2013

1a) a is a hard magnet: It has a high remanence and coercivity meaning it emits a relatively strong field in the absence of an external field.

Whereas b is a soft magnet as it has a low remanence and coercivity so it is easily magnetised and demagnetised and doesn't hold onto field well.

b) $M = \frac{m}{V}$

$\rho = \frac{mass}{V}$ $\Rightarrow V = \frac{Mass}{\rho}$

$M = \frac{m}{Mass} N$

$V = 4/3 \pi r^3 = \frac{4}{3}\pi \times (22.5\times10^{3})^3 = 4.7713\times10^{13} m^3$

$N = \frac{5.5\times10^{30}}{1.67\times10^{-27}} = 3.293\times10^{57}$

$M = \frac{9.6\times10^{-27}}{4.77\times10^{13}} \times 3.293\times10^{57} = 6.63\times10^{17}\ Am^{-1}$

2 a) ~~P represents the~~

P = Polarisation

M = Magnetisation

n = directional unit vector / surface ~~…~~ are considered

σ_p = surface current polarisation ~~density~~ Density

ρ_p = volume current ~~…~~ Polarisation Density

j_m = Bound surface magnetisation current density

J_M = Bound volume magnetisation current density.

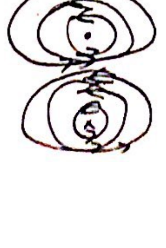
b) • j_m arises due to microscopic current loops (DIPOLE MOMENTS.) within a bulk of a material which cancel out within the material but have a net surface contribution.

• non uniform magnetised bulk

~~…~~

→ net current

• J_M arises due to intrinsic dipole moments being aligned in the presence of a magnetic field which result in an overall volume magnetisation contribution.



3a) i) k = Wavenumber $= \frac{2\pi}{\lambda}$

ω = Angular Frequency $= 2\pi f$

ω_p = Plasma Frequency

c = Speed of light $\simeq 3\times10^8 ms^{-1}$

ii) $\cancel{\frac{\omega^2}{k^2} = V_p^2}$ $\quad V_p = \frac{\omega}{k}$

$\cancel{\frac{\omega^2}{k^2} = \frac{\omega_p^2}{k^2} + c^2}$

$\omega^2 = \omega_p^2 + k^2c^2$

$2\omega d\omega = 2k\,dk\,c^2$

$\frac{d\omega}{dk} = \frac{k}{\omega}c^2$

$V_g = \frac{c^2}{V_p}$

$V_pV_g = c^2$

iii) ◦ $V_p = V_g = c^2$

OR

◦ $V_g << c$ and $V_p > c$, phase velocity travels faster than speed of light. Though information travels at v_g. doesn't violate laws of physics as Energy and information. Also implies $n_{plasma} < 1$.

b) $k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$

$W << \omega_p$

k is imaginary thus doesn't propagate.

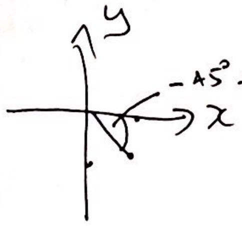
4a) ~~False, paramagnet. dipole moments are randomly aligned (unless exposed to a magnetic field, whereas in a ferromagnet~~

True, Paramagnet moments are of similar magnitude to a ferromagnet for atoms, however a Ferromagnet has aligned moments where a paramagnet doesn't, without a field.

b) False, the magnetic moments do not remain for a paramagnetic material as random alignment ~~[illegible]~~ is more energetically favourable (due to the available energy. A Ferromagnet would retain magnetisation.

