Yue Sun

NetID: ysun276

***Homework2: Written Assignment***

1. ***Proof of the equations:***

We have already known . To calculate min value of E, we need consider the situation that the derivative of E is 0, where we can get

Based on that we can visualize that:

b

a-c

Since we know the two right-angle sides can be b and a-c, the hypotenuse can be . Then it is easy to get .

Additional, based on rules of trigonometric function: , and . We can apply them in .

Then we get:

Apply above:

**Therefore:**

We have already known that

By trigonometric transform principle:

Then replicate the whole process above, we can easily get:

,

**which is equal with**

1. ***Argue E is real and non-negative:***

Since E is moment of inertia of the binary image which is defined as the products of label value (always equal or greater than 0 in practice) times distance of the axis of rotation squared (always positive as well), E must be real and non-negative. Alternatively, we can observe through the equation of E: , where is the characteristic function we use. must be non-negative and so is. Therefore, the result of double integral is also real and non-negative.

As E is non-negative, the minimal value of E, must be non-negative. By what we have proven in 1), we know that .

Therefore, we have:

**Therefore, we can prove that:**

1. ***When is E equal to zero:***

Based on the function: in the course package, there are only two conditions that E could be 0: equals to 0, or equals to 0.

However, in practice of image processing (2D binary image), our characteristic function is normally defined as non-negative (background is 0, object label value greater than 0). Therefore, the only condition is  **equals to 0**. stands for the square of distance of a point to the axis of rotation. When , all the points should lie on the axis of rotation (note: in the practice of 2D binary image). Therefore, when all the point whose lies on the axis of rotation, E is equal to zero.