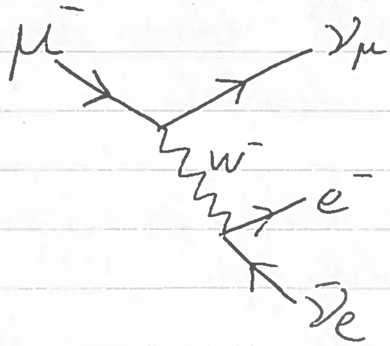
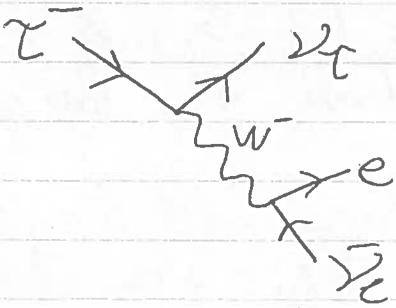
NPP - 2015

1. e^-, μ^-, τ^-
Stable unstable unstable





2. a) The ratio of the cross section for $e^+e^- \to$ Hadrons to the cross section of $e^+e^- \to \mu^+\mu^-$ (Excluding ~~..~~ $t\bar{t}$):

$$\frac{\sigma(u\bar{u}) + \sigma(d\bar{d}) + \sigma(c\bar{c}) + \sigma(b\bar{b}) + \sigma(s\bar{s})}{\sigma(\mu^+\mu^-)} \approx N_c$$

$$= N_c \; \frac{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2}{1^2} = \frac{11}{9} N_c$$

this is very to $\frac{33}{9}$ $\Rightarrow$ there are 3 colour charge states

b) The strong force is a short range force because it is mediated by gluons. Gluons carry a colour charge and therefore self interact. In QCD, coupling increases with increasing distance.

3. a) ~~Zm_p term from~~

$Zm_p + (A-Z)m_n - a_V A + a_S A^{\frac{2}{3}} + a_c Z^2 A^{-\frac{1}{3}}$ terms all are from the liquid ~~gg~~ drop model

$+a_a\left(Z-\frac{A}{2}\right)^2 A^{-1} \pm \delta a_p f(A)$ terms ~~or~~ come from the fermi gas model.

b) for an odd A nuclei, the nucleon pairing term vanishes
Z, N = even, odd or odd, even

for an even A nuclei, there are two parabolas due to the nuclear pairing, Z, N = even, even and Z, N = odd, odd. even, even pairings are usually stable since they have two additional pairings compared to odd, odd. Hence ~~ee~~ even, even is the lower parabola.

4. i) electrons are charged ∴ will be deflected by ~~the~~ a magnetic field in a detector and will have a curved path. Whereas a photon will not be deflected.

ii) positrons and electrons have opposite charges, therefore they will be deflected in opposite directions by a magnetic field.

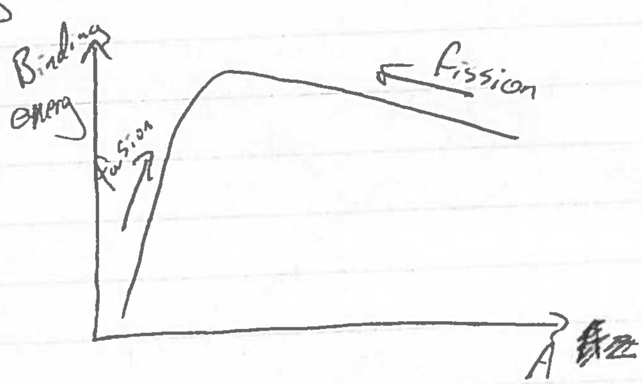
iii) muons typically pass through a detector without decaying and will normally ~~pen~~ ~~penetrat~~ penetrate through the calorimeters ~~a~~ ~~until~~ and reach the muon spectrometer.

Charged pions will deposit energy in the hadronic calorimeter and leave a track

iv) Electrons deposite energy in the em calorimeter and leave a track, whereas jets contain many hadrons and are identified by their ~~tr~~ tracks in the hadronic calorimeter.

5. Fission releases more energy per neucleus.
Fusion releases more energy per neucleon.

This is because fission involves very heavy neuclei whereas fusion involves light nuclei. ~~Since~~ This means that the energy per nucleon is low for fission since there are many neucleons.

