Candidate No. ....

Seat No. ....

UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**PHAS3225**
ASSESSMENT PATTERN	:	**PHAS3225A**
MODULE NAME	:	**Solid State Physics**
DATE	:	**12 May 2017**
TIME	:	**10:00 am**
TIME ALLOWED	:	**2 hours 30 mins**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

2015/2016, 2016/2017

<u>Under no circumstances</u> are the attached papers to be removed from the examination by the candidate.

2016/17-PHAS3225A-001-EXAM-114

TURN OVER

Answer EVERY question from section A, ONE question from section B1 and ONE question from section B2.

The numbers in square brackets in the right-hand margin indicate a provisional allocation of maximum possible marks for different parts of each question.

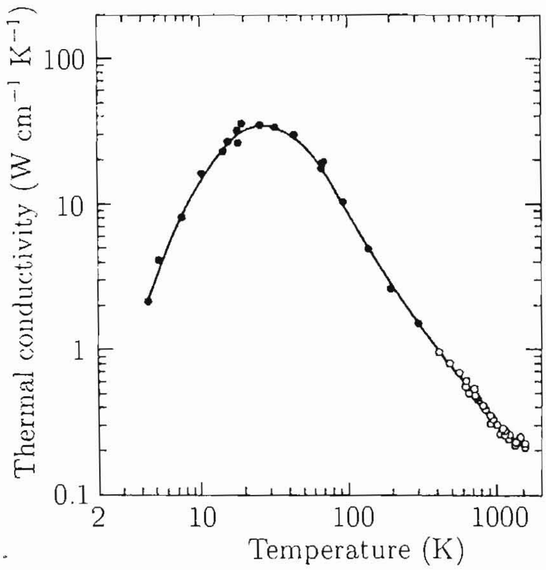
The following may be assumed, if required:

Electron rest mass	m_e	$=$	9.109×10^{-31} kg
Elementary charge	e	$=$	1.602×10^{-19} C
Permittivity of free space	ϵ_0	$=$	8.854×10^{-12} F m^{-1}
Boltzmann constant	k_B	$=$	1.381×10^{-23} J K^{-1}
Planck constant/2π	$\hbar$	$=$	1.055×10^{-34} J s
Speed of light	c	$=$	2.998×10^{8} m s^{-1}
Bohr magneton	μ_B	$=$	9.274×10^{-24} J T^{-1}
Avogadro constant	L	$=$	6.022×10^{23} mol^{-1}
Atomic mass unit	u	$=$	6.661×10^{-27} kg

Section A

(Answer ALL SIX questions from this section)

1. The graph below shows the thermal conductivity of silicon as a function of temperature.



(a) The thermal conductivity may be written as a product of three physical quantities. What are they? [2]

(b) Explain why the thermal conductivity increases with temperature at low temperatures. [2]

(c) Explain why the thermal conductivity decreases with temperature at high temperatures. [2]

2. In the free electron theory of metals the Fermi energy E_F is given by

$$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$$

where n is the number of electrons per unit volume.

Calculate the Fermi energy in units of eV for a monovalent metal that crystallises in the body centred cubic structure with lattice constant $a = 3.49$ Å. [3]

Explain how, given the thermal conductivity of this metal, you could estimate its electrical conductivity at a given temperature. State any assumptions you would be making and explain briefly why they may not be valid in the general case. [4]

3. (a) Write down an expression for a reciprocal lattice vector $\mathbf{G}_{hkl}$ in terms of the primitive reciprocal lattice vectors $\mathbf{a}_1^*$, $\mathbf{a}_2^*$ and $\mathbf{a}_3^*$, and the integers h, k and l. Write down also the general expression for the spacing between lattice planes with Miller indices (hkl) in terms of $\mathbf{G}_{hkl}$. [2]

(b) Consider a crystal with a tetragonal structure defined by primitive lattice vectors $\mathbf{a}_1 = (a, 0, 0)$, $\mathbf{a}_2 = (0, a, 0)$ and $\mathbf{a}_3 = (0, 0, c)$. By writing down the primitive reciprocal lattice vectors for this structure, derive an expression for the spacing between lattice planes with Miller indices (hkl) in terms of a and c. [5]

