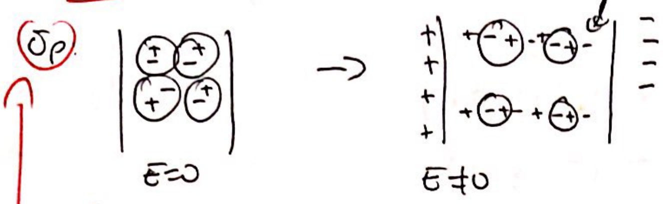
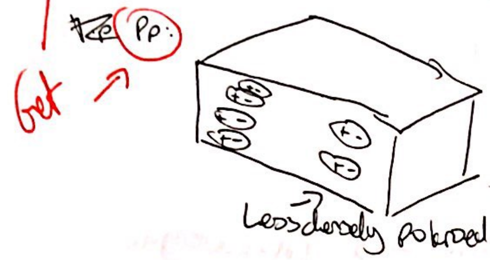
1a) P is the polarization

- σ_p is the surface polarization ~~current~~ charge density due to a polarised dielectric, the potential outside the field is equal to the potential inside the field. Due to bound charges, ~~polar~~ along with (aligned) a field to be polarised → in atoms
- $\nabla \cdot P$ ~~is the~~ $= \rho_p$ is the volume polarisation charge density due to a non-uniformly polarised material non-electric field.

Charges accumulate on surface, this is equal to the charge outside





Less densely polarised, so there is ~~net~~ non-uniform polarisation.

b) $\nabla \cdot P = 0$ implies there are no volume polarisation current density, only bound current density and the material is uniformly polarised

2a) $\underline{B} = \underline{\nabla} \times \underline{A}$

b) $\underline{\nabla} \cdot \underline{B} = \underline{\nabla} \cdot (\underline{\nabla} \times \underline{A}) = 0$

- This result implies that magnetic monopoles cannot exist in nature

c)

$\nabla \times B = \mu_0 J$

$\nabla \times \nabla \times A = \mu_0 J$

$\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J$

∵ Coulomb's Gauge : $\underline{\nabla} \cdot \underline{A} = 0$

$-\nabla^2 A = \mu_0 J$

$\nabla^2 A = -\mu_0 J$

3a) $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

- J is the volume current density
- ρ is the volume charge density

This equation implies that the total charge/current density flowing into a point is equal to the charge/current density leaving that point. There is a strong sense of conservation of charge

b) Ampere's Equation: $\oint B \cdot dl = \mu_0 I$ → no current passes through consistent with Ampere's law. implies that the magnetic field around a closed current loop is ~~equal~~ proportional to the current flowing through it. This implied that magnetic fields are ~~are~~ created by current loops. Maxwell rectified this to $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ which includes the varying electric field. implies a ~~chan~~ time changing electric field may also induce a magnetic fields and

Original Eq: $\nabla \times B = \mu_0 J$

$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

$\nabla \cdot (\nabla \times B) = \mu_0 (\nabla \cdot J)$

$= \cancel{\mu_0 (-\frac{\partial \rho}{\partial t})}$ We know ~~$\rho = \epsilon E$~~

(from $\nabla E = \frac{\rho}{\epsilon_0}$)

$\rho = \epsilon_0 \nabla \cdot E$

$\nabla J \rightarrow \nabla J + \frac{\partial \rho}{\partial t}$

$= \nabla \cdot J + \frac{\partial}{\partial t}(\epsilon_0 \nabla \cdot E)$

$= \nabla \cdot (J + \frac{\partial}{\partial t} \epsilon_0 E)$

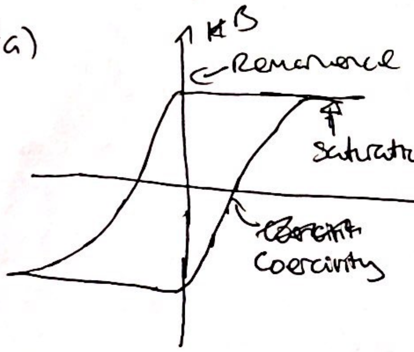
$= \nabla \cdot (J + \epsilon_0 \frac{\partial E}{\partial t})$

$\therefore \nabla \times B = \mu_0 (J + \epsilon_0 \frac{\partial E}{\partial t})$

$\nabla \times B = \mu_0 J + \underbrace{\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}_{\text{Additional Term}}$

Additional Term → The varying E field produces a B field.