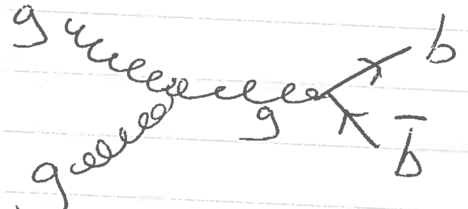


Fission event causes more energy to be released than a fusion event since the binding energies are typically higher. Hence ~~fi~~ ~~nucle~~ fission has a higher energy per nucleus.

6. a) The Higg's boson couples with particles that ~~to~~ have mass. ~~The Higgs couples more with mass~~. The heaviest particle with a mass that is less than half of the Higgs mass is the bottom quark. And generally ~~with~~ with Higgs decay, the larger the mass of the products the larger the branching ration.

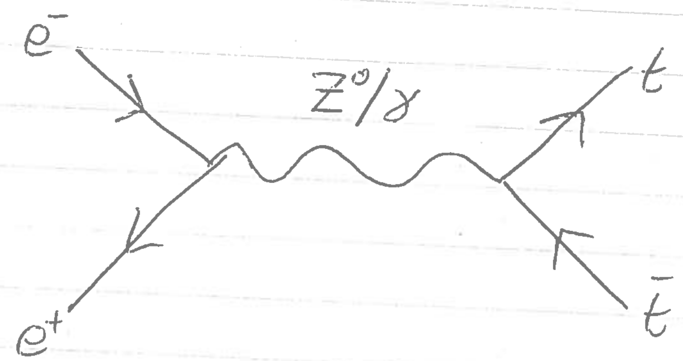
b) Decay to a W^+W^- pair is allowed through virtual particles, but this ~~is~~ decay mode is surpressed. Becuse $M_H < 2M_W$ the virtual particles are needed. If the ~~Bo~~ higgs mass was large so that $M_H > 2M_W$, since $M_W > M_t$ the branching fraction for W^+W^- would be larger ~~be~~ because

c) $H \to b\bar{b}$ is hard to detect because of a very large background from:



which is ~10 million times more likely to occur than a $H \to b\bar{b}$ event.
There is also a poor m_{bb} resolution, even the $Z \to bb$ peak is difficult to see.

7. a)



$$E_{cm} \geq 2m_t$$
$$E_{cm} \geq 346 \text{ GeV}$$

b)i) $t \to W + b$

$$E^2 = m^2c^4 + p^2c^2$$
$$\Rightarrow p = \frac{1}{c}\sqrt{E^2 - m^2c^4}$$

Energy: $m_t c^2 = E_W + E_b$

momentum: $0 = p_W - p_b$

$$\Rightarrow 0 = \frac{1}{c}\sqrt{E_W^2 - m_W^2c^4} - \frac{1}{c}\sqrt{E_b^2 - m_b^2c^4}$$
$$0 = \frac{1}{c}\sqrt{(E_b - m_tc^2)^2 - m_W^2c^4} - \frac{1}{c}\sqrt{E_b^2 - m_b^2c^4}$$
$$(E_b - m_tc^2)^2 - m_W^2c^4 = E_b^2 - m_b^2c^4$$
$$\cancel{E_b^2} - 2m_tc^2E_b + m_t^2c^4 - m_W^2c^4 = \cancel{E_b^2} - m_b^2c^4$$
$$E_b = \frac{m_b^2 + m_t^2 - m_W^2}{2m_t}c^2 = \frac{4.5^2 + 173^2 - 80^2}{2 \times 173}c^2 = 68 \text{ GeV}$$

ii) $E_B \approx 0.7 \times 68 = 47.6 \text{ GeV}$

$\tau = 1.5 ps = 1.5 \times 10^{-12} s$

$\gamma = \frac{E}{mc^2} = \frac{47.6}{5.0} = 9.52$

$d = \gamma c t = 9.52 \times (3 \times 10^8) \times (1.5 \times 10^{-12}) = 4.3 \times 10^{-3} m$
$= 4.3 mm$

Lorentz do-da thing

iii) $W \rightarrow \nu_l + l^{\pm}$
$W \rightarrow q + \bar{q}$

⇒ there will be 2 jets because of the quark and antiquark.
⇒ there ~~w~~ will be 1 charged lepton

iv) hadronic jets are identified by the tracks (direction of curve) and the energy deposits in the calorimeters.

