Electrostatic field : Field due to a stationary charge
Induced field : Due to a changing magnetic field.

Divergence theorem: $\int_S E.dS = \int_V \nabla . E dV$

Stoke's theorem : $\oint_C E.dl = \int_S \nabla \times E.dS$

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EMT 2016

SECTION A

(1) $E = \frac{1}{2}(x^3\hat{x} + y^3\hat{y} + z^3\hat{z})$

(a) Electrostatic fields are conservative $\rightarrow$ $E = -\nabla V$

Curl of conservative fields are 0.

$$\nabla \times E = \frac{1}{2}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$$

$\therefore$ $E = \frac{1}{2}(x^3\hat{x} + y^3\hat{y} + z^3\hat{z})$ could represent an electrostatic field.

(b) Surface integral $\oint_S E.n\,da = \frac{q}{\epsilon_0}$

$$\oint_S E.n\,da = \int \frac{\rho\,dV}{\epsilon_0}$$

$$\int_V \nabla.E\,dV = \frac{1}{\epsilon_0}\int_V \rho\,dV$$

$$\boxed{\epsilon_0 \int_V \nabla.E\,dV = Q}$$

$$\nabla.E = \frac{\rho}{\epsilon_0}$$

$$\nabla.E = \frac{3}{2}x^2 + \frac{3}{2}y^2 + \frac{3}{2}z^2$$

$$= \frac{3}{2}(x^2+y^2+z^2)$$

$$Q = \frac{3}{2}\epsilon_0\int_0^{\pi} d\theta \int_0^{2\pi} d\phi \int_0^{a} r^4 \sin\theta\,dr$$

$$= \frac{3\epsilon_0}{2}\int_0^{\pi} d\theta\; 2\pi \left[\frac{r^5}{5}\right]_0^a \sin\theta$$

$$= \frac{3\pi a^5 \epsilon_0}{5}\int_0^{\pi} \sin\theta\,d\theta$$

$$= \frac{3\pi a^5 \epsilon_0}{5}\left[-\cos\theta\right]_0^{\pi}$$

$$\boxed{Q = \frac{6\epsilon_0 \pi a^5}{5}}$$

$\nabla.E = \frac{\rho}{\epsilon_0}$

$\oint_C E\,da = \int \nabla.E\,dV$

(2) (a) Invariant quantity: Quantity that remains unchanged under a certain transformation.
Lorentz invariant is an invariant quantity that remains unchanged under Lorentz tran

(I) The charge of an electron : Invariant
(II) The speed of light in a vacuum: Invariant
(III) The charge density of a cloud of electrons : Not invariant
(IV) E.B : Invariant.

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Stoke's theorem: $\oint_C E.d\ell = \int_S (\nabla\times E).\hat{a}da$

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(b) Interval between two events $(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - c^2(dt)^2$

(I) Spacelike: $(ds)^2 > 0$

(II) Timelike: $(ds)^2 < 0$

$\int_S (\nabla\times E).\hat{a}da = -\frac{\partial}{\partial t}\int_S B.\hat{a}da$

(3) Gauss's Law: } in differential form: $\nabla.E = \frac{\rho}{\epsilon_0}$

Faraday's law: $\nabla\times E = -\frac{\partial B}{\partial t}$

$\oint_C E.d\ell = -\frac{\partial}{\partial t}\underbrace{\int_S B.\hat{a}da}_{\Phi}$

$\oint_C E.d\ell = -\frac{\partial \Phi}{\partial t}$

$\nabla.E = \frac{\rho}{\epsilon_0}$

$\nabla\times E = -\frac{\partial B}{\partial t}$

$\int_V \nabla.E\, dV = \frac{1}{\epsilon_0}\int_V \rho\, dV$

Taking surface integral

Using Divergence theorem: $\int_S F.dS = \int_V \nabla.F\, dV$

$\int_S (\nabla\times E).dS = -\frac{\partial}{\partial t}\int_S B.dS$

$\int_S E.dS = \frac{Q}{\epsilon_0}$

Using Stoke's theorem: $\oint_C E.d\ell = \int_S \nabla\times E.dS$

$\nabla.E = \frac{\rho}{\epsilon_0}$ GAUSS $\int_S E.\hat{a}da = \int_V \nabla.E\, dV$

