Chapter 3

Electroweak Interactions: Quark Mixing

3.1 Introduction

In section1.2 the different quark flavours u, d, s, c, b, t were introduced. The strong and electromagnetic interactions conserve flavour, while W boson exchange of the weak interaction changes flavour. That's why flavour transitions in general involve a change of electric charge (the $W^{\pm}$ boson carries electric charge). Both leptons and quarks come in three generations of doublets (see eq. 1.1 and 1.29). For quarks to first order the flavour transitions via W boson exchange couple an "up-type" quark (u,c,t) with electric charge +2/3 to a "down-type" (d,s,b) quark with electric charge $-1/3$. Couplings to the anti-quarks are as always implied. The exchange of a weak Z Boson preserves flavour and so called neutral flavour changing currents are suppressed. This can be easily seen from the associated Feynman diagrams (see lecture slides).

3.2 Quark Mixing

3.2.1 Evidence

The decays of hadrons showed that for quarks a mixing across generations is possible. For example Kaon decays ($K^- \to \mu^- \bar{\nu_\mu}, K^0 \to \pi^+ \pi^-$) show a transition from the second generation into the first, a $s \to u$ transition. Experiments found however, that the decay across generations is suppressed with respect to decays within a doublet of a generation.

3.2.2 The Cabbibo Angle and GIM mechanism

Niccolo Cabbibo found an elegant solution to the problem. At the time only three quarks (u,d & s) where known. He proposed that the weak eigenstates of the two known "down-type" quarks (d' s') were actually a linear superposition of the quark mass eigenstates (d s):

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \tag{3.1}$$

$$\begin{aligned} d' &= +d\cos\theta_C + s\sin\theta_C \\ s' &= -d\sin\theta_C + s\cos\theta_C \end{aligned}$$

As for the neutrino mixing we can think of this as a rotation into a different basis using a rotation matrix. That "down-type" quarks are rotated and not "up-type" is pure convention, as the Cabbibos solution also helped to solve another problem. Certain Kaon decays were not observed. These decays proceeded through both an u-s and s-d

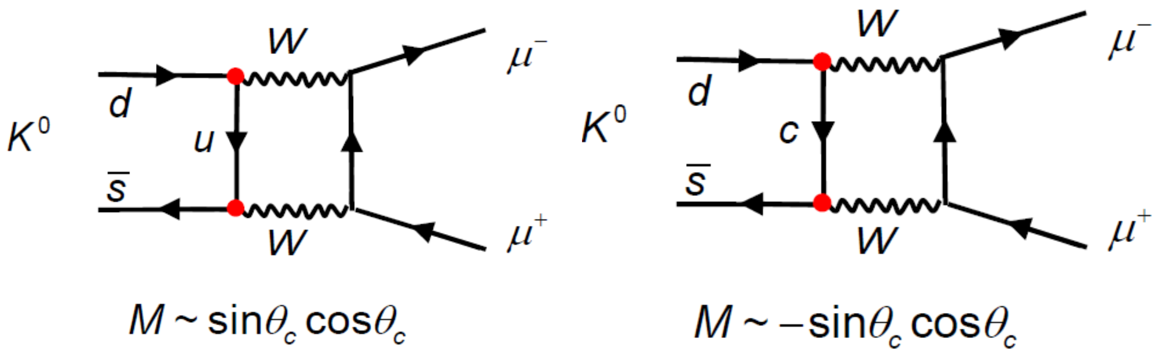

Figure 3.1: An application of the GIM mechanism. The two diagrams pick up an opposite sign and cancel out.

transition. The suppression of these decays could be explained by a cancellation if a fourth quark was involved (with a resulting u-c and c-d transition). This was proposed in 1970 by Sheldon Lee Glashow, John Iliopoulos and Luciano Maiani and is known as the GIM mechanism. The charm quark was eventually discovered in 1974.

3.2.3 The CKM mixing matrix

The Cabbibo-Kobayashi-Maskawa (CKM) matrix is an extension of the mixing formalism to three families or generations.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{3.2}$$

Unitary conditions

$$V_{CKM}V_{CKM}^{\dagger} = V_{CKM}^{\dagger}V_{CKM} = \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.3}$$

The matrix equation 3.3 leads to 9 constraints. In particular the diagonals of the identity matrix yield:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \tag{3.4}$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \tag{3.5}$$
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 \tag{3.6}$$

This guarantees that the total size of the coupling is kept the same. The zeros in the identity matrix lead to constrains that can be expressed as a triangle in the complex plane. For example the bottom left corner reads:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{3.7}$$

The equation 3.7 can be visualized as the triangle in the complex plane as seen in Fig. 3.2.

