

TMA 4265 Stochastic Modelling

Exercises week 37

Exercise 1: Consider the Markov chain $\{X_n\}$ with state space $\{1, 2, 3, 4, 5, 6\}$ and transition matrix

$$\mathbf{P} = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}.$$

- a) Is this chain reducible or irreducible? If it is reducible, specify its equivalence classes.
- b) What is the period of each state?
- c) Which states are transient and which are recurrent? Are there any absorbing states?

Exercise 2: Consider the Markov chain $\{X_n\}$ with transition probability matrix

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0.05 & 0.8 & 0.15 \\ 0 & 0 & 1 \end{pmatrix}.$$

- a) Is the Markov chain reducible or irreducible?
- b) Given that you start in state 2, what is the limiting distribution of states 1, 2 and 3?
- c) Write a code to simulate this Markov chain. Verify the answer you got in question b.

Exercise 3: We repeatedly throw a fair coin. Let n denote the n -th throw in the sequence and define a stochastic process for $n = 0, 1, \dots$

$X_n =$ Number of consecutive throws of H, including throw n .

- a) Explain why this is a Markov process. Find the transition probability matrix of the process and specify the state space.

- b) Is this Markov chain irreducible?
- c) For each state, calculate its period and explain whether it is recurrent or transient.
- d) Calculate the long run proportion of time that the sequence of throws ends in 3 or more heads.