TMA4265 Stochastic Modeling

Week 42 Exercises

Exercise 1.
Biathlon commonly refers to the winter sport that combines cross-country skiing and rifle shooting. Assume that the inhabitants of Oslo want to improve their Biathlon skills and go to a popular skiing area to train. There is a stadium with three public shooting stands available. Skiers arrive at the shooting stands according to a Poisson process with rate 5 skiers per minute, i.e. $\lambda = 1/12$ skier per second. If a shooting stand is free an entering skier immediately starts to shoot and then leaves directly the stadium. If all stands are occupied he waits in line and then goes to the first free shooting stand. The time a skier spends at either of the shooting stands is independent of the other skiers and exponentially distributed with mean 30 seconds, i.e. with rate $\mu = 1/30$.

Let $X(t)$ denote the number of skiers in the stadium at time t, i.e. skiers who are either shooting or waiting in line until a shooting stand becomes free. We assume that $X(0) = 0$.

i) Explain briefly why $X(t)$ is a birth-death process and give all birth and death rates.

ii) If $X(t) = 3$, what is the expected time until all three skiers have finished shooting.

iii) Starting at time 0, what is the expected time until $X(t) = 3$ for the first time.

 In the remaining answers, first express answers as functions of λ and μ. Thereafter, compute the numerical answer for the parameter values given.

iv) Derive the limiting probabilities for $X(t)$.

v) In the long run what proportion of skiers can start shooting immediately after arrival (i.e. without first waiting until a shooting stand becomes available)?

vi) Compute the expected number of skiers in the stadium after a long time has passed.

vii) Use Little's formula to find the average amount of time each skier spends at the stadium.

Exercise 2.

A tourist guide is stationed at a port and hired to give sightseeing tours with his boat. If the guide is free, it takes an interested tourist a certain time, exponentially distributed with mean $1/\mu_1$, to negotiate the price and type of tour. Assume there is a probability α that there is no agreement and the tourist leaves. If the tourist and the guide reach an agreement, they start directly the tour. The duration of the tour is exponentially distributed with mean $1/\mu_2$. Suppose that interested tourists arrive at the guide's station according to a Poisson process with rate λ and request service only when the guide is free, i.e. the guide is neither negotiating nor on tour with another tourist. Suppose further that the Poisson process, the negotiation time, the duration of the tour and whether the tourist and the guide achieve an agreement are independent.

Set up the set of balance equations and derive the proportion of time (in the long run) that the guide is free.