A popular parametrisation of the CKM matrix was given by Wolfenstein:

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{3.8}$$

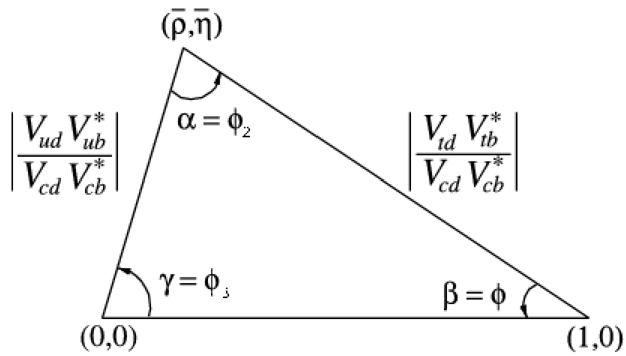

Figure 3.2: The unitarity triangle in the complex plane, visualizing equation 3.7.

Where $\lambda = \sin\theta_{cabbibo} \sim 0.2$ and $A \sim 0.8$. It can be seen, that transitions within the same generation are favoured, while transitions across generations are highly suppressed. Or in other words $V_{ud} \sim V_{cs} \sim V_{tb} \sim O(1) >> V_{us} \sim V_{cd} \sim V_{cb} \sim V_{ts} >> V_{ub} \sim V_{td}$. The 3x3 case with three generations is special as it is the smallest number of generations where the mixing parameters contain a non-removable complex phase. All possible transitions between quark flavours are summarised in Fig. 3.3. Each transition amplitude has a vertex factor of the corresponding CKM matrix element. We can see that for example charm decays into a down quark are less likely than those into a strange quark. The ratio of these decays is given by ratio of the squared amplitudes $|V_{cd}|^2/|V_{cs}|^2 = |-\lambda|^2/|1-\lambda^2|^2 \sim |0.2|^2/|1-0.04|^2 \sim 0.04$.

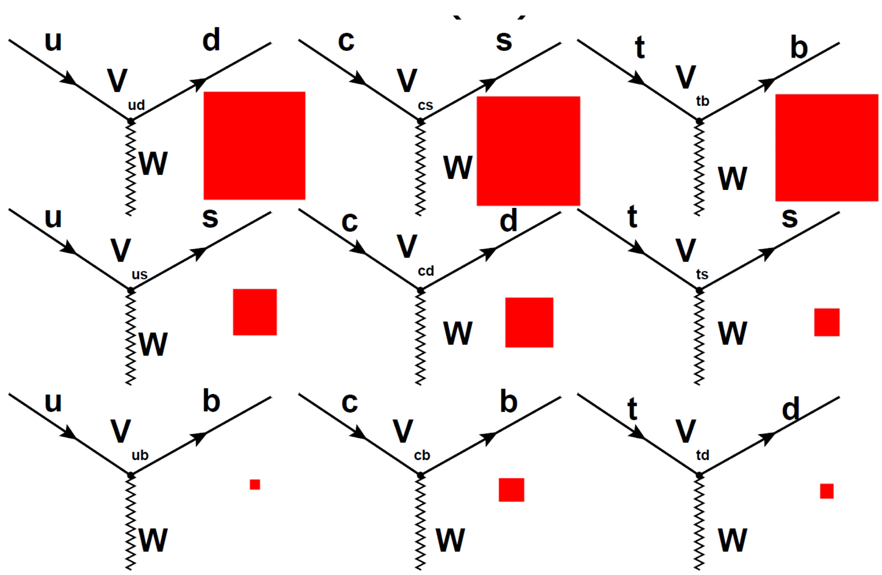

Figure 3.3: The possible transitions and their vertex factor.

3.2.4 Kaon states and CP violation

The neutral Kaon K^0 a $\bar{s}d$ state and its anti-particle the $\bar{K^0}$ a $s\bar{d}$ state form an interesting system. Charge conjugation turns each particle into its anti-particle, hence neither K^0 nor $\bar{K^0}$ are eigenstates of the charge conjugation operator

$\hat{C}$ nor the combined operator of parity and charge conjugation operator $\hat{CP}$:

$$\hat{C}|K^0\rangle = -|\bar{K^0}\rangle; \;\; \hat{C}|\bar{K^0}\rangle = -|K^0\rangle \tag{3.9}$$
$$\hat{P}|K^0\rangle = -|K^0\rangle; \;\; \hat{P}|\bar{K^0}\rangle = -|\bar{K^0}\rangle \tag{3.10}$$
$$\rightarrow \hat{CP}|K^0\rangle = +|\bar{K^0}\rangle; \;\; \hat{CP}|\bar{K^0}\rangle = +|K^0\rangle \tag{3.11}$$