, to conserve energy, the electric dipole moment aligns itself to oppose the applied field ~~...~~

5a) $\underline{E} = \cancel{E_0} e^{i\left(k\left(\frac{\hat{\underline{x}} + \underline{y} + \hat{\underline{z}}}{3}\right) - \omega t\right)} \left(E_{0x}\,\hat{\underline{x}} + E_{0y}\, e^{-\frac{\pi}{4}\hat{\underline{\phi}}}\right)$



$-45° = \alpha \quad \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

$= \hat{E}_0 \cdot e^{i\left(\frac{k}{3}(\hat{\underline{x}} + \underline{y} + \hat{\underline{z}}) - 2\pi f t\right)} \cancel{\left(E_{0x}\,\hat{\underline{x}} + E_{0y}\, e^{-\frac{\pi}{4}}\hat{\underline{y}}\right)} \underbrace{\left(\frac{\hat{x} - \hat{y}}{\sqrt{2}}\right)}_{\text{Polarisation}}$

~~b) Take the cross product with~~

b) ~~Take the Take the cross pro~~

Take the dot product with the direction of travel and the propagation direction. It should only be propagating in one direction and none other. Zero for all perpendicular directions.

6a) $\underline{\nabla} \cdot \underline{J} + \frac{\partial \rho}{\partial t} = 0$

Continuity Equation: The charge (current) entering a point is equal to the current leaving (current is conserved)

bi) $J^\mu = (c\rho, J_x, J_y, J_z)$

$\cancel{\partial^\mu} \quad \partial x^\mu = \cancel{(\ldots)} \left(\frac{1}{c}\partial t, \partial x, \partial y, \partial z\right)$

$\partial x^\mu \cdot J^\mu = \frac{1}{c}\frac{\partial}{\partial t}(c\rho) + \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y + \frac{\partial}{\partial z} J_z$

$= \frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$

ii) This means that the current flowing into a point is equal to the current leaving and we have continuity at this point, even as a four vector. We can also rewrite Maxwell's equations using these.

(a)

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$$

$$\underline{\nabla} \times \underline{E} = i\underline{k} \times \underline{E}$$

$$\frac{\partial B}{\partial t} = -i\omega \underline{B}$$

$$\cancel{i}\underline{k} \times \underline{E} = \cancel{+i}\omega \underline{B}$$

$$\underline{B} = \frac{\underline{k} \times \underline{E}}{\omega}$$

(b)

$$N = \frac{1}{\mu_0}(E \times B)$$

$$= \frac{1}{\mu_0}\left(\underline{E} \times \frac{(\underline{k} \times \underline{E})}{\omega}\right)$$

$k \perp E$

$$= \frac{1}{\mu_0}\frac{k}{\omega}\bar{E}_0^2$$

$$\frac{k}{\omega} = \frac{\cancel{\varepsilon}n}{c} = \frac{1}{v}$$

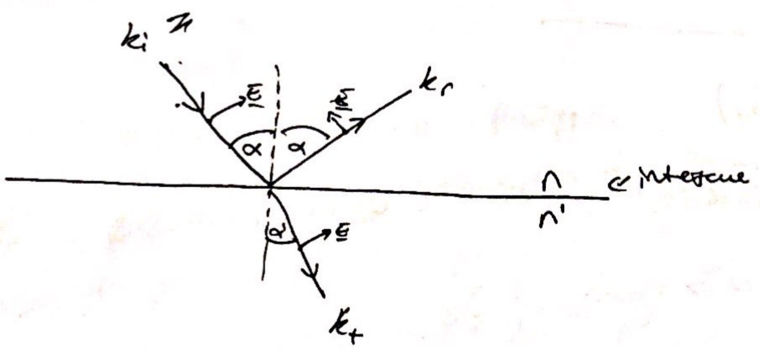
$$= \frac{1}{\mu_0}\frac{n}{c}\bar{E}_0^2$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$= \frac{1}{\mu_0}\sqrt{\frac{\mu_0\varepsilon_0}{1}} \cdot n\bar{E}_0^2$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot n\bar{E}_0^2$$

(c)



