Faraday law is $\nabla\times E = -\frac{\partial B}{\partial t} \rightarrow \int_s \nabla\times E\cdot n\, da = -\frac{\partial}{\partial t}\left(\int B\cdot da\right)$

$\cancel{\int \nabla\cdot B\cdot n\, da = -\int \frac{\partial E}{\partial t}\cdot da}$

$\cancel{\oint F\cdot d \quad \oint B\cdot dL = -\int \frac{dE}{\partial t}\cdot da}$

$\oint_s E\cdot dl = -\frac{\partial}{\partial t}\int_s \underbrace{B\cdot da}_{\phi}$

$\oint_c E\cdot dL = -\frac{d\phi}{dt}$

4a) $t \cancel{\frac{t_r}{r}}$ $\boxed{t_r = t - \frac{r}{c}}$ ← Retarded time

Retarded time tells you the status of the source at an earlier time t_r

Retarded time is the time when the field began to propagate from its source. 'Retarded' is used in the sense of propagation delay.

b) $t' = t - r/c$

$\frac{\partial t'}{\partial t} = -\cancel{r/c}\ 1$

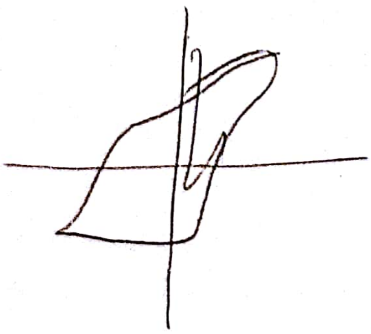
$\frac{\partial t'}{\partial t'} = -\cancel{r/c}\ 1$

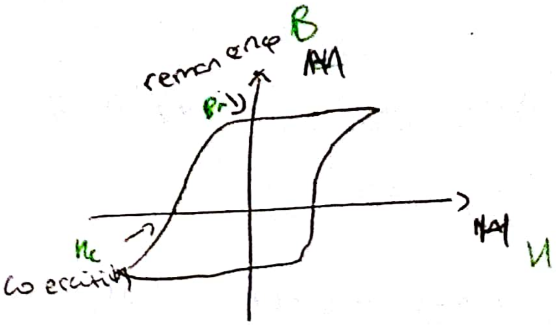
$\frac{\partial F(t')}{\partial r} = -\frac{1}{c}$

$\frac{\partial t'}{\partial t} = -\cancel{r/c}\ 1$

$\frac{1}{c}\frac{\partial F(t')}{\partial r} = \frac{\partial t'}{\partial t}$

5) a)





b) BC as it has the ~~lower~~ higher H_c and B_r values thus best describes a hard magnet

6) a) $r_{\parallel}$ → parallel component of wave reflected at boundary

$r_{\perp}$ → Perpendicular component of wave reflected at boundary

n → Refractive index

b) $\sin\theta_i = \sin\theta_r = n \sin\theta_t$

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$n = 1.2 + (4.1\times10^{-11})i$

$\omega = 9.4\times10^{14}$

7

i)

$\frac{k}{\omega} = \frac{n}{c} \rightarrow \boxed{v = \frac{c}{n}} \rightarrow \frac{\omega}{k} = \frac{c}{n}$

$k = \frac{n}{c}\omega$

$k = \frac{1.2 + (4.1\times10^{-11})i}{3\times10^{8}} \times (9.4\times10^{14}) = 3.76\times10^{6}\ (1.28\times10^{-4})i$

ii)

$E = E_0 e^{i}$

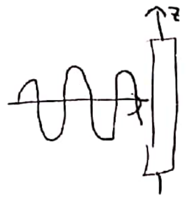
$E = E_0 e^{-i(kz - \omega t)}$

$E = E_0 e^{-i(1.28\times10^{-4})z \ - \ (9.4\times10^{14})t)}$

$E = E_0 e^{i\,3.76\times10}$

$E = E_0 e^{-(1.28\times10^{-4})z}\ e^{i(3.76\times10^{6}z - 9.4\times10^{14}t)}$

Sub in k and i's cancel out



iii)

$e^{-\lambda} \ E e^{-z/\delta} \qquad \delta = 1/k$

$E_0 e^{-\frac{1000}{1/4.1\times10^{-11}}} = 0.87E_0$ ✓

iv)

$V_p = \frac{\omega}{k} = \frac{9.4\times10^{14}}{3.76\times10^{6}} + \frac{9.4\times10^{14}}{1.28\times10^{-4}}i = $?