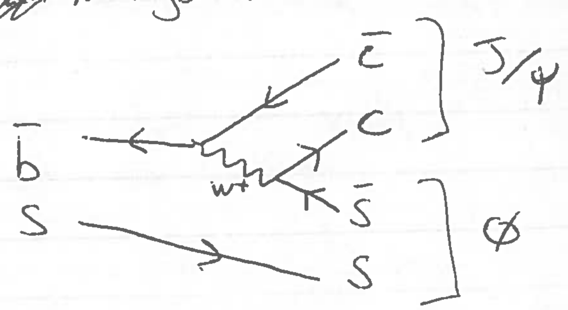
c) i) $n \rightarrow \bar{p} + e^+ + \nu_e$
$udd \quad \bar{u}\bar{u}\bar{d}$

Q	0	→	-1	+1	
L_e	0	→	0	-1	+1
B	+1	→	-1	0	0

Forbidden because the ~~p~~ baryon number is not conserved

ii) $B_s \rightarrow J/\psi + \phi$
$\bar{b}s \quad c\bar{c} \quad s\bar{s}$

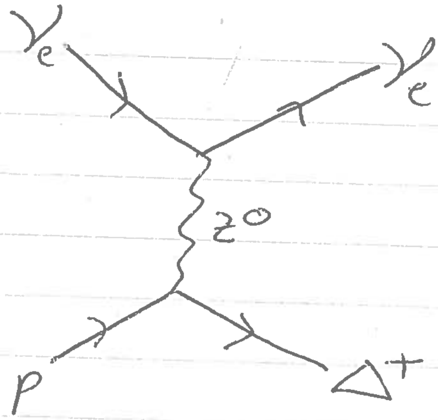
Q	0	→	0	+	0
B	0	→	0	+	0



$c = +\frac{2}{3}$
$s = -\frac{1}{3}$
$\ldots - 1$

iii) $\nu_e + p \rightarrow \nu_e + \Delta^+$

	p (uud)		Δ^+ (uud)
Q	$+1$	$\rightarrow$	$+1$
L_e $+1$		$\rightarrow$	$+1$
B	$+1$	$\rightarrow$	$+1$



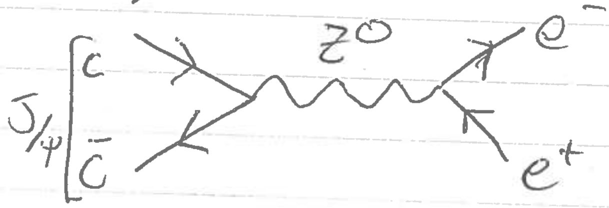
iv) $J/\psi \rightarrow e^+ + e^-$

$c\bar{c}$

Q $0 \rightarrow +1 \; -1$

B $0 \rightarrow 0 + 0$

L_e $0 \rightarrow -1 \; +1$



v) $J/\psi \rightarrow \tau^+ + \tau^-$

not allowed because $m_{J/\psi} < 2m_\tau$

$3.1\,GeV < 3.6\,GeV$

vi) $J/\psi \rightarrow \nu_e + \bar{\nu}_\mu$

L_e $0 \rightarrow +1 + 0$

L_μ $0 \rightarrow 0 \; -1$

Not allowed because electron and muon lepton numbers are not conserved.

8. a) i) $E_\nu = \sqrt{m_\nu^2 c^4 + p_\nu^2 c^2}$

$= p_\nu c$ for massless neutrinos

$\Rightarrow p_\nu E_\nu = E_\nu^2$ in natural units

since $E_0 = E_e + E_\nu$

$\Rightarrow E_\nu^2 = (E_0 - E_e)^2$

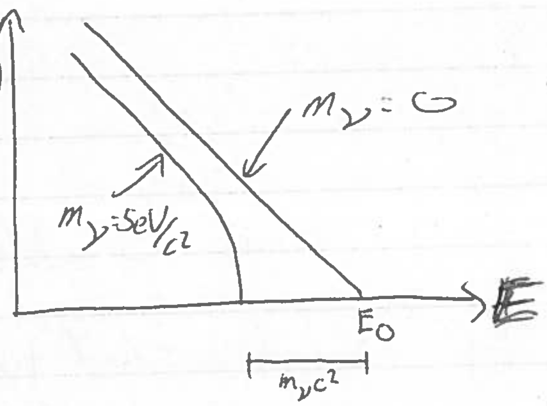
$\Rightarrow \frac{d\omega}{dE_e} = (E_0 - E_e)^2$

ii) 3 Body decay; $E_0 = E_e + E_\nu + E_{recoil} \simeq E_e + E_\nu$. Because the masses are very small the recoil energy is small $\therefore$ negligible