- n = Refractive Index of ~~the~~ incident media (vacuum)
- n' = Refractive Index of transmitted media (surface)
- α = angle of that incident ~~wave~~ wave makes with normal to boundary
- α' = angle that transmitted waves makes with normal to boundary
- $r_{\parallel}$ Ratio of reflected amplitude component parallel to surface
- $r_{\perp}$ reflected amplitude perpendicular to surface
- $t_{\parallel}$ transmitted amplitude parallel to surface

$E_{\parallel}\cos\alpha - E''_{\parallel}\cos\alpha'' = E'_{\parallel}\cos\alpha'$

$\alpha = \alpha''$

$E_{\parallel}\cos\alpha - E''_{\parallel}\cos\alpha = E'_{\parallel}\cos\alpha'$

$H_{\parallel} + H''_{\parallel} = H'_{\parallel}$

$B = \frac{k}{\omega} E$

$\mu_0 H = \frac{E}{c}$

$H = \sqrt{\frac{\varepsilon}{\mu}}\, E$

$\sqrt{\frac{\varepsilon}{\mu}}\, E_{\parallel} + \sqrt{\frac{\varepsilon}{\mu}}\, E''_{\parallel} = \sqrt{\frac{\varepsilon'}{\mu'}}\, E'_{\parallel}$

$\mu \simeq \mu'$

$\sqrt{\varepsilon} \simeq n$

$nE_{\parallel} + nE''_{\parallel} = n'E'_{\parallel}$

$\cancel{\frac{n(E_{\parallel}+E''_{\parallel})}{n'} = E'_{\parallel}}$ $\quad \frac{n'E'_{\parallel} - nE_{\parallel}}{n} = E''_{\parallel}$

$E_{\parallel}\cos\alpha - \left(\frac{n'E'_{\parallel} - nE_{\parallel}}{n}\cos\alpha\right) = E'_{\parallel}\cos\alpha'$

$nE_{\parallel}\cos\alpha - n'E'_{\parallel}\cos\alpha + nE_{\parallel}\cos\alpha = E'_{\parallel}\cos\alpha'\, n$

$E'_{\parallel}(n\cos\alpha' + n'\cos\alpha) = 2nE_{\parallel}\cos\alpha$

$\frac{E'_{\parallel}}{E_{\parallel}} = t = \frac{2n\cos\alpha}{n'\cos\alpha + n\cos\alpha'}$ As required.

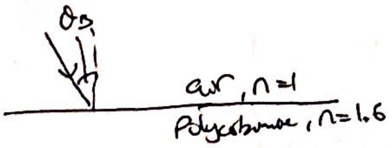
d) Normal to the interface: $\alpha = \alpha' = 0$

$\cos\alpha = \cos\alpha' = 0$

$r_{\parallel} = \frac{n'-n}{n'+n}$

$r_{\parallel} = \left(\frac{n'-n}{n'+n}\right)^2 = R_{\parallel}$

ii)



$n_i\sin\theta_B = n_t\sin(90-\theta_B)$

$\frac{n_t}{n_i} = \tan\theta_B$

$\theta_B = \tan^{-1}\left(\frac{n_t}{n_i}\right) = \tan^{-1}(1.6) = 58°$

$R_{\parallel} = \left(\frac{1.6-1}{1.6+1}\right)^2 = \frac{9}{196}$

$\theta_B + \theta_t = \frac{\pi}{2}$

$\Rightarrow \sin\theta_t = \frac{n_i\sin\theta_B}{n_t} = \frac{\sin(58)}{1.6} = 0.53$

$\theta_t = 32°$

? $(r_{\perp})^2 = \left(\frac{\cos(58) - (1.6)\cos(32)}{\cos 58 + (1.6)\cos(32)}\right)^2 = 0.142$

98%

8) $t' = t - \frac{r}{c}$

Retarded time uses the concept that energy and information travels at a finite speed and therefore takes time to propagate. Thus an electric field felt at r and time t must have been emitted from another point r' at earlier time t'.