4. (a) Sketch low-field magnetic susceptibility versus temperature curves for (i) an ideal paramagnet and (ii) a Pauli paramagnet. In each case state what type of solid might be expected to show this behaviour. [4]

(b) The Mn^{2+} ion with S = 5/2 has a spin-only magnetic moment. Calculate the ratio $\langle \hat{\mu}_z \rangle / \sqrt{\langle \hat{\mu}^2 \rangle}$ where $\langle \hat{\mu}_z \rangle$ is the maximum projection of the moment on a weak applied field, and $\langle \hat{\mu}^2 \rangle$ is the mean square magnetic moment. [3]

5. (a) Sketch a typical magnetisation versus field curve for a type-I superconductor at fixed temperature, marking the critical field. [2]

(b) Sketch a field-temperature phase diagram for a material showing type-II superconducting behaviour, identifying three main regions. For each region draw figures to illustrate the approximate distribution of magnetic flux when a sample of this material is subject to an applied magnetic field. [5]

6. (a) Use the example of phosphorus (P) doping to explain what is meant by a *donor bound level* in silicon (Si). Illustrate your answer with an appropriately labelled energy level diagram. [3]

(b) Explain briefly what is meant by a *p-n junction* in terms of doping and majority carriers. Make a sketch of its typical current-voltage characteristic. [3]

Section B1
(Answer *ONE* question from this Section)
Note: only one Section B1 answer will be marked

7. This question is about a two dimensional metal with a square lattice of atoms of spacing a for which the electrons may initially be considered to be free.

(a) Draw a sketch of the reciprocal lattice of this solid, and identify the area of the first Brillouin zone. [3]

(b) Using the free-electron approximation, determine the expressions for the kinetic energies of the least and most energetic electrons on the Brillouin zone boundary and write down one wavevector corresponding to each case labelling them as $\mathbf{k}_1$ and $\mathbf{k}_2$. [3]

(c) By evaluating the area of the first Brillouin zone and the density of states in reciprocal space, show that, if spin degeneracy is taken into account, the first Brillouin zone contains exactly $2N$ electron states, where N is the number of atoms. [5]

(d) For the case that the metal is monovalent, calculate the magnitude of the Fermi wavevector k_F as a fraction of the shortest distance from the origin to the nearest Brillouin zone boundary, and make a sketch of the Fermi surface within the first Brillouin zone. [5]

(e) It is now assumed that the metal is divalent, and that the electrons are affected by the periodic potential of the ion cores. Discuss with the aid of sketches how the shape of the Fermi surface evolves as the strength of the periodic potential is increased from zero. [7]

(f) Deduce the condition in terms of the energy gaps at $\mathbf{k}_1$ and $\mathbf{k}_2$ for the solid to be an insulator and sketch the corresponding band dispersion along $\mathbf{k}_1$ and $\mathbf{k}_2$. [7]

8. This question is about X-ray diffraction and phonon dispersion curves in nickel (Ni), which crystallises in a face centred cubic (FCC) structure with a lattice parameter of 3.52 Å.

(a) Write down the general expression for the unit cell structure factor ensuring that you define any terms you introduce. [3]

(b) By adopting the conventional cubic unit cell, describe using an annotated figure how the FCC structure can be decomposed into a simple cubic lattice and a basis. Identify the basis atoms and write down their coordinates. [3]

(c) Obtain an expression for the unit cell structure factor of the FCC structure using the conventional unit cell and deduce the restrictions on values of (hkl) for the observation of diffraction peaks. [3]

(d) Give the (hkl) values and determine the corresponding scattering angles 2θ (in degrees) for the first three Bragg peaks that would be observed in an X-ray scattering experiment using a monochromatic beam of wavelength 0.1540 nm on a powdered sample of nickel. [6]

(e) Consider the neighbouring planes of atoms with inter-plane spacing d and perpendicular to the [100] crystallographic direction in a crystal of nickel. Assume that the longitudinal vibrations of these atomic planes can be modelled as the vibrations of a one-dimensional chain of identical masses m with spacing d, connected by springs with force constant C. The dispersion relation for the vibrations of such a chain is given by

$$\omega^2 = \frac{4C}{m}\sin^2(kd/2)$$

where ω is the frequency of vibration and k is the corresponding wavevector.

