7.1.3

(i) $\psi = \frac{1}{\sqrt{2}}(\psi_{1x}\,\psi_{2y})$

(ii) $\psi = \frac{1}{\sqrt{2}}(\psi_{1L}\,\psi_{2L})$

7.1.4

γ_1 zuerst

$$\psi = \frac{1}{\sqrt{2}}(\psi_{2L}\,\psi_{2L}^{+})\,\psi_{1x}\,\psi_{2y}$$

$$= \frac{1}{\sqrt{2}}(\psi_{2L}\,\psi_{2L}^{+})(\psi_{1L}+\psi_{1R})(\psi_{2L}-\psi_{2R})$$

$$= \frac{1}{\sqrt{2}}(\psi_{1L}+\psi_{1R})\,\psi_{2L}$$

mit:

$$\psi_{1L} = \frac{1}{\sqrt{2}}(\psi_{1x} - i\,\psi_{1y})$$

$$\psi_{1R} = \frac{1}{\sqrt{2}}(\psi_{1x} + i\,\psi_{1y})$$

$$\psi_{2L} = \frac{1}{\sqrt{2}}(\psi_{2x} + i\,\psi_{2y})$$

$$\psi_{2R} = \frac{1}{\sqrt{2}}(\psi_{2x} - i\,\psi_{2y})$$

$$\longrightarrow \psi = \frac{1}{\sqrt{2}}\,\psi_{1x}(\psi_{2x} + i\,\psi_{2y})$$

$$= \frac{1}{2}\,\psi_{1x}\,\psi_{2L}$$

γ_2 zuerst

$$\psi = \frac{1}{\sqrt{2}} \psi_{1L} \psi_{2L}$$

$$= \frac{1}{\sqrt{2} \cdot 2} \psi_{1x} (\psi_{2x} + i \psi_{2y})$$

$$= \frac{1}{\sqrt{2}} (\psi_{1L} + \psi_{1R}) \psi_{2L}$$

$$= \frac{1}{2} \psi_{1x} \psi_{2L}$$

7.2.2

Legendre-Trafo: $H = \vec{P} \dot{\vec{x}} - L$

$$= m \dot{\vec{x}}^2 + q A \dot{\vec{x}} - \frac{1}{2} m \dot{\vec{x}}^2 + q (V - \dot{\vec{x}} \vec{A})$$

$$= \frac{1}{2} m \dot{\vec{x}}^2 + q V \qquad \Big| \dot{\vec{x}} = \frac{1}{m} (\vec{P} - q \vec{A})$$

$$= \frac{1}{2m} (\vec{P} - q \vec{A})^2 + q V$$

7.2.3

$$\dot{\vec{x}} = \frac{\partial H}{\partial \vec{p}} = (\vec{P} - q \vec{A}) \frac{1}{m} \Rightarrow \quad T = \frac{1}{2} m \dot{\vec{x}}^2 = \frac{1}{2m} (\vec{P} - q \vec{A})$$

$$U = q V$$

$$\Rightarrow \quad H = T + U$$

7.2.4

$$\dot{p}_j = -\frac{\partial H}{\partial x_j}$$

$$= -\frac{1}{m}\underbrace{(\vec{p} - q\vec{A})_k}_{\vec{P}_k} \frac{\partial}{\partial x_j}(\vec{p} - q\vec{A})_k - q\frac{\partial}{\partial x_j} V$$

$$= -\frac{1}{m}\vec{P}_k\left(-q\frac{\partial}{\partial x_j}\vec{A}_k\right) - q\frac{\partial}{\partial x_j} V$$

$$= \frac{q}{m}\vec{P}_k\frac{\partial}{\partial x_j}\vec{A}_k - q\frac{\partial}{\partial x_j} V$$

$$\Rightarrow \dot{\vec{p}} = q\vec{\nabla}(\vec{v}\cdot\vec{A}) - q\vec{\nabla}V \qquad (*)$$