$(2.5\times10^{8} +$

$\frac{a}{b+ci} \cdot \frac{b-ci}{b-ci}$

$= \frac{ab - aci}{b^2 - c^2}$

b) i) LIH signifies the material is linear, Isotropic

Homogeneous. This means $M = \chi H$?

$D = \varepsilon_0 E$ $\quad B = \mu H$

ii) $\int B \cdot da = \mu_0 I \times N$

$\nabla \times B = \mu_0 J$

$\int_v \nabla \times B \cdot da = \int \mu_0 J \, dv$

$\nabla \times \int B \cdot da = N \int \mu_0 J \cdot da$

$= \int B \cdot da = N \mu_0 I$

$\oint = B \cdot 2\pi R = N \mu_0 I$

$B = \mu_0 H$

$H \mu_0 2\pi R = N \mu_0 I$

$H = \frac{NI}{2\pi r} \hat{\phi}$

iii) $B = \frac{N I \mu_0}{2\pi r} \hat{\phi}$

$B = \mu_0 (H + M)$

$M = \frac{1}{\mu_0} B - H$

$M = B - \mu H$

$M = \chi H$

$M = 0$?

iv) $U_{mag} = \frac{B^2}{2\mu_0}$ magnetic feild density $= \int \frac{B^2}{2\mu_0} dv$

$\int_0^{2\pi} d\phi \int_R^{R+a} r \, dr \left(\frac{N^2 I^2 \mu_0 (\omega)}{8\pi^2 r^2} \right) = \cancel{\int_0^{2\pi} d\phi \int_R^{R+a} \left(N^2 I^2 \right.}$

$\frac{N^2 I^2 \mu_0 a}{8\pi^2} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \int_R^{R+a} \frac{1}{r} dr = \frac{N^2 I^2 \mu_0 \omega a}{4\pi} \ln \left(\frac{R+a}{R} \right)$

$\sim 9.4\times10^{14}$ (2)

A) $\omega \to 0$

$$\frac{\mu_0 I_0 \delta L}{4\pi}\sin\theta\left[\frac{1}{r^2}\right]\underbrace{e^{ikr}}_{\text{small } r}\hat{\phi} \qquad e^{ik.r}=1$$

$$B(r)=\frac{\mu_0 I}{4\pi}\frac{\delta L\sin\theta\,\hat{\phi}}{r^2}=\frac{\mu_0 I}{4\pi}\frac{\delta L\times\hat{r}}{r^2}$$

B) Large r

$$\cancel{\frac{\mu_0 I}{4\pi}\frac{\delta L\times\hat{r}}{r^2}e^{ik.r}} \qquad \frac{\mu_0 I\delta L}{4\pi}\sin\theta\left[\frac{-i\omega}{rc}\right]e^{ik.r-\omega t}\hat{\phi}$$

$1/r^2$ term $\to 0$ at large r

consistent with radiation field as only includes a r^{-1} term

i) r then to be large such that $\frac{1}{r^2}\to 0$

$v=f\lambda \qquad c=f\lambda$

ii)

iii) Time averaged power flow $\Rightarrow$ <u>poyntings vector</u>

$$\bar{N}=\frac{1}{\mu_0}(E\times B)$$

$$E\times B=\left(\frac{\mu_0 I_0\delta L}{4\pi}\right)^2\sin^2\theta\, e^{i2k.r-\omega t}\left[\frac{-i\omega}{rc}\cdot-\frac{i\omega}{r}\right]$$

$$\bar{N}=\frac{1}{\mu_0}\left(\frac{\mu_0 I\delta L}{4\pi}\right)^2\sin^2\theta\,\underbrace{e^{2i(kr-\omega t)}}_{\text{take real part!}}\left(-\frac{\omega^2}{r^2c}\right)$$

$$\frac{1}{\mu_0}\left(\frac{\mu_0 I\delta L}{4\pi}\right)^2\sin^2\theta\cos(kr-\omega t)^2\left(-\frac{\omega^2}{r^2c}\right)$$

$$\begin{array}{cccc} & i & j & k \\ ~~\nabla\times E~~ = \\ E\times B = & \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ & E_r & E_\theta & E_\phi \\ & B_r & rB_\theta & r\sin\theta\, B_\phi \end{array}$$

$$E\times B=\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{\phi}\\ E_r & E_\theta & E_\phi\\ B_r & B_\theta & B_\phi\end{vmatrix}$$

