TMA4265 Stochastic Processes
Exercises week 36

Exercise 1

Consider a Markov chain $\{X_n, n = 0, 1, 2, ...\}$ with state space $\Omega = \{A, B, C\}$ and stationary transition matrix

$$\mathbf{P} = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0.1 & 0.7 & 0.2 \\ 0.5 & 0.1 & 0.4 \\ 0.3 & 0.6 & 0.1 \end{pmatrix} \end{array}.$$

The initial distribution is given by $P(X_0 = A) = 0.2$, $P(X_0 = B) = 0.5$ and $P(X_0 = C) = 0.3$. Compute

a) $P(X_3 = A)$, $P(X_3 = B)$ and $P(X_3 = C)$.
b) $P(X_3 = A|X_1 = B, X_0 = A)$
c) $P(X_3 = A|X_2 = C, X_1 = B, X_0 = A)$
d) $P(X_6 = A|X_3 = C)$
e) $P(X_3 = C|X_6 = A)$

Exercise 2

Suppose that the probability of rain tomorrow only depends on whether it rains today. If it rains today, the probability of rain tomorrow is 0.70, and if it does not rain today, it will rain tomorrow with probability 0.25. This process can be modeled as a Markov chain with two states:
State 1: It rained today.
State 2: It did not rain today.
Thus, the transition matrix is given by $\mathbf{P} = \begin{pmatrix} 0.70 & 0.30 \\ 0.25 & 0.75 \end{pmatrix}$.

a) Assume that it rained today and that it is Monday. What is the probability of rain on Friday?
b) What fraction of days can be expected to be rainy in the long-run?
c) Make a script in Python/R/Matlab where you simulate the weather for one year (365 days) according to the above Markov chain. You can assume that it rains January 1. Visualize the results.
What is the probability of rain for a random day during the year according to your simulations? Compare this with your answer in **b)**.
Hint: The function `random.uniform()` in Python can be useful in your simulations. Equivalent functions in R and Matlab are `runif` and `rand` respectively.

Exercise 3

Consider a Markov chain with state space $\Omega = \{0, 1, 2, 3\}$ whose transition probability matrix given by

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.4 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0.6 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

a) Starting in state 1, determine the probability that the Markov chain ends in state 0.

b) Determine the mean time of absorption given that the process starts in state 1.