Trading Volatility in EM FX markets

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Abstract

We investigate if markets price future volatility efficiently ...

Keywords: keyword1

JEL classification: ABC123

Contents

Fredag agenda:

- Flytte rundt på trading strat / data afsnit?
- Prøve at regne profit på trading strat, se om det giver mening.
- Udfyld sum stats
- Plan for resten af projektet

Spørgmål til Philipp:

- Is the scope of the paper suffient for a seminar paper in Financial Econometrics? Thorough theory section, 3 different GARCH models applied for trading (incl. mis-specification tests etc.), evaluating performance (RMSE) along with performance in PnL.
- We follow trading strategy proposed by Dunis: Forecast 1m ahead volatility and go long straddle if (conditional volatility forecast / implied vol)-ratio ¿ threshold (short straddle vice versa). Hold to maturity and only one trade at a time.
- modellere enkelte serier? eller kan vi kaste 3 modeller ned over hovedet på RUB,ZAR,BRL?

1 Introduction

In April 2019, the international foreign exchange (FX) markets saw a daily turnover of USD 6.6 trillion with most of the volume constituted by swaps and spot trades (Bank for International Settlements, 2019). FX markets consist of various players participating with different objectives. Investors ranging from retail to banks and institutionals trade FX speculating on price fluctuations, while others trade mechanically, e.g. central banks carrying out monetary policy or corporations trading FX for operations and cash flow hedging purposes.

The trading of FX, and its derivatives, is conducted on an over-the-counter (OTC) basis via a network of banks rather than via centralized exchanges. The magnitude of the market combined with the absence of regulation would suggest that the FX markets resemble a close proxy for a “perfect” market. Such a market should, according to the efficient market hypothesis, incorporate all available information in prices, which implies that no available information can provide more accurate forecasts of future prices than what is currently reflected in the market prices.

Our paper investigates this proposition in the emerging markets FX universe, namely whether one may profit by explicitly modelling available information with non-linear time series models. Specifically, inspired by Dunis and Huang (2002), we aim at backtesting an options trading strategy based on long and short “straddles”, which combines call and put options to profit from future up- or down movements in market volatility. This strategy may be profitable when current market pricing does not correctly forecast future volatility. We find that ...

There exists an extensive literature on the study of financial market volatility, which was intensified with the seminal work of **Engle (1982)**, who formulated the autoregressive conditional heteroskedasticity (ARCH) model for explicit conditional variance modelling. Focusing on class of non-linear conditional heteroskedastic ARCH models, we here highlight some central conclusions from the literature on FX market volatility modelling. In an extensive study of the ”alphabet soup” of GARCH-type models, Hansen and Lunde (2005) evaluate the out-of-sample forecasting performance of 330 GARCH-type models for the conditional volatility of the US Dollar versus Deustche Mark exchange rate. Their main conclusion is that there is no evidence of the parsimonious GARCH(1,1) specification being inferior to more elaborate models such as the IGARCH, NGARCH, etc. Moreover, in their study of exchange rate data ranging from 1987 through 1992, they also conclude that estimating the models with an imposed t-distribution for the innovation process yields more precise estimates.

If the GARCH(1,1) provides a good measure of the conditional variance, can we use the available information at time t profit at time $t+1$? One such way would, as suggested by Dunis and Huang (2002), appear if the GARCH conditional volatility forecast is a better predictor of future volatility than what is actually priced by derivatives markets. Therefore, they suggest constructing an FX options trading strategy that is market (”delta”) neutral but long or short the ”vega”. In short, a trader can profit by buying (writing) an at-the-money call and put options with the same strike K, if the realized volatility at expiry turns out to be higher (lower) than what is priced into the options at time of entering the trade.

To investigate whether one such strategy is feasible, Pilbeam and Langeland (2014) investigate four highly liquid developed market dollar parities, namely the USD against the Japenese Yen (JPY), Swiss Franc (CHF), British Pound (GBP), and Euro (EUR). They find that for the period 2002-2012 the GARCH(1,1), EGARCH(1,1), and GJR(1,1) models are all inferior in terms of in-sample forecast precision versus the market implied volatilties. As a result, they conclude that such models are ”not particularly useful for forecasting exchange rate volatility”, and that markets efficiently price expected future volatility. However, what is not accounted for in their study, is that even though the market-implied volatilities are better predictors than GARCH-predicted volatilities on average, there may be few, but substantial occurences of positive profits from the strategy which may exceed the losses of other periods.

That conclusion that GARCH-type models are not useful for forecasting and trading exchange rate volatility is in contrast to the conclusions of Dunis and Huang (2002). They find that even though the GARCH(1,1) prediction is less inferior to the market-implied volatilities in terms of forecasting accuracy, they are able to generate profit estimates of roughly 21.7 percent on average across algorithm calibrations. However, as we will also discuss in Section 5, the profitability is difficult to backtest precisely due to actual (reliable) traded option quotes not being widely available.

Our paper is structured as follows. In Section 2, we outline the central theory with respect to option pricing and trading, as well as the econometric foundation for estimation of non-linear time series models for the conditional variance. In Section 3 we present the data that we use for our application along with descriptional statistics and a brief discussion of structural breaks. In Section 4 we present and analyse the findings of the implemented strategy, which are subsequently discussed and concluded upon in Section 5.