$\oint_C E.d\ell = -\frac{\partial}{\partial t}\int_S B.dS$ → Φ : FLUX

$\nabla\times E = -\frac{\partial B}{\partial t}$ FARADAY $\int_S E.\hat{a}da = \frac{1}{\epsilon_0}\int_V \rho\, dV$

$\oint_C E.d\ell = -\frac{\partial \Phi}{\partial t}$

$\int_S E.\hat{a}da = \frac{Q}{\epsilon_0}$ ✓

(4) (a) Retarded time: $t' = t - r/c$

The retarded time is the time when the field began to propagate from the point where it was emitted to an observer. "Retarded" is used ~~to~~ in the sense of propagation delays.

A wave takes time $\frac{r}{c}$ to reach observer due to finite speed. Therefore, an observer experiences a charge distribution at a time retarded by $\frac{r}{c}$.

(b) Show that the differentials of a function of retarded time, t', wrt time & position are:

$$\frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'} \qquad \frac{\partial f(t')}{\partial r} = -\frac{1}{c}\frac{\partial f(t')}{\partial t'}$$

$$\frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'}\frac{\partial t'}{\partial t}$$

$$\cancel{\frac{\partial f(t')}{\partial r} = \frac{\partial f(t')}{\partial r}\frac{\partial r}{\partial r}}$$

using $t' = t - r/c$

$$\frac{dt'}{dt} = 1$$

$$\frac{\partial f(t')}{\partial r} = \frac{\partial f(t')}{\partial t'}\frac{\partial t'}{\partial r} \qquad \frac{\partial t'}{\partial r} = -\frac{1}{c}$$

$$= -\frac{1}{c}\frac{\partial f(t')}{\partial t'}$$

$$\therefore \frac{\partial f(t')}{\partial t} = \frac{\partial f(t')}{\partial t'}$$

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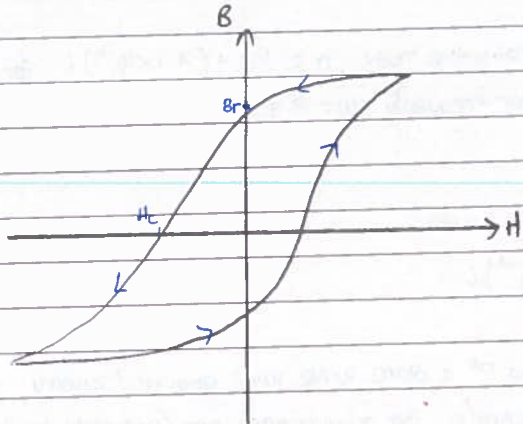
(5) (a) standard ~~hysteresis~~ hysteresis loop



Remanence, B_r : value of B when H is returned to 0.

Coercivity, H_c : value of H required to reduce B to 0 after saturation.

(b) For permanent magnets → must use hard magnetic material (large coercivity and remanence)

C → Because it is hard to demagnetise once magnetised.

(6) Fresnel equations for reflection of an electromagnetic wave that is incident in air ($\epsilon_r = 1$, $\mu_r = 1$) on the plane surface of a dielectric material are given by

$$r_{\parallel} = \frac{\cos\theta_t - n\cos\theta_i}{\cos\theta_t + n\cos\theta_i} \qquad r_{\perp} = \frac{\cos\theta_i - n\cos\theta_t}{\cos\theta_i + n\cos\theta_t}$$

θ_i : The angle of incidence

θ_t : The angle of transmission

(a) Physical significance of the symbols $r_{\parallel}$, $r_{\perp}$ and n in these equations.

$r_{\parallel}$: Gives amplitude reflection of parallel component of wave

$r_{\perp}$: Gives amplitude reflection of ~~transverse~~ transverse component of wave.

n : Refractive index of dielectric ; $n = \sqrt{\epsilon_r}$

(b) $\theta_i = \theta_r$

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$\sin\theta_i = n_2 \sin\theta_t$.

