

**Written exam, January 24th, 2025. Page 1 of 18 pages**

**Course name:** Mathematical Programming Modelling

**Course number:** 42112

**Allowed aids:** All aids and materials permitted. Use of the internet is not allowed.

**Duration:** 3 hours

**Weights of questions:** The exam consists of 7 regular multiple-choice questions, a modeling assignment specified through 4 sets of multiple choice questions (Q8-Q20); and one assignment for which 2 julia programs are needed.

Each correct answer to a regular multiple-choice question gives 7 points (49 in total). A wrong answer gives 0 points. Only one answer is allowed per multiple-choice question. The modeling part is worth 23 points. The programming part is worth 28 points.

If you think a question is ambiguous, then point out the ambiguity in the comment section of your first julia file, mentioning the question number, your answer, and explain how you have chosen to interpret the question.

- Please answer the **20** multiple-choice questions by clicking the right options on the exam that is named: "42112 - 42112 Mathematical Programming Modelling (1/2)"
- Please hand in your julia code for the assignment as an answer to "42112 - 42112 Mathematical Programming Modelling (2/2)" (upload your responses).

### Question 1

You are in charge of making sure there is enough toiletpaper in your unit of 12 students. Once a month you drive to a wholesaler to pick up cheap and eco-friendly packages containing 12 rolls each. You are an early riser, so we will assume there is no demand while you buy the toilet paper. You want to minimize inventory cost (toiletpaper takes up a lot of space), while making sure you always have enough on stock.

Please select the correct inventory constraint to complete the below model. (Hint: you can always review e.g. the MicroBrewery assignments for inspiration).

Parameters:

$Cost$	The cost of having a single roll in inventory for one month.
$M$	Number of months you want to plan for.
$Demand_m$	Demand for toilet paper roles per month $m \in \{1, \dots, M\}$ .
$InitialStorage$	Number of rolls on stock at the start of your planning period (e.g. at the end of month 0).
$Rolls$	Number of toilet rolls per package bought.

Variables:

$x_m$  the number of packages that you will buy at the start of month  $m$ ,  $m \in \{1, \dots, M\}$ .

$y_m$  number of rolls on stock at the end of month  $m$ ,  $m \in \{0, 1, \dots, M\}$

$$\text{Minimize: } \sum_{m \in \{1, \dots, M\}} Cost \cdot y_m$$

Subject to:

$$y_0 = InitialStorage$$

$$x_m \in Z^+ \quad \forall m \in \{1, \dots, M\}$$

$$y_m \in R^+ \quad \forall m \in \{0, \dots, M\}$$

Select the correct inventory constraint to complete the above model from the options below:

**1A)**  $y_m = Demand_m - x_m \cdot Rolls + y_{m-1} \quad \forall m \in \{1, \dots, M\}$

**1B)**  $y_m = Demand_m + y_{m-1} \quad \forall m \in \{1, \dots, M\}$

**1C)**  $y_m = x_m \cdot Rolls + y_{m-1} \quad \forall m \in \{1, \dots, M\}$

**1D)**  $y_m = x_m \cdot Rolls - Demand_m + y_{m-1} \quad \forall m \in \{1, \dots, M\}$

## Question 2

You try to solve the Micro brewery 2 assignment in the book on a *very* old computer. You stop it after 10 minutes. It has not solved your model to optimality, but returns to you the following information:

- The LP solution
- The best found integer solution
- The MIP Gap

What does the MIP Gap represent?

**2A)** The relative difference between the current best lower bound and the current best found integer solution.

**2B)** The relative difference between the optimal LP solution (allowing any number of beers brewed per month), and the optimal integer solution (allowing to brew only one type of beer per month).

**2C)** The relative difference between the computation time required for the LP solution, and the computation time required for the current best integer solution.

**2D)** The difference between the current best known solution and the optimal solution.

### Question 3

You have identified a really exciting optimization problem at your current student job. However, you are struggling to solve it within reasonable time.

Which of the below is *not* a way that could possibly speed up your model?