2 Theory

2.1 A Primer on Foreign Exchange as an Asset Class

Foreign exchange (forex) markets are a central element of international finance since it allows for the transfer of capital between countries with different currency denominations. The bilateral nominal rate of exchange is defined as the relative price of two currencies. There are different approaches for quoting the exchange rate and in this paper, we adopt the so-called price quotation system, which implies that if the ZARUSD price is 0.067, one may trade one South African Rand for 0.067 US Dollar.[1] This has the modelling implication that an increase in the ZARUSD price should be interpreted as an appreciation of the USD relative to the ZAR.

The assumption of efficient forex markets and thereby the absence of arbitrage opportunities implies the existence of equilibrium relationships between the forward exchange rates and nominal interest rate differential between countries. To establish one of the key equilibrium conditions, namely the covered interest rate parity (CIP), consider the following trading strategy:

1. Letting USD being the domestic currency, borrow 1 foreign currency unit, say EUR, for one period at the prevailing interest rate of $r_{t,f}$. At the same time, enter a forward

[1] See, e.g., Leon-Ledesma and Mihailov (2022) for a discussion of quotation methodologies.

contract to receive $F_t^{t+1}(1+r_{t,f})$ EUR one period from now in exchange for $(1+r_{t,f})$ USD.

2. With the proceeds from borrowing in EUR, deposit S_t USD in a one-period deposit yielding the US interest rate of $r_{t,d}$.

At the end of the period, the investor has accumulated liabilities of $(1+r_t^*)F_t^{t+1}$ and assets of $S_t(1+r_t)$. Assuming that that the deposits are equally risky, the CIP thereby states that

$$S_t(1+r_{t,d}) = (1+r_{t,f})F_t^{t+1} \Leftrightarrow 1+r_{t,d} = (1+r_{t,f})\frac{F_t^{t+1}}{S_t} \tag{1}$$

Thus, under CIP, the return from two equally risky deposits with no exchange rate risk should equate. Next, assuming that investors are risk-neutral, it holds that the forward rate F_t^{t+1} is equal to the expected future spot rate, $E[S_{t+1}]$. Using this in the CIP leads to the uncovered interest rate parity (UIP), which states that

$$1+r_t = (1+r_{t,f})\frac{E[S_{t+1}]}{S_t}$$

Taking logarithms leads to

$$r_{t,d} - r_{t,f} = E[s_{t+1}] - s_t \tag{2}$$

From (2), we see that, under UIP, the expected depreciation is equal to the interest rate differential. Thus, a higher domestic interest rate implies an expected depreciation of the exchange rate. However, the consensus in the literature seems to be that the UIP is rejected when assuming the risk neutrality and/or rational expectations, see e.g. **Burnside et al. 2006**. A possible explanation to the literature rejecting the UIP may be the existence of a time-varying risk premium as suggested by **Fama 1970)**.

Given the failure of the standard UIP relation, it should be possible to earn risk-free (arbitrage) profit: If high-interest currencies do not depreciate in value on average, one could buy high-interest currency bonds and sell low-interest rate currency bonds, since there would be an expected currency gain as well as a positive interest rate differential from the bond coupons, also known as a ”carry trade”. We choose not to account for equilibrium relationships in our model due to the literature not showing substantial support for these in the short run.

2.2 Foreign Exchange Options

The valuation of FX options is somewhat analogous to the formulae known from Black and Scholes (1973) but extended by Garman and Kohlhagen (1983) to cope with the presense of both a domestic and a foreign interest. The underlying asset is the spot exchange rate, defined as S_0, i.e. the value of one unit of foreign currency in domestic currency. As the holder of foreign currency receives the the risk-free yield in the foreign currency, the asset can be thought of as being similar to a stock with a known dividend yield.[2]

Assuming that the exchange rate is a log-normal process following a geometric brownian motion with a drift equal to the domestic-foreign risk-free interest rate differential, $dS_t = (r_d - r_f)S_t dt + \sigma S_t dW_t$, the pricing formulas for European calls and puts are given by equations (3) and (4) respectively:

$$c = S_0 e^{-r_f T} N(d_1) - K e^{-r_d T} N(d_2) \tag{3}$$

$$p = K e^{-r_d T} N(-d_2) - S_0 e^{-r_f T} N(-d_1) \tag{4}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r_d - r_f + \sigma^2/2)\,T}{\sigma\sqrt{T}} \tag{5}$$

$$d_2 = \frac{\ln(S_0/K) + (r_d - r_f - \sigma^2/2)\,T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{6}$$

with K being the strike price[3], $N(x)$ being the normal cumulative distribution function and T being *time to maturity* calculated in accordandance to relevant day count conventions. To obtain the lower bounds for the price of the European currency option, say a call, we realize that the same probability distribution for the exchange rate at time T is obtained from 1) the exchange rate starting at S_0 providing a continuously compounded risk-free yield of r_f

[2]Hull (2021).

