

TMA4265 Stochastic Modeling

Exercises week 38

Exercise 1:

from the book

- a) (5.1.1) Defects occur along the length of a filament (according to a Poisson process) at a rate $\lambda = 2$ per foot.
 - (i) Calculate the probability that there are no defects in the first foot of the filament.
 - (ii) Calculate the conditional probability that there are no defects in the second foot of the filament, given that the first foot contained a single defect.
- b) (5.1.2) Let $p_k = \Pr\{x = k\}$ be the probability mass function corresponding to a Poisson distribution with parameter λ . Verify that $p_0 = \exp\{-\lambda\}$, and that p_k may be computed recursively by $p_k = (\lambda/k)p_{k-1}$.
- c) (5.1.3) Let X and Y be independent Poisson distributed random variables with parameters α and β , respectively. Determine the conditional distribution of X , given that $N=X+Y=n$.
- d) (5.3.1) A radioactive source emits particles according to a Poisson process of rate $\lambda = 2$ particles per minute. What is the probability that the first particle appears after 3 min?
- e) (5.3.2) A radioactive source emits particles according to a Poisson process of rate $\lambda = 2$ particles per minute.
 - (i) What is the probability that the first particle appears some time after 3 min but before 5 min?
 - (ii) What is the probability that exactly one particle is emitted in the interval from 3 to 5 min?
- f) (5.3.3) Customers enter a store according to a Poisson process of rate $\lambda = 6$ per hour. Suppose it is known that only a single customer entered during the first hour. What is the conditional probability that this person entered during the first 15 min?

Exercise 2: Football

The number of goals scored by Vålerenga IF during a football match is Poisson distributed with an average of $\lambda_v = 1.2$ goals per match while the number of goals scored by Rosenborg BK is Poisson distributed with an average of $\lambda_r = 2$ goals per match. The number of goals scored by Vålerenga is independent of the number of goals scored by Rosenborg. Assume that a football match lasts for exactly 90 minutes (2 x 45 minutes) and that Vålerenga plays a match against

Rosenborg:

- a) What is the distribution for the total number of goals scored in this match?
- b) What is the probability that there are no goals during the first half of the match?
- c) What is the probability that the final result is 2-2?
- d) What is the expected time until the first goal in this match?
- e) Assume that no goals are scored the first 15 minutes of the match. What is the probability that Vålerenga score at least one goal before the break at 45 minutes?
- f) Verify your theoretical answers by simulations in R, Matlab or Python. Some useful functions are `rpois()` and `rexp()` in R, `numpy.random.poisson()` and `numpy.random.exponential()` in Python or `poissrnd()` and `expnrnd()` in Matlab.