4a)



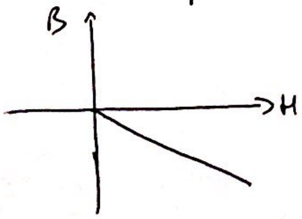
Saturation - Point at which magnet is at maximum magnetisation. Magnet no longer responds linearly to an increase in the applied field. Highest possible magnetisation → intensity

Remanence - The strength of magnetic field emitted by the material in the absence of any applied field. $(H = 0)$

Coercivity - the ease of which it is to magnetise & demagnetise a material $(B = 0)$

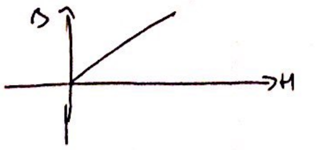
b) Diamagnetism: No intrinsic dipole moments and aligns its dipole moments ~~to~~ when an electric field is applied to induce an electric field which opposes any external electric field.

$\chi_m < 0$



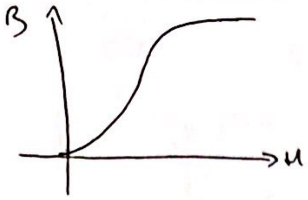
Paramagnetism: Intrinsic dipole moments but randomly aligned, it aligns itself when a magnetic field is applied.

$0 < \chi_m \ll 1$



~~etism~~ - Intrinsic dipole moments uniformly aligned in domains and align to any applied field & ~~retain~~ remains magnetized when the applied field is removed

$\chi_m > 1$



~~C~~ Cat

5) $B_{\perp 1} = B_{\perp 2}$

$\cancel{H}_{\parallel 1} = H_{\parallel 2}$

$B_1 \cos 60 = B_2 \cos\beta$

$H_1 \sin 60 = H \sin\beta$

$\mu_1 H_1 \cos 60 = \mu_2 H_2 \cos\beta$

$$\frac{\tan 60}{\mu_1} = \frac{\tan\beta}{\mu_2}$$

$$\tan\beta = \frac{\mu_2}{\mu_1}\tan 60$$

$$\beta = \tan^{-1}\left(\frac{\mu_2}{\mu_1}\tan 60\right) = \tan^{-1}\left(\frac{2.1}{5}\tan 60\right)$$

$$= 36.0^\circ$$

b) $\cancel{H_1 \sin}$ $H_{u1} = H_{u2}$

$H_1 \sin 60 = H_2 \sin\beta$

$$H_2 = \frac{H_1 \sin 60}{\sin\beta}$$

$$= 3\times10^3 \times \frac{\sin(60)}{\sin(36)}$$

$$= 4.42\times10^3 \text{ A/m}$$

6) a) $n = \frac{c}{v} = \frac{c}{(c/3)} = 3$

$n = 3$

b) $\cancel{\varepsilon}$ $n \simeq \sqrt{\varepsilon_r}$

$n^2 \simeq \varepsilon_r$

$\varepsilon_r \simeq 3^2 = 9$

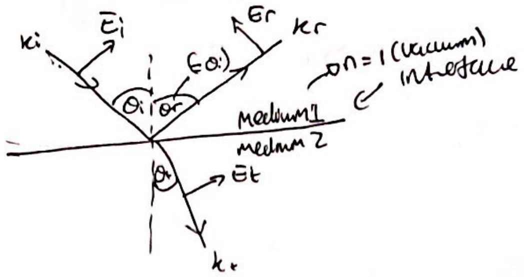
c) $\omega = v_s k$

$2\pi f = v_s \cdot \frac{\cancel{2\pi}}{\lambda}$

$$\lambda = \frac{v_s}{f} = \frac{1}{3}c \cdot \frac{1}{300\times10^9} = \frac{\cancel{3}\times10^8}{\cancel{3}\times300\times10^9} = \frac{1}{300\times10} = \frac{1}{3000}$$

$$= 3.3\times10^{-4}\text{ m}$$

7) a)



6

n_1 : Refractive index of incident wave medium

n_2 : Refractive index of transmitted wave medium

θ_i = Angle incident wave makes with normal to surface

θ_t = Angle transmitted wave " "

$r_{\parallel}$: Ratio of reflected amplitude to incident amplitude parallel to the surface

$r_{\perp}$: " " "perpendicular" "

$t_{\parallel}$: Ratio of transmitted amplitude to incident amplitude parallel to the surface

$t_{\perp}$: "Ratio "perpendicular" "

b) ~~k d = k d' = k d''~~

Get.

$\underline{k} \cdot \underline{d} = \underline{k}' \cdot \underline{d} = \underline{k}'' \cdot \underline{d}$

As k & k'' are in the same medium, $k = k''$

$kd\cos\alpha = kd\cos\alpha_r$

$\alpha_i = \alpha_r$

$\therefore \boxed{\theta_i = \theta_r}$ → Angle of incidence is equal to the angle of reflection.

