Computational Physics - Problem Set 3

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1 Matrix Multiplication

The matrix product of two NxN matrices is computed using nested for loops and np.dot method respectively. See Figure 1.

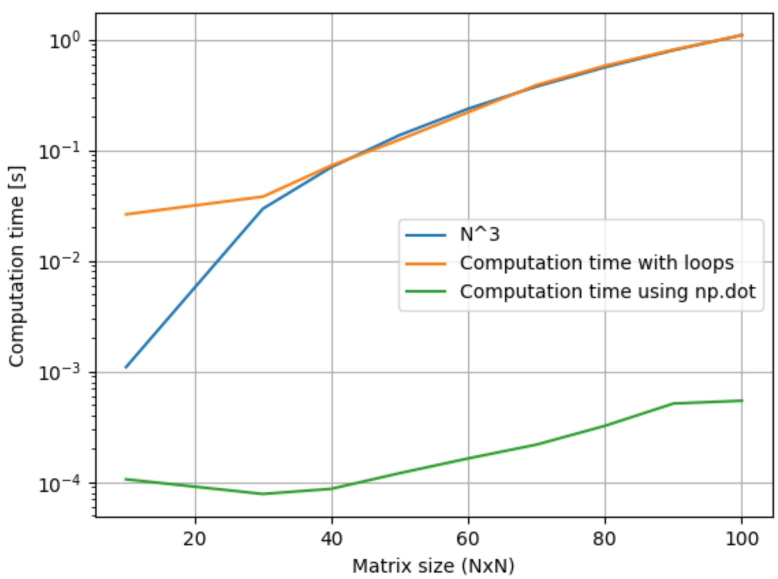

Figure 1: The computation time is plotted in log scale against the matrix size N, so that the matrices has sizes NxN. The matrix elements are random numbers. The N^3 graph is normalized against the nested for-loop computation and shows very good agreement. One notices how the np.dot method is much faster than the nested loop method.

2 Decay

The decay of Bi-213 to Bi-209 has been simulated and the populations over a time interval of 20,000 seconds are shown in figure 2.

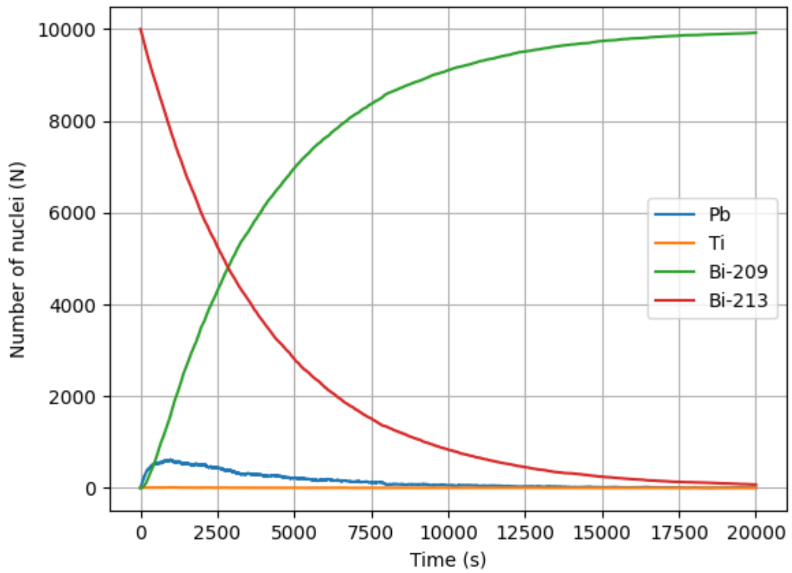

Figure 2: Simulation of the population of the four different populations, Ti, Bi-213, Bi-209 and Pb, for a series of decays where Bi-213 can decay to Bi-209 either via Ti-Pb-Bi or directly via Pb-Bi.

3 Decay 2: Non-uniform distribution

We use a non-uniform distribution,

$$P(t)dt = 2^{-t/\tau}\frac{\ln 2}{\tau}dt$$

to calculate the decay of a sample of N atoms, as a different and more effective method than the one shown above. We use a uniform distribution of random numbers from 0 to 1, z and map those onto the non-uniform distribution x. Using the transformation method presented in Newman ch. 10, we have the mapping between the two distributions x and z to be:

$$x = -\frac{1}{\mu}\ln(1-z)$$

After sorting the values, we ask for each time interval (from 0 to 2000 seconds) how many atoms haven't decayed yet. This defines the decay graph shown in figure 3.