[3]The value of the strike must also be quoted in units of domestic currency per unit of foreign currency.

and 2) the exchange rate starting at $S_0 e^{-r_f T}$ paying no yield. Considering two portfolios:

$$Portfolio\ 1 : \text{One European call option and one cash-account worth } Ke^{-r_d T}$$
$$Portfolio\ 2 : e^{-r_f T} \text{ units of foreign currency with yield continuously being re-invested}$$

we know that Portfolio 1 is worth $\max\{S_0 e^{-r_f T}, K\}$ at time T, whereas Portfolio 2 is worth $S_0 e^{-r_f T}$ at time T. By a no-arbitrage argument, Portfolio 1 must also be worth at least as much as Portfolio 2 at time t and thus we obtain the following lower bound:

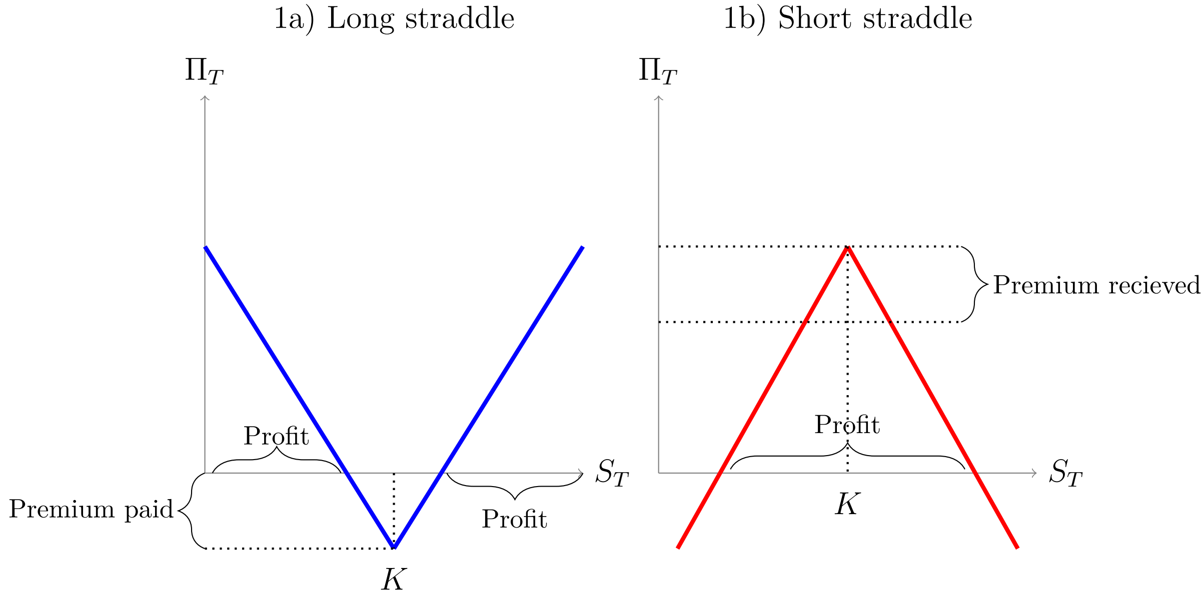
$$\begin{aligned} &c + Ke^{-r_d T} \geq S_0 e^{-r_f T} \\ &c \geq S_0 e^{-r_f T} - Ke^{-r_d T} \\ &c \geq \max\left\{S_0 e^{-r_f T} - Ke^{-r_d T}, 0\right\} \end{aligned} \tag{7}$$

with the last expression coming from the fact that the value of a call option expiring worthless cannot be negative. Analogously, the lower bound for a European put option is given by $p \geq \max\left\{Ke^{-rT} - S_0 e^{-rf}, 0\right\}$ and from the combination of the two, the put-call parity can be derived from similar argument replication argument:

$$c + Ke^{-r_d T} = p + S_0 e^{-r_f T} \tag{8}$$

which must hold for European currency calls and puts with same strike and maturity for there to be no arbitrage oppotunities. Figure 1 below shows the payoff structure from being long versus short the straddle trade.

Figure 1: Payoff structures from long and short straddles



2.2.1 Implied volatility

The extended Black-Scholes model outlined gives a closed-form solution for the price of a European FX option per unit of notional given the input parameters.[4] All of the input parameters apart from future volatility are observable in the market, and hence implied volatility is calculated from inverting the model and solving for future volatility using the known market prices for that specific option. As such, implied volatility can be seen as a price-based proxy for market risk, i.e. what the market expects the volatility of the price of the underlying to be over the life of the option. An option with a high implied volatility is gauged as having a bigger probability of ending "in-the-money" and therefore has a higher premium.

If the assumptions of the Black-Scholes model hold, the implied volatility would be identical for all European options on a given FX rate, i.e. over the diffent strikes and times to maturity. In reality, however, the believe is that returns are extreme than a log-normal process would prescribe which gives rise to the well-known volatility surface. For forex options in particular, implied volatility as a function of "moneyness" usually takes the form of a

[4]Today, many industry practitioners use more sophisticated models like SABR or a wide range of local volatility models. **Citation:** Risk metrics can differ across models, but as they are mostly in line when it comes to prices, we just stick to keeping it simple.

"smile" due to the fact that the price is merely a ratio, e.g. $S_t = EUR/USD$:

$$-log(S_t) = -log\left(\frac{EUR_t}{USD_t}\right) = log\left(\frac{USD_t}{EUR_t}\right) = log(1/S_t). \tag{9}$$

As such, high skews on one side will automatically be accompanied by a high skew on the other side. Although they don't have to be perfectly symmetric, "smirks" to either side are generally unsustainable for longer time periods.

