Compulsory assignment 1 [unfinished!]

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Problem 1

a)

We first determine $\boldsymbol{\mu}_{\boldsymbol{Y}}$. Since $E(A\mathbf{X}) = AE\mathbf{X}$, we have

$$\boldsymbol{\mu}_{\boldsymbol{Y}} := E\mathbf{Y} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \end{pmatrix}$$

To find the covariance matrix Σ, we use the property that $\mathrm{Cov}(A\mathbf{X}) = A\mathrm{Cov}\mathbf{X}A^T$. We calculate this in R:

```
pr1_A <- matrix(c(1/sqrt(2), -1/sqrt(2), 1/sqrt(2), 1/sqrt(2)), nrow=2, byrow=TRUE)
pr1_Sigma_X <- rbind(c(3,1), c(1,3))
pr1_Sigma_Y <- pr1_A %*% pr1_Sigma_X %*% t(pr1_A)
pr1_Sigma_Y
```

```
##      [,1] [,2]
## [1,]    2    0
## [2,]    0    4
```

Thus, the covariance matrix

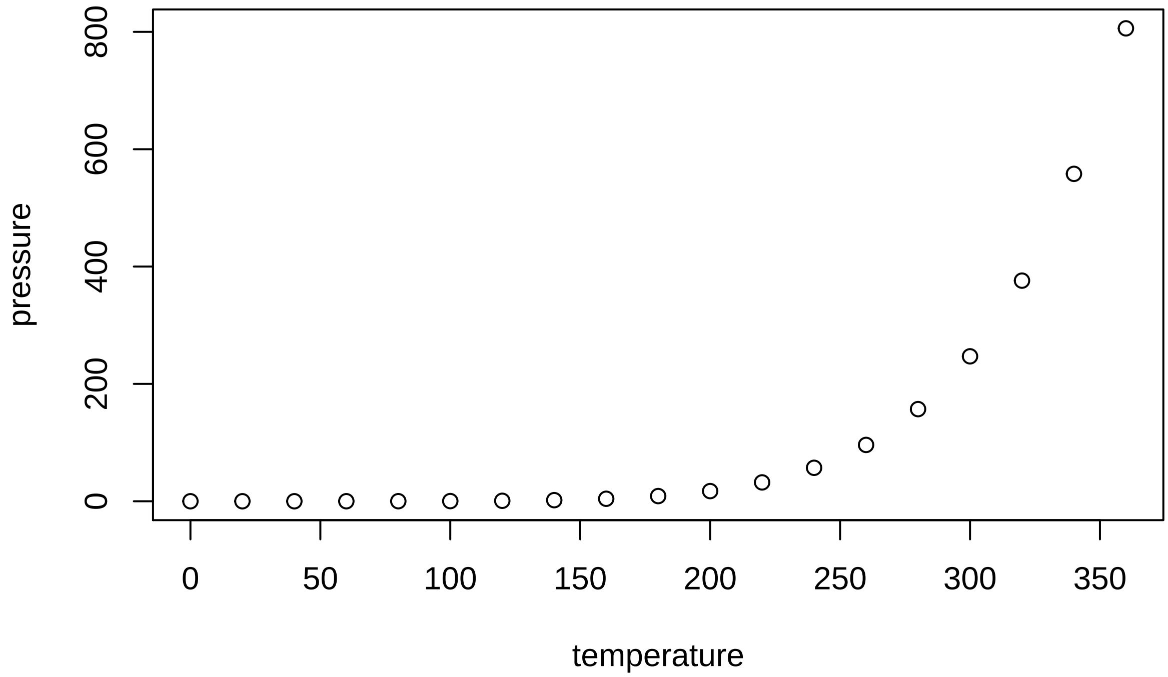
$$\Sigma_Y = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}.$$

The random variable $\mathbf{Y}$ has a bivariate normal distribution, since it is a linear combination of the bivariate normal distribution $\mathbf{X}$. From Σ_Y, we also see that $\mathrm{Cov}(Y_1, Y_2) = 0$, hence the variables are independent.

b)

A little stuck on this

You can also embed plots, for example:



Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.