Patipol Chiammunchit Livrable : Measuring the activity of community by Δ-density

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1 Introduction

Nowadays, there are two kinds of social relationship, social link and interaction. Social link is the relationship between two people which is explicit e.g. Facebook friends, Twitter following, etc., and interaction is implicit relationship e.g. forum post and comment.

In explicit social link, it can be models as a network. Although, modelize the dynamic tion is an issue in this context. The reason is that defining social links by observing interactions is challenge. So, the solution of that is representation of link stream.

2 Definition

In the project, the interaction of community is transformed into link stream. Link stream is like a list of interaction between two people at a point of time. Then, Link stream is used to compute Δ-density of link, density of interaction between two people, for every pair of people. After that, Δ-density of streams, density of interaction in one community, is computed by averaging Δ-density of link for every possible pair of people.

2.1 Link stream

Modelizing the interaction suggests link stream[3]. Link stream is defined as $L = \{l_i\}_{i=[1,k]}$ where k equal to number of link in link stream and $l_i = (u_i, v_i, t_i)$ refers to the interaction between two nodes, u_i and v_i, at time t_i. Link stream started at time α and stopped at time ω, so $\alpha \leq t_i \leq \omega$ for all links.

Considering link stream which contains only two nodes, u and v, τ_i is defined as time difference between l_i and l_{i+1}: $\tau_i = l_{i+1} - l_i$ for $i = [1, k-1]$. Moreover, the first link have to calculate time difference with started time and the last link with stopped time. So, τ_0 equals to $t_1 - \alpha$ and τ_k equals to $\omega - t_k$.

2.2 Δ-density

The concept and notation of the Δ-density was defined by [3]. The goal of Δ-density is to measure . The concept is the probability that every pair of nodes interacts every Δ time. The first thing to do is to calculate Δ-density of link for every possible link, then it is used to calculate Δ-density of stream (community).

2.2.1 Δ-density of links

The Δ-density of a pair of nodes u and v, written in $\delta_\Delta(u, v)$. Δ-density of (u, v), is the probability that a randomly chosen time-interval of size Δ contains at least one link between u and v. In other words, the Δ-density of (u, v) is the probability that u and v interact every Δ time. Although, directly computing Δ-density of link is quite hard. So, the easier way to compute Δ-density of link is computing the probability that two nodes using more than Δ time to interact, then use it to minus from one. This lead to the following equation:

$$\delta_\Delta(u, v) = 1 - \frac{\Sigma_i max(\tau_i - \Delta, 0)}{\omega - \alpha - \Delta}$$

The Δ-density of (u, v) reaches maximum value, that is 1, if and only if u and v interact within Δ time. That means $\tau_{i=[0,k]}$ are less than zero and $\Sigma_i max(\tau_i - \Delta, 0)$ equals to zero , and so Δ-density of (u, v) equals to 1. However, if some τ_i are more than Δ, some links do not interact within Δ time, the Δ-density of (u, v) will not reach maximum value. Finally, Δ-density of (u, v) equals to zero if and only if there are no link between u and v.

2.2.2 Δ-density of streams

The Δ-density of stream is extended from the notion of Δ-density of link that is average of Δ-density of link for every possible pair of nodes. So, the equation of the Δ-density of stream is

$$\delta_\Delta(S) = \frac{\Sigma_{(u,v) \in S} \delta_\Delta(u, v)}{|S|}$$

where S is set of link in link stream. Although, this equation presents only temporal aspects. So, it have to combine with structural aspects.

In structural aspects, the density of interaction in community is the probability that every node interact to all others. So, the equation is $\delta(G) = \frac{2 \cdot m}{n \cdot (n-1)}$ where n is a number of nodes and m is a number of links. These two aspects are combined into one equation which is

$$\delta_\Delta(L) = \frac{2 \cdot \Sigma_{(u,v) \in V \times V} \delta_\Delta(u, v)}{|V| \cdot (|V| - 1)}$$

where $|v|$ is a number of nodes in the link stream L.

