CS 162 Programming languages

Lecture 7: Operational Semantics I

Yu Feng

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Putting all together

Substitution $c[v \mapsto e]$	**Alpha-Renaming Necessary?**	**Output**
$(x\ (\lambda x.\ y))[y \mapsto x]$	(JR) Yes	(JR) First rename to $x\ (\lambda a.y)$. Then output: $x\ (\lambda a.x)$
$((\lambda y.\ x)\ (\lambda z.\ y))[y \mapsto x]$	(JR) No	(JR) $((\lambda y.\ x)\ (\lambda z.\ x))$
$(\lambda z.\ ((\lambda y.\ y)\ y))[y \mapsto \lambda x.(y\ z)]$	(JR) Yes	(JR) First rename to $\lambda a.\ ((\lambda b.\ b)\ y)$. Then output: $\lambda a.((\lambda b.\ b)\ (\lambda x.(y\ z)))$

$$[x \mapsto s]x = s$$

$$[x \mapsto s]y = y (x \neq y)$$

$$[x \mapsto s]\lambda y\,.\,t_1 = \lambda y\,.\,[x \mapsto s]t_1 (y \neq x \wedge y \notin FV(s))$$

$$[x \mapsto s]t_1\ t_2 = [x \mapsto s]t_1\ [x \mapsto s]t_2$$

What does a program mean?

- We have learned how to specify syntax.
    - Example: let x = *lambda lambda* is not a valid λ^+ program
    - But we have not yet talked about what the meaning of a program is.
- First question: What is the meaning of a program in λ^+?
    - Answer: The value the program evaluates to
    - Example: let x=3 in x Value:3

How to specify meaning of programs

- Option 1: Don't worry too much
- Developer of language has some informal concept of the intended meaning, implement a compiler/interpreter that does whatever the language designers believe to be reasonable.
- Then, declare the meaning to be whatever the compiler produces
- A terrible idea

How to specify meaning of programs

- Why is this such a bad idea?
- This approach promotes bugs/inconsistencies to expected behavior.
- Hides specification of language in many implementation details
- Makes it almost impossible to implement another compiler that accepts the same language
- Unfortunately, this is (still) a very common approach
- Languages designed this way: C, C++ (to some extent), Perl, PHP, JavaScript, ...

How to specify meaning of programs

- Option 2: Try to write out precisely the meaning of each language construct in documentation, then follow this description in implementation
- Example: Describe the meaning of $e_1 + e_2$ in the λ^+ language:
- First attempt: “This evaluates to the sum of e_1 and e_2”
- What if e_1 or e_2 is not a number?
- Second attempt: “This evaluates to the sum if both e_1 and e_2 evaluate to numbers, and is stuck if either of them evaluate to a list”
- What if e is lambda?…

How to specify meaning of programs

- Written language is, by nature, ambiguous. It is very difficult to fully specify the meaning of all language constructs this way

- Easy to miss cases

- Results in long, complicated and difficult to understand specifications, but an improvement over no specification

Written specification in practice

- Let's look at the ISO C++ standard: page 34:

2 A declaration is a *definition* unless it declares a function without specifying the function's body (8.4), it contains the **extern** specifier (7.1.1) or a *linkage-specification*[25] (7.5) and neither an *initializer* nor a *function-body*, it declares a static data member in a class definition (9.2, 9.4), it is a class name declaration (9.1), it is an *opaque-enum-declaration* (7.2), it is a *template-parameter* (14.1), it is a *parameter-declaration* (8.3.5) in a function declarator that is not the *declarator* of a *function-definition*, or it is a **typedef** declaration (7.1.3), an *alias-declaration* (7.1.3), a *using-declaration* (7.3.3), a *static_assert-declaration* (Clause 7), an *attribute-declaration* (Clause 7), an *empty-declaration* (Clause 7), a *using-directive* (7.3.4), an explicit instantiation declaration (14.7.2), or an explicit specialization (14.7.3) whose *declaration* is not a definition.

[*Example:* all but one of the following are definitions:

```
int a;                          // defines a
extern const int c = 1;         // defines c
int f(int x) { return x+a; }    // defines f and defines x
struct S { int a; int b; };     // defines S, S::a, and S::b
struct X {                      // defines X
  int x;                        // defines non-static data member x
  static int y;                 // declares static data member y
  X(): x(0) { }                 // defines a constructor of X
};
int X::y = 1;                   // defines X::y
enum { up, down };              // defines up and down
namespace N { int d; }          // defines N and N::d
namespace N1 = N;               // defines N1
X anX;                          // defines anX
```

whereas these are just declarations:

```
extern int a;                   // declares a
extern const int c;             // declares c
int f(int);                     // declares f
struct S;                       // declares S
typedef int Int;                // declares Int
extern X anotherX;              // declares anotherX
using N::d;                     // declares d
```

Machine model

- To study the operational semantics, we must understand what our machine model will require.