? b) $\frac{d\omega}{dE_e} \propto (E_0 - E_e)^2$

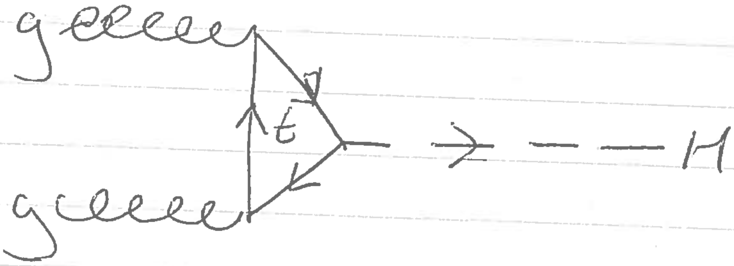
c) i) cant do this without b)

ii)

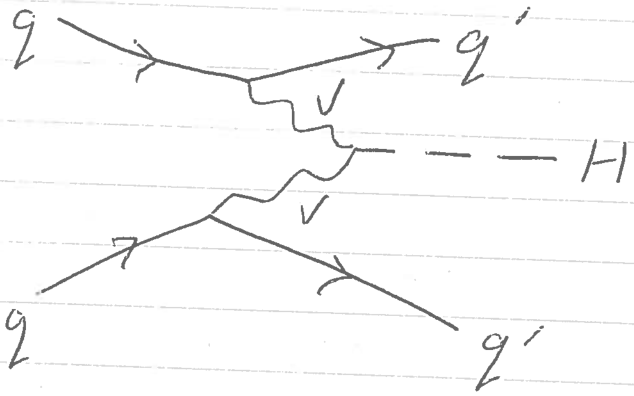


? d)

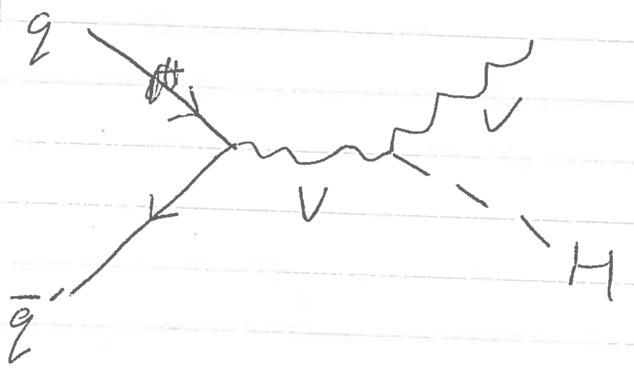
e) $^{185}_{75}Re \quad J^P = \frac{5}{2}^+$

10.a) 

gluon-gluon fusion (via top quark loop)

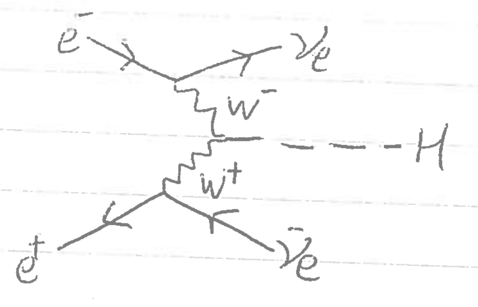


Weak Boson fusion ($V = W^{\pm}$ or Z^0)



Higgs-Strahlung (Associated W/Z H production)

b) Weak Boson fusion and Associated W/Z ~~H~~ H production are possible ~~at~~ in a e^-e^+ collider when V is a Z^0 Boson in the Associated H production mechanism and V is a $W^{\pm}$ Boson in the Weak boson fusion mechanism

ie. 

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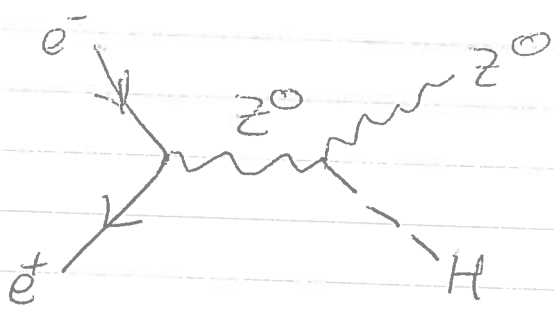


c) The E_{cm} can be lower for an e^+e^- collider ~~than~~ for producing heavy particles because they do not need to overcome a coulomb barrier in order to collide. ~~Here~~ Proton-Proton collisions need to overcome this since they have the same charge. Protons are also much more massive than electrons, so naturally E_{cm} will be larger.