**3A)** Use tighter bounds on your big-M variables

**3C)** Add additional constraints to cut-off additional non-integer points in the solution space

**3B)** Reduce the number of integer variables

**3D)** Change continuous variables to integer variables

**Question 4**

Given two decision variables  $x \in \mathbf{R}^+$  and  $y \in \{0, 1\}$ , which of the following constraints correctly, and linearly, ensures that when  $y$  is equal to 1, then  $x$  should be larger or equal to  $b$ .

**4A)**  $y \leq bx$

**4C)**  $xy \geq by$

**4B)**  $x \geq by$

**4D)**  $x \geq b(y - 1)$

### Question 5

Given a bi-objective MIP model with decision variables  $x$  and two linear objective functions  $F^1(x)$  and  $F^2(x)$  each possible solution  $x$  has two objective values  $z^1 = F^1(x)$  and  $z^2 = F^2(x)$ . Given a solution  $z_1 = (F^1(x), F^2(x)) = (z_1^1, z_1^2)$ , what is the definition that this solution is Pareto optimal, assuming minimization ?

**5A)** No other solution  $x_2$  with the objective values  $z_2 = (z_2^1, z_2^2)$  exist for which  $z_2^1 < z_1^1$  and  $z_2^2 < z_1^2$

.

**5B)** No other solution  $x_2$  with the objective values  $z_2 = (z_2^1, z_2^2)$  exist for which  $z_2^1 \leq z_1^1$  and  $z_2^2 \leq z_1^2$  and where either  $z_1^1 \neq z_2^1$  or  $z_1^2 \neq z_2^2$  or both.

**5C)** A value  $\alpha \in ]0, 1[$  exist such that no other solution  $x_2$  with the objective values  $z_2 = (z_2^1, z_2^2)$  exist where  $\alpha z_2^1 + (1 - \alpha) z_2^2 < \alpha z_1^1 + (1 - \alpha) z_1^2$

**5D)** No other solution  $x_2$  with the objective values  $z_2 = (z_2^1, z_2^2)$  exist for which  $z_2^1 < z_1^1$  or  $z_2^2 < z_1^2$  and a value  $\alpha \in ]0, 1[$  exist such that  $\alpha z_2^1 + (1 - \alpha) z_2^2 < \alpha z_1^1 + (1 - \alpha) z_1^2$

### Question 6

Recall the stamp assignment from the coursebook where the aim was to maximize the profit you can get by selling stamps. The correct MIP model for this problem is as follows:

$$\begin{aligned}
 \text{Maximize: } & \sum_{b \in B} BidPrice_b \cdot x_b \\
 \text{Subject to: } & \sum_{b \in B} BidSets_{b,s} \cdot x_b \leq 1 \quad \forall s \in S \\
 & x_b \in \{0, 1\} \quad \forall b \in B
 \end{aligned}$$

A stamp-collector association offers you to buy all remaining stamps at a fixed value of  $StampRestPrice$  crowns for each stamp remaining, provided that at least 30% of their members bids are accepted.

Define  $b \in B^*$  all bidders part of this association, and define  $y_s$  a binary variable indicating on whether a stamp is sold to the association.

Which of the following models is correct ?

A)

$$\begin{aligned}
 \text{Maximize: } & \sum_{b \in B} BidPrice_b \cdot x_b + \sum_{s \in S} StampRestPrice \cdot y_s \\
 \text{Subject to: } & \sum_{b \in B} y_s + \sum_{b \in B} BidSets_{b,s} \cdot x_b \leq 1 \quad \forall s \in S \\
 & \sum_{b \in B^*} x_b \geq 0.3|B^*| \cdot y_s \quad \forall s \in S \\
 & x_b \in \{0, 1\} \quad \forall b \in B \\
 & y_s \in \{0, 1\} \quad \forall s \in S
 \end{aligned}$$