3 Methodology

In this chapter, I draw the evolution of Δ-density of community according to Δ time. Then, I find the KPI of the community from that evolution, the maximum Δ-density of community and the smallest Δ where Δ-density is maximum.

Figure 1 shows the link stream of the toy dataset. The link stream has 5 nodes, $[a, b, c, d, e]$, interact with each others. The link stream shows that node a mostly interacts with node b. However, node e does not often interact with the others.

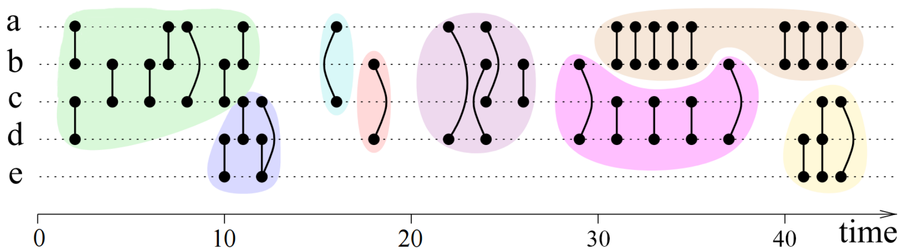

Figure 1: The toy dataset of link stream with 5 nodes $[a, ..., e]$. Each of the 35 links represent an interaction between two individuals at a time t in $[0, 44]$. [2]

3.1 Analysis of several communities

In this section, I try to calculate Δ-density of community from several Δ. Then, plot the list of Δ-density on the figure which X-axis is Δ and Y-axis is Δ-density.

Figure 2 show the evolution of Δ-density according to Δ of communities [1, 2], [1, 4], [2, 3, 4], [3, 4, 5], [1, 2, 3, 4]. I choose these communities because the appearance of each communities are difference. Communities [1, 2] and [2, 3, 4] have many links in this link stream but [2, 3, 4] has member more than [1, 2]. Community [1, 4] has only one link. Community [3, 4, 5] takes short time to interact to each others in community but it does not interact in the community for a long time. And, community [1, 2, 3, 4] is the combination of short time interaction community (community [1, 2]) and long time interaction community (community[1, 3, 4]).

Community [1, 4] slightly rose at the first and then sharply increased before reached maximum.

Communities [2, 3, 4], [3, 4, 5], and [1, 2, 3, 4] are dramatically grew at beginning and gently went up at the end of the line but it reaches maximum Δ-density at different Δ value.

Community [1, 2] sharply rose at the beginning. At the middle of the line, it slightly went up and suddenly grew again at the end of the line.