- Need to know when our machine is “done” executing a program: the final expressions are values.

Values in λ^+

- We define a value in an inductive way:
  - Any integer i is a value.
  - Boolean constants true and false are values
  - Any lambda expression lambda x. e is a value.
  - Nil is a value.
  - If v_1, v_2 are values, then $v_1::v_2$ are values.
  - No other expression is a value.

Values in λ^+

- Those expressions are values:

$$10$$
$$\textsf{lambda}\, x.\ 1 + 2$$
$$\textsf{true}$$
$$\textsf{Nil}$$
$$10 :: \textsf{lambda}\, y.\ y$$

- Those expressions are NOT values:

$$1 + 2$$
$$(\textsf{lambda}\, x.\ 1 + 2)10$$
$$\textsf{if Nil then}\ 10\ \textsf{else}\ 20$$
$$(1 + 2) :: \textsf{Nil}$$

Inference rules

$$\frac{\begin{array}{l}\text{Hypothesis 1}\\ \dots\\ \text{Hypothesis N}\end{array}}{\vdash \text{Conclusion}}$$

- This means“given hypothesis1,…N, the conclusion is provable”

$$\frac{\begin{array}{l}\text{Miterm 1 grade} >= 70\\ \dots\\ \text{Final grade} >= 140\end{array}}{\vdash \text{Final grade: A}}$$

Operational Semantics

- Operational semantics: define how program states are related to final values

- The ***big-step*** evaluation relation asserts that *we can prove for any expression of the form e that the meaning of this expression will evaluate to v*

$$e \Downarrow v$$

Operational Semantics

$$\frac{}{i \Downarrow i}\ \textsc{Int}$$

Any integer constant i will evaluate to itself

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2}{e_1 + e_2 \Downarrow i_1 + i_2}\ \textsc{Add}$$

if e_1 and e_2 both evaluate to integers, then $e_1 + e_2$ evaluates to the sum of those integers

$$\cfrac{\textsc{Add}\ \cfrac{\textsc{Int}\ \cfrac{}{1 \Downarrow 1} \qquad \cfrac{}{2 \Downarrow 2}\ \textsc{Int}}{1 + 2 \Downarrow 3} \qquad \cfrac{}{4 \Downarrow 4}\ \textsc{Int}}{(1 + 2) + 4 \Downarrow 7}\ \textsc{Add}$$

Operational Semantics

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2 \qquad i_1 \odot i_2 \text{ holds}}{e_1 \odot e_2 \Downarrow \mathsf{true}} \textsc{PredTrue}$$

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2 \qquad i_1 \odot i_2 \text{ does not hold}}{e_1 \odot e_2 \Downarrow \mathsf{false}} \textsc{PredFalse}$$

The predicate operators $\odot \in \{=, <, >\}$ evaluate to false if the predicate does not hold and to true otherwise

$$\frac{e_1 \Downarrow \mathsf{true} \qquad e_2 \Downarrow v}{\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \textsc{IfTrue}$$

$$\frac{e_1 \Downarrow \mathsf{false} \qquad e_3 \Downarrow v}{\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \textsc{IfFalse}$$

Operational Semantics

$$\frac{}{\textsf{lambda}\ x.\ e \Downarrow \textsf{lambda}\ x.\ e}\ \textsc{Lambda}$$

Lambda abstractions just evaluate to themselves

$$\frac{e_1 \Downarrow \textsf{lambda}\ x.\ e_1' \qquad e_2 \Downarrow v \qquad [x \mapsto v]e_1' \Downarrow v'}{(e_1\ e_2) \Downarrow v'}\ \textsc{App}$$

To evaluate the application (e_1 e_2), we first evaluate the expression e_1. The operational semantics "get stuck" if e_1 is not a lambda abstraction. This notion of "getting stuck" in the operational semantics corresponds to a runtime error. Assuming the expression e_1 evaluates to a lambda expression, and e_2 evaluates to a value v, we evaluate the application expression by binding v to x and then evaluating the expression $[x \rightarrow v]e'_1$ as in β-reduction in lambda calculus.

Operational Semantics

$$\frac{e_1 \Downarrow v_1 \qquad [x \mapsto v_1]e_2 \Downarrow v_2}{\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Downarrow v_2}\ \textsc{Let}$$

First evaluate the initial expression e_1, which yields value v_1.
Next, we substitute x with v_1 in e_2, and evaluate it to v_2,
which becomes the result of evaluating the entire let expression.