B)

$$\begin{aligned}
 \text{Maximize: } & \sum_{b \in B} BidPrice_b \cdot x_b + StampRestPrice \cdot y_s \\
 \text{Subject to: } & \sum_{s \in S} y_s + \sum_{b \in B} BidSets_{b,s} \cdot x_b \leq 1 \\
 & \sum_{b \in B^*} x_b \geq 0.3|B^*| \cdot y_s \quad \forall b \in B \\
 & x_b \in \{0, 1\} \quad \forall b \in B \\
 & y_s \in \{0, 1\} \quad \forall s \in S
 \end{aligned}$$

C)

$$\begin{aligned}
 \text{Maximize: } & \sum_{b \in B} \text{StampRestPrice} \cdot y_s \cdot \text{BidPrice}_b \cdot x_b \\
 \text{Subject to: } & \\
 & \textcolor{blue}{y_s + \sum_{b \in B} \text{BidSets}_{b,s} \cdot x_b = 1} \quad \forall s \in S \\
 & \sum_{b \in B^*} x_b \geq 0.3|B^*| \cdot y_s \quad \forall s \in S \\
 & x_b \in \{0, 1\} \quad \forall b \in B \\
 & y_s \in \{0, 1\} \quad \forall s \in S
 \end{aligned}$$

D)

$$\begin{aligned}
 \text{Maximize: } & \sum_{b \in B} \text{BidPrice}_b \cdot x_b + \text{StampRestPrice} \cdot y_s \\
 \text{Subject to: } & \\
 & \textcolor{blue}{y_s + \sum_{b \in B} \text{BidSets}_{b,s} \cdot x_b = 1} \quad \forall s \in S \\
 & \sum_{b \in B^*} x_b \geq 0.3|B^*| \cdot y_s \quad \forall b \in B^* \\
 & x_b \in \{0, 1\} \quad \forall b \in B \\
 & y_s \in \{0, 1\} \quad \forall s \in S
 \end{aligned}$$

### Question 7

Recall the Santas Workshop assignment, where workshops for a number of families  $f \in F$  were planned for a number of days  $d \in D$ . The model to minimize the daily costs is given below:

$$\begin{aligned}
 \text{Minimize: } & \sum_{f \in F, d \in D} DayVisitCost_{f,d} \cdot x_{f,d} \\
 \text{Subject to: } & \sum_{d \in D} x_{f,d} = 1 \quad \forall f \in F \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \geq 125 \quad \forall d \in D \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \leq 300 \quad \forall d \in D \\
 & x_{f,d} \in \{0, 1\} \quad \forall f \in F, d \in D
 \end{aligned}$$

The families are of different size, with up to 8 persons. Santa decides that it is probably nice when small families (i.e. with 3 or less members) can meet at the event. Then, the kids may be more open to play together as they may have no or few other siblings to play with. Hence, he decides to have certain "small family days" where there have to be at least 5 families with 3 or less members. All small families need to be assigned to "small family days", while larger families may be assigned to both small family days as well as other days.

Let us define  $F_{\leq 3}$  as the set of small families, and  $F_{\geq 4}$  as the set of large families. Moreover, we introduce a new binary decision variable  $y_d$ , that is equal to 1 when day  $d$  is a small family day, and zero otherwise.

Which of the following models correctly ensures the small family days ?

A)

$$\begin{aligned}
 \text{Minimize: } & \sum_{f \in F, d \in D} DayVisitCost_{f,d} \cdot x_{f,d} \\
 \text{Subject to: } & \sum_{d \in D} x_{f,d} = 1 \quad \forall f \in F \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \geq 125 \quad \forall d \in D \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \leq 300 \quad \forall d \in D \\
 & \sum_{f \in F_{\geq 4}} x_{f,d} \leq |F| \cdot (1 - y_d) \quad \forall d \in D \\
 & x_{f,d} \in \{0, 1\} \quad \forall f \in F, d \in D \\
 & y_d \in \{0, 1\} \quad \forall d \in D
 \end{aligned}$$