$$\dot{\vec{P}} = \frac{d}{dt}(\vec{p} - qA)$$

$$\overset{(*)}{=} q\vec{\nabla}(\vec{v}\cdot\vec{A}) - q\vec{\nabla}V - q\underbrace{\left(\frac{\partial\vec{A}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{A}\right)}_{\frac{d\vec{A}}{dt}}$$

$$\frac{d\vec{A}}{dt} = \frac{\partial\vec{A}}{\partial\vec{x}}\frac{\partial\vec{x}}{\partial t} + \frac{\partial\vec{A}}{\partial t}$$

$$= (\vec{\nabla}A)\vec{v} + \frac{\partial A}{\partial t}$$

$$= \vec{\nabla}(\vec{A}\vec{V}) - \vec{v}\times(\vec{\nabla}\times A) + \frac{\partial\vec{A}}{\partial t}$$

$$\Rightarrow \dot{\vec{P}} = q\left(-\vec{\nabla}V - \frac{\partial A}{\partial t} + \vec{v}\times(\vec{\nabla}\times\vec{A})\right)$$

7.2.5

$$\vec{p} = \vec{\pi}$$

$$\{x_i, \pi_j\} = \frac{\partial x_i}{\partial x_k}\frac{\partial \pi_j}{\partial \pi_k} - \frac{\partial x_i}{\partial \pi_k}\frac{\partial \pi_j}{\partial x_k} = \delta_{ij}$$

$$\{x_i, p_j\} = \frac{\partial}{\partial \pi_k}(\pi_j - qA_j)\,\delta_{ki} = \delta_{ij}$$

7.2.6

$$\{\pi_i, \pi_j\} = \frac{\partial \pi_i}{\partial \pi_k}\frac{\partial \pi_j}{\partial \pi_k} - \frac{\partial \pi_i}{\partial \pi_k}\frac{\partial \pi_j}{\partial \pi_k} = 0$$

$$\{p_i, p_j\} = \frac{\partial}{\partial x_k}(\pi_i - qA_i)\frac{\partial}{\partial \pi_k}(\pi_j - qA_j)$$
$$- \frac{\partial}{\partial \pi_k}(\pi_i - qA_i)\frac{\partial}{\partial x_k}(\pi_j - qA_j)$$

$$= \delta_{kj}\frac{\partial}{\partial x_k}(-qA_i) - \delta_{ki}\frac{\partial}{\partial x_k}(-qA_j)$$

$$= -q\left(\frac{\partial}{\partial x_j}A_i - \frac{\partial}{\partial x_i}A_k\right) \neq 0$$

7.2.7

$$H = \frac{1}{2m}(-i\hbar\vec{\nabla} - q\vec{A})^2 + qV$$

7.2.8

$$[H, \pi_k] = [H, -i\hbar \frac{\partial}{\partial x_k}]$$

mit $\quad H = \frac{1}{2m}(\partial_{x_i}^2 \hbar^2 + i\hbar \partial_{x_i} A_i + i\hbar A_i \partial_{x_i} + q^2 A_i^2) + qV$

$$\longrightarrow = \left(-\frac{i\hbar}{2m}(\partial_{x_i}\partial_{x_k} A_i) - \frac{i\hbar}{2m}(\partial_{x_k} A_i)\partial_{x_i} - \frac{q^2}{2m}(\partial_{x_k} A_i^2) - q\partial_{x_k} V\right) \cdot (-i\hbar)$$

$$\frac{d\hat{x}}{dt} = \frac{i}{\hbar}[H, \hat{x}] + \underbrace{\frac{\partial x}{\partial t}}_{0} = \frac{\hat{p}_x}{m}$$

$$\frac{d\hat{p}_x}{dt} = \frac{i}{\hbar}[H, \hat{p}_x] = -\frac{\partial V}{\partial x}$$

$$H = \frac{1}{2m}\hat{p}^2 + V$$