In practice, it is the convention that options are quoted in terms of implied volatility. The industry standard of quoting in units of the common IV model input ensures that market participants with different models and inputs can agree on the terms and trade. It is important to note that the implied volatility of an FX option depends on the numeraire of the purchaser, i.e. the currency in which the option is valued. An option on EURUSD gives a USD value linear in the pair using USD as the numeraire and a non-linear value in EUR due to the non-linearity of the inversion operation: $x \mapsto 1/x$.

2.3 Modelling and Forecasting Volatility

In this section, we outline the GARCH framework as well as the approach for forecasting conditional variances based on the GARCH model. The main idea is that, in the long run and in the absence of innovations, the conditional variance will converge to the unconditional variance of the underlying data generating process.

Thus, the complicated part of modelling conditional variance is to i) obtain a reasonable estimate for the current conditional variance and ii) obtain a reasonable estimate for how fast the conditional variance converges. The GARCH literature is enormous, and we therefore rely on a few of the most common specifications as extensions to the baseline model.

2.3.1 The GARCH framework

The general and widely applied GARCH(p, q) model is defined as a joint model for the conditional mean and conditional variance equations as follows:

$$r_t = \varepsilon_t \tag{10}$$

$$\varepsilon_t = \sigma_t z_t \tag{11}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{12}$$

with r_t being the log return at time t, x_0, σ_0^2 given, and the parameter restrictions $\sigma_t^2 > 0$ and $\alpha, \beta \geq 0$. In this general specification, the innovation z_t is an i.i.d. process drawn from some known distribution, e.g., the Gaussian or the Student's t-distribution. For the standard asymptotic properties of the GARCH(p, q) to apply, one needs to establish a law of large numbers and a central limit theorem. For the GARCH(1,1) process we apply, which is the workhorse model in the GARCH literature, such properties hold for $\alpha + \beta < 1$, where the process has a finite second order moment.

We estimate the GARCH(1,1) model in Equations (10)-(12) with the Quasi Maximum Likelihood Estimator, i.e., despite that the true data generating process for z_t is fat-tailed, we superimpose the Gaussian distribution. This leads to inefficient, but consistent and unbiased estimators. The log-likelihood function, following the assumption of Gaussianity, is (up to a scaling constant) given by:

$$\ell_T(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left(\log \sigma_t^2(\theta) + \frac{r_t^2}{\sigma_t^2(\theta)} \right) \tag{13}$$

The QMLE is then obtained by maximizing Equation (13) with respect to θ under the aforementioned parameter restrictions, such that

$$\hat{\theta}_T = \arg\max_{\theta \in \Theta} \ell_T(\theta)$$

which is in practice done via numerical procedures due to the non-linearity of the optimization problem.

2.3.2 Model extensions

To allow for other types of return dynamics we consider extensions to the model for the conditional variance in the appendix, which thus serves as a robustness check for our results in the empirical analysis. Although Hansen and Lunde (2005) find no evidence that extensions to the GARCH(1,1), it is interesting to compare the results against these models. Should the ex post trading profit from such models be higher, it would be inefficient not to consider these, as we seek to maximize out-of-sample profit, not in-sample goodness-of-fit.

The literature on conditional volatility modelling has led to what Bollerslev has pointed out to be a "perplexing alphabet-soup of acronyms and abbreviations"[5], which is also evident in the paper by Hansen and Lunde (2005) who investigate 330 different specifications of the model. In particular, all model extensions are constructed based on the need for alternative dynamics and parameter restrictions. We choose to evaluate the trading performance with two different extensions to the baseline model to capture these alternative dynamics.

First, to circumvent the potential drawbacks from the symmetry of how past innovations affect current volatility, we first consider the model by Glosten et al. (1993) known as the GJR-GARCH model. In this model, past negative shocks have a larger impact on current volatility than past positive shocks. In particular, consider the conditional variance specification given by:

$$\sigma_t^2 = \begin{cases} \omega + (\alpha + \kappa)\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, & \text{if } \varepsilon_{t-1} < 0 \\ \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, & \text{if } \varepsilon_{t-1} \geq 0 \end{cases}$$

This model was originally motivated for modelling equity volatility due to the so-called leverage effect: A negative return implies that the market value of equity decreases which, in turn, increases the leverage ratio. A higher leverage ratio implies a higher default risk and therefore the volatility of the stock price should increase. Note that for foreign exchange, there is no such default risk, and the model is motivated by its ability to capture the fat-tailedness of the innovations.

Secondly, we consider the exponential GARCH(1,1), known as the EGARCH(1,1), sug-

[5]**Verbeek**, p. 327

gested by by **Nelson (1991)** which is a model of the log-variance given by:

$$\ln \sigma_t^2 = \omega + \alpha g(z_{t-1}) + \beta \ln \sigma_{t-1}^2 \tag{14}$$

where $\sigma_t > 0$ and the functional $g(z_{t-1})$ is given by

$$g(z_{t-1}) = \theta z_{t-1} + \xi(|z_{t-1}| - E[|z_{t-1}|]) \tag{15}$$

with $\omega, \alpha, \beta, \theta$ and $\xi \in \mathbb{R}$. We choose to include the EGARCH in our estimation approach since it mitigates the restrictions on the return dynamics that arise as a result of parameter restrictions in the GARCH(1,1) and GJR-GARCH(1,1). In particular, σ_t^2 will always be positive since we can obtain the conditional variance as $\sigma_t^2 = \exp \ln \sigma_t^2$

