2 Bernoulli distribution

given by $P(k;p) = \begin{cases} p & \text{if } k=1 \\ q = 1-p & \text{if } k=0 \end{cases} \quad x \in \mathbb{N}_0$

2.1 Normalization

- Probability for success : r
- Probability for failure : $1-r$

$$P(X=0) + P(X=1) = (1-r) + r) = 1$$

2.2 Mean (Expected value)

$$E(X) = \sum_x x \cdot P(X=x) \qquad \begin{aligned} P(X=0) &= 1-p \\ P(X=1) &= p \end{aligned}$$

$$\mu = 0 \cdot (1-p) + 1 \cdot p = p$$

2.3 Variance

$$Var(X) = E(X^2) - [E(X)]^2 \quad , \; E(X) = p$$

$$E(X^2) = \sum_x x^2 \cdot P(X=x)$$

$$E(X^2) = 0^2 \cdot (1-p) + 1^2 \cdot p = p$$

$$Var(X) = p - p^2 = p(1-p)$$

3 Poisson distribution

3.1 Mean

$$E(x) = \lambda$$

$$E(X) = \sum_{x \in X} x \cdot f_X(x)$$

$$E(X) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_{x=1}^{\infty} x \, \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \cdot \sum_{x=1}^{\infty} \frac{x}{x!} \lambda^x$$

$$= \lambda e^{-\lambda} \cdot \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \cdot \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} \qquad \left(e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \right)$$

$$= \lambda e^{-\lambda} \cdot e^{\lambda}$$

$$= \underline{\underline{\lambda}}$$

3.2 Variance

$$Var(X) = \lambda$$

$$Var(X) = E(X^2) - E(X)^2$$

$$E(X) = \lambda$$

$$E[X(X-1)] = \sum_{x \in X} x(x-\ ) \cdot f_X(x)$$

$$= \sum_{x=0}^{\infty} x(x-1 \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_{x=2}^{\infty} x(x-1) \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \cdot \sum_{x=2}^{\infty} x(x-1) \cdot \frac{\lambda^x}{x \cdot (x-1) \cdot (x-2)!}$$

$$= \lambda^2 \cdot e^{-\lambda} \cdot \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}$$

$(z = x-2)$ $$= \lambda^2 \cdot e^{-\lambda} \cdot \sum_{z=0}^{\infty} \frac{\lambda^2}{z!}$$ $\left(e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}\right)$

$$= \lambda^2 e^{-\lambda} \cdot e^{\lambda}$$

$$= \lambda^2$$

$$E(X(X-1)] = E(X^2 - X) = E(X^2) - E(X)$$

$$E(X^2) - E(X) = \lambda^2 \Rightarrow E(X^2) = \lambda^2 + \lambda$$

$$Var(X) = \lambda^2 + \lambda - \lambda^2 = \underline{\underline{\lambda}}$$