~~kd~~ $\cos\alpha_i = \cos(90 - \theta_i) = \sin\theta_i$

$\cos\alpha_t = \cos(90 - \alpha_t) = \sin\theta_t$

$k \cancel{d} \sin\theta_i = kd\sin\theta_t$

$k = \frac{\omega n}{c}$

$\frac{\cancel{\omega} n_i}{\cancel{c}} \sin\theta_i = \frac{\omega n_t}{c} \sin\theta_t$

$\boxed{n_i \sin\theta_i = n_t \sin\theta_t}$ → Snell's Law

c) $\nabla \times \underline{E} = -\frac{\partial B}{\partial t}$

$\underline{E} = E_0 e^{i(kr - \omega t)}$

$\nabla \times \underline{E} = i\,\underline{k} \times \underline{E}$

$\underline{B} = B_0 e^{i(kr - \omega t)}$

$\frac{\partial B}{\partial t} = -i\omega \underline{B}$

$\cancel{i}\underline{k} \times \underline{E} = -(-\cancel{i}\omega B)$

$B = \frac{\underline{k} \times \underline{E}}{\omega} = \mu_0 \underline{H}$

$\underline{H} = \frac{1}{\mu_0} \frac{k \times E}{\omega}$

$S \equiv \frac{1}{\mu_0}(\underline{E} \times \underline{B}) = (\underline{E} \times \underline{H})$

$= E \times \frac{1}{\mu_0 \omega}(k \times E)$

$= \frac{1}{\mu_0 \omega} E \times k \times E$

$= E_0^2 \frac{k}{\mu_0 \omega} \quad (k \perp E)$

0 Reasoning!

$\frac{k}{\omega} = \frac{n}{c}$

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ (in a vacuum)

$S = E_0^2 \cdot \frac{n}{\mu_0} \sqrt{\epsilon_0 \mu_0}$

$= E_0^2 n \sqrt{\frac{\epsilon_0}{\mu_0}}$

e) $\boxed{\frac{\langle S \rangle_r}{\langle S \rangle_i} = \frac{E_{0r}^2 n \sqrt{\epsilon_0/\mu_0}}{E_{0i}^2 n \sqrt{\epsilon_0/\mu_0}} = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2}$

$r^2 = r_{\parallel}^2 = r_{\perp}^2$ for $\theta_t = \theta_i = 0$

where: $r_{\parallel}^2 = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$

f) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \sin\theta_B = n_2 \sin\theta_2$

$\theta_2 + \theta_B = \pi/2$

$n_1 \sin\theta_B = n_2 \sin(\pi/2 - \theta_B)$

$= n_2 \cos\theta_B$

$\tan\theta_B = \frac{n_2}{n_1}$

$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.5}{1}\right) = 56.31°$

$r_{\parallel} = 0$

$n_1 \sin\theta_B = n_2 \sin\theta_2$

$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2} \sin\theta_B\right)$

$= \sin^{-1}\left(\frac{1}{1.5} \sin(56.31)\right)$

$= 33.69°$

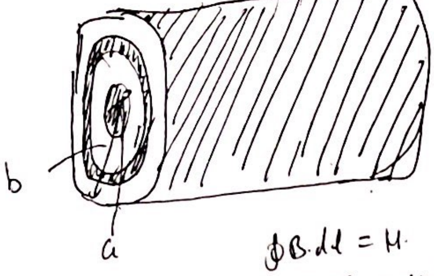
$R = |r_\perp|^2 = \left| \frac{\cos(56.31) - (1.5)\cos(33.69)}{\cos(56.31) + (1.5)\cos(33.69)} \right|^2$

$= \frac{0.148}{2} \rightarrow 7.4\%$

~~$t_\perp = \frac{2\cos\theta_B}{\cos\theta_B + (1.5)\cos\theta_t} = 0.615$~~

~~$t_\parallel = \frac{2\cos\theta_B}{1.5\cos\theta_B + \cos\theta_t} = 0.666..$~~

8a)



$\oint B \cdot dl = \mu \cdot$

$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$

$\nabla \times H = J + \frac{\partial D}{\partial t}$

$\oint_C H \cdot dl = \int J \cdot ds = I$

$H \cdot 2\pi r = I$

$H = \frac{I}{2\pi r} \hat{i}_\phi$ ← B in $\hat{\phi}$ direction.

ii) $E = -\nabla V$

$= -\left(\hat{i}_r \frac{\partial V}{\partial r} + \hat{i}_\phi \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{i}_z \frac{\partial V}{\partial z} \right)$