② To find electric field, ~~First you to find~~ and find r_1 and r_2 ~~Now~~ ~~at~~ projection on z axis ~~Ee~~

$z = r\cos\phi \rightarrow 3\times10^{-3}$

For $r_1 \rightarrow z = 5\cos(60^\circ) = 5/2$

$r_2 \rightarrow z = \underbrace{20\cos\theta}_{\text{maximum value at } \cos\theta = 1} = 20$

$\frac{5/2}{20} = \frac{1}{8} \times (3\times10^{-3})$

$= 3.75\times10^{-4}\ E_0$

$\bar{N} = \frac{1}{2} \quad \bar{N} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\,(3.75\times10^{-4})^2$

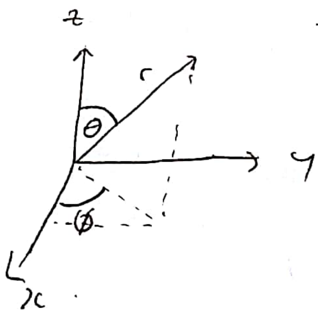
b) $\bar{N} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\,E_0^2\,\hat{k}$

Learn it

$$\boxed{\begin{aligned} x &= r\sin\phi\cos\theta \\ y &= r\sin\phi\sin\theta \\ z &= r\cos\phi \end{aligned}}$$

learn!

✗ don't need to ~~to~~ met



$r_1 = \left(15/4,\ \frac{5\sqrt{3}}{4},\ 5/2\right)$

$r = r$ dumbass!

$|r_1| = \sqrt{\left(\frac{15}{4}\right)^2 + \left(\frac{5\sqrt{3}}{4}\right)^2 + \left(\frac{5}{2}\right)^2} = 5\,m$

at this point $\bar{N} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\,(3\times10^{-3})^2$

at 20 m away ~~divide~~ by 4 ? $E = 5/20\ E_0$

$N = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\left(\frac{3\times10^{-3}}{4}\right)^2$

$N =$

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(2)

B) $z = 0.5$

h)

9a) i) plasma is ~~slow moving~~ positive ions surr by at cloud of ~~ele~~ electron such that the overall charge is neutral

ii) $\omega_p = \sqrt{\frac{N_e e^2}{m_e \varepsilon_0}}$

$\omega_p = \sqrt{\frac{10^{12} \times (1.6 \times 10^{-19})^2}{(9.11 \times 10^{-31})(8.85 \times 10^{-12})}}$

iii) $k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$

$k^2 = \frac{\omega^2}{c^2}(1 - \omega_p^2)$

A) If $\omega << \omega_p$

→ electromagnetic wave is completely reflected off plasma →

↳ k imaginary wave decays on enter plasma

B) If $\omega >> \omega_p$ $\quad k^2 = \frac{\omega^2}{c^2}($

↳ sheet becomes dissipationless and does not dissipate the wave at all → no decay

$\omega_p = 2\pi \times 10^8$

$\frac{e^{kz}}{1}$

iv)

A) $E_0 e^{ikz}$

$\left(\frac{e^{i(kz - \omega t)}}{e^{i(kz - \omega t)}}\right)$

$\frac{e^{\cancel{ikz}-i\omega t}}{\cancel{e^{ikz}}\, e^{-i\omega t}} = \cancel{e^{ikz}}\, e$

?