B)

$$\begin{aligned}
 \text{Minimize: } & \sum_{f \in F, d \in D} DayVisitCost_{f,d} \cdot x_{f,d} \\
 \text{Subject to: } & \sum_{d \in D} x_{f,d} = 1 \quad \forall f \in F \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \geq 125 \quad \forall d \in D \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \leq 300 \quad \forall d \in D \\
 & \sum_{f \in F_{\leq 3}} x_{f,d} \leq |F| \cdot y_d \quad \forall d \in D \\
 & \sum_{f \in F_{\leq 3}} x_{f,d} \cdot y_d \geq 5 \quad \forall d \in D \\
 & x_{f,d} \in \{0, 1\} \quad \forall f \in F, d \in D \\
 & y_d \in \{0, 1\} \quad \forall d \in D
 \end{aligned}$$

C)

$$\begin{aligned}
 \text{Minimize:} \quad & \sum_{f \in F, d \in D} DayVisitCost_{f,d} \cdot x_{f,d} \\
 \text{Subject to:} \quad & \sum_{d \in D} x_{f,d} = 1 \quad \forall f \in F \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \geq 125 \quad \forall d \in D \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \leq 300 \quad \forall d \in D \\
 & \sum_{f \in F_{\leq 3}} x_{f,d} \leq |F| \cdot y_d \quad \forall d \in D \\
 & \sum_{f \in F_{\leq 3}} x_{f,d} \geq 5 \cdot y_d \quad \forall d \in D \\
 & x_{f,d} \in \{0, 1\} \quad \forall f \in F, d \in D \\
 & y_d \in \{0, 1\} \quad \forall d \in D
 \end{aligned}$$

D)

$$\begin{aligned}
 \text{Minimize:} \quad & \sum_{f \in F, d \in D} DayVisitCost_{f,d} \cdot x_{f,d} \\
 \text{Subject to:} \quad & \sum_{d \in D} x_{f,d} = 1 \quad \forall f \in F \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \geq 125 \quad \forall d \in D \\
 & \sum_{f \in F} FamilySize_f \cdot x_{f,d} \leq 300 \quad \forall d \in D \\
 & \sum_{f \in F_{\leq 3}} x_{f,d} \leq |F| \cdot y_d \quad \forall d \in D \\
 & \sum_{f \in F_{\geq 4}} x_{f,d} \cdot (1 - y_d) \geq 5 \quad \forall d \in D \\
 & x_{f,d} \in \{0, 1\} \quad \forall f \in F, d \in D \\
 & y_d \in \{0, 1\} \quad \forall d \in D
 \end{aligned}$$

## A new Rolling Stock Model

All following questions will be about adjusting the below formulation (1)–(8) of the Rolling Stock Schedule formulation. Below we provide the formulation as discussed in Lecture 6, as well as in Schrijver; with the addition of the variables  $\delta_a^{1st}, \delta_a^{2nd}$  that count the number of seat shortages per ride arc  $a \in A^r$ .

$$\min \sum_{a \in A^o} (c^I x_a^I + c^{II} x_a^{II}) \quad (1)$$

$$s.t. \quad \sum_{a \in \delta^+(v)} x_a^I - \sum_{a \in \delta^-(v)} x_a^I = 0 \quad \forall v \in V \quad (2)$$

$$\sum_{a \in \delta^+(v)} x_a^{II} - \sum_{a \in \delta^-(v)} x_a^{II} = 0 \quad \forall v \in V \quad (3)$$

$$3x_a^I + 4x_a^{II} \leq u_a \quad \forall a \in A^r \quad (4)$$

$$38x_a^I + 65x_a^{II} + \delta_a^{1st} \geq l_a^{1st} \quad \forall a \in A^r \quad (5)$$

$$163x_a^I + 218x_a^{II} + \delta_a^{2nd} \geq l_a^{2nd} \quad \forall a \in A^r \quad (6)$$

$$x_a^I, x_a^{II} \in \mathbb{Z}_+ \quad \forall a \in A \quad (7)$$

$$\delta_a^{1st}, \delta_a^{2nd} \in \mathbb{R}_+ \quad \forall a \in A^r \quad (8)$$

(9)

Notation (as in Lecture 6):

*Parameters:*

$c^I, c^{II}$  cost for usage of a rolling stock unit of type I, respectively type II.