- $\frac{\partial V}{\partial z} = 0$ (uniform through z)
- $\frac{dV}{\partial \phi} = 0$, uniform distribution

$E = -\frac{dV_{ab}}{dr} \hat{i}_r$

← energy per unit time per unit ~~area~~ transported by EM fields.

iii) $S = |E \times H| = |-\nabla V \times H|$

$= \left| \frac{-dV_{ab}}{dr} \right| \times H$

$= \left| \frac{dV}{dr} \right| \times \frac{I}{2\pi r}$

$= \frac{I}{2\pi r} \left| \frac{dV}{dr} \right|$

~~The direction is the~~ ~~to $\hat{i}_r$ direction~~

$\hat{i}_\phi$ direction. As $\hat{i}_r \times \hat{i}_\phi = \hat{i}_z$

iv) There is no current flowing as this is a dielectric medium so no free currents flow. As a result $I = 0$ & $S = 0$

$P = \oint s \cdot da = \int \frac{I}{2\pi r} \left| \frac{dV}{dr} \right| \hat{z} \cdot \hat{r} \, da = 0$

~~v) $dP = \frac{I}{2\pi r} \left| \frac{dV}{dr} \right| \cdot r dr d\phi$~~

~~vi) $P = \frac{I}{2\pi r} \left| \frac{dV}{dr} \right| \cdot \frac{r^2}{2} = \frac{I}{2} \frac{dV}{dr} = IV$~~

$\frac{}{dt}$ is the rate

v) $dP = dS = \frac{-I}{2\pi r^2}\left|\frac{dV}{dr}\right| dr$

$P = \frac{-I}{2\pi r^2}\int_0^r r\,dr \int_0^{2\pi} d\phi \cdot \left|\frac{dV}{dr}\right|$

$= \frac{-I}{\cancel{2\pi r^2}} \cdot \frac{r^2}{2} \cdot 2\pi \left|\frac{dV}{dr}\right| = \frac{-I}{2}\left|\frac{dV}{dr}\right|$

Consider both ends: $P = IV$

$S = \frac{I}{2\pi r}\left|\frac{dV}{dr}\right|$

$dP = \frac{I}{2\pi r}\left|\frac{dV}{dr}\right| \int_0^{2\pi} r\,dr\,d\phi$

$dP = \frac{I}{2\pi} \cdot 2\pi\, dV$

$P = IV$

bi). $\frac{dW}{dt}$ = The total rate of change of energy flowing through (out) of a surface and volume due to static fields and EM radiation → Energy change in a system

• $-\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)dV$ = the rate of change of energy loss from the energy stored in EM fields → Energy stored in fields

• $-\frac{1}{\mu_0}\oint_S (E\times B)\,da$ = the rate of energy loss due to ~~radiation~~ radiation out of a surface → Rate at which energy per unit time transported by fields out of surface

iii) $\frac{dW}{dt} = \int_V (E\cdot J)\,dV$

$\nabla\times B = \mu_0 J + \mu_0\varepsilon_0 \frac{\partial E}{\partial t}$

$\frac{\nabla\times B}{\mu_0} - \varepsilon_0\frac{\partial E}{\partial t} = J$ → When work is done on charge distribution by EM Force. = Decrease in energy by stored fields + energy per time transported out of surface

$E\cdot\left(\frac{\nabla\times B}{\mu_0} - \varepsilon_0\frac{\partial E}{\partial t}\right)$

$\boxed{\text{Get}}$ $\nabla\cdot(F\times G) = (\nabla\times F)\cdot G - (\nabla\times G)\cdot F$

$\nabla(F\times G) + (\nabla\times G)F = (\nabla\times F)\cdot G$

$\frac{1}{\mu_0}\left(-\nabla(E\times B) + (\nabla\times E)B\right) - \varepsilon_0 E\frac{\partial E}{\partial t}$

$\frac{1}{\mu_0}\left(\nabla(E\times B) + \left(-\frac{\partial B}{\partial t}\right)B\right) - \varepsilon_0 E\frac{\partial E}{\partial t}$

$\Rightarrow \int_V -\frac{1}{\mu_0}\nabla(E\times B)\,dV - \left(\int_V \frac{1}{\mu_0}\frac{\partial(B^2)}{\partial t}\cdot\frac{1}{2} + \varepsilon_0\frac{\partial(E^2)}{\partial t}\cdot\frac{1}{2}\,dV\right)$

(Div. Th)

$= -\frac{1}{\mu_0}\oint_S (E\times B)\,da - \frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)dV$