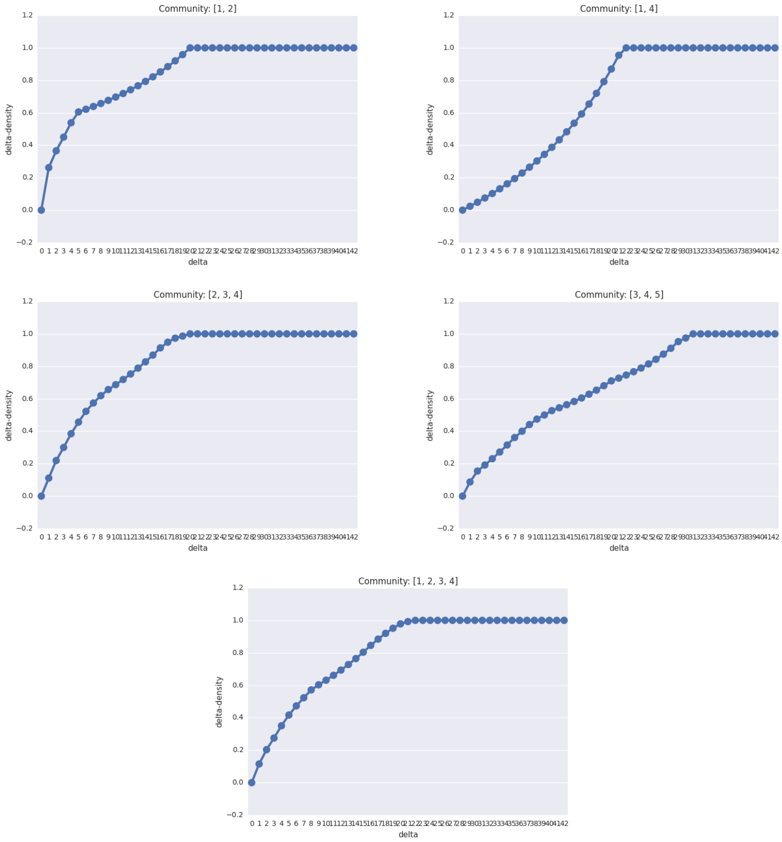

Figure 2: The evolution of Δ-density according to Δ of several communities.

3.2 Analyzing dynamics of interaction inside the link stream

In the previous chapter, I plot the evolution of Δ-density of community according to Δ. I define the KPI for using in this chapter, the maximum Δ-density and the smallest Δ where Δ-density is maximum. Then, I plot these two KPI on the figure. The X-axis is the smallest Δ where Δ-density is maximum and the Y-axis is the maximum Δ-density. Community which has high Δ-density means many node in community interact with many node, and, community which has low Δ means interaction occur within Δ time.

In figure 3, every community has the same Δ-density at 1. However, the smallest Δ where Δ-density is maximum is different. At point A, communities [1, 2] and [2, 3, 4], takes small Δ to reach maximum Δ-density so these communities take short time to interact to each others. Move to point B, communities [1, 4], [1, 2, 3, 4] takes Δ more than point A to reach maximum Δ-density therefore it takes longer time than A to interact to each others. Finally, point C, community [3, 4, 5], takes highest Δ to reach maximum Δ-density. Thus, it takes long time to interact with each others.

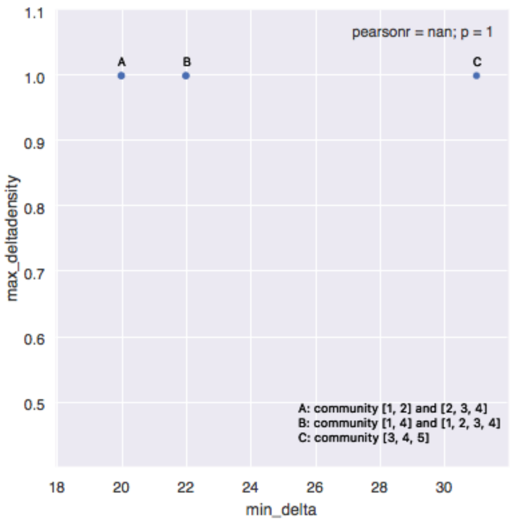

Figure 3: The distribution of every possible community. the X-axis is the smallest Δ where Δ-density of community is maximum and the Y-axis is the maximum Δ-density of the community.

4 Acknowledgement

This project is inspired by [1] and improve the code by adding some features.

References

[1] Projet dyngraph. `https://www-complexnetworks.lip6.fr/~magnien/DynGraph/Software/Delta_Density/`.

[2] Noé Gaumont, Tiphaine Viard, Raphaël Fournier-S'niehotta, Qinna Wang, and Matthieu Latapy. Analysis of the temporal and structural features of threads in a mailing-list. In *Complex Networks VII*, pages 107–118. Springer, 2016.

[3] Jordan Viard and Matthieu Latapy. Identifying roles in an ip network with temporal and structural density. In *Computer Communications Workshops (INFOCOM WKSHPS), 2014 IEEE Conference on*, pages 801–806. IEEE, 2014.