TMA4265 Stochastic Modeling

Exercise week 35

Exercise 1: Coin Toss

We have two coins in our pocket. One is a normal coin with a head and a tail. The other coin is biased and has two heads. We select one coin randomly from our pocket and flip it once.
a) Assume the outcome of the flip was a head. Compute the probability that we flipped the biased coin.
b) We flip the coin a second time and again the outcome is a head. Compute the probability that we flipped the biased coin.
c) The function `random.uniform()` in Python can be used to draw random numbers between 0 and 1 from a continuous uniform distribution. A similar function in Matlab is `rand()` and a similar function in R is `runif()`. Explore how you can use one of the above functions to simulate the outcome of a coin toss. Next, write code that simulates that you pick one of the coins from your pocket and flip it once (as in 1a) or twice (as in 1b). Use the code to verify the results in **a)** and **b)** by simulating the situations N times, where N is a large number of your choice.

Exercise 2: Insurance claims

Let N denote the number of claims (in hundreds) received by an insurance company during one specific year. The number of claims N is Poisson distributed with expected value $\lambda = 6$. Let the claim amounts be denoted by $C_1, \dots, C_N$. Every claim amount (in mill. kr.) has a log-Gaussian distribution with parameters $\mu = -2$ and σ^2=1, i.e., $C_i = \exp(Y_i)$ where $Y_i \sim \mathcal{N}(\mu, \sigma^2)$ for $i = 1, ...N$. Each claim amount is also independent of the other claim amounts.

a) Show that for the log-Gaussian distribution $\mathrm{E}(C_i) = \mu_c = \exp(\mu + \sigma^2/2)$ and $\mathrm{Var}(C_i) = \sigma_c^2 = \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$. *Hint: Check out the moment generating function of the normal distribution.*

b) Use the law of iterated expectation and the law of total variance to compute the expected total sum of claim amounts and the variance of this total claim amount, i.e., find $\mathrm{E}(\sum_{i=1}^N C_i)$ and $\mathrm{Var}(\sum_{i=1}^N C_i)$.

c) Write code in R/Matlab/Python that simulates the situation described in this exercise. The code should be used to verify the answers from **b)** and **c)** by simulating B years of insurance claims where B is a large number of your choice. In Python you can use the functions `numpy.random.poisson()` and `numpy.random.lognormal()` from the `numpy` package to generate random values from the Poisson distribution and the log-Gaussian distribution respectively. Equivalent functions in Matlab are `poissrnd()` and `lognrnd()`, and in R you can use `rpois()` and `rlnorm()`.