(i) Calculate the spacing d between these planes. [2]

(ii) Sketch the dispersion relation $\omega(k)$ for values of k within the first Brillouin zone. Sketch also the atomic displacements corresponding to the longitudinal vibrations with the highest and lowest energy. [5]

(iii) Use the dispersion relation above to derive an expression for the speed of sound for longitudinal waves in nickel travelling along the [100] direction and calculate its value given that the maximum energy of longitudinal phonons along this direction is 35 meV. [8]

Section B2

(Answer *ONE* question from this Section)

Note: only one Section B2 answer will be marked

9. In this question parts (a-c) relate to superconductors and parts (d-f) relate to semiconductors. Answer all parts.

(a) Explain what is meant by the superconducting *energy gap* and how evidence of the energy gap may be found in specific heat measurements. Explain briefly how BCS theory naturally accounts for the energy gap. [4]

(b) Explain briefly what is meant by the *Meissner effect* in a superconductor. Show how the London equation

$$\mathbf{j} = -\frac{1}{\mu_0 \lambda_L^2}\mathbf{A}$$

leads to the Meissner effect. Hence explain why λ_L is called a 'penetration depth' and why electrical currents in a superconductor are confined to a surface layer. [8]

(c) The critical current flowing parallel to the axis of a cylindrical wire of radius r may be written $I_c = 2\pi r H_c$. Starting with Ampère's law, derive this equation and note any assumptions. If the zero-temperature critical current of a cylindrical superconducting wire is 1.5 A, calculate its value at 90% of the critical temperature. [5]

(d) For a semiconductor, write down the relationship between the electron effective mass m_e^* and the band curvature. Hence argue that the smaller the energy gap, the smaller is $|m_e^*|$ near the gap. [3]

(e) State what is meant by a *hole* in the context of semiconductors. Compare the energy, wavevector, velocity, effective mass and effective charge of a hole with that of the electron orbitals from which it is derived. [5]

(f) In silicon at 900 K. the election mobility is a factor of 3 larger than the hole mobility. At this temperature an impure sample of silicon is found to have electrical conductivity that is a factor a factor of 12 larger than that of pure silicon. Calculate the ratio of hole concentrations between the impure sample and pure silicon and briefly explain the number of solutions you find. [5]

10. In this question (a-d,g) relate to magnetic materials and (e-f) relate to superconductors. Answer all parts.

(a) Briefly explain what are meant by the following: *ferromagnetic interactions*, *antiferromagnetic interactions*, the *exchange constant*, and the *Curie-Weiss constant* θ. State how these concepts relate to each other. [6]

(b) The differential work done on a magnetic sample may be written:

$$đW = B_0 \, \mathrm{d}m_z.$$

Carefully define and explain the terms B_0 and m_z on the right hand side of this equation including any assumptions made. [5]

(c) State what is meant by a *diamagnet* and how one may be identified by subjecting it to a magnetic field gradient. Explain why all chemical substances have a diamagnetic component to their susceptibility. [4]

(d) Define the *demagnetising factor* of an ellipsoidal sample and summarise its main properties. Hence explain why it is an advantage to formulate the magnetic susceptibility ($\chi = M/H$) in terms of the internal H-field rather than the applied H-field. [4]

(e) Show that the internal H-field in a spherical sample of a type-I superconductor is a factor (3/2) larger than the applied H-field. [3]

(f) By considering the H- and B-fields near the equator of a spherical sample, argue for the existence of the *mixed state* of a type-I superconductor at applied fields $B_0 > (2/3)B_c$ where B_c is the critical field. [4]

(g) A measurement of the Curie-Weiss constant on a spherical sample found $C = 3.3$ K, $\theta = 4.5$ K, but no demagnetising factor correction was made. Calculate the value of θ that would have been obtained if the demagnetising factor correction had been applied. [4]

END OF PAPER