2.3.3 Out-of-sample volatility forecasts

Having obtained a vector of parameter estimates, one may conduct an out-of-sample volatility forecast using the recursive structure of the GARCH models. For the purpose of the exposition, recall that the GARCH(1,1) may be written as an ARMA(∞) model by recursive substitution:

$$\begin{aligned}
\sigma_t^2 &= \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \\
&= \omega + \alpha\varepsilon_{t-1}^2 + \beta(\omega + \alpha\varepsilon_{t-2}^2 + \beta\sigma_{t-2}^2) \\
&\;\vdots \\
&= \frac{\omega}{1-\beta} + \alpha \sum_{j=0}^{\infty} \beta^{j-1}\varepsilon_{t-j}^2
\end{aligned}$$

Recall that the process is weakly mixing if $\alpha + \beta < 1$, which implies that the unconditional variance is $\sigma^2 = \frac{\omega}{1-\alpha-\beta} > 0$. Rewriting the unconditional variance to $\omega = \sigma^2(1 - \alpha - \beta)$, we may rewrite (12) with $p = 1, q = 1$ as:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$
$$\Leftrightarrow \sigma_t^2 = \sigma^2(1-\alpha-\beta) + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$
$$\Leftrightarrow \sigma_t^2 = \sigma^2 + \alpha(\varepsilon_{t-1}^2 - \sigma^2) + \beta(\sigma_{t-1}^2 - \sigma^2)$$

From the last equation, it is clear that the conditional variance is given by the unconditional variance plus two "noise" terms adjusted by α and β. Now, we may utilize a forward recursion of the conditional variance equation, as $E[\sigma_{T+1}^2|\mathcal{I}_T]$ can be calculated directly with the information at time T. Doing so yields the forecasting equation for period $T+h$, $h \in \mathbb{Z}$:

$$E[\sigma_{T+h}^2|\mathcal{I}_t] = \omega\sum_{i=0}^{h-1}(\alpha+\beta)^i + (\alpha+\beta)^{h-1}\hat{\sigma}_{T+1}^2 \tag{16}$$

where $\hat{\sigma}_{T+1}^2|\mathcal{I}_t = \hat{\omega} + \hat{\alpha}r_T + \hat{\beta}\sigma_T^2$. From the forecasting equation, we may interpret the parameter sum $\alpha+\beta$ as the speed of convergence towards to unconditional variance. Here, the weakly mixing assumption for the return process is crucial. If $\alpha+\beta > 1$, the sum does not converge, which implies that the volatility becomes explosive and the second order moment would be infinite. Moreover, we emphasize that for longer horizons, the GARCH model becomes uninformative as the forecast converges towards the unconditional variance, which is seen by letting h tend to infinity and using $\omega = \sigma^2(1-\alpha-\beta)$, such that we obtain:

$$\begin{aligned} E[\sigma_{T+\infty}^2|\mathcal{I}_t] &= \lim_{h\to\infty}\left[\omega\sum_{i=0}^{h-1}(\alpha+\beta)^i + (\alpha+\beta)^{h-1}\hat{\sigma}_{T+1}^2\right] \\ &= \sigma^2(1-\alpha-\beta)\frac{1}{1-\alpha-\beta} \\ &= \sigma^2 \end{aligned}$$

For the empirical analysis, we are interested in the forecasted annualized volatility 21 trading days ahead. This is therefore, based on equation (16) given by:

$$\sqrt{252}\hat{\sigma}_{T+21} = \sqrt{252\left[\omega\sum_{i=0}^{20}(\alpha+\beta)^i + (\alpha+\beta)^{20}\hat{\sigma}_{T+1}^2\right]}$$

The 21-day realized (annualized) volatility that we use to calculate the performance is given by the equation

$$\sqrt{252}\sigma_t = \sqrt{252}\frac{1}{21}\sum_{t-20}^{t} |s_t|, \tag{17}$$

where we have used that returns have an unconditional mean of zero (see Table 1) such that the average of the absolute returns are equal to the standard deviation.

3 Data

We use daily historical data for 6 different currencies measured against the US Dollar: Russian Rubles (RUB), South African Rands (ZAR), Brazilian Reals (BRL), Euros (EUR), British Pounds (GBP), and Japenese Yen (JPY). All data is sourced via Bloomberg, and the sample period is January 15, 2000 to February 22, 2022 for all currencies, however, the individual series are only modelled for the periods where both returns and implied volatility are accessible (see Table 1 for the specific ranges). We calculate the log return of each currency pair as:

$$x_t = s_t - s_{t-1}, \tag{18}$$

where $s_t = \log S_t$ with S_t being the nominal exchange rate measured in USD. Hence, a positive return is associated with a relative USD appreciation. This means that EURUSD and GBPUSD prices have been inverted switching bid and ask prices.

Table 1 below presents defines the sample and presents the summary statistics.