(c) $\underline{k} \cdot \underline{d} = \underline{k}' \cdot \underline{d} = \underline{k}'' \cdot \underline{d}$ $\qquad$ $\underline{k}$ is in direction of travel for each of the 3 waves (incident, reflect and transmitted waves).

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SECTION B

(7) (a) Amorphous silica : complex refractive index, $n = 1.2 + (4.1\times10^{-11})i$ for infrared radiation with an angular frequency, $\omega = 9.4\times10^{14}\ s^{-1}$.

(I) $k = \frac{n\omega}{c}$

$k = (3.76\times10^{6}) + (1.2846\times10^{-4})i$

(II) Expression for the electric field of a plane wave with angular frequency $9.4\times10^{14}\ s^{-1}$ travelling through amorphous silica (in the x-direction) and (polarized in the z-direction)

$\underline{E} = E_0 e^{i(kx-\omega t)}\hat{z}$

$= E_0 e^{i([(3.76\times10^{6})+(1.2846\times10^{-4})i]x-(9.4\times10^{14})t)}\hat{z}$

$= E_0 e^{-(1.2846\times10^{-4})x} e^{i[(3.76\times10^{6})x-(9.4\times10^{14})t]}\hat{z}$

(III) Amplitude of the electric field → after travelling 1.0 km.

Subs. $x = 1\times10^{-3}$

$= E_0 e^{-(1.2846\times10^{-4})(10^{-3})}$

$= 0.87944\ldots E_0$

$= 0.88 E_0$

(IV) Phase velocity ⇒ $v_p = \frac{\omega}{k}$

$= \frac{\cancel{(2.5\times10^{8})}\ 9.4\times10^{14}}{(3.76\times10^{6})+(1.2846\times10^{-4})i} \times \frac{(3.76\times10^{6})-(1.2846\times10^{-4})i}{(3.76\times10^{6})-(1.2846\times10^{-4})i}$

DON'T FORGET THE UNITS!

$= (2.5\times10^{8}) - i(8.541\times10^{-3})\ ms^{-1}$

(b) Toroidal ring : internal radius, R

square cross section → side length a.

LIH material — Coil N turns → Carries a current I.

has relative permeability, μ_r.

Assume: All of the magnetic flux is restricted to the ring.

(I) LIH ⇒ Linear: χ_e is independent of $\underline{E}$ (or χ_m of $\underline{B}$) $\underline{P}$ depends linearly on $\underline{E}$

Isotropic: $\underline{P}$ is parallel to $\underline{E}$ (or $\underline{M}$ to $\underline{H}$)

Homogeneous: χ_e does not vary with position

For LIH material, we can write $\underline{P} = \epsilon_0\chi_e\underline{E}$

since $\underline{D} = \epsilon_0\underline{E} + \underline{P}$

$= \epsilon_0(1+\chi_e)\underline{E}$

$\epsilon_0\epsilon_r = \epsilon$

$\epsilon_r = (1+\chi_e)$

$\nabla \times \underline{H} = J_f + \frac{\partial D}{\partial t}$ $\quad$ $\nabla \times \underline{E} = -\frac{\partial B}{\partial t}$ $\quad$ $\nabla \cdot \underline{D} = \rho_f$ $\quad$ $\nabla \cdot \underline{B} = 0$

↓
Ampere-Maxwell

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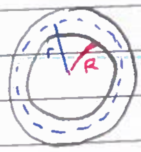
(I) Show the magnetic intensity within the material of the ring:

$$\underline{H} = \frac{NI}{2\pi r}\hat{\phi}$$

$\nabla \times \underline{H} = \underline{J}_f + \frac{\partial \underline{D}}{\partial t}$

$\nabla \times \underline{H} = \underline{J}$



Ampere's law: $\nabla \times \underline{H} = \underline{J}_f$

Using Stoke's theorem

$$\oint_C \underline{H} \cdot d\underline{l} = \int_S \underline{J}_f \cdot d\underline{S} = I$$

$\int_S (\nabla \times \underline{H}) \cdot n\, da = \int_S \underline{J} \cdot n\, da$

