

## Summary of Notation

Symbol	Meaning
$\exists$	there exists
$\forall$	for all
$A \cup B$	the union of sets $A$ and $B$
$A \cap B$	the intersection of sets $A$ and $B$
$\bigcup_i A_i$	the union of the sets $A_i$
$A \subseteq B$	$A$ is a subset of $B$
$a \in A$	element $a$ is a member of set $A$
$\neg$	logical not
$\vee$	logical or
$\wedge$	logical and
$\sum_{i=1}^n x^{(i)}$	$x^{(1)} + x^{(2)} + \dots + x^{(n)}$
$\prod_{i=1}^n x^{(i)}$	$x^{(1)} \times x^{(2)} \times \dots \times x^{(n)}$
$\mathbb{N}$	the set of natural numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	the set of $n$ -dimensional vectors over $\mathbb{R}$
$\mathbb{R}^{n \times m}$	the set of $(n \times m)$ -dimensional matrices over $\mathbb{R}$

Symbol	Meaning
$\{x^{(i)}   \text{Condition}\}_{i=1}^n$	a set of objects $x^{(1)}, x^{(2)}, \dots, x^{(n)}$ , defined such that <b>Condition</b> is fulfilled
$f : \mathbb{X} \rightarrow \mathbb{V}$	a function $f$ which maps from the space $\mathbb{X}$ to the space $\mathbb{V}$
$f : x \mapsto f(x)$	a function $f$ which maps an input $x$ to an output $f(x)$
$\mathbb{I}[\text{Condition}]$	indicator function (equals 1 if <b>Condition</b> is true, and 0 otherwise)
$\delta_{ij}$	Kronecker delta (equals 1 if $i = j$ , and 0 otherwise)
$\mathbf{x} = \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$	column vector comprising elements $x_1, x_2, \dots, x_n$
$\mathbf{x}^T = [x_1, x_2, \dots, x_n]$	row vector comprising elements $x_1, x_2, \dots, x_n$
$x_i$ or $[\mathbf{x}]_i$	the $i$ th element of a vector $\mathbf{x}$
$\langle \mathbf{x}, \mathbf{v} \rangle$ or $\mathbf{x} \cdot \mathbf{v}$ or $\mathbf{x}^T \mathbf{v}$	inner or scalar or dot product: $\sum_{i=1}^n x_i v_i$
$\ \mathbf{x}\ _2$ or $\ \mathbf{x}\ $	$\ell_2$ -norm or two-norm of $\mathbf{x}$ : $\sqrt{\sum_{i=1}^n x_i^2}$
$\ \mathbf{x}\ _1$ or $ \mathbf{x} $	$\ell_1$ -norm or one-norm of $\mathbf{x}$ : $\sum_{i=1}^n  x_i $
$\mathbf{A} = \underline{A} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nm} \end{bmatrix}$	matrix comprising elements $\{A_{ij}\}_{i=1, j=1}^{n, m}$
$A_{ij}$ or $[\mathbf{A}]_{ij}$	the $i, j$ th element of a matrix $\mathbf{A}$
$\mathbf{I}_n$	the $n$ by $n$ identity matrix

Symbol	Meaning
$\mathbf{A}^{-1}$	the inverse matrix of matrix $\mathbf{A}$
$\mathbf{A}^T$	the transpose of matrix $\mathbf{A}$
$\det(\mathbf{A})$ or $ \mathbf{A} $	the determinant of matrix $\mathbf{A}$
$\text{tr}(\mathbf{A})$	the trace of matrix $\mathbf{A}$
$\text{diag}(\mathbf{x})$	the diagonal matrix with diagonal elements corresponding to the elements of the vector $\mathbf{x}$
$\mathbf{A} \succ 0$	the matrix $\mathbf{A}$ is positive definite
$\mathbf{A} \succeq 0$	the matrix $\mathbf{A}$ is positive semi-definite
$\mathbf{A} \prec 0$	the matrix $\mathbf{A}$ is negative definite
$\mathbf{A} \preceq 0$	the matrix $\mathbf{A}$ is negative semi-definite
$\frac{df(x)}{dx}$ or $f'(x)$	the derivative of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ at $x$
$\frac{d^2f(x)}{dx^2}$ or $f''(x)$	the second derivative of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ at $x$
$\frac{\partial f(\mathbf{x})}{\partial x_i}$	the partial derivative of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at $\mathbf{x}$ with respect to $x_i$
$\nabla_{\mathbf{x}}f(\mathbf{x}) = \left[ \frac{\partial f(\mathbf{x})}{\partial x_1}, \frac{\partial f(\mathbf{x})}{\partial x_2}, \dots, \frac{\partial f(\mathbf{x})}{\partial x_n} \right]^T$	the gradient of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at $\mathbf{x}$
$\nabla_{\mathbf{x}}^2f(\mathbf{x})$ or $\mathcal{H}(\mathbf{x})$	the Hessian of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at $\mathbf{x}$
$\int f(x)dx$	the integral of a function $f : \mathbb{R} \rightarrow \mathbb{R}$
$\min_x f(x)$	the minimal value of $f(x)$
$\max_x f(x)$	the maximal value of $f(x)$

Symbol	Meaning
$\operatorname{argmin}_x f(x)$	the set $\{x f(x) = \min_z f(z)\}$
$\operatorname{argmax}_x f(x)$	the set $\{x f(x) = \max_z f(z)\}$
$x \sim \mathcal{D}$	$x$ is the outcome of some random variable (say, $\mathcal{X}$ ) drawn from a probability distribution $\mathcal{D}$
$\mathcal{S} \sim \mathcal{D}^n$	$\mathcal{S} = \{x^{(i)}\}_{i=1}^n$ is a data set, the elements of which are the outcome of some random variable (say, $\mathcal{X}$ ) drawn i.i.d. from a probability distribution $\mathcal{D}$
$\mathbb{P}(\mathcal{X} = x)$ or $\mathbb{P}_{\mathcal{D}}(\mathcal{X} = x)$ or $\mathbb{P}_{x \sim \mathcal{D}}(x)$	the probability that an outcome associated with a random variable, $\mathcal{X}$ , drawn from a probability distribution, $\mathcal{D}$ , assumes a value $x$
$\mathbb{E}[f(\mathcal{X})]$ or $\mathbb{E}_{\mathcal{D}}[f(\mathcal{X})]$	the expected value or mean of a random variable, $f(\mathcal{X})$ , generated from the action of a function, $f$ , on some random variable, $\mathcal{X}$ , with outcomes $x \sim \mathcal{D}$
$\mathbb{E}_{\mathcal{S}}[f(\mathcal{X})] = \frac{1}{n} \sum_{i=1}^n f(x^{(i)})$	the sample mean of the output of some function, $f$ , evaluated on a data set, $\mathcal{S} = \{x^{(i)}\}_{i=1}^n$ , the elements of which are the outcomes associated with a random variable, $\mathcal{X}$