$\rightarrow$ the $\frac{r}{c}$ to reach observer from

$t' = t - r/c$

b) $$\frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'} \frac{\partial t'}{\partial t}$$

$$= \frac{\partial f(t')}{\partial t'} \cdot 1 = \frac{\partial f(t')}{\partial t'}$$

$$\frac{\partial f(t')}{\partial r} = \frac{\partial f(t')}{\partial t'} \frac{\partial t'}{\partial r} = \frac{\partial f(t')}{\partial t'} \cdot -\frac{1}{c} = -\frac{1}{c} \frac{\partial f(t')}{\partial t'}$$

c) $$E = -\nabla \cancel{\phi} V - \frac{\partial A}{\partial t}$$

$$-\nabla V = -\nabla(0) = 0$$

$$-\frac{\partial A}{\partial t} = \cancel{+}\frac{\partial}{\partial t}\left(\cancel{+}\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r} \sin\omega t' \hat{\phi} \right)$$

$$= -\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r} \cdot \omega \cos\omega t'$$

$$= \frac{\mu_0 m_0 \omega^2}{4\pi c^2} \frac{\sin\theta}{r} \cos\omega t' \hat{\phi}$$

$$B = \nabla \times A$$

$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \partial r & \partial\theta & \partial\phi \\ 0 & 0 & -\cancel{r\sin\theta}\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin^2\theta}{\cancel{r}} \sin\omega t' \end{vmatrix}$$

$$= -\frac{1}{r^2 \sin\theta} \cdot \frac{\mu_0 m_0 \omega}{4\pi c} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \partial r & \partial\theta & \partial\phi \\ 0 & 0 & -\sin^2\theta \sin\omega t' \end{vmatrix}$$

$$= \cancel{+}\frac{1}{r^2 \cancel{\sin\theta}} \frac{\mu_0 m_0 \omega^2}{4\pi c}\left[\cancel{r} (2\sin\theta\cos\theta \sin\omega t') \hat{\theta} \right]$$

$$= -\frac{1}{r} \frac{\mu_0 m_0 \omega^2}{4\pi c} \cancel{\ } \cdot \frac{\cancel{\sin\theta\cos\theta}}{\cancel{c}} \cancel{\ }) \sin\omega t') \hat{\theta}$$

$$= -\frac{\mu_0 m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cancel{\ } \cos\omega t' \hat{\theta}$$

$$= \frac{1}{\mu_0}(\underline{E} \times \underline{B})$$

$$= \frac{1}{\mu_0} \frac{\mu_0 M_0 \omega^2}{4\pi c} \frac{\sin\theta}{r}(\cos\omega t') \frac{-\mu_0 M_0 \omega^2}{4\pi c^2} \frac{\sin\theta}{r} \cos\omega t' \, \phi(\hat{\underline{\theta}} \times \hat{\underline{\phi}})$$

$$= -\cancel{4\pi_0}\left(\frac{\mu_0 \omega^2}{4\pi c^2}\right)^2 \left(\frac{\sin\theta}{r}\right)^2 (\cos\omega t')^2 \, \hat{\underline{r}}$$

$$= -\frac{\mu_0 M_0^2 \omega^4}{16\pi^2 c^3} \frac{\sin^2\theta}{r^2} \cos^2\omega t' \, \hat{\underline{r}}$$

~~Yes, there is radiation~~ As $r \gg c/\omega$, there is no radiation as $s \to 0$.

$$\hat{\underline{\theta}} \times \hat{\underline{\phi}} = \hat{\underline{r}}$$

e) $\hat{\underline{E}} \times \hat{\underline{B}} = |\underline{E}||\underline{B}| \, \hat{\underline{\theta}} \times \hat{\underline{\phi}} = |\underline{E}||\underline{B}| \, \hat{\underline{r}}$

so radiate towards the same direction.