$\oint_C \underline{H} \cdot d\underline{l} = \int_S \underline{J} \cdot n\, da$

since there are N turns,

$$\oint_C \underline{H} \cdot d\underline{l} = NI$$

$d\underline{l} = dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{z}$

$$2\pi r \underline{H} = NI\hat{\phi}$$

$$\underline{H} = \frac{NI}{2\pi r}\hat{\phi}$$

(GET THE PROPER WAY TO INCLUDE ANGLE)

(II) $\underline{M} = \chi_m \underline{H}$ $\qquad$ $\underline{B} = \mu_0(\underline{H} + \underline{M})$

$\underline{M} = \frac{\chi_m NI}{2\pi r}\hat{\phi}$ $\qquad$ $\underline{B} = \mu_0(\underline{H} + \chi_m \underline{H})$

$$= \mu_0(1 + \chi_m)\underline{H}$$

$1 + \chi_m = \mu_r$

$$\underline{B} = \mu_0 \mu_r \underline{H}$$

$$\underline{B} = \mu_0 \mu_r \frac{NI}{2\pi r}\hat{\phi}$$

(IV) Energy density associated with a magnetic field → $U = \frac{1}{2}\underline{B} \cdot \underline{H}$

$$U = \frac{1}{2}\left(\frac{\mu_0 \mu_r NI}{2\pi r}\hat{\phi}\right) \cdot \left(\frac{NI}{2\pi r}\hat{\phi}\right)$$

$$= \frac{1}{2}\left[\mu_0 \mu_r \left(\frac{NI}{2\pi r}\right)^2\right]$$

$dV = r\,dr\,d\phi\,dz$

Magnetic energy stored within the ring $= \frac{1}{2}\mu_0\mu_r\left(\frac{NI}{2\pi}\right)^2 \int \frac{1}{r^2}\, dV$

→ BECAUSE OF THE SQUARE CROSS SECT

$$= \frac{1}{2}\mu_0\mu_r\left(\frac{NI}{2\pi}\right)^2 \int_0^a dz \int_0^{2\pi} d\phi \int_R^{R+a} \frac{1}{r}\, dr$$

$$= \frac{1}{2}\mu_0\mu_r\left(\frac{NI}{2\pi}\right)^2 2\pi a \left[\ln r\right]_R^{R+a}$$

$$= \frac{\mu_r \mu_0 N^2 I^2 a}{4\pi} \ln\left(\frac{R+a}{R}\right) \quad \#$$

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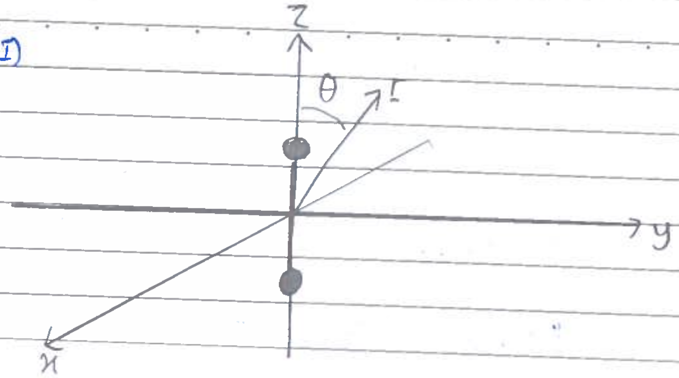
(8) (a) (I)



- The current dipole has strength $I_0 \delta l$ and oscillates with angular frequency ω.
- the magnetic field that it produces is given by the real part of the expression

$$\underset{\sim}{B}(\underset{\sim}{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left[-\frac{i\omega}{rc} + \frac{1}{r^2} \right] e^{i(kr-\omega t)} \hat{\phi}$$