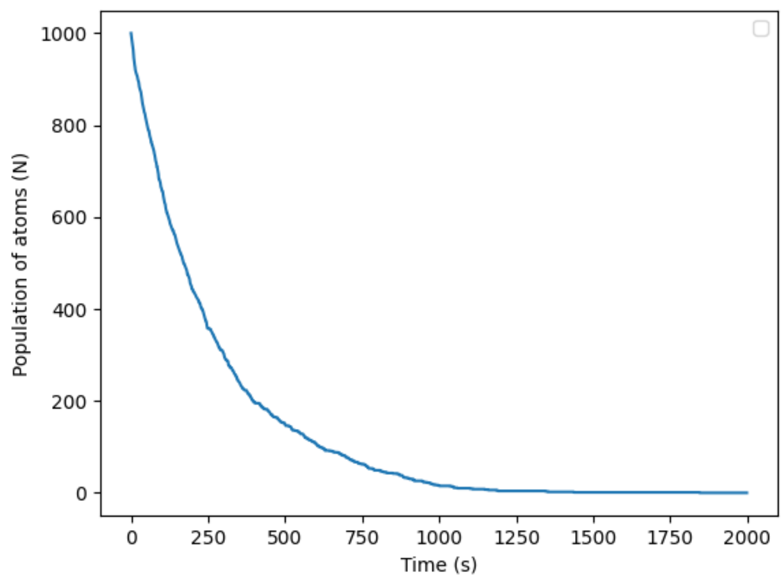

Figure 3: Simulation of the population decay of Ti-208 made using the transformation method of generating 1000 random numbers from a nonuniform distribution. The graph shows how many atoms have not decayed yet at a given point in time.

4 The Central Limit Theorem

First we carry out the derivations: The theoretical mean of y is found by noting that the mean of the x_i is equal to one:

$$E[y] = N^{-1} \sum_{i=1}^{N} E[x_i] \tag{1}$$

$$= N^{-1} N = 1 \tag{2}$$

For finding the variance, we use that $Var[x_i] = 1$ and that the variance of a constant gives the square of that constant:

$$Var[y] = \frac{1}{N^2} \sum_{i=1}^{N} Var[x_i] \tag{3}$$

$$= \frac{1}{N^2} N = N \tag{4}$$

Then we use a sample size of 1000 for each of the N values. For every N, we compute the distribution of y and see that in the limit of large N, the distribution becomes a Gaussian, see Figure 4. For a plot of how the mean, skewness, variance and kurtosis varies with N, see figure 5.

The skewness becomes less than 1 percent of the case for $N = 1$ at $N = 211$ and likewise for the kurtosis at $N = 41$.

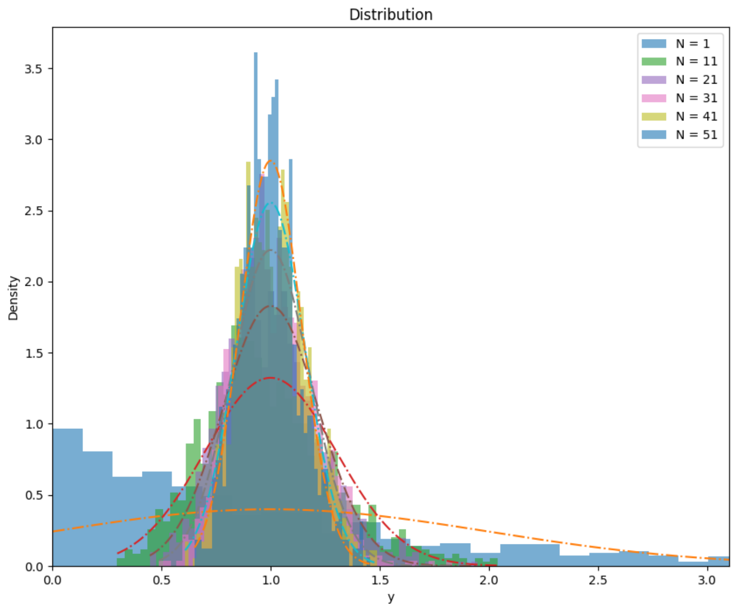

Figure 4: For larger and larger N, a histogram (bin number = 50), is plotted. For every distribution, the mean is 1. For large N, the distribution is in good agreement with the Gaussian (dashed line).

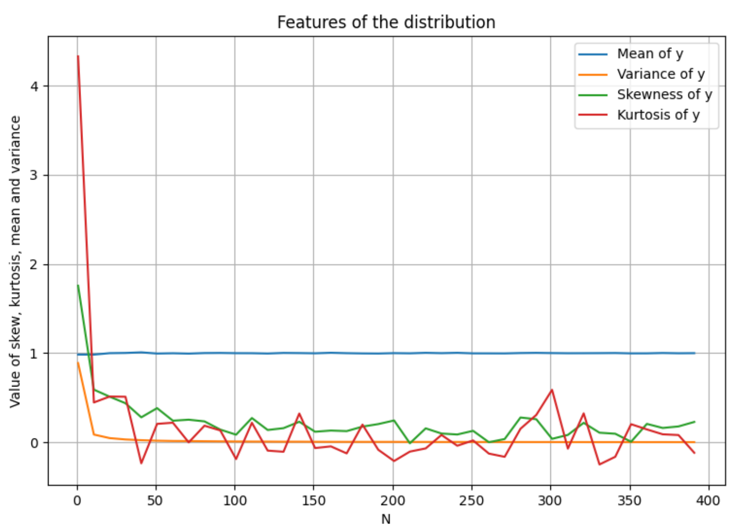

Figure 5: Each graph shows the N-depedence of mean, variance, kurtosis and skewness respectively. We see that mean=1 as expected, and that the other three vanishes for large N.