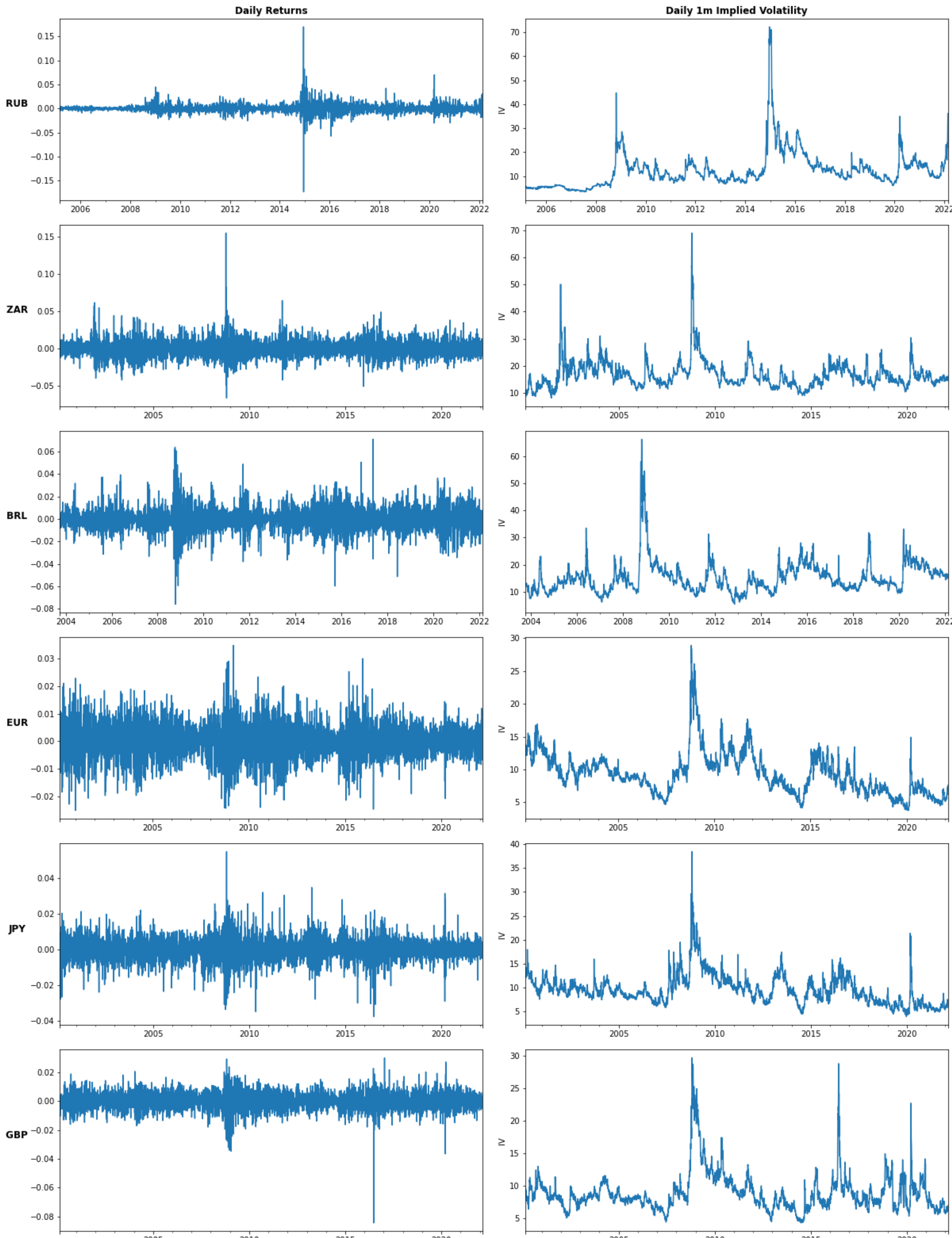
Table 1: Summary statistics of the log return in USD

	RUB	ZAR	BRL	EUR	JPY	GBP
Mean return	0.000	0.000	0.000	0.000	0.000	0.000
Std. dev.	0.008	0.011	0.010	0.006	0.006	0.006
Min return	-0.173	-0.066	-0.103	-0.025	-0.038	-0.084
Max return	0.170	0.155	0.071	0.035	0.055	0.030
Skewness	0.555	0.838	0.062	0.042	-0.045	-0.755
Exc. kurt.	85.732	6.938	2.813	-1.210	1.524	7.436
Mean bid-ask[6]	0.001	0.003	0.001	0.000	0.000	0.000
Observations	4429	5740	4800	5725	5740	5740
Sample start	2005-03-03	2003-02-23	2003-10-01	2000-03-15	2000-02-23	2000-02-23
Sample end	2022-02-22	2022-02-22	2022-02-22	2022-02-22	2022-02-22	2022-02-22

Note: All exchange rates are against the USD. Increase = USD appreciation.

