TMA4265 Stochastic Modeling

Week 43 Solution

Exercise 1

1. We can see that the five variables are independent. Then, the probability is given by:

   $$P(s_1 > 1, s_2 > 1, s_3 > 1, s_4 > 1, s_5 > 1) = \prod_{i=1}^{5} P(s_i > 1) = \prod_{i=1}^{5}(1 - P(s_i \leq 1))$$

   We standardize each s_i and we will see that this is equal to:

   $$= (1-P(z \leq -0.6))(1-P(z \leq -0.5))(1-P(z \leq -0.2))(1-P(z \leq 0))(1-P(z \leq -0.1))$$

   We can find these values in an statistical table since $z \sim \mathcal{N}(0,1)$. Then we have:

   $$= 0.726 \times 0.691 \times 0.579 \times 0.5 \times 0.54 = 0.0784$$

2. If $\text{Corr}[x_i, x_j] = 0.25$ and $\text{Var}[x_i] = 1.5 = \sigma_i^2$, and from the GP note we know that $\text{Corr}[x_i, x_j] = \text{Cov}[x_i, x_j]/(\sigma_i \sigma_j)$. Then:

   $$\text{Cov}[x_i, x_j] = 0.25 \times \sqrt{1.5} \times \sqrt{1.5} = 0.375$$

   And the covariance matrix is:

   $$\Sigma_E = \begin{bmatrix} 1.5 & 0.375 & 0.375 & 0.375 & 0.375 \\ 0.375 & 1.5 & 0.375 & 0.375 & 0.375 \\ 0.375 & 0.375 & 1.5 & 0.375 & 0.375 \\ 0.375 & 0.375 & 0.375 & 1.5 & 0.375 \\ 0.375 & 0.375 & 0.375 & 0.375 & 1.5 \end{bmatrix}$$

3. We know the values of $\boldsymbol{\mu} = (1.6, 1.5, 1.2, 1.0, 1.1)$ and Σ_E.

   Then $(x_2, x_5) \sim \mathcal{N}_2(\boldsymbol{\mu}^*, \Sigma_{\text{E}}^*)$ with:

   $$\mu^* = (1.5, 1.1)$$

   and

   $$\Sigma_{\text{E}}^* = \begin{bmatrix} 1.5 & 0.375 \\ 0.375 & 1.5 \end{bmatrix}$$

Exercise 2

1. Following the formulas of the GP note, we have:

$$E(x_2|x_1 = 175) = \mu_2 + \Sigma_{1,2}\Sigma_1^{-1}(x_2 - \mu_2) = 180 + 15 \times \frac{1}{16} \times (175 - 180) = 175.3125$$

$$Var(x_2|x_1 = 175) = \Sigma_2 - \Sigma_{1,2}\Sigma_1^{-1}\Sigma_{2,1} = 25 - 15 \times \frac{1}{16} \times 15 = 10.9375$$

2. We can standardize and obtain:

$$P(x_2|x_1 = 175 < 170) = P(z < -1.606) = 0.054$$

Exercise 3.

In R:

```
t_A = c(3,5,9,10,13,20)
t_B = c(3.5,5.2,7.8,12.1)

H_A = dist(t_A, diag = TRUE)
H_B = dist(t_B, diag = TRUE)

H_AB = matrix(nrow=6, ncol=4)
for (i in 1:6){
  for (j in 1:4){
    H_AB[i,j] = dist(rbind(as.matrix(t_A[i]),as.matrix(t_B[j])))
  }
}
```