3 contributions to the fields : produced by a Hertzian dipole

r^{-3} : Electrostatic field

r^{-2} : Induction field

r^{-1} : Radiation field

A. limit of small distances r and zero angular frequency, with $\omega \to 0$ faster than r

$$\underset{\sim}{B}(\underset{\sim}{r}) = \frac{\mu_0 I_0 \delta l \sin\theta}{4\pi} \left[\frac{-i\omega}{rc} + \frac{1}{r^2} \right] \left\{ \cos(kr-\omega t) + i\sin(kr-\omega t) \right\} \hat{\phi}$$

$$= \frac{\mu_0 I_0 \delta l \sin\theta}{4\pi} \left\{ -\frac{i\omega}{rc} \cos(kr-\omega t) + \frac{\omega}{rc} \sin(kr-\omega t) + \frac{1}{r^2} \cos(kr-\omega t) + \frac{i}{r^2} \sin(kr-\omega t) \right\} \hat{\phi}$$

ignore

$$Re[\underset{\sim}{B}(\underset{\sim}{r})] = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left(\frac{\omega}{rc} \sin(kr-\omega t) + \frac{1}{r^2} \cos(kr-\omega t) \right) \hat{\phi}$$

$$\underset{\sim}{B}(\underset{\sim}{r}) = \frac{\mu_0 I_0 \delta l \sin\theta}{4\pi} \left[\frac{1}{r^2} \right] \left\{ 1 \right\} \hat{\phi}$$

$$\underset{\sim}{B}(\underset{\sim}{r}) = \frac{\mu_0 I}{4\pi} \frac{\delta \underset{\sim}{l} \times \hat{r}}{r^2}$$

$$\delta \underset{\sim}{l} \times \hat{r} = |\delta l| |\hat{r}| \sin\theta \hat{\phi}$$

$$= \underbrace{\delta l \sin\theta \hat{\phi}}_{\text{replace these with}}$$

No: .... $\frac{1}{r^2}$ term will disappear: Date: ....

(B) Limit of large r: $\underline{B}(\underline{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left[\frac{-i\omega}{rc} + \frac{1}{r^2} \right] e^{i(kr-\omega t)} \hat{\phi}$

$$\underline{B}(\underline{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left[\frac{-i\omega}{rc} \right] \underbrace{e^{i(kr-\omega t)}}_{} \hat{\phi}$$

$$\underline{B} \propto \frac{1}{r}$$

r here is ?

(II) Condition for the radiation term in the expression for the field B to dominate

$$\frac{-i\omega}{rc} \gg \frac{1}{r^2}$$

$$\frac{-i\omega}{c} \gg \frac{1}{r} \qquad \omega = 2\pi f \qquad c = f\lambda$$

$$\omega = \frac{2\pi c}{\lambda} \qquad \frac{c}{\lambda} = f$$

$$\frac{-i2\pi}{\lambda} \gg \frac{1}{r}$$

$$r \gg \lambda i 2\pi$$

Not including $-i$ $\Rightarrow$ $\frac{\omega}{rc} \gg \frac{1}{r^2}$

$$\frac{2\pi}{\lambda} \gg \frac{1}{r}$$

$$\boxed{r \gg \frac{\lambda}{2\pi}}$$

(III) Limit of large distances, the electric field that is associated with the radiation magnetic field is given by the real part of

$$\underline{E}(\underline{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left(\frac{-i\omega}{r} \right) e^{i(kr-\omega t)} \hat{\underline{\theta}}$$

Find the real part: $\frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left(\frac{-i\omega}{r} \right) \left\{ \cos(kr-\omega t) + i\sin(kr-\omega t) \right\} \hat{\underline{\theta}}$

$$Re[\underline{E}(\underline{r})] = \frac{\mu_0 I_0 \delta l}{4\pi} \left(\frac{\omega}{r} \right) \sin\theta \sin(kr-\omega t) \hat{\underline{\theta}}$$

$S = \frac{1}{\mu_0}(E \times B)$

$\langle S \rangle = Re\left(\frac{1}{2\mu_0} \underline{E} \times \underline{B}^* \right)$