$a \in A$  arc in graph defined by the timetable, where

- $a = ((i, w), (i', w')) \in A^r$  (i.e.,  $a$  is a ride arc)
- $a = ((i, w_k), (i, w_{k+1})) \in A^s$  (i.e.,  $a$  is a stay arc)
- $a = ((i, w_{|W_i|}), (i, w_1)) \in A^o$  (i.e.,  $a$  is an overnight arc)

$u_a$  maximum length of composition on arc  $a \in A$

$l_a^{1st}, l_a^{2nd}$  demand for seats for arc  $a \in A^r$  for first and second class, respectively

*Variables:*

$x_a^I \in \mathbb{Z}_+$  for each arc  $a \in A$  with the following meaning

- If  $a \in A^r$  (i.e.,  $a$  is a ride arc),  $x_a^I$  is the number of train units of type I deployed to travel from station  $i$  at time  $w$  to station  $i'$  at time  $w'$
- If  $a \in A^s$  (i.e.,  $a$  is a stay arc),  $x_a^I$  is the number of train units of type I parked at station  $i$  between time  $w_k$  and time  $w_{k+1}$
- If  $a \in A^o$  (i.e.,  $a$  is an overnight arc),  $x_a^I$  is the number of train units of type I parked overnight at station  $i$

The definition for variables  $x_a^{II} \in \mathbb{Z}_+$  representing the usage of type II units is equivalent to the definition of the above  $x_a^I \in \mathbb{Z}_+$ .

$\delta_a^{1st}, \delta_a^{2nd} \in \mathbb{R}_+$  represent the number of seat shortages in first and second class, respectively, per ride arc  $a \in A^r$

In the following four sets of multiple choice questions you are going to select those options that together formulate the correct answers. They are formulated as multiple choice questions so you do not need to worry about how to write the math or hand it in. The sets of multiple choice questions mean that you can get partial points for each of the below questions. Below is an example(!) question to illustrate how this works. The answer is provided below.

#### **EXAMPLE question set: defining a budget constraint (0 points!!!!!!)**

Let there be a budget  $B$  on operating costs alone, that is, the cost for the number of required rolling stock units may not exceed  $B$ . The cost for a unit of type 1 is  $c^I$  and the cost for a unit of type 2 is  $c^{II}$ . Select below such that the following equation is correct:

$$\sum_{T1} T2 \quad T3 \quad B \quad \forall T4$$

T	A	B	C
$T1$	$a \in A$	$a \in A^r$	$a \in A^o$
$T2$	$(c^I x_a^I + c^{II} x_a^{II})$	$c^I x_a^I$	$c^{II} x_a^{II}$
$T3$	$\leq$	$=$	$\geq$
$T4$	$a \in A$	$a \in A^0$	single term

Correct answer:  $T1=C$ ,  $T2=A$ ,  $T3=A$ ,  $T4=C$ , which specifies:  $\sum_{a \in A^o} (c^I x_a^I + c^{II} x_a^{II}) \leq B$   
Note that this is a single constraint, and hence we selected option C in T4.

#### **First question set: defining a new decision variable(2 points)**

We need to count all arcs that have any shortage in seats. Let us introduce variable  $y_a$  to represent arcs which have a seat shortage. Specify the type of decision variable (Q8), as well as for which set of arcs it should be defined (Q9), to allow to count arcs with seat shortages in the model defined by equations (1)-(8).

Q	A	B	C
$Q8$	continuous	integer	binary
$Q9$	$a \in A^o$	$a \in A^r$	$a \in A$

#### **Second Question set: specifying the objective function (7 points)**

We want to change the objective to minimize (only!) the costs of total seat shortages, where seat shortage cost are defined as:

- $\phi_a$  for every arc that has a seat shortage
- $\xi_a^{1^{st}}$  per seat shortage in the first class
- $\xi_a^{2^{nd}}$  per seat shortage in the second class.