ii) $P_{\text{electron}} = \cancel{\frac{\mu_0 I}{4\pi c}} \frac{\mu_0^2 I_0^2}{16\pi^2 c} \frac{\sin^2\theta}{r^2} (d\omega)^2 (\cos\omega t')^2 \, \hat{\underline{r}}$

$$\frac{P_{\text{wave}}}{P_{\text{photon}}} = \frac{\cancel{\mu_0} M_0^2 \omega^4}{\cancel{16\pi^2 c^3 r^2}} \cancel{\frac{\sin^2\theta}{r^2}} \cancel{\cos^2\omega t'} \cdot \frac{\cancel{r^2}\,\cancel{16\pi^2 c}}{\mu_0^2 I_0 \cancel{\sin^2\theta} (d\omega)^2 \cancel{\cos\omega t}}$$

$$= \frac{M_0^2 \omega^4}{c^2} \cdot \frac{1}{\mu_0 I_0 (d\omega)^2}$$

$$= \frac{M_0^2}{c^2} \cdot \frac{\omega^4}{\underbrace{\mu_0 I_0 (d\omega^2)^2}_{P_0^2}}$$

$$= \frac{M_0^2}{c^2} \cdot \frac{1}{P_0^2}$$

$$\boxed{\begin{aligned} I &= q_0 \omega \\ P_0 &= q_0 d \end{aligned}}$$

iii) +

$$\frac{M_0^2}{c^2} \cdot \frac{\omega^2}{\mu_0 q^2 \omega^2 d^2} = \frac{m_0^2}{c^2}$$

$$M = IA = I_0 \pi a^2$$

$$\Rightarrow \frac{M_0^2}{P_0^2 c^2} = \frac{m_0^2 \omega^2}{c^2 I_0^2 d^2} = \frac{a^2 \omega^2}{c^2}$$

9(a i) $\underline{J} = g\underline{E}$

ii) $\nabla \times E = -\frac{\partial B}{\partial t}$

$\cancel{\nabla \times B}$ $\nabla \times \nabla \times E = -\frac{\partial(\nabla \times B)}{\partial t}$

$\nabla(\nabla \cdot E)^{\to 0} - \nabla^2 E = - \frac{\partial}{\partial t}\left(\mu_0 J + \frac{\partial E}{\partial t}\mu_0\varepsilon_0\right)$

$\nabla^2 E = \frac{\partial}{\partial t}\left(\mu_0 g E + \mu_0\varepsilon_0 \frac{\partial E}{\partial t}\right)$

$\nabla^2 E = \mu_0 g \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$

$\nabla^2 \underline{E} - g\mu \frac{\partial \underline{E}}{\partial t} - \varepsilon\mu \frac{\partial^2 \underline{E}}{\partial t^2} = 0$

iii) Plane wave

$\underline{E} = E_0 e^{i(kr - \omega t)}$

$\nabla^2 E = -k^2 \underline{E}$

$\frac{\partial E}{\partial t} = -i\omega \underline{E}$

$\frac{\partial^2 E}{\partial t^2} = -\omega^2 \underline{E}$

$\Rightarrow -k^2\cancel{\underline{E}} + g\mu(\cancel{-}i\omega\cancel{\underline{E}}) + \varepsilon\mu(\cancel{-}\omega^2\cancel{\underline{E}}) = 0$

$k^2 = g\mu i\omega + \varepsilon\mu\omega^2$

$= \varepsilon\mu\omega^2\left(1 + \frac{\mu i g\omega}{\varepsilon\mu\omega^2}\right)$

$k^2 = \varepsilon\mu\omega^2\left(1 + \frac{ig}{\varepsilon\omega}\right)$

iv) Poor conductor: $ig \ll \varepsilon\omega$ $\left(\frac{ig}{\varepsilon\omega} \to 0\right)$

Good conductor: $\varepsilon\omega \gg ig$ $\left(\frac{ig}{\varepsilon\omega} \to \infty\right)$

b i) Skin Depth is the depth which a wave travels through a material before being attenuated by a factor of $\frac{1}{e}$

ii) Good conductor: $k^2 = \mu\varepsilon\omega^2 \cdot \frac{ig}{\varepsilon\omega} = \mu\omega g i$