$e^{kz} = e^{-\left(\frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)\right)^{1/2}}$

$= e^{-\frac{\omega\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}}{c}}$

$= e$

9a) :)

B) $z = 0.5$ $~~0.99\omega_p \sqrt{1-\frac{\omega^2}{0.99^2\omega_p^2}}~~$

$e^{-\cancel{\omega_p}}$

$$e^{-\omega_p \frac{\sqrt{1-\frac{(0.99)^2\omega_p^2}{\omega_p^2}}}{c} \times 0.5}$$

$\omega = 0.99\omega_p$

$$e^{-\omega_p \frac{\sqrt{1-(0.99)^2}}{2c}}$$

at $\frac{\omega}{\omega_p} << 1$

$$e^{-\omega_p \frac{\sqrt{1-\cancel{0}^2}}{2c}}$$

$$\frac{e^{-\frac{\sqrt{\omega_p \times 0.199}}{2c}}}{e^{-\frac{\omega_p}{2c}}} = e^{\sqrt{0.0199}} = 1.15$$

5) i) Skin depth is the distance at which a wave will propogate through (in a material) ~~without~~ till in amplitude is reduced by $\frac{1}{e}$ e^{-1}

ii) $E = E_0 e^{i(k \cdot r - \omega t)}$

$$\frac{k^2}{\omega^2} = \mu_0 \mu_r \varepsilon_0 \varepsilon_r \left(1 + i \frac{g}{\varepsilon_0 \varepsilon_r \omega}\right)$$

good conductor

$\sigma \to g >> 1$

$\delta = \frac{1}{k}$

$$\frac{k^2}{\omega^2} = \left(i \frac{g \mu_0 \mu_r}{\omega}\right)$$

$\Rightarrow \sqrt{i} = \frac{1+i}{\sqrt{2}}$

$$k^2 = i \frac{g \mu_0 \mu_r \omega}{\cancel{\omega}}$$

$$k = \sqrt{g \mu_0 \mu_r \omega} \times \frac{1+i}{\sqrt{2}}$$

$$= \sqrt{\frac{g \mu_0 \mu_r \omega}{2}} \leftarrow \text{take imaginary part}$$

iii) $v = \frac{\omega}{k} = \frac{8 \times 10^{+8}}{(3 \times 10^{-6})^{-1}}$

$= 2400\,000\,m?$

$P = |\frac{}{r^2}|$

$r_o = 8\,cm$

4) Band surface charge

$\rho_b = -\nabla \cdot P$ ×

Surface charge

$\boxed{\sigma_b = P \cdot n}$

Band volume charge

$\boxed{\rho_b = -\nabla \cdot P}$

i) inner surface charge

Inner:
$\sigma_i = -\frac{A}{r_i^2}$

$\cancel{2\pi r_i L}\left(-\frac{A}{r_i}\right) =$

$q_{inner} = \underline{2\pi r L}\left(-\frac{A}{r_i^2}\right) = (-)1.68 \times 10^{-8}\,C$
?

outer surface charge

$\sigma = \frac{A}{r_o^2}$

$q_{out} = 2\pi r l\left(\frac{A}{r_i^2}\right) = 6.29 \times 10^{-9}\,C$

ii) $\cancel{\rho_b = -}\nabla \cdot P$ No Net charge thus

$q_{inner} + q_{outer} + q_{volume} = 0$

$q_{volume} = 1.052 \times 10^{-8}\,C$

iii) $D = \varepsilon E + P$

$E = 0$?

P at $r = 4.5$ cm

$D = \left(\frac{4 \times 10^{-10}}{0.0045}\right) = 1.98 \times 10^{-7}\,C/m^2$

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(9)

b) $M(r) = 500{,}000(a-r)\hat{z}$ A/M

i) Surface magnetization current density : $\boxed{J_M = M\times n}$

or

volume magnetization current density : $\boxed{J_M = \nabla \times M}$

Surface current density

$500{,}000(a-r)\hat{z}\times\hat{r} = 0$

volume magnetization current density

$$\nabla \times M = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ M_r & rM_\phi & M_z \end{vmatrix}$$

$$= \frac{1}{r}\left(-\frac{\partial}{\partial r}\left(500{,}000(a-r)\right)\hat{z}\right.$$

$$= -\frac{500000}{a}\hat{\phi} = -25\times10^7 \text{ A/M}$$

ii) $B = \mu_0 M$

$\cancel{B = \mu_0 J}$ $\cancel{\int B \cdot dl} = \int \mu_0 J$

$\oint B \cdot dl = \mu_0 I$

B inside magnet

$\boxed{B = \mu_0 J}$

B outside = 0