1.3.4 Bedingung an Spaltbreite

$\Rightarrow \quad d \approx \lambda$

1.3.5

$$\psi_j = A \exp\left(i\left(k(r+j\Delta) - \omega t\right)\right)$$

$$\sum \psi_j = A\, e^{i(kr-\omega t)}\left(1 + e^{i\Delta} + e^{i2\Delta} + \ldots + e^{i(N-1)\Delta}\right)$$

$$= A\, e^{i(kr-\omega t)} \cdot \frac{e^{iN\Delta}-1}{e^{i\Delta}-1}$$

1.3.6

$$\frac{e^{iN\Delta}-1}{e^{i\Delta}-1} = \frac{e^{i\frac{N}{2}\Delta}}{e^{i\Delta/2}} \cdot \frac{\left(e^{i\frac{N\Delta}{2}} - e^{-i\frac{N\Delta}{2}}\right)}{\left(e^{i\frac{\Delta}{2}} - e^{-i\frac{\Delta}{2}}\right)}$$

$$= e^{\frac{i}{2}(N-1)\Delta} \cdot \frac{\left(\cos\frac{N\Delta}{2} + i\sin\frac{N\Delta}{2}\right) - \left(\cos\frac{N\Delta}{2} - i\sin\frac{N\Delta}{2}\right)}{\left(\cos\frac{\Delta}{2} + i\sin\frac{\Delta}{2}\right) - \left(\cos\frac{\Delta}{2} - i\sin\frac{\Delta}{2}\right)}$$

$$= e^{\frac{i}{2}(N-1)\Delta} \cdot \frac{\sin\frac{N\Delta}{2}}{\sin\frac{\Delta}{2}}$$

1.3.7

$$\Delta = \frac{d}{N}\sin\alpha$$

$$\psi = \tilde{A}\,\frac{\sin\left(\frac{N}{2}\frac{d}{N}\sin\alpha\right)}{\sin\left(\frac{d}{2N}\sin\alpha\right)}$$

$$\tilde{A} = A\, e^{i(kr-\omega t)}\, e^{\frac{i}{2}(N-1)\Delta}$$

$$= A\, e^{-i\omega t}\, e^{i\left(kr + d\frac{(N-1)}{2N}\sin\alpha\right)}$$

$$N \to \infty \quad \Rightarrow \quad \frac{d}{N} \to 0$$

$\hookrightarrow \quad \sin\left(\frac{d}{2N} \cdot \sin\alpha\right) \to \frac{d}{2N} \sin\alpha$

$\Rightarrow \quad \tilde{A} \to A e^{i\omega t} e^{i(kr + \frac{d}{2}) \sin\alpha}$

$\Rightarrow \quad \psi = A e^{i\omega t} e^{i(kr + \frac{d}{2}) \sin\alpha} \cdot N \frac{\sin(\frac{d}{2} \sin\alpha)}{\frac{d}{2} \sin\alpha}$

1.3.8

$$I = |\psi|^2 \sim \frac{\sin^2(\frac{d}{2} \sin\alpha)}{(\frac{d}{2} \sin\alpha)^2}$$