Using the usual conventions. It turns out that a process involving the so called box diagram shown in Fig.3.4 can turn a K^0 into a $\bar{K^0}$ and vice versa. This means that the observed particles can be super positions of K^0 and $\bar{K^0}$ states. We can chose these super positions to be $\hat{CP}$ eigenstates as shown in eq. 3.15. The signs become obvious, if we note, that $\hat{CP}$ simply turns a K^0 into a $\bar{K^0}$ and vice versa.

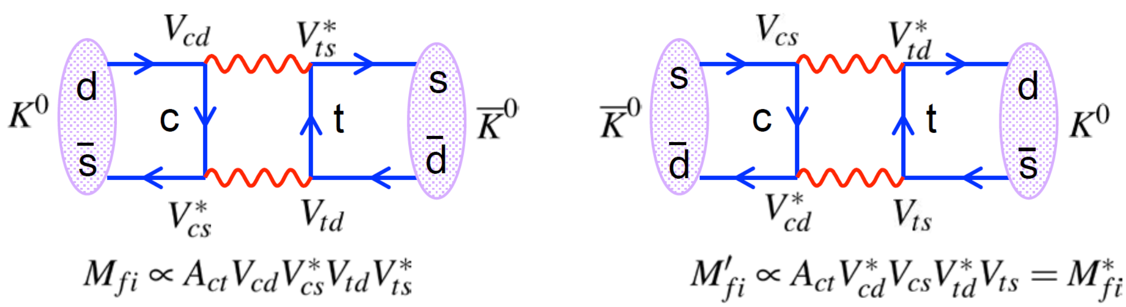

Figure 3.4: The box diagram exchanging 2 $W-$bosons that convert a K^0 into a $\bar{K^0}$ and vice versa.

$$|K^0_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K^0}\rangle\right) \tag{3.12}$$
$$|K^0_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K^0}\rangle\right) \tag{3.13}$$
$$\hat{CP}|K^0_2\rangle = \frac{1}{\sqrt{2}}\left(\hat{CP}|K^0\rangle - \hat{CP}|\bar{K^0}\rangle\right) = \frac{1}{\sqrt{2}}\left(|\bar{K^0}\rangle - |K^0\rangle\right) \tag{3.14}$$
$$\rightarrow \hat{CP}|K^0_1\rangle = +|K^0_1\rangle; \;\; \hat{CP}|K^0_2\rangle = -|K^0_2\rangle \tag{3.15}$$

Lets now look at possible decays of these two states K^0_1 and K^0_2. Both K^0 and $\bar{K^0}$ can decay into $\pi\pi$ states as shown in diagram 3.5. Such two pion states are $\hat{CP}$ even: $\hat{CP}(\pi\pi) = +1$.

$$CP(\pi^0\pi^0) = (P_{\pi^0})^2(-1)^L(C_{\pi^0})^2 = (-1)^2(-1)^0(1)^2 = +1 \tag{3.16}$$

Where we have used the parity ($P(\pi^0) = -1$) and charge conjugation $C(\pi^0) = 1$ values for the π^0 as derived in the $\theta - \tau$ puzzle. For the $\pi^+\pi^-$ state one can also use the observation that both $\hat{P}$ and $\hat{C}$ interchange the π^- with the π^+ and hence the combination $\hat{CP}$ leaves the $\pi^-\pi^+$ state unchanged.

If $\hat{CP}$ is conserved on the $\hat{CP}$ even K^0_1 state should be able to decay into two pions. The $\hat{CP}$ odd K^0_2 state will have to decay into a $\hat{CP}$ odd final state. A state formed of three pions $\pi\pi\pi$ is such a state.

$$CP(\pi^0\pi^0\pi^0) = (P_{\pi^0})^3(-1)^L(C_{\pi^0})^3 = (-1)^3(-1)^0(1)^3 = -1 \tag{3.17}$$

The decay into $\pi\pi\pi$ is suppressed by phase space ($Q = m(K^0) - 3 \times m(\pi) \sim 498$ MeV $- 3 \times (135$ MeV$) = 94$ MeV) (see Fig. 3.6). We hence expect the K^0_2 state to live longer than the K^0_1 state. The observed states are aptly named $K^0_{long} \sim K^0_2$ with $\tau_l \sim 5.2 \times 10^{-8}s$ and $K^0_{short} \sim K^0_1$ with $\tau_s \sim 8.9 \times 10^{-11}s$