$$E_{cm} = \left[2E_1E_2 + (m_1^2+m_2^2)c^4 + 2\sqrt{E_1^2-m_1^2c^4}\sqrt{E_2^2-m_2c^4} \right.$$

d) $M_H = 125\,GeV/c^2 = 0.125\,TeV/c^2$

$E_{cm} = 8\,TeV$

$$p + p \rightarrow H$$

Energy $2E_p \rightarrow 8\,TeV$

~~momentum $2xp_p \rightarrow$~~

$$\cancel{p_p = \frac{1}{c}\sqrt{E_p^2 - m_p^2c^4}}$$

$$\cancel{p_H = \frac{x2}{c}\sqrt{E_p^2 - m_p^2c^4}}$$

$$\cancel{E^2 = m^2c^4 + p^2c^2}$$

~~for H: $8^2 = 0.125^2 + p_H^2c^2$~~

$$\cancel{p_H = \frac{1}{c}\sqrt{8^2 - 0.125^2}}$$

~~$p_H \approx 7.999\,TeV/c$~~

Higgs in CM

$$\Rightarrow m_H^2c^4 = (E_{p_1}x_1 + E_p x_2)^2 - (p_{p_1}x_1 + p_{p_2}x_2)^2c^2$$

$x_1 = x_2 = x$

in C.O.M. frame $p_1 = -p_2$

$E_1 = E_2$

$$\Rightarrow m_H^2c^4 = 4E_p^2x^2 - \cancel{(x - x)^2c^2}^{\,0}$$

$$E_p = \frac{E_{cm}}{2} = \frac{8}{2} = 4\,TeV$$

$$\Rightarrow x = \sqrt{\frac{m_H^2c^4}{4E_p^2}} = \frac{m_Hc^2}{2E_p}$$

$$x = \frac{0.125}{2 \cdot 4} = 0.0156$$

e) ~~B R~~ overall: $p + p \rightarrow H + Z^0$

$$(m_H^2 + m_Z^2)c^4 = 4E_p^2 x^2$$

$$p = \frac{1}{c}\sqrt{E^2 - m^2c^4}$$

$$m_H^2 c^4 + E_Z^2 = 4E_p^2 x^2$$

energy $m_H c^2 + E_Z = 2E_p x$

momentum $p_Z = x p_{p_1} - x p_{p_2}$
$= 2x(0)$

$$E_Z^2 = 2E_p^2 - m_H^2$$

$$\frac{1}{c}\sqrt{E_Z^2 - m_Z^2 c^4} = \frac{2x}{c}\sqrt{E_p^2 x^2 - m_p^2 c^4}$$

$$(2E_p x - m_H c^2)^2 - m_Z^2 c^4 = 4x^2(E_p^2 x^2 - m_p^2 c^4)$$

$$4E_p^2 x^2 - 4E_p m_H c^2 x + m_H^2 c^4 - m_Z^2 c^4 = 4x^4 E_p^2 - 4m_p^2 c^4 x^2$$

$$E_p^2 x^2 - E_p m_H c^2 x = E_p^2 x^4 - m_p^2 c^4 x^2$$

$$p_H = p_Z \qquad E_H^2 - m_H^2 c^4 = E_Z^2 - m_Z^2 c^4$$

$$E_Z^2 = (2E_p x - E_H)^2$$
$$= 4E_p^2 x^2 - 4E_H E_p x + E_H^2$$

$$E_H^2 = 4E_p^2 x^2 - 4E_Z E_p x + E_Z^2$$

$$m_H^2 c^4 = -4E_H E_p x + 4E_p^2 x^2 + m_Z^2 c^4$$

$$(m_H c^2 + m_Z c^2)^2 = 4E_p^2 x^2$$

$$x = \frac{m_H c^2 + m_Z c^2}{2E_p} = \frac{0.125 + 0.091}{2 \cdot 4} = 0.027$$

? f) ~~By plot~~ $Z^0 \rightarrow l^+ l^-$ $(l = e/\mu)$

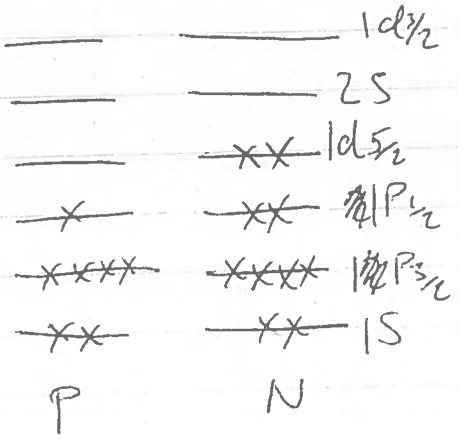
$\Rightarrow$ $2Z^0 \rightarrow l^+ l^- l^+ l^-$

