TMA4265 Stochastic Modeling
Week 43 Exercises

Exercise 1

SINTEF researchers are considering five sites near Stavanger for a possible CO_2 injection and storage project. Assume the CO_2 saturations in these locations can be modelled as a multivariate Gaussian: $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5) \sim \mathcal{N}_5(\boldsymbol{\mu}, \Sigma)$, with $\boldsymbol{\mu} = (1.6, 1.5, 1.2, 1.0, 1.1)$ and $\Sigma = \mathbf{I}_5$.

1. Calculate the probability that the saturation values in the locations simultaneously exceed 1.

2. After careful investigations, the researchers have found that a more accurate model is an exchangeable model, where $\text{Var}[x_i] = 1.5$ for $i = 1, 2, \ldots, 5$ and $\text{Corr}[x_i, x_j] = 0.25$ for $i \neq j$. Calculate the resulting covariance matrix Σ_{E} in this case.

3. The researchers have discovered that only sites 2 and 5 are of interest, i.e., x_2 and x_5. What is the joint distribution of (x_2, x_5) if the joint distribution of $\boldsymbol{x} \sim \mathcal{N}_5(\boldsymbol{\mu}, \Sigma_{\text{E}})$?

Exercise 2

Consider the stock price of Telenor tomorrow. Let x_1 be the value at 10:00 and x_2 be the value at 11:00. Assume that a reasonable model for the stock prices is $(x_1, x_2) \sim \mathcal{N}_2(\boldsymbol{\mu}, \Sigma)$, where

$$\boldsymbol{\mu} = (180, 180) \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 16 & 15 \\ 15 & 25 \end{bmatrix}$$

We observe that the stock price at 10:00 is 175 kr.

1. Find the conditional mean and variance of the stock price at 11:00, $x_2 | x_1 = 175$.

2. Calculate the conditional probability that the stock price at 11:00 is less than 170.

Exercise 3.

Exercise D from the GP note.