↓

~~From Wiki~~

$\underline{B} = \mu \underline{H}$

$$Re[\underline{B}(\underline{r})] = \frac{\mu_0 I_0 \delta l}{4\pi} \left(\frac{\omega}{rc} \right) \sin\theta \sin(kr-\omega t) \hat{\underline{\phi}}$$

$$\frac{1}{\mu_0} \left(\frac{\mu_0 I_0 \delta l}{4\pi} \right)^2 \left(\frac{\omega^2}{r^2 c} \right) \sin^2\theta \overbrace{\sin^2(kr-\omega t)}^{1/2} \; ?$$

$$\frac{1}{2\mu_0} \left(\frac{\mu_0 I_0 \delta l}{4\pi} \right)^2 \left(\frac{\omega^2}{r^2 c} \right) \sin^2\theta .$$

$$\underline{B}(\underline{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left(\frac{i\omega}{rc} \right) e^{-i(kr-\omega t)} \hat{\phi}$$

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$$B(\underline{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left(-\frac{i\omega}{rc} \right) e^{i(kr-\omega t)} \hat{\underline{\phi}}$$

$$E(\underline{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left(\frac{-i\omega}{r} \right) e^{i(kr-\omega t)} \hat{\underline{\theta}}$$

$$\frac{1}{2\mu_0} E \times \underline{B}^* = \frac{1}{2\mu_0} \left(\left(\frac{\mu_0 I_0 \delta l}{4\pi} \right)^2 \sin^2\theta \left(\frac{\omega^2}{r^2 c} \right) \hat{\underline{r}} \right)$$

(e) Time averaged Poynting vector:

$$\bar{N} = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\, E_0^2 \,\hat{\underline{k}}$$

$\hat{\underline{k}}$: Direction of propagating energy.

E_0 : electric field amplitude

ϵ, μ : permitivity and permeability.

- Oscillating at 925 MHz
- Located in a vacuum at the origin of spherical coordinates (r, θ, ϕ)

– point $\underline{r}_1$: $r_1 = 5.0\,m$

$\theta_1 = 30°$

$\phi_1 = 60°$

- Time-averaged value of the Poynting vector has a magnitude $3.0 \times 10^{-3}\ Wm^{-2}$.

– Calculate the max possible value of the magnitude of the time-averaged Poynting vector at a distance of 20 m from the dipole.

NOT SURE WHETHER TO USE REAL COMPONENT OR THE WHOLE EQUATION.

① $Re[E(\underline{r})] = \frac{\mu_0 I_0 \delta l}{4\pi} \left(\frac{\omega}{r} \right) \sin\theta \, \sin(kr - \omega t) \, \hat{\underline{\theta}}$

OR

② $E(\underline{r}) = \frac{\mu_0 I_0 \delta l}{4\pi} \sin\theta \left(-\frac{i\omega}{r} \right) e^{i(kr-\omega t)} \hat{\theta}$

Assuming ① is the proper way: Then, $E_0 = \frac{\mu_0 I_0 \delta l}{4\pi} \left(\frac{\omega}{r} \right) \sin\theta$

$$\bar{N} = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}} \left(\frac{\mu_0 I_0 \delta l}{4\pi} \left(\frac{\omega}{r} \right) \sin\theta \right)^2$$

$\mu_0 = 4\pi \times 10^{-7}\ H/m$

$$\bar{N} = \sqrt{\frac{\epsilon}{\mu}} (I_0 \delta l)^2 \frac{1}{2} \left(\frac{\mu_0}{4\pi} \left(\frac{\omega}{r} \right) \sin\theta \right)^2$$

$\omega = 2\pi f = 2\pi(925 \times 10^6)\ Hz$

$$\bar{N} = \underbrace{\frac{1}{2}\sqrt{\frac{\epsilon}{\mu}} \left(\frac{\mu_0 I_0 \delta l}{4\pi} \right)^2}_{\text{let } A \text{ this equal } A} \omega^2 \, \frac{\sin^2\theta}{r^2}$$

$A = 0.3$

r = 20 m

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The max. possible → ∴ $\sin\theta = 1$ // $\theta = 90^0$

$$\bar{N} = 0.3 \frac{\sin^2\theta}{r^2}$$

$$\bar{N} = \frac{0.3}{20^2}$$

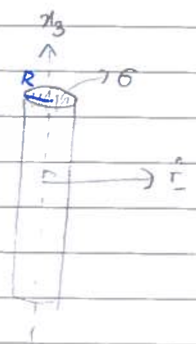
$$\bar{N} = 7.5\times10^{-4}\ Wm^{-2}$$

(c) space-time x_μ a long straight cylindrical insulating rod lying along the x_3-axis ~~has~~

crossed sectional area, σ

Uniformly charged - charge per unit length, λ defined in its rest frame.