What should be the objective value? Select the right combination for Q10, Q11 and Q12 such that the new objective function is specified as:

$$\min Q10 + \sum_{Q11} Q12$$

Q	A	B	C
$Q10$	$\sum_{a \in A^o} (c^I x_a^I + c^{II} x_a^{II})$	(ommit term)	$\sum_{a \in A^o} (c^I \delta_a^I + c^{II} \delta_a^{II})$
$Q11$	$a \in A^o$	$a \in A^r$	$a \in A$
$Q12$	$\delta_a^{1^{st}} \xi_a^{1^{st}} + \delta_a^{2^{nd}} \xi_a^{2^{nd}} + \phi_a y_a$	$\phi_a y_a$	$\delta_a^{1^{st}} \xi_a^{1^{st}} + \delta_a^{2^{nd}} \xi_a^{2^{nd}}$

### Third Question set: counting arcs with a seat shortage (7 points)

Let's define  $w_a$  as a binary variable that is equal to 1 when an arc has a seat shortage that is above a value  $\Delta$  (e.g. more than 200), and is zero otherwise. Select the below combination such that the new constraint specified as follows is correct:

$$Q13 \ Q14 \ Q15 \ \forall Q16$$

$Q$	A	B	C
$Q13$	$\Delta w_a$	$\delta_a^{1st} + \delta_a^{2nd}$	$\delta_a^{1st} + \delta_a^{2nd} - \Delta$
$Q14$	$\leq$	$=$	$\geq$
$Q15$	$Mw_a$	$M(\delta_a^{1st} + \delta_a^{2nd})$	$\Delta$
$Q16$	$a \in A$	$a \in A^r$	$a \in A^o$

### Fourth Question set: limiting the number of arcs with seat shortages (7 points)

Finally, we want to add a constraint such that the number of arcs with a seat shortage above  $\Delta$  (e.g. 200 seats) cannot be more than  $\sigma$  (e.g. 5 arcs). Select the below combination such that the new constraint specified as follows is correct:

$$\sum_{Q17} Q18 \leq Q19 \ \forall Q20$$

$Q$	A	B	C
$Q17$	$a \in A$	$a \in A^r$	no sum, single term
$Q18$	$\sigma w_a$	$w_a$	$\delta_a^{1st} + \delta_a^{2nd}$
$Q19$	$\sigma$	$\Delta w_a$	$\Delta$
$Q20$	$a \in A^r$	$a \in A$	no range, single constraint

### Question 21

This is an implementation exercise. Therefore, you will hand in one(!) julia file per assignment below, and thus 2 in total. You may use comments in your file to make clear what you intend to program. Use of the same names as in the question is preferred.

You are in charge of planning the schedule of the written exams at a university. At the university there are a number of written exams  $E$ , each with a number of students  $ExamStudents_e$ . Each exam has to be scheduled to one of the  $R$  rooms in one of the  $T$  timeslots. There can be several exams per timeslot per room, as long as the total number of students expected at these exams does not exceed the room capacity  $RoomCap_r$ , i.e. the number of seats. If one or more exams are planned for a room in one timeslot, it entails the fixed cost  $RoomCost_{t,r}$  for using room  $r \in R$  at timeslot  $t \in T$  (i.e. for exam personnel, cleaning, lights etc.). This cost has to be paid, independently of how many seats are used. The room costs are the real costs, which have to be paid.

In addition you want to optimize exam room and time suitability for each course. Therefore, you have asked the course responsible professor for each exam to rate how well suited a room is for an exam on a scale from 1 to 100, where 1 is perfect and 100 is totally unacceptable, i.e. the rate is a penalty for using that room. These numbers are given in the matrix  $ExamRoomPenalty_{e,r}$  where each row represents an exam and each column represents a room. In addition, you have asked them to rate the suitability of each timeslot, also on a scale from 1 to 100, which is given in  $ExamTimeslotPenalty_{e,t}$ , where each row represents an exam and each column represents a timeslot. Even though these penalties are not real costs, and much lower than the real room costs, these penalties are simply added together with the room cost, to try to create an exam schedule which is both cheap, but also attempts to allocate the exams to the best rooms and timeslots. All the data are correctly scaled and further scaling in the objective is not necessary. All data necessary for the assignment is given in the file **ExamPlanningData.jl**. Notice that all models have to be **LINEAR**.