9a) A plasma is a slow moving collection of ~~gases~~ positive ions surrounded by an dissociated electron cloud. This is homogenous on the macroscopic level ~~&~~ and has no net charge distribution. Though on the microscopic level, the electron cloud cancels out any positive ions. fluctuations result in Polarization : $\underline{P} = -N_e e \underline{z}$.

b)

N_e = Plasma Density

e = charge

M_e = electron mass

ε_0 = permittivity of free space

c) $D = \varepsilon_0 E + P = \varepsilon_0 \varepsilon_r E$

$$\frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t}$$

$$\frac{\partial P}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t} (\varepsilon_r - 1)$$

$$= \varepsilon_0 \frac{\partial E}{\partial t} \left(\cancel{1} - \frac{\omega_p^2}{\omega^2} \cancel{-1}\right)$$

$$= \varepsilon_0 \frac{\partial E}{\partial t} \left(-\frac{\omega_p^2}{\omega^2}\right)$$

$$E = E_0 e^{i(kr - \omega t)}$$

$$\frac{\partial E}{\partial t} = -i\omega E$$

$$\frac{\partial P}{\partial t} = \varepsilon_0 (-i\omega E) \left(\frac{N_e e^2}{M_e \varepsilon_0}\right) \cdot \frac{1}{\omega^2}$$

$$\therefore \underline{J}_p = i \left(\frac{N_e e^2}{M_e \omega}\right) \underline{E}$$

d) $V_p = \frac{\omega}{k} = \frac{c}{n}$

$$k = \frac{\omega n}{c} = \frac{\omega \sqrt{\varepsilon_r \mu_r}}{c} = \omega \frac{\sqrt{\varepsilon_r}}{c} = \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

$$\therefore k^2 = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

e) • If $\omega \to \omega_p$, $\varepsilon_r \to 0$, as a result, ~~D would~~ $D \to \infty$ as $D = \frac{E}{\varepsilon_r}$ so ~~the~~ oscillations would increase and waves ~~wouldn't~~ propagate. The medium will become dispersionless.

5

$5\times10^{12}\,m^{-3}$

$$\omega_p = \left(\frac{5\times10^{12}\times(1.6\times10^{-19})^2}{9.11\times10^{-31}\times8.85\times10^{-12}}\right)^{1/2} = 1.26\times10^{8}\ \cancel{Hz}\ s^{-1}$$

$$f = \frac{1.26\times10^{8}}{2\pi} = 2.01\times10^{7}\,s^{-1}$$

We want $\omega >> \omega_p$, so frequencies should be greater than $2.01\times10^{7}s^{-1}$.

ii) $f = 3.88\,GHz$

$$\frac{(2\pi f)^2 M_e \varepsilon_0}{e^2} = n$$

$$n = \frac{(2\pi\times3.88\times10^{9})^2\times9.11\times10^{-31}\times8.85\times10^{-12}}{(1.6\times10^{-19})^2}$$

$$= 1.87\times10^{17}\,m^{-3}$$

iii) $n = 2\times1.87\times10^{17} = 3.7435\times10^{17}\,m^{-3}$

$\cancel{E_0}\cdot10^{-3} = \cancel{E_0}\,e^{i(kr-\omega t)}$

~~Alternate~~ plasma frequency is split into real & imaginary components.

$$\boxed{\frac{1}{k_i} = \delta = \frac{c\sqrt{2}}{\cancel{\sqrt{2}}\,\omega_p}}$$

$$k_i = \frac{k}{\sqrt{2}} = \frac{1}{\delta}$$

$$\frac{1}{\delta} = \frac{c\sqrt{2}}{\omega_p}$$

$$e^{-d/\delta} = 10^{-3}$$

$$\ln(10^{-3}) = -d/\delta$$

$$3\ln(10) = d/\delta$$

$$d = 3\ln(10)\times\delta$$

$$= 3\ln(10)\times\frac{c\sqrt{2}}{\cancel{\sqrt{2}}\,\omega_p}$$

$$= 3\times\ln(10)\times\frac{c}{\sqrt{2}}\cdot\sqrt{\frac{M_e\varepsilon_0}{ne^2}}$$

$$= 3\times\ln(10)\times\frac{3\times10^{8}}{\sqrt{2}}\times\sqrt{\frac{9.11\times10^{-31}\times8.85\times10^{-12}}{3.7435\times10^{17}\times(1.6\times10^{-19})^2}} = 0.0425\,m$$

$= 4.25\,cm$

$\times2 = 8.5$

10a) $a^\mu = (A_x, A_y, A_z, i\phi/c)$

--