External electric field → $\underline{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{\underline{r}}$



The electric potential relative to the surface of the rod is

$$\phi(r) = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)$$

R: radius of rod.

(I) Quoting the formulas from the front of the page

- The Lorentz transform for the space-time x_μ from $S \to S'$ moving with v in the x_3 direction

$$x_1' = x_1$$
$$x_2' = x_2$$
$$x_3' = \gamma x_3 + i\beta\gamma x_4$$
$$x_4' = \gamma x_4 - i\beta\gamma x_3$$

~~WHAT HAPPEN TO~~ HE imaginary i

- $\partial_\mu = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{-i}{c}\frac{\partial}{\partial t}\right)$ $x_\mu = (x, y, z, ict)$ $a_\mu = (A_x, A_y, A_z, \frac{i\phi}{c})$ $j_\mu = (J_x, J_y, J_z, ic\rho)$

⇒ • The four-current density inside is $j_\mu = (J_x, J_y, J_z, ic\rho)$

(ASK WHY) • The J values are 0.

(Because it is an insulating rod)

• $\rho = \frac{\lambda}{\sigma}$ The charge in length ℓ is $\ell\lambda$, we ~~de~~ divide by volume $\ell\sigma$ to obtain the charge density.

• The 4-potential outside the rod $a_\mu = (A_x, A_y, A_z, \frac{i\phi}{c})$

(ASK WHY) The A values are 0. ϕ is given in the question.

$$j_\mu = \left(0, 0, 0, \frac{c\lambda}{\sigma}\right) \quad a_\mu = \left(0, 0, 0, \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r}\right)\right)$$

inside rod

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Date: ....

No: ....

II

Ponro'j

No: .... Date: ....

(9) (a) (I) Plasma : A condition of matter containing an appreciable fraction of freely moving charged particles. There are sufficient no. of these charged particles to cause the electromagnetic properties of the medium to be significantly different from those of solids, liquids and gas.

Plasma is an ionised gas consisting of positive ions and free electrons such that there is no overall charge

(II) $$w_p = \sqrt{\frac{N_e e^2}{m_e \epsilon_0}}$$ For $N_e = 10^{12}$,

$5.63 \times 10^7\ s^{-1}$ (CHECK UNIT)

(III) $$k^2 = \frac{w^2}{c^2}\left(1 - \frac{w_p^2}{w^2}\right)$$

(A) $w \ll w_p$

$$k^2 = \frac{w^2}{c^2}\left(-\frac{w_p^2}{w^2}\right)$$

$$k^2 = -\frac{w_p^2}{c^2}$$

$k^2 < 0$, we have absorption of energy and damping over some attenuation length, L.

(B) $w \gg w_p$

$$k^2 = \frac{w^2}{c^2}$$

$k^2 > 0$, so k is real and there is no attenuation.

★ (IV) Electromagnetic plane wave → . angular frequency, w
- travelling in the z-direction
- normally incident on a dielectric window in a chamber containing a collisionless isotropic plasma that has plasma frequency, $w_p = 2\pi \times 10^8 s^{-1}$.

(a) Show that the ratio of the amplitude of the electric field in the plasma at a distance z from the window to the amplitude just inside the window when $w < w_p$ is given by

$$e^{-\frac{w_p\sqrt{1-\frac{w^2}{w_p^2}}}{c} z}$$

Assume that the field is constant across a plane with constant z within a 'beam' defined by the area of the window.

$$k^2 = \frac{w^2}{c^2}\left(1 - \frac{w_p^2}{w^2}\right)$$

$$k = \frac{w}{c}\left(1 - \frac{w_p^2}{w^2}\right)^{1/2}$$

Electromagnetic plane wave
↳ $E(z,t) = E_0 e^{i(kz - wt)}$

For $w < w_p$: $k = \frac{w}{c}(-1)^{1/2}\left(\frac{w_p^2}{w^2} - 1\right)^{1/2}$ ⇒ $k = \frac{wi}{c}\left(\frac{w_p^2}{w^2} - 1\right)^{1/2}$. ⇒ $k = \frac{wi}{c}\left(\frac{w_p}{w}\right)\left(1 - \frac{w^2}{w_p^2}\right)^{1/2}$.