$k = \sqrt{\frac{\mu\omega g}{2}}(1+i)$

$$E_0 e^{ikr} e^{-i\omega t} = E_0 e^{-i\omega t} e^{ik_{R}e} e^{-k_{im}} = E_0^{-d/\delta}$$

$$k_{im} = \frac{1}{\delta}$$

$$k_{im} = \sqrt{\frac{\mu\omega g}{2}}$$

$$\delta = \sqrt{\frac{2}{\mu\omega g}}$$

iii) $\cancel{k^2 = \varepsilon\mu\omega^2}$ $\cancel{k = \sqrt{\varepsilon\mu}\,\omega}$

$$k^2 = \mu\varepsilon\omega^2\left(1+\frac{ig}{\varepsilon\omega}\right)$$

$$k = \sqrt{\mu\varepsilon}\,\omega\left(1+\frac{ig}{\varepsilon\omega}\right)^{\frac{1}{2}}$$

$$= \sqrt{\mu\varepsilon}\,\omega\left(1+\frac{1}{2}\frac{ig}{\varepsilon\omega}+\dots\right)$$

$$k_i = \frac{\sqrt{\mu\varepsilon}\,\omega g}{2\varepsilon\omega} = \frac{g}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\delta = \frac{1}{k_i} = \frac{2}{g}\sqrt{\frac{\varepsilon}{\mu}}$$

iv) Non-Magnetic metal, good conductor.

$$\delta = \sqrt{\frac{2}{\mu g\omega}}$$

$$\delta^2 = \frac{2}{\mu g\omega}$$

$$g = \frac{2}{\mu\delta^2\omega}$$

$$= \frac{2}{4\pi\times10^{-7}\times(2.1\times10^{-6})^2\times(2\pi\times10^{11})} = \underline{5.74\times10^{5}\ \Omega\, m^{-1}}$$

$$J = g E$$

J: Am^{-2}, E: Vm^{-1}

$$\frac{Am^{-2}}{V^{-1}\,m}$$

$\delta = \frac{1}{k}$

$$k^2 = \sqrt{\frac{\mu\omega g}{2}}\,(1+i)$$

$$\boxed{\omega\delta = \frac{\omega}{k} = v_p}$$

$$2\pi\times10^{11}\times2.1\times10^{-6}$$

$$= \underline{1.32\times10^{6}\ ms^{-1}}$$

10a) $c =$ speed of light

$J_1 =$ current Density in the x-direction

$J_2 =$ " " y-direction

$J_3 =$ " " z-direction

$\rho =$ Charge Density

$a_1 =$ magnetic vector potential in the x-direction

$a_2 =$ " " y-direction

$a_3 =$ " " z-direction

$\phi =$ ~~Sd~~ Electric Scalar potential.

b i) $x_1 = x$

$x_2 = y$

$x_3 = z$

$x_4 = ict$

ii) $x_1 = x_1'$

$x_2 = x_2'$

$x_3 = \gamma x_3' - i\beta\gamma x_4'$

$x_4 = \gamma x_4' + i\beta\gamma x_3'$

c) $\underline{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$ long straight cylindrical insulating rod

$\underline{E} = \frac{\lambda}{2\pi\epsilon_0}\hat{r}$, $\phi = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)$

$J_\mu = (J_1, J_2 J_3, ic\rho)$, $a_\mu = (a_1, a_2, a_3, \frac{i\phi}{c})$ → Rod at rest $v=0$

$J_\mu = (0,0,0, ic\frac{\lambda}{\sigma})$ $\rho = \frac{\lambda}{\sigma}$ ↑ Free charge Density

Potential only outside the surface of the rod.

$a_\mu = \left(0,0,0, \frac{i}{c}\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)\right)$

d) Apply ~~vector potential~~ Lorentz transformation for J_μ'