By plotting the invariant mass of the 4 detectable leptons against energy there will be a peak at ~125 GeV that is from the $H \rightarrow 2Z^0 \rightarrow l^+l^-l^+l^-$ decay process.

There will be be background signals at energies corresponding to $2m_q c^2$ but since the masses of individual quarks are far less than that of the Higgs these backgrounds invariant masses will be very low.

The $Z^0 \rightarrow e^+ e^-$ decay channel will be easiest to distinguish because e^-'s can be found in the em calorimeter

g) $^{15}_{7}N$



$J = |j_p - j_n| \rightarrow (j_p + j_n)$

$\Rightarrow j_p = \frac{1}{2}$ $\quad j_n = 0$

$P_p = (-1)^1 = -1$

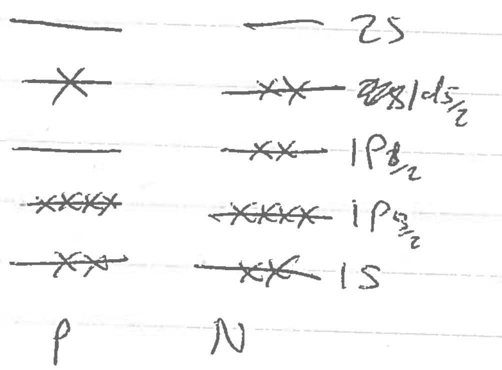
$P_N = (-1)^2 = +1$

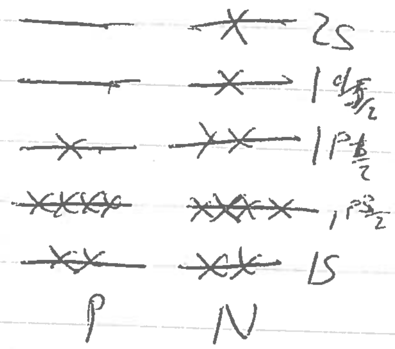
$P_{nucleus} = -1 \cdot +1 = -1$

$\Rightarrow J = \frac{1}{2}$

$\Rightarrow J^P = \frac{1}{2}^-$

h) Excited states:





$P_{nucleus} = (-1)^2 \cdot (-1)^2$
$= 1 \cdot 1$
$= 1$

$j = \frac{5}{2}$

$\Rightarrow J^P = \left(\frac{5}{2}\right)^+$

$P_{nucleus} = (-1)^0 \times (-1)^1$
$= 1 \times -1$
$= -1$

$j_p = \frac{1}{2}$, $j_n = 1$

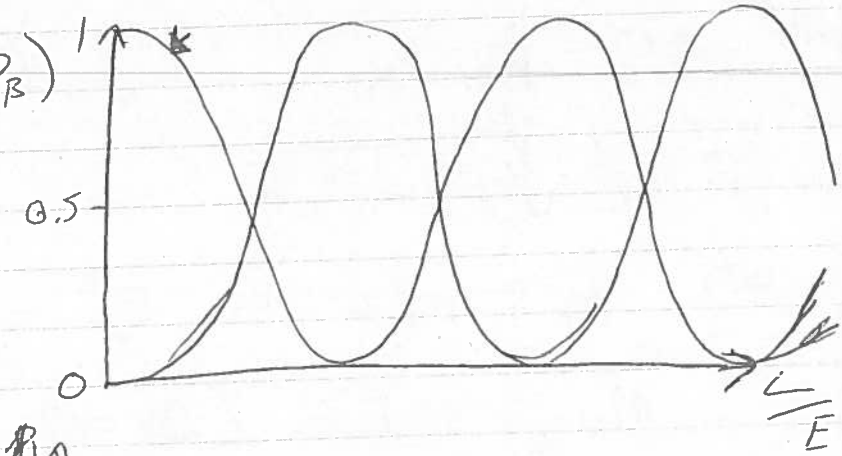
$J = \left|\frac{1}{2} - 1\right| \rightarrow \left(\frac{1}{2} + 1\right)$
$= \frac{1}{2} \rightarrow \frac{3}{2}$