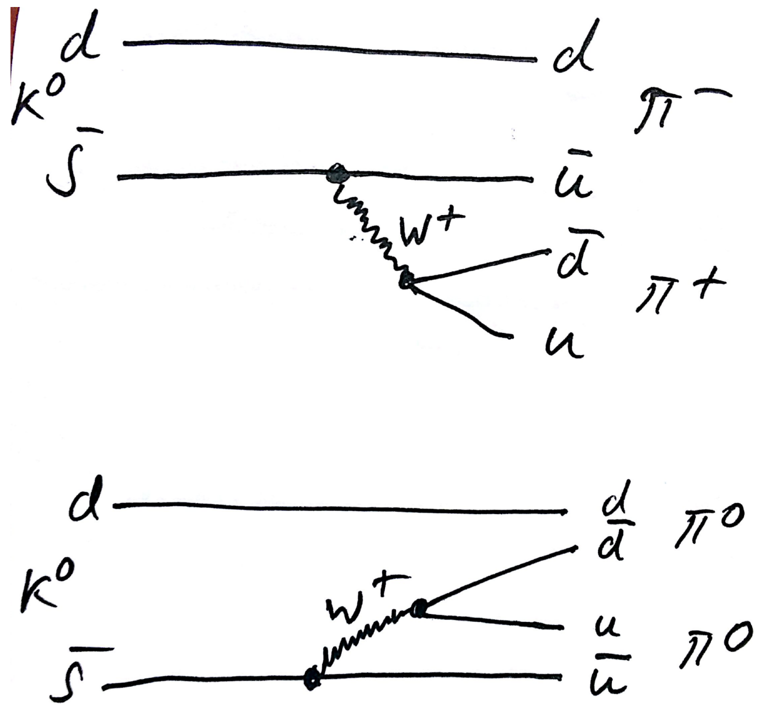

Figure 3.5: The Feynman diagram for the decay $K^0 \to \pi\pi$.

The different lifetimes of these states mean that an originally created K^0 will with time oscillate into a $\bar{K^0}$. This can be seen if we use eq. 3.15 to show K^0 as a superposition of K_2^0 and K_1^0 states:

$$|K^0\rangle \quad = \quad \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right) \sim \frac{1}{\sqrt{2}}\left(|K_s^0\rangle(t) + |K_L^0\rangle(t)\right) \tag{3.18}$$

$$\sim \quad \frac{1}{\sqrt{2}}\left(|K_s^0\rangle e^{-im_s - \frac{t}{2\tau_s}} + |K_L^0\rangle e^{-im_l - \frac{t}{2\tau_l}}\right) \tag{3.19}$$

$$\tag{3.20}$$

The time evolution will progress in a similar fashion as for neutrino oscillations, but now the lifetime difference has to be also taken into account. Meson oscillation phenomena have been observed in many neutral mesons:
$K^0 = |d\bar{s}\rangle$ $D^0 = |\bar{u}c\rangle$ $B_d^0 = |\bar{b}d\rangle$ $B_s^0 = |\bar{b}s\rangle$
$\bar{K^0} = |\bar{d}s\rangle$ $\bar{D^0} = |u\bar{c}\rangle$ $\bar{B_d^0} = |b\bar{d}\rangle$ $\bar{B_s^0} = |b\bar{s}\rangle$

The corresponding mass and lifetime configurations for these systems are shown in Fig. 3.7.

So far we have assumed $\hat{CP}$ is conserved. E.g that the $K_{short}^0 \sim K_1^0$ can only decay into the $\hat{CP}$ even $\pi\pi$ state and $K_{long}^0 \sim K_2^0$ only into the $\hat{CP}$ odd $\pi\pi\pi$ state. However Chronin and Fitch found that a small fraction of the longer lived K_L^0 do decay via two pions. This was a clear sign of CP violation. There are two possible explanations for this. First the physical states $|K_s^0\rangle$ and $|K_L^0\rangle$ might not be perfectly equal to the CP eigenstates K_1^0 and K_2^0 but contain a small contamination ϵ of the "wrong" CP state. This possibility is shown in eq. 3.22.

$$|K_L^0\rangle \quad = \quad \frac{1}{\sqrt{1+\epsilon^2}}\left(|\epsilon K_1^0\rangle + |K_2^0\rangle\right) \tag{3.21}$$