COMMENT ON SUMMARY STAT TABLE

Figure 2: Daily returns and 1-month market-implied volatility



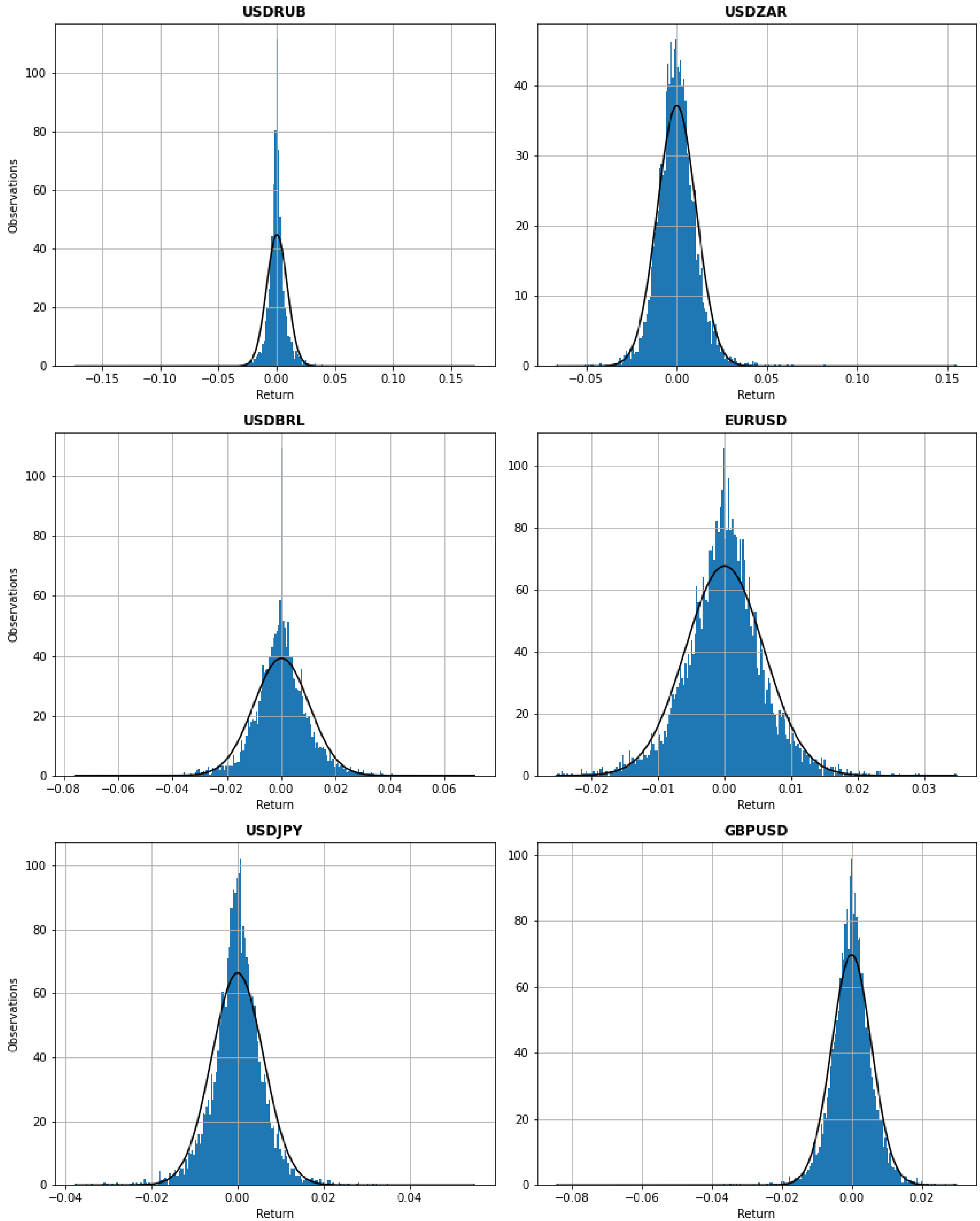
Note: Positive return = Nominal USD appreciation

As can be seen from Figure 2, returns are fat-tailed for all series but EURUSD and volatility tends to cluster, which motivates modelling them within a conditionally heteroskedastic framework, where the conditional variance is allowed to spike and exert a persistent retraction towards the unconditional variance.

The figure also indicates that currency risk is driven by both common and idiosyncratic factors. The common factor, measured by a general market price of risk, is particularly evident in times of market turmoil. In particular, the global financial crisis of 2008 seems to have driven the spike in conditional variance seen in that period. A key insight from 2 is, however, that the market price of risk differs between emerging and developed markets. Comparing the implied volatility following the crisis between the top and bottom three countries in the figure shows that the level of implied volatility is about twice as high in emerging versus developed currencies. This may be explained by the depth of the markets, and the fact that emerging markets are heavily reliant on the developed market business cycle.

In terms of idiosyncratic currency risk, many events can be highlighted: The Russian annexation of Crimea in 2014 spiked USDRUB implied volatility to a level of 70% percent p.a., while the outcome of the Brexit referendum in 2016 spiked the USDGBP implied volatility to almost 30% p.a. This is obviously difficult to model ex ante, but allowing for a fat-tailed innovation distribution may alleviate some of the effects from these so-called tail events. Using the Student's t-distribution instead of the Gaussian distribution is supported by the plots in Figure 3 below, which shows the return distributions against the standard normal distribution.

Figure 3: Empirical Return Distributions



Note: 250 bins.

4 Implementation and Empirical Analysis

As established, the determination of option prices hinges critically on expectations of future volatility. Kroner et al. (1995) points out that more accurate forecasts of volatility can help option traders identifying rich and cheap options. This is the main idea of the volatility trading strategy that we follow based on the methodology proposed by Dunis and Huang (2002): if a given volatility model can produce accurate forecasts of conditional volatility and the market is inefficient in pricing in future volatility, then a profitable trading strategy emerges where the trader should go long volatility in so far the forecast of future volatility is higher than the prevailing implied volatility in the market and vice versa.