$\Rightarrow J^P = \left(\frac{3}{2}\right)^-$

9. a) i) for $\Theta = 45°$ $\Rightarrow 2\Theta = 90°$

$\Rightarrow \sin^2(2\Theta) = 1$

$\therefore P(\cancel{\alpha}\nu_\alpha \to \nu_\beta)$



$P(\nu_\alpha \to \nu_\alpha)$

flat at 0
no osscillation
because $\Delta m^2 = 0$

$\Rightarrow P(\nu_\alpha \to \nu_\alpha) = 0$

ii) $L = 1300\,km$ $\Delta m^2 = 2.43 \times 10^{-3}\,eV^2$

Maximized when $\sin^2\left(1.27 \frac{\Delta m^2 L}{E}\right) = 1$

$\Rightarrow 1.27 \frac{\Delta m^2 L}{E} = 90° = \frac{\pi}{2},\ 270° = \frac{3\pi}{2}$ etc

Largest energy: $E = \frac{1.27 \times (2.43 \times 10^{-3}) \times 1300}{\left(\frac{\pi}{2}\right)} = 2.55\ GeV$

next largest energy: $E = \frac{1.27 \times (2.43 \times 10^{-3}) \times 1300}{\left(\frac{3\pi}{2}\right)} = 0.85\ GeV$

b) $\pi \rightarrow \mu + \nu$

Energy $m_\pi c^2 = E_\mu + E_\nu$ in π rest frame

momentum $0 = p_\mu - p_\nu$

$E^2 = m^2c^4 + p^2c^2$

$$p = \frac{1}{c}\sqrt{E^2 - m^2c^4}$$

$$\Rightarrow E_\mu^2 - m_\mu^2 c^4 = E_\nu^2 - m_\nu^2 c^4$$

m_ν is negligable [$= 0$ in standard model]

$E_\mu = m_\pi c^2 - E_\nu$

$$\Rightarrow (m_\pi c^2 - E_\nu)^2 - m_\mu^2 c^4 = E_\nu^2$$

$$m_\pi^2 c^4 - 2E_\nu m_\pi c^2 + E_\nu^2 - m_\mu^2 c^4 = E_\nu^2$$

$$m_\pi^2 c^2 - 2E_\nu m_\pi - m_\mu^2 c^2 = 0$$

$$2E_\nu m_\pi = m_\pi^2 - m_\mu^2$$

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

μ and ν in same direction:

hence: $p_\pi = p_\mu + p_\nu$

in natural units: $\Rightarrow \sqrt{E_\pi^2 - m_\pi^2} = \sqrt{E_\mu^2 - m_\mu^2} + \sqrt{E_\nu^2 - m_\nu^2}$

m_ν is negligable

$E_\mu = m_\pi c^2 - E_\nu$

$E_\pi = E_\mu + E_\nu$

$E_\mu = E_\pi - E_\nu$

$$\Rightarrow \sqrt{E_\pi^2 - m_\pi^2} = \sqrt{(m_\pi - E_\nu)^2 - m_\mu^2} + E_\nu$$

$$\sqrt{E_\pi^2 - m_\pi^2} = \sqrt{m_\pi^2 - 2m_\pi E_\nu + E_\nu^2 - m_\mu^2} + E_\nu$$

~~$E_\pi^2 - m_\pi^2 = m_\pi^2 - 2m_\pi E_\nu + E_\nu^2 - m_\mu^2 + E_\nu^2 + 2E_\nu \sqrt{\dots}$~~

$$\sqrt{E_\pi^2 - m_\pi^2} = \sqrt{E_\pi^2 + E_\nu^2 - 2E_\pi E_\nu - m_\mu^2} + E_\nu$$

$$(E_\pi^2 - m_\pi^2)^{\frac{1}{2}} = E_\pi\left(1 - \frac{m_\pi^2}{E_\pi^2}\right)^{\frac{1}{2}}$$
$$= E_\pi\left(1^{\frac{1}{2}} - \frac{1}{2}\frac{m_\pi^2}{E_\pi^2} + \dots\right)$$

c) i) $$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2\left(E_\pi - \sqrt{E_\pi^2 - m_\pi^2}\right)}$$

$E_\pi \gg m_\pi$ $$\Rightarrow \quad E_\nu = \frac{m_\pi^2 - m_\mu^2}{2\left(E_\pi - E_\pi\left(1 - \frac{m_\pi^2}{E_\pi^2}\right)^{\frac{1}{2}}\right)}$$

$$\cancel{= \frac{m_\pi^2 - m_\mu^2}{2\left(1 - \frac{1}{2}m_\pi\right)}}$$

Using Taylor Expansion

$$= \frac{m_\pi^2 - m_\mu^2}{2\left(E_\pi - E_\pi\left(1 - \frac{1}{2}\frac{m_\pi^2}{E_\pi^2} + \dots\right)\right)}$$

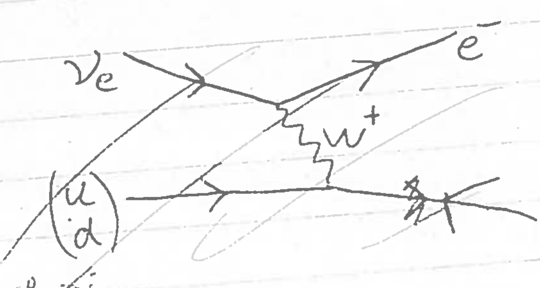
$$= \frac{m_\pi^2 - m_\mu^2}{2\left(+\frac{m_\pi^2}{2E_\pi^2}\right)}$$

$$= \frac{m_\pi^2 - m_\mu^2}{\left(m_\pi^2/E_\pi\right)}$$

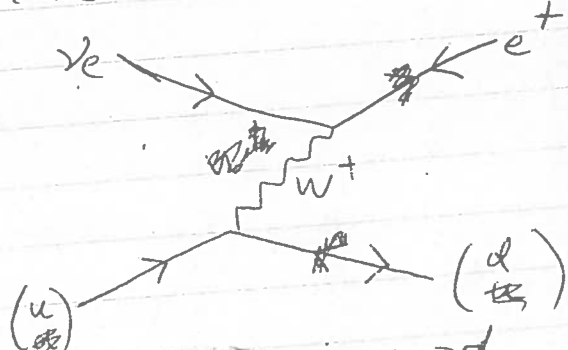
$$\Rightarrow \quad \frac{E_\nu}{E_\pi} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2}$$

ii) $E_\nu = 3\,\mathrm{GeV}$ $$\Rightarrow E_\pi = 7.03\,\mathrm{GeV}$$

d)

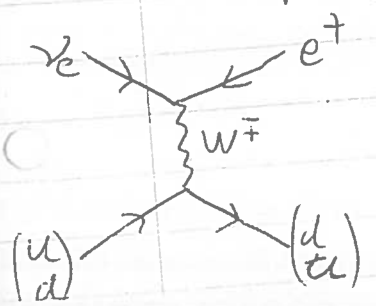


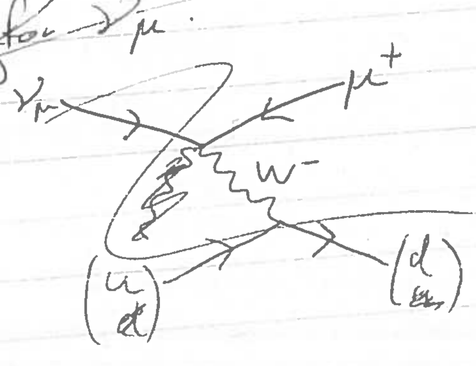
Since nuclei

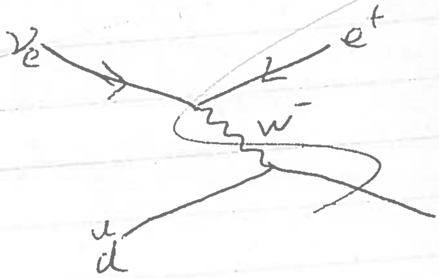


~~ie electron on proton~~ ie $u \to d$, $d \to u$

~~corresponding for ν_μ~~







4 ii) ~~By~~ ~~Instead~~ Electrons will leave a track in the em calorimeter and will cause an em Shower in the calorimeter.

Muons on the other hand will generally pass through ~~&~~ the ~~extent~~ entire detector and will ~~be observed~~ ~~by the layers~~ interact with the scintillator that makes up the ~~the~~ outer muon detector.