$$k = \frac{i\omega_p}{c}\left(1 - \frac{\omega^2}{\omega_p^2}\right)^{1/2}$$

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$$E(z,t) = E_0\, e^{i(kz - \omega t)}$$

subs. $k = \frac{\omega i}{c}\left(\frac{\omega_p^2}{\omega^2} - 1\right)^{1/2}$

$$\cancel{E(z,t) = E_0\, e^{i\left[\left(\frac{\omega i}{c}\left(\frac{\omega_p^2}{\omega^2}-1\right)^{1/2}\right)z - \omega t\right]} = E_0\, e^{-\frac{\omega}{c}\left(\frac{\omega_p^2}{\omega^2}-1\right)^{1/2} z}\, e^{-i\omega t}}$$

$$k = \frac{i\omega_p}{c}\left(1 - \frac{\omega^2}{\omega_p^2}\right)^{1/2}.$$

$$E(z,t) = E_0\, e^{i\left[\left(\frac{i\omega_p}{c}\left(1 - \frac{\omega^2}{\omega_p^2}\right)^{1/2}\right)z - \omega t\right]}$$

$$= \underbrace{E_0\, e^{-\frac{\omega_p}{c}\left(1 - \frac{\omega^2}{\omega_p^2}\right)^{1/2} z}}_{\text{amplitude}}\; e^{-i\omega t}$$

The ratio of the amplitude : $\exp\left[-\frac{\omega_p\left(1 - \frac{\omega^2}{\omega_p^2}\right)^{1/2}}{c}\, z\right]$

(B) • $z = 0.5\,m$ $\quad \omega = 0.99\,\omega_p \quad \omega_p = 2\pi \times 10^8\, s^{-1}$

Ratio $\Rightarrow$ 0.86266 . . .

• Limiting value, $\frac{\omega}{\omega_p} \ll 1$

Ratio $\Rightarrow$ 0.350919 . . .

(b) (I) Skin depth: $\delta = \frac{1}{k_i}$ (Write proper explanation)

(II) $\underline{E}(\underline{r}, t) = \underline{E}_0 \exp\left[i(\underline{k}\cdot\underline{r} - \omega t)\right]$

medium → linear
→ finite conductivity, g
→ No free charge

$$\frac{k^2}{\omega^2} = \mu_0\mu_r\epsilon_0\epsilon_r\left(1 + i\frac{g}{\epsilon_0\epsilon_r\omega}\right)$$

Good conductor : $\boxed{g \gg \epsilon\omega}$

$$\frac{k^2}{\omega^2} = \mu_0\mu_r\epsilon\left(1 + i\frac{g}{\epsilon\omega}\right)$$

$$\frac{k^2}{\omega^2} = i\mu_0\mu_r\frac{g}{\omega}$$

$$k^2 = i\mu_0\mu_r g\omega$$

$$k = +\sqrt{i\mu_0\mu_r g\omega}$$

$$\sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$$

$$\therefore k = \frac{1}{\sqrt{2}}(1+i)\sqrt{\mu_0\mu_r g\omega}$$

$$k = \sqrt{\frac{\mu_0\mu_r g\omega}{2}} + i\sqrt{\frac{\mu_0\mu_r g\omega}{2}}$$

$$k = k_r + i k_i$$

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since skin depth => $\delta = \frac{1}{k_i}$

$= \sqrt{\frac{2}{\mu_0 \mu_r g \omega}}$

(III) $v_p = \frac{\omega}{k}$ phase velocity in non-magnetic field

$= \omega \delta$

$= \omega \sqrt{\frac{2}{\mu_0 \mu_r g \omega}}$ $\delta = 3.0 \mu m$

$= (8.0 \times 10^{10})(2\pi)(3 \times 10^{-6})$

$v_p = 1.5079.. \times 10^6 \ ms^{-1}$

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