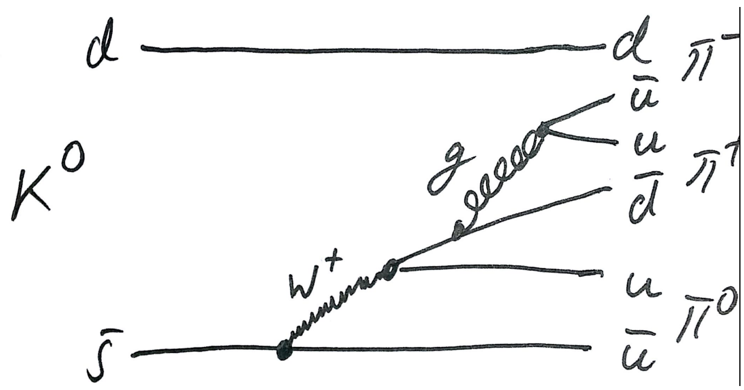

Figure 3.6: The Feynman diagram for the decay $K^0 \to \pi\pi\pi$.

$$|K_s^0\rangle = \frac{1}{\sqrt{1+\epsilon^2}}\left(|K_1^0\rangle - \epsilon|K_2^0\rangle\right) \tag{3.22}$$

The second possibility is that CP is not conserved in the decay of the CP eigenstates K_1^0 and K_2^0. This is shown in eq. 3.23 and known as direct CP violation.

$$\langle\pi\pi|K_2^0\rangle = \epsilon' \neq 0 \tag{3.23}$$

Measurements show that both effects exist, but are small. It was found that $\epsilon = (2.232 \pm 0.007) \times 10^{-3}$ and $\left(\frac{\epsilon'}{\epsilon}\right) = (14.7 \pm 2.2) \times 10^{-4}$.

CP violation effects have also been found in bottom mesons and baryons. Precision measurements of B-hadron decays have greatly improved the understanding of CP violation and the CKM matrix. Unfortunately all CP violating effects found so far are too small to explain the baryon asymmetry of the universe. The origin of the known CP violation is understood to be the complex phase $\rho - i\eta$ of the CKM matrix. A look at the box diagrams for K^0 and $\bar{K^0}$ (fig. 3.4) reveals a difference between the transition matrix elements for these particles if $V \neq V^*$. The Jarlskog invariant $J = Im(V_{ij}V_{kl}V_{ij}^*V_{kl}^*)$, named after Swedish scientist Cecilia Jarlskog, which is a phase-convention-independent measure of CP violation is given by the area of the CKM triangles.

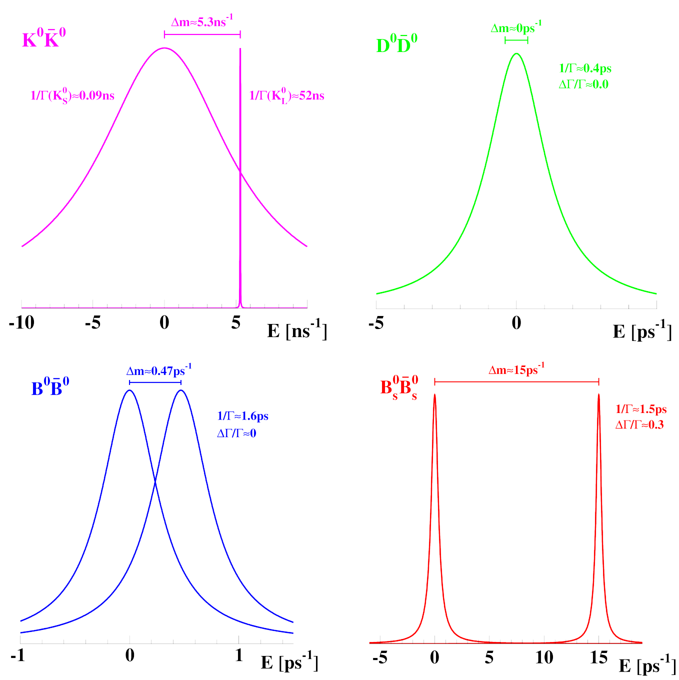

Figure 3.7: Shown are the mass and lifetime configurations in form of a Breit-Wigner resonance for $K^0 = |d\bar{s}\rangle$ $D^0 = |\bar{u}c\rangle$ $B^0_d = |\bar{b}d\rangle$ $B^0_s = |\bar{b}s\rangle$ mesons. The large lifetime difference between $|K^0_s\rangle$ and $|K^0_L\rangle$ is visible in the top left. For B-mesons in the bottom row, the mass difference is more important and the corresponding states are referred to as B_{Light} and B_{Heavy}.