**Assignment 1** Create a mathematical model and implement it in Julia/JuMP which minimize the combined Room cost, ExamRoomPenalty and ExamTimeslotPenalty and which creates a schedule where all exams are scheduled and the room capacities are respected.

Optimal Objective Range: [25000 - 28000]

Some of the exams include the same students. To create a feasible schedule where all students can attend all of their exams, one needs to ensure that exams of courses that have an overlap in signed-up students cannot take place at the same time. The incidence matrix  $Collision_{e1,e2}$  is equal to 1 if two courses **can** take place at the same time, and 0 if they cannot.

**Assignment 2** Create a mathematical model and implement it in Julia/JuMP which minimize the combined Room cost, ExamRoomPenalty and ExamTimeslotPenalty and which creates a schedule where all exams are scheduled, the room capacities are respected **AND** which ensure that exams with shared students are not planned in the same timeslots.

Optimal Objective Range: [28000 - 31000]

**Data** The data are available in the file ExamPlanningData.jl. You do **NOT** have to type in the data yourself. In the file the following data are given:

Sets:

- Exams:  $E = 12$
- Rooms:  $R = 3$
- Timeslots:  $T = 4$

No of students at exams:  $ExamStudents = [40 \ 97 \ 100 \ 18 \ 73 \ 55 \ 96 \ 82 \ 82 \ 55 \ 38 \ 93]$

Room capacity of rooms:  $RoomCap = [96 \ 88 \ 190]$

Cost of using a room for one or more exams,  $RoomCost_{t,r} =$

Room	Timeslot 1	Timeslot 2	Timeslot 3	Timeslot 4
1	3510	3563	3511	3582
2	3473	3420	3491	3550
3	5407	5464	5522	5463

Penalty for using specific timeslots for exams:  $ExamTimeslotPenalty_{e,t} =$

Exam	Timeslot 1	Timeslot 2	Timeslot 3	Timeslot 4
1	87	20	30	84
2	43	43	47	50
3	96	89	48	48
4	83	48	90	82
5	66	76	71	38
6	75	57	62	56
7	26	24	52	67
8	90	58	15	63
9	52	85	80	12
10	11	82	57	89
11	40	38	58	56
12	18	13	37	75

Penalty for using specific rooms for exams:  $ExamRoomPenalty_{e,r} =$

Exam	Room 1	Room 2	Room 3
1	96	97	37
2	96	97	37
3	96	97	37
4	96	97	37
5	96	97	37
6	96	97	37
7	96	97	37
8	96	97	37
9	96	97	37
10	96	97	37
11	96	97	37
12	96	97	37

Collision matrix, to avoid same time allocation of two exams which includes one or more of the same students. Collision[e1,e2]=0 means that exam e1 and exam e2 **cannot** take place in the same timeslot.  $\text{Collision}_{e1,e2} =$

1	0	1	0	1	0	0	0	0	1	1	0
0	1	1	0	1	1	1	1	1	0	0	0
1	1	1	1	1	0	0	0	1	0	1	0
0	0	1	1	0	1	0	1	1	1	1	1
1	1	1	0	1	1	0	1	0	0	1	1
0	1	0	1	1	1	1	0	1	1	0	1
0	1	0	0	0	1	1	0	0	1	0	1
0	1	0	1	1	0	0	1	0	1	0	1
0	1	1	1	0	1	0	0	1	0	1	1
1	0	0	1	0	1	1	1	0	1	0	0
1	0	1	1	1	0	0	0	1	0	1	1
0	0	0	1	1	1	1	1	1	0	1	1

Again: The data are available in the file ExamPlanningData.jl. You do **NOT** have to type in the data yourself.