The volatility trading strategy should refrain from taking a stance on the direction of the underlying exchange rate and therefore utilize at-the-money (ATM) straddles. A straddle is a combination of a call and a put with the same strike so as to offset and neutralize the opposite deltas and thus hold no forward risk at inception. As can be seen from the profit zones in the payoff profiles in Figure 1, offset by the call (put) premium payed (received), a long (short) straddle profits from high (low) price volatility independent of the direction. Also, as noted in Hull (2021), ATM calls and puts have the same gamma and vega sensitivities, i.e. sensitivities to changes in delta and volatility respectively, and the ATM straddle should therefore be free of directional bias. In addition, using at-the-money forward volatilities also avoids introducing bias caused from volatility smile effects.

4.1 Volatility Trading Methodology

To obtain the trading signals, we carry out 21-days ahead forecasts of conditional volatility on a rolling basis and compare with the prevaling 1-month implied volatility level.[7,8] For a given FX pair, we use the FX returns data up until 2021-01-01 to fit the GARCH model and then roll the window forward to obtain a total of 298 daily out-of-sample forecasts of volatility 1 month ahead thus utilizing the recursive structure of the conditional variance equation. In this way, the fitting period may vary between the different pairs based on

[7]The volatility forecasts are obtained using the `arch` python-library by Kevin Sheppard.

[8]Assuming 21 business days per month.

data-availability, but the out-of-sample forecasting window is identical.

To compare with the implied volatility, we annualize the conditional volatility forecasts and calculate the forecast-to-implied ratio. We set the threshold for when to long the straddle to be 1.1 and 0.9 symmetrically for when to short the straddle. When a trade is initialized it is held to maturity and no other trades are executed during that period. Volatility clustering would cause trading signals to cluster as well, so this restriction prevents the strategy from building up unreasonably large positions in either direction. We note that the straddle position only holds no forward risk at inception and that one should optimally gamma scalp on a continuous basis to keep the position neutral during the period to maturity. For simplicity, we refrain from rebalancing the initial straddle position and let it run for both calls and puts without adjustments noting the asymmetry interms of loss between the two.

4.2 Performance Calculations

It is complicated to conduct an accurate backtesting procedure of a trading strategy for FX options, and retrieve reliable historical performance measures, due to the decentralized structure of trading. That is, retrieving historical quotes across currency crosses, maturities, and strikes is not feasible on a large scale. Therefore, we approximate the return from each trade in terms of volatility points, following the approach of Dunis and Huang (2002). We approximate the net return on a long straddle as the realized volatility at expiry, given by Equation (17) minus the implied volatility at inception, given by the observed market prices, less transaction costs. Conversely, the return on a short straddle is given by the implied volatility at inception minus the realized volatility at expiry less transaction costs.

In the market microstructure theory, transaction costs in trading are functions of different factors, e.g. the liquidity and volatility of prices. In Dunis and Huang (2002), all trades are penalized by 25 basis points (0.25%) regardless of the specific currency cross considered. We choose to extend this approach by allowing for currency-specific and time-varying bid-ask spreads. This way, all trades are penalized by the prevailing bid-ask spread in the FX spot market, such realized trading returns do not become artificially high. In the Appendix, we have included summary statistics as well as plots of the individual bid-ask spread series. We note that our approach is an extension to Dunis and Huang (2002), however, it is not flawless; an options trader would not pay the spot market bid-ask spread directly, but the spread should be included in the option premia, hence this approximation should resemble actual trading costs.

As Dunis and Huang (2002) point out, the approximation of returns by volatility points will overestimate the potential losses, since the long straddle has a lower payoff bound given by the premium paid. On the other hand, the net payoff of a short straddle is given by the market-implied volatility at inception minus the market-implied volatility at expiry less transaction costs.

4.3 Forecasting and Trading Results

4.3.1 Forecasting Ability - RMSE

4.3.2 Trading Profitability

5 Discussion

List of potential discussion points:

- The precision of the straddle return approximation
- Specification of trading costs
- Specification of thresholds - lookback bias
- Robustness checks - Alternative GARCH specifications

6 Conclusion

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A Misspecification Tests

Table A.1: Estimation results

GARCH(1,1)

	RUB	ZAR	BRL	EUR	JPY	GBP
ω	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
α	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
β	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
ν	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
AIC						
Log L						

GJR-GARCH(1,1)

	RUB	ZAR	BRL	EUR	JPY	GBP
ω	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
α	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
β	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
κ	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
ν	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
AIC						
Log L						

EGARCH(1,1)

	RUB	ZAR	BRL	EUR	JPY	GBP
ω	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
α	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
β	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
θ	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
ξ	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
ν	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)	0.01 (1.96)
AIC						
Log L						

Note: Robust t-statistics in parentheses.
The estimation period is ... to .... We apply a rolling-window estimation, hence only the first estimation is reported here.

Table A.2: Misspecification tests

			GARCH(1,1)			
Test	RUB	ZAR	BRL	EUR	JPY	GBP
Autocor						
JB						
No ARCH						
			GJR-GARCH(1,1)			
	RUB	ZAR	BRL	EUR	JPY	GBP
Autocor						
JB						
No ARCH						
			EGARCH(1,1)			
	RUB	ZAR	BRL	EUR	JPY	GBP
Autocor						
JB						
No ARCH						

Note: The table shows the p-values for each test and currency pair.

B Bid-ask spreads

Summary statistics (mean, min, max, std. dev) for all bid-ask spread series as well as plots of them (Frederik B will do this :))