DAT110 - Exercise 7

1 Poisson revisited

1.1

When k (number of observations) is zero the Poisson distribution function simplifies to: e^-lamda, because dividing the expression by zero would give an undefined answer.

$$P(x=0) = e^{-\lambda}$$
$$= e^{-2.2}$$
$$= 0{,}1108.031583623$$
$$= \underline{\underline{11{,}1\%}}$$

1.2

Here I we can sum up the probabilities of each situation occurring.

$$\sum_{n=1}^{3} P(n) = \sum_{n=1}^{3} \frac{2.2^n e^{-2.2}}{n!}$$
$$= \left(2.2\, e^{-2.2}\right) + \left(\frac{2.2^2 e^{-2.2}}{2!}\right) + \left(\frac{2.2^3 e^{-2.2}}{3!}\right)$$
$$= 0.905187935048$$
$$= \underline{\underline{90{,}5\%}}$$

1.3

The straight forward solution would probably be to recommend do increase the mean to 4.4, because it would be logical for the mean of bags lost per day to double as the number of flights doubles. However there could be a lot of different factors effecting this number, therefore it would be hard to tell which of these situations is the most realistic one.

2 Poisson revisited

2.1
Here I would use the Poisson distribution, because it is commonly used to calculate number probability of number of occurrences in a given time space.

2.2
The mean is 75 customers per hour in the morning. This is given in the task description. The standard deviation is the square root of this number, which is approximately 8.66 customers per hour.

2.3
Using the Poisson equation we can calculate the probability to be approximately 1.02% for there being 60 customers per hours in these morning hours. Therefore we can conclude that 60 customers per hours at this time of the day would be highly unlikely.

$$p(60) = \frac{75^{60} e^{-75}}{60!}$$

$$= 0{,}01026632033 86$$

$$= \underline{\underline{1{,}02\%}}$$

2.4
I had some troubles with this task. First I plugged the different values into the Poisson equation, but got about 4% probability. I found this a bit low, therefore I calculated the z-score to be 0.577 and looked this up in a table and found the probability to be 71.57%. I find this answer a lot more plausible.

3 Combinatorics

3.1
The number of ways = n!

3.2

Distinguishable objects

With k distinguishable objects the number of ways they can be placed without repetition is given by:

$$\text{Number of ways: } \frac{n!}{(n-k)!}$$

Where n! is number of possibilities and (n-k)! represents number of places k can be placed without caring about the order.

Undistinguishable objects

For indistinguishable objects we can not set them apart and therefore the number of ways is given by:

$$\text{Number of ways: } \frac{n!}{k!(n-k)!}$$

3.3

To solve this I'll use the binominal coefficient formula, which is given by:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\binom{49}{7} = \frac{49!}{7!(49-7)!}$$

$$= \underline{\underline{85900584 \text{ combinations}}}$$

3.4

$$\underbrace{** \cdots **}_{k_1} \mid \underbrace{** \cdots **}_{k_2} \mid \underbrace{** \cdots **}_{k_3}$$

$$\binom{n+r}{r} = \binom{171}{2} = \frac{171!}{2!(171-2)!} = \frac{171 \cdot 170}{2} = 14535$$

There are 14535 ways the politicians can sit.

4 Vehicle speeds on interstate 5 freeway

4.1

To do this we first have to calculate the z-value:

$$z_{80} = \frac{x-\mu}{\sigma} = \frac{80-72{,}6}{4{,}78} \approx 1{,}5469$$

I looked up the z-value in a table and found out that 93.8% of the vehicles travel under the speed of 80 miles per hour.

4.2

We then have to find the z-value for 60 miles per hour and subtract the 60 miles per hours percentile from the 80 miles per hour percentile.

$$z_{60} = \frac{60-72{,}6}{4{,}78} = -2{,}636$$

$$z_{80} = 1{,}5469$$

$$\int_{60}^{80} p(x) = 0{,}9382 - 0{,}0044$$

$$= 0{,}9338 = \underline{93{,}4\%}$$

93,4% of the vehicles lie between 60 and 80 miles per hour.

4.3

I solved it by reversing with z-values. I looked up the z-value for 95% and used it to solve for x

$$1.65 = \frac{x - 72{,}6}{4{,}78}$$

$$x = 7{,}887 + 72{,}6 = 80{,}487 \frac{\text{miles}}{\text{hour}}$$

4.4

Here we have to calculate the z-value and look up the cdf and take (1 – cdf(70)).

$$z = \frac{70 - 72.6}{4{,}78} = 0{,}29460$$

$$\% \text{ over } 70 = (1 - 0{,}29460) \cdot 100\% = \underline{\underline{70{,}54\%}}$$

4.5

Here I'll use the binominal distribution formula to calculate the probability.

$$\binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$= \frac{\cancel{5}!}{\cancel{5}!\underbrace{(5-5)!}_{=0}} p^5 \cancel{(1-p)^{5-5}}$$

$$= p^5$$

$$= 0